VISUAL PHYSICS

SHORT NOTES

CHAPTER

Work, Power & Energy

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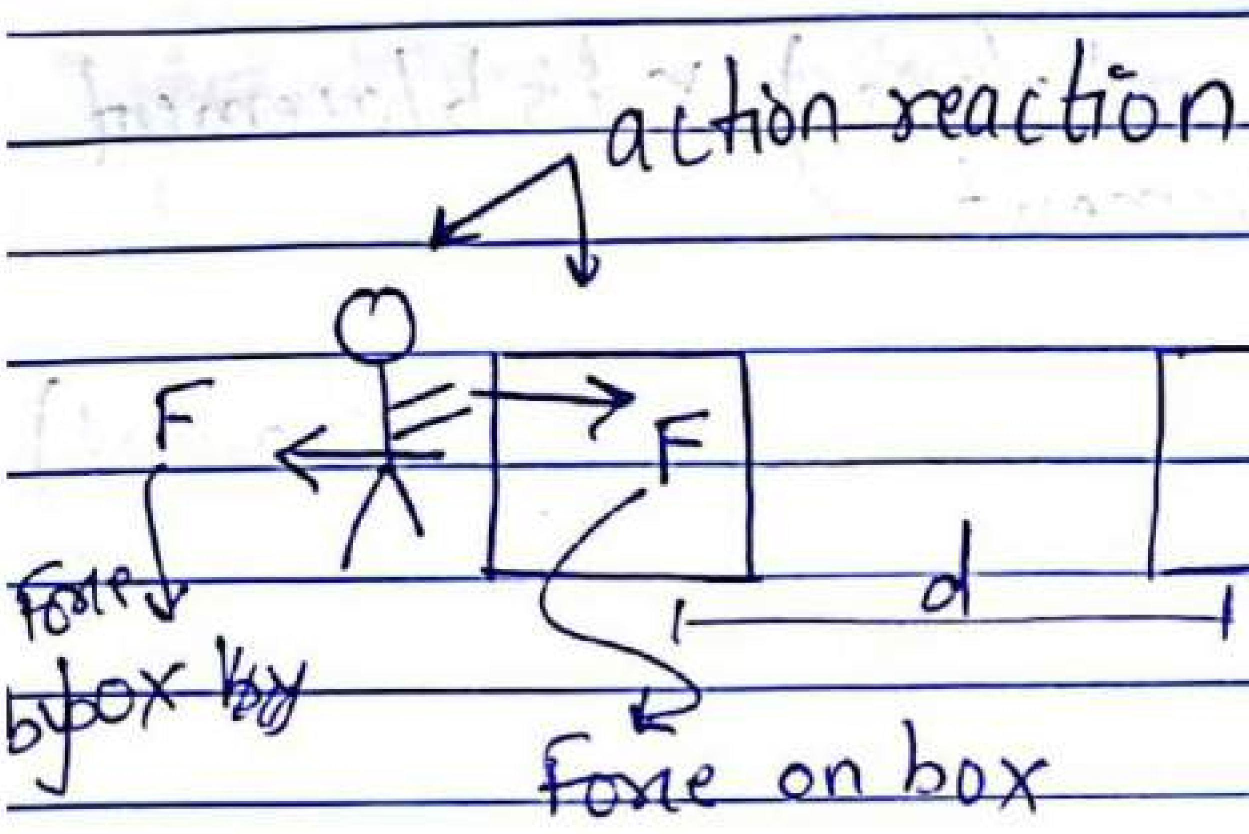
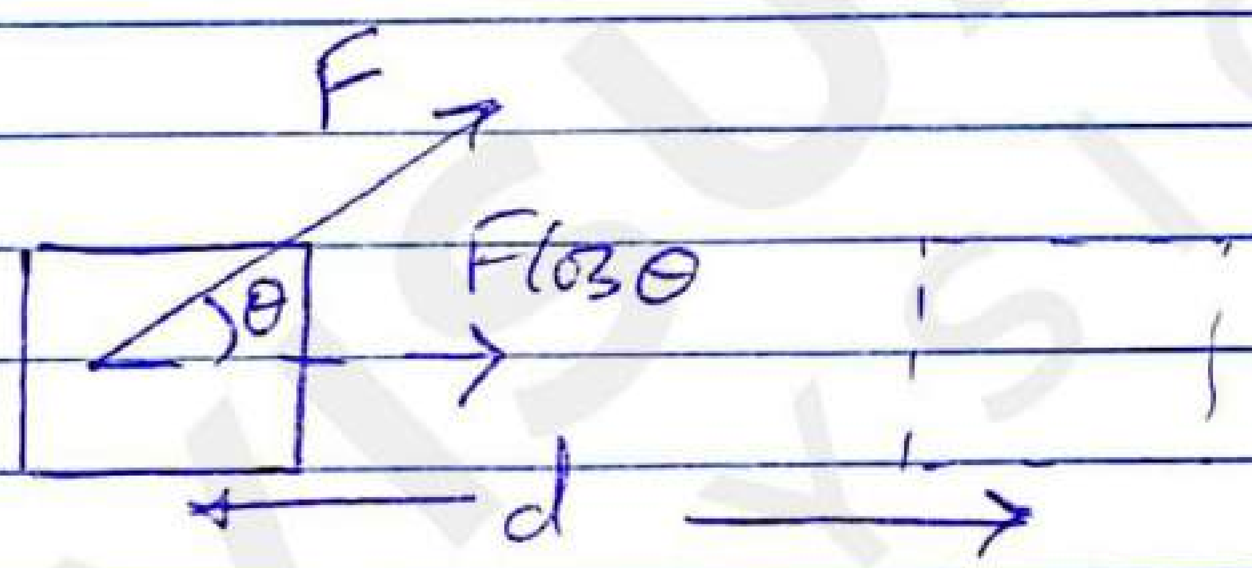


'F' Force needed to displace the box by d.



Here F cos θ amount of force need to displace object by 'd'.

$$\text{Work} = \vec{F}\cdot\vec{d} = F\cos\theta\, d$$

(N) (m) → so SI unit (N-m)

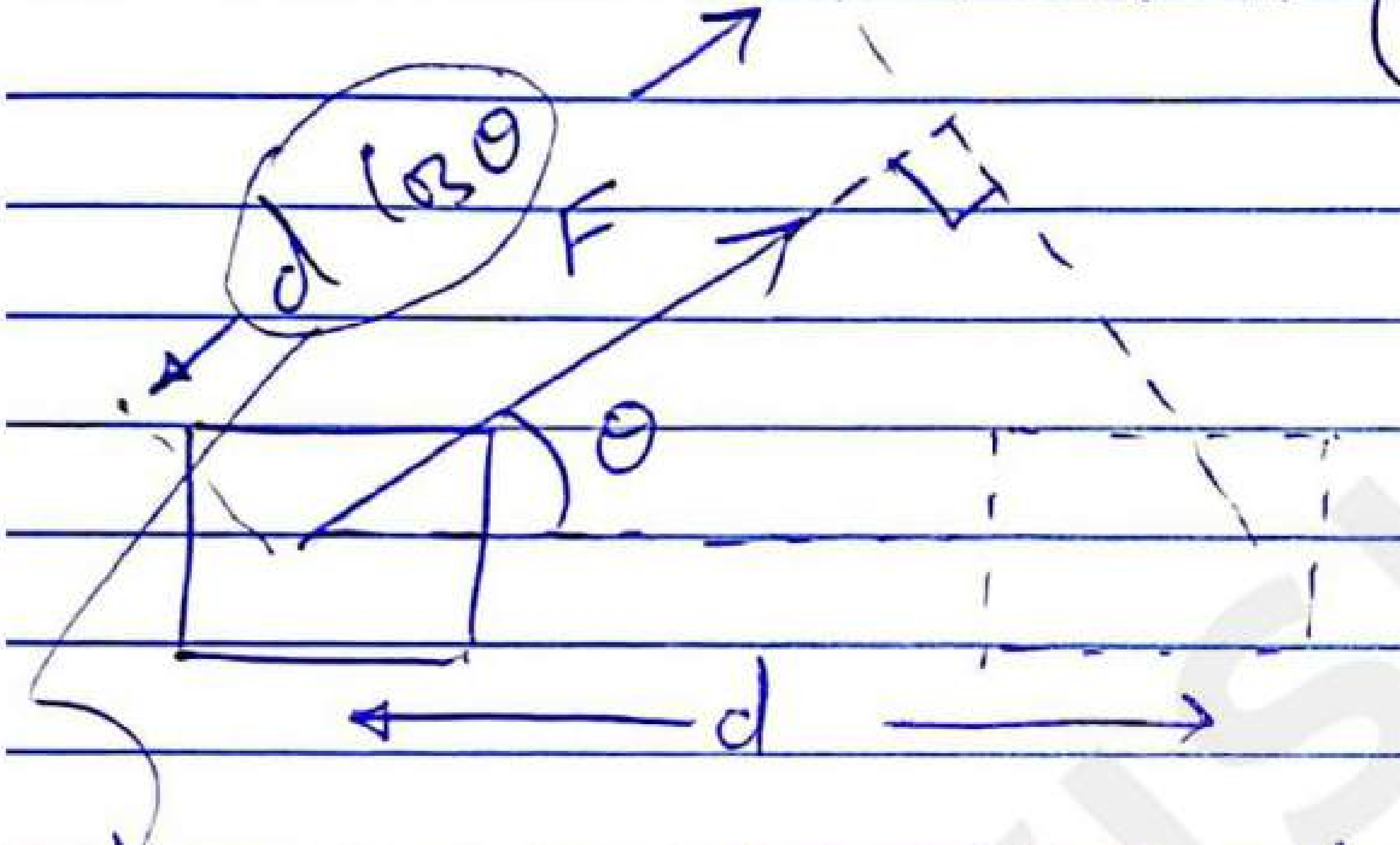
Force on object — displacement

Now $F\cos\theta$ is component along 'd'

& $d\cos\theta$ is component along 'F'

So, $W = \begin{pmatrix}\text{Force component along} \\ \text{displacement}\end{pmatrix} \times \text{displacement}$

$= \text{Force} \times \begin{pmatrix}\text{displacement component} \\ \text{along Force}\end{pmatrix}$



displacement component along force.

Vector

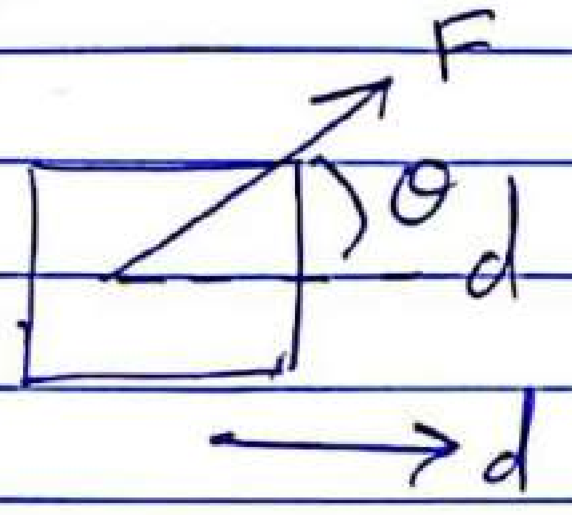
If in terms of vector

$$W = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k})$$

$$W = \underbrace{F_x X + F_y Y + F_z Z}_{\text{scalar}}$$

scalar
no direction

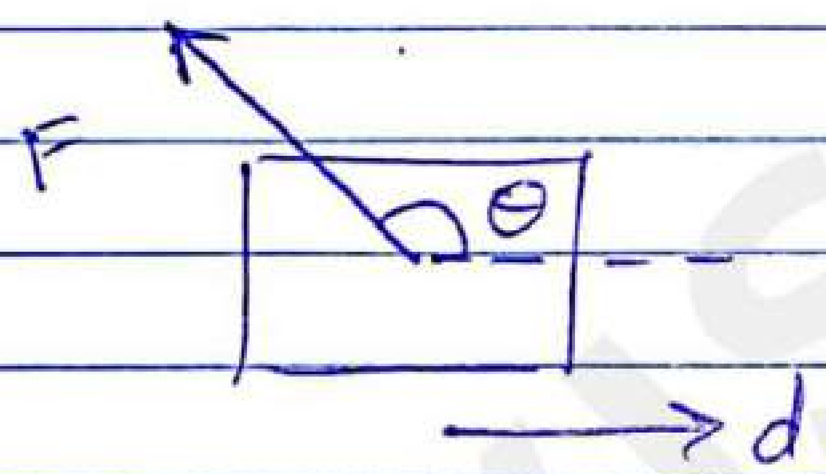
positive work $(0 < \theta < 90)$



$W = Fd\cos\theta$

Work is done <u>on</u> the object

negative work $(90 < \theta < 180)$

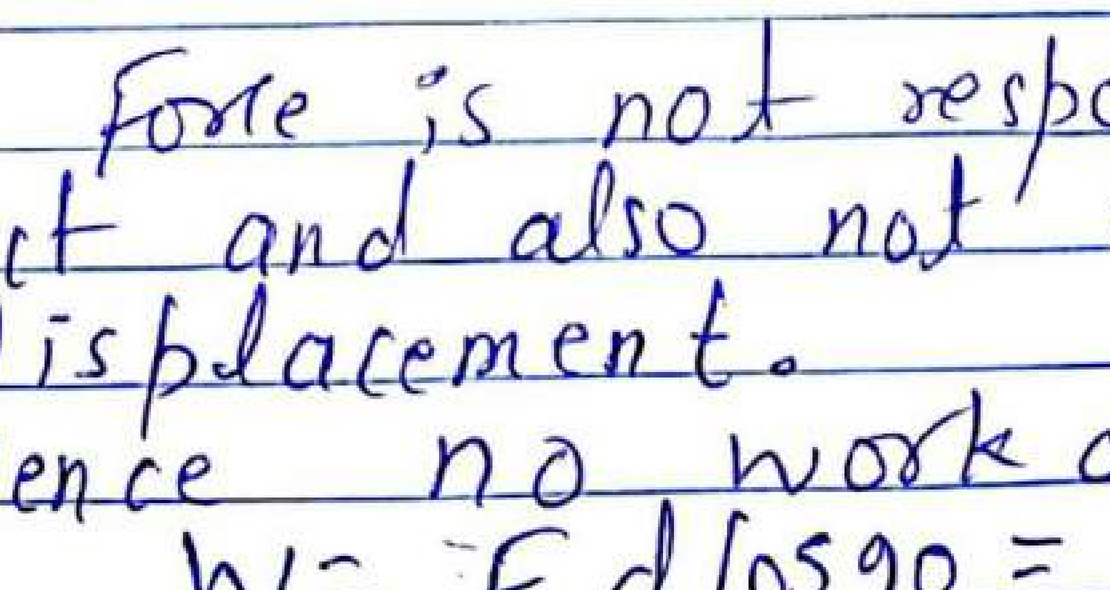


$W = Fd\cos\theta$

negative as $\cos \rightarrow -ve$ when $90 < \theta < 180$

Here work is done <u>against</u> the object

<u>Zero work</u>:

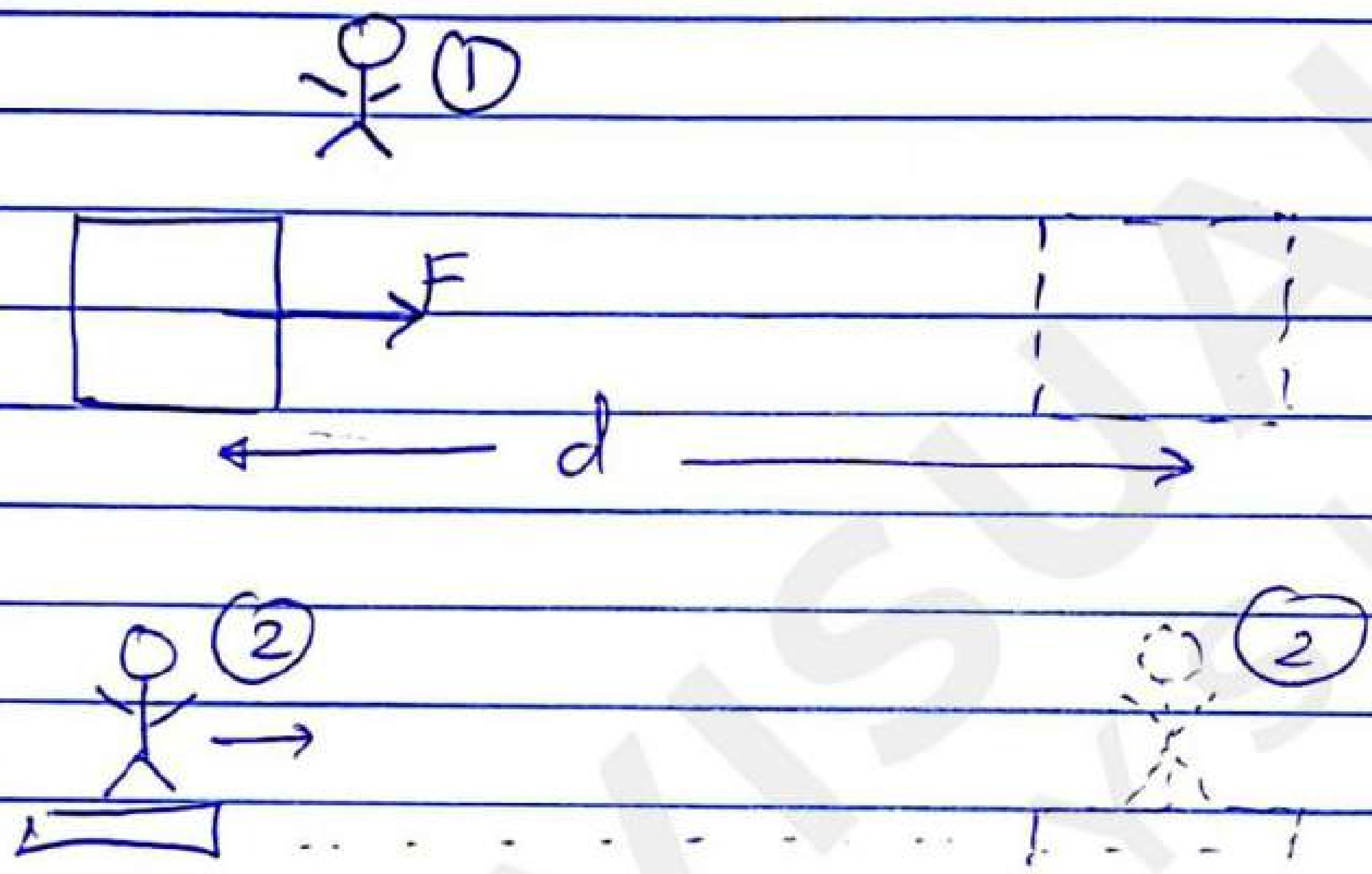


As here Force is not responsible in displacing the object and also not restricting the object displacement.

Hence no work done

$W = Fd\cos 90 = 0$

Work Depends on frame of reference :-



for 1 , block move by d

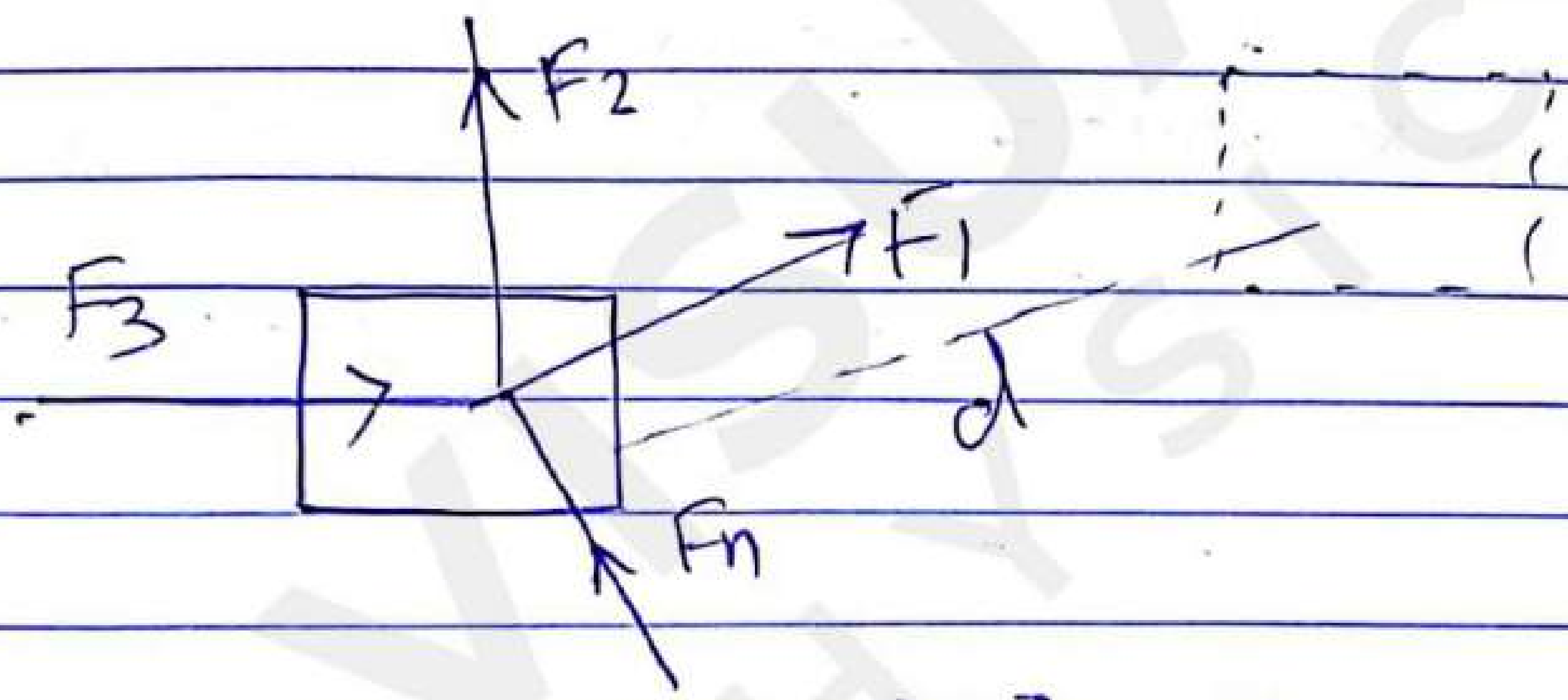
so $W_1 = Fd$

for observer 2 , block did not move

so

$W_2 = F(0) = \underline{0}$

so work depends on frame of reference.



In this case work = $\vec{F}_{net} \cdot \vec{d}$

$$\vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots \vec{F_n}$$

Now

$$W = \vec{F} \cdot \vec{d}$$

when the force is constant throughout 'd'

Now, what if, F is not constant throughout?

We calculate small work 'dw'.

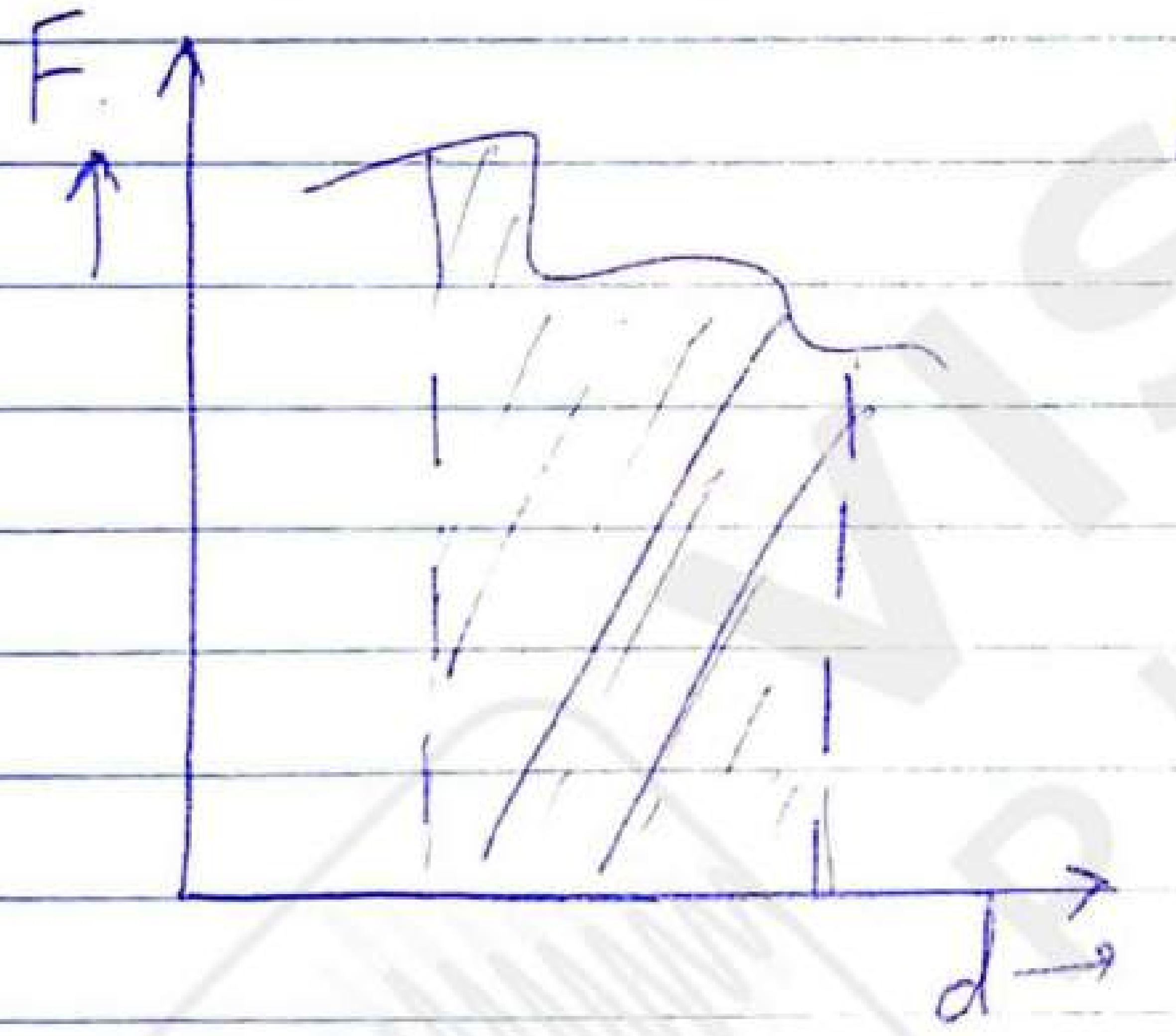
for small displacement 'ds' ↳ displacement

for which F is constant.

so, $dw = \vec{F} \cdot \vec{ds}$

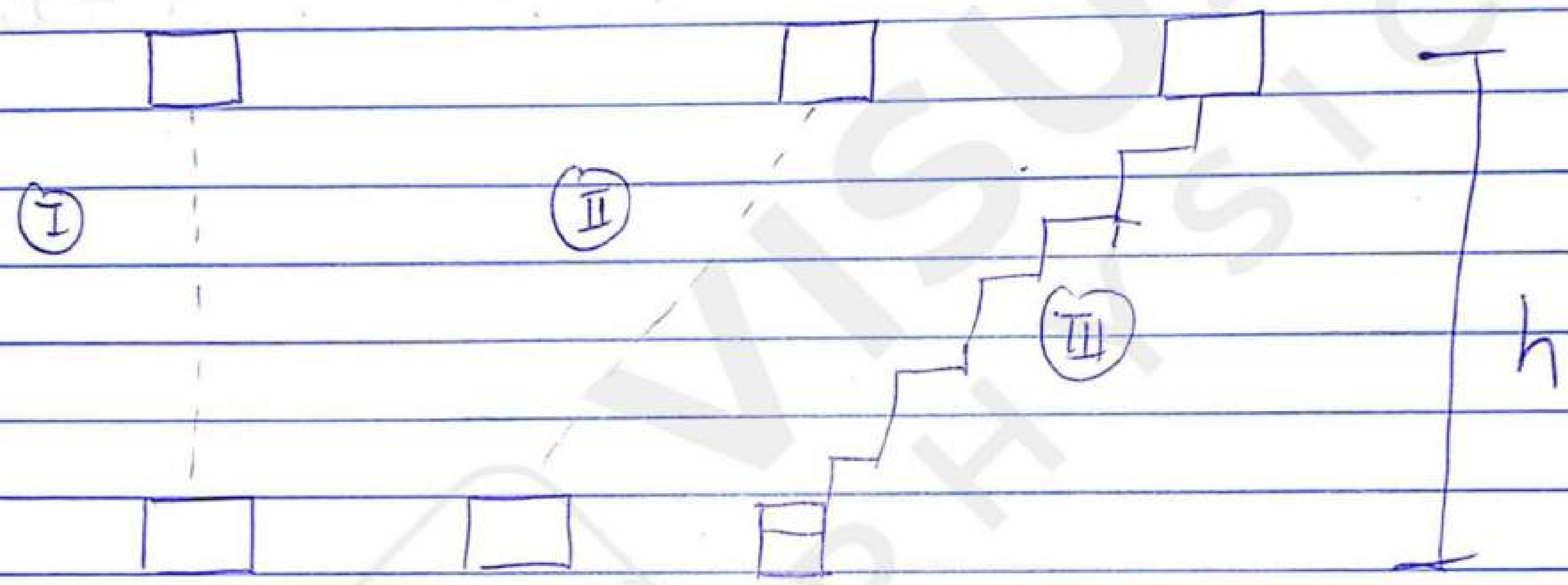
so net work ⇒ $\int dw = \int \vec{F} \cdot \vec{ds}$

Graphical Interpretation of work:



Area under F-d graph gives work done.

Work done by gravity:

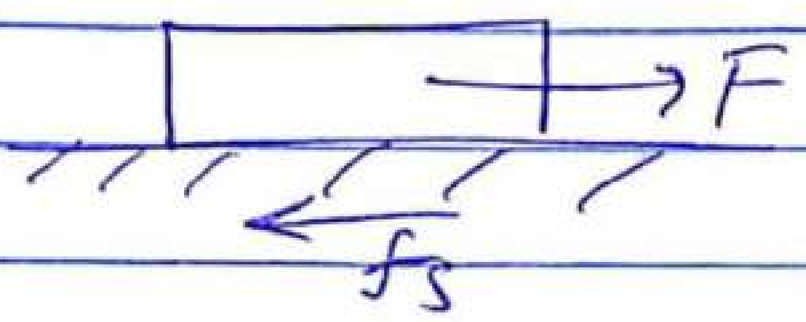


In all three cases work is same $(mg)(h)$

Work done by friction

Static friction:

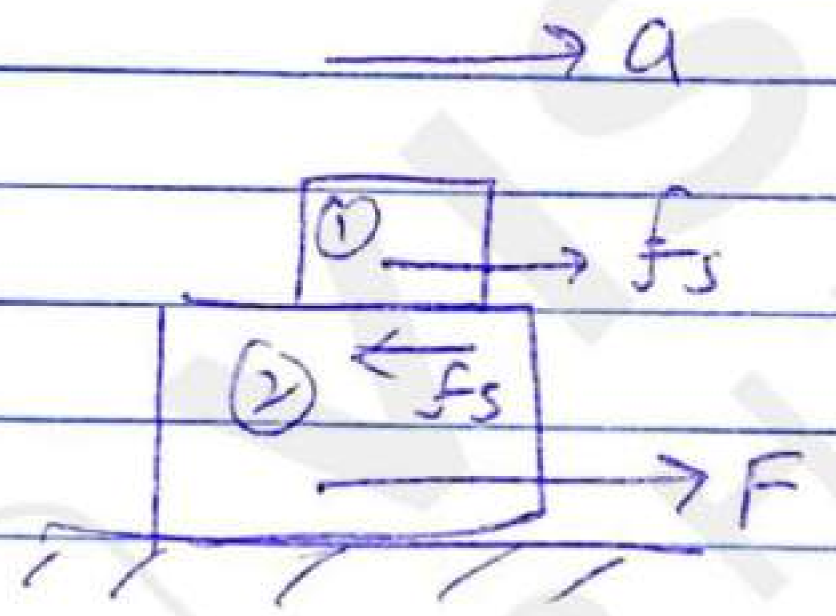
Case I :

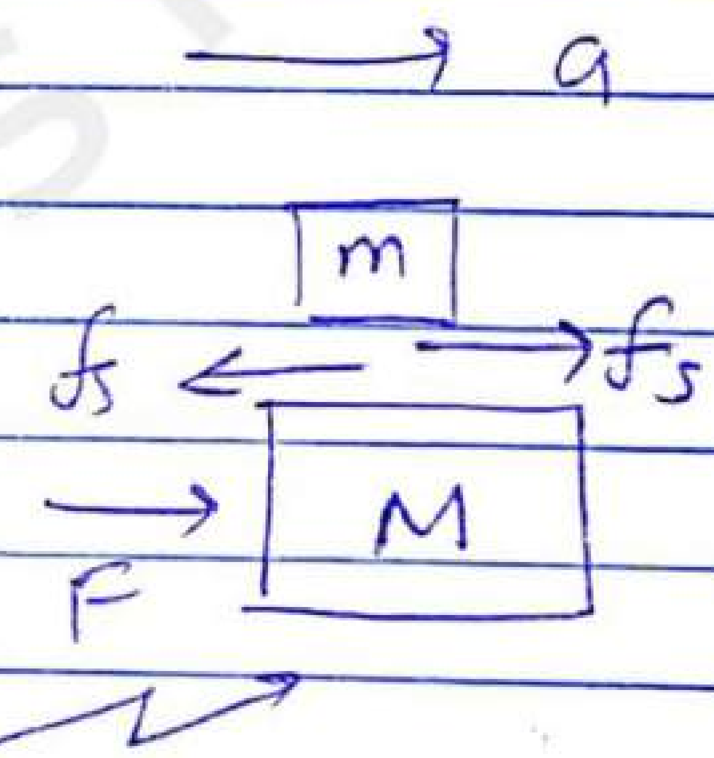


body is not moving

$$\text{work} = f_s(0) = 0$$

Case II :





both blocks moving together

So work done by static friction of small block

$$= (f_s)(d) = +f_s d$$

on M (big block) $= (f_s)\cdot(d)$ — f_s is opposite to d

$$= -f_s d = f_s d \cos 180^\circ$$

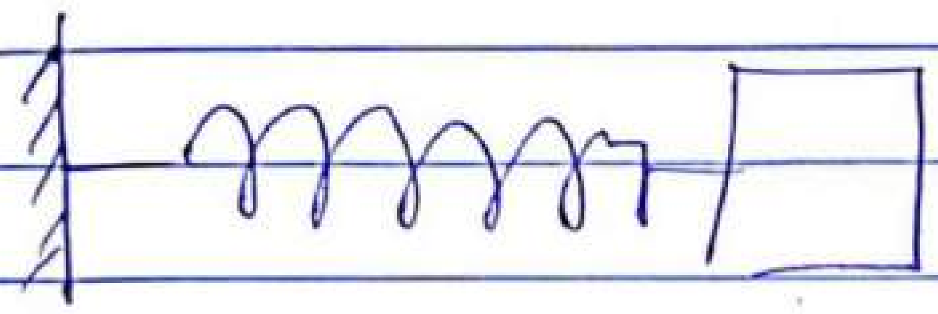
so, static friction work can be positive negative and zero depending on condition

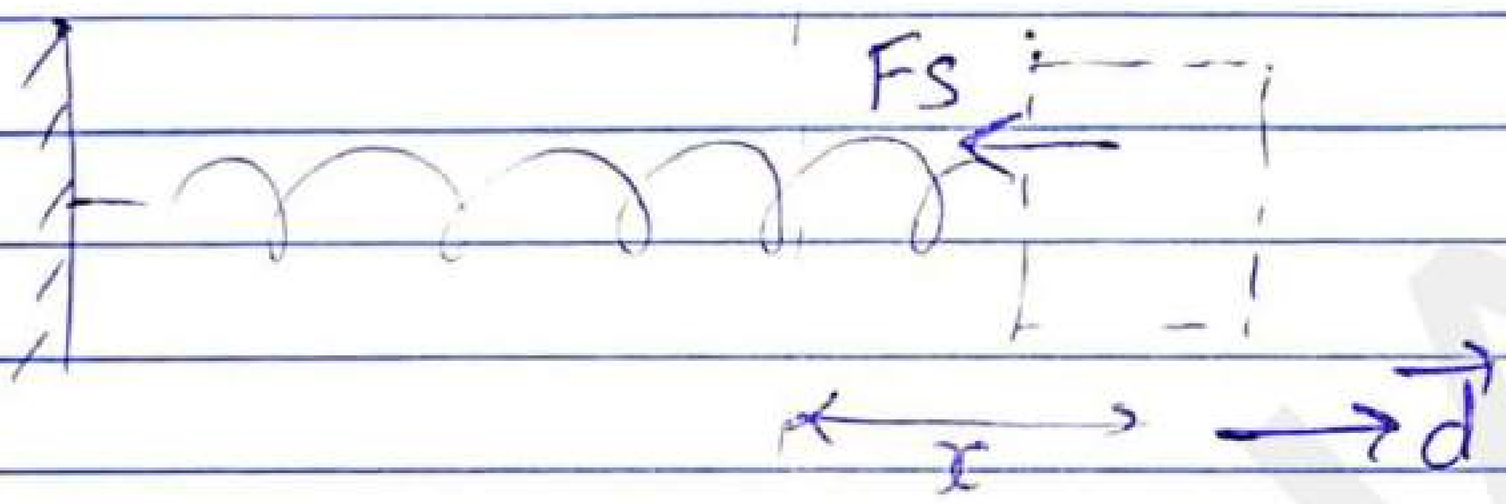
Kinetic friction:

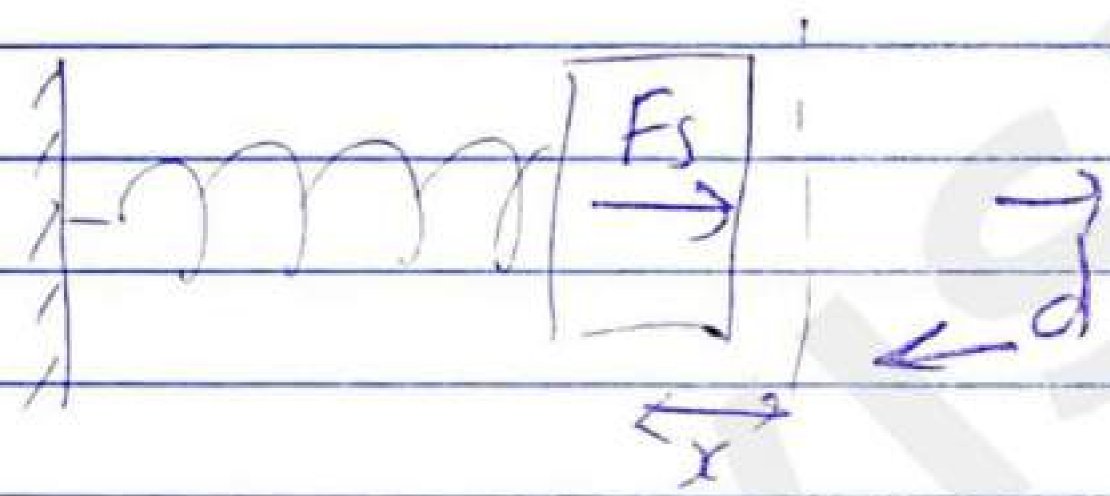
Kinetic friction is there when slipping occurs

so $\boxed{W = (\vec{f_k}) \cdot \vec{d}}$

Work done by spring







So spring force is always opposite to displacement.

Hence Spring always do negative work

$$W = -\frac{1}{2} k x^2$$

Spring Constant

displacement

magnitude

kinetic energy:

$$K.E = \frac{1}{2} m v^2$$

velocity

mass of body

Energy possesed because of motion

Also $K.E = \frac{1}{2} m v^2 \times \frac{m}{m}$

momentum

$$K.E = \frac{1}{2} \frac{p^2}{m}$$

Work-Energy theorem:

Net work done on body = change in K.E

All types of work i.e. gravitational, Spring, friction etc.

$$= \frac{1}{2} m [v_f^2 - v_i^2]$$

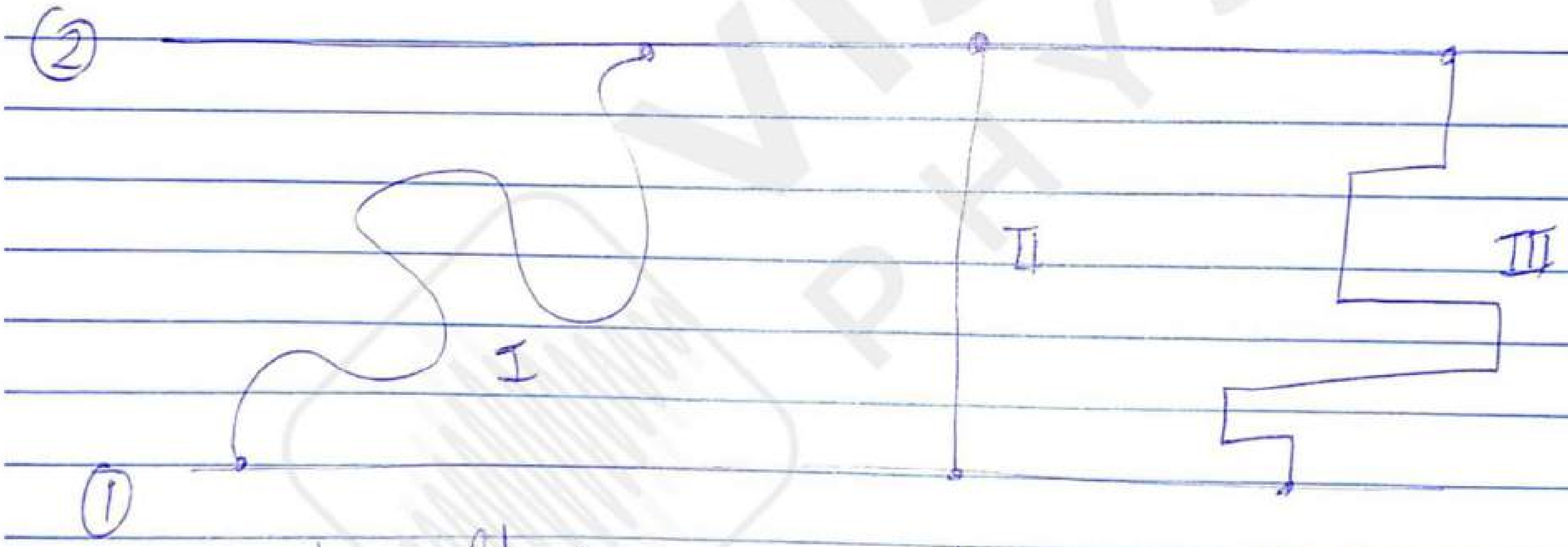
final velocity

initial velocity

$$\Sigma W = \Delta K$$

Conservative And non-Conservative forces

Conservative force :- work does not depend on path taken. only depends on final and initial point

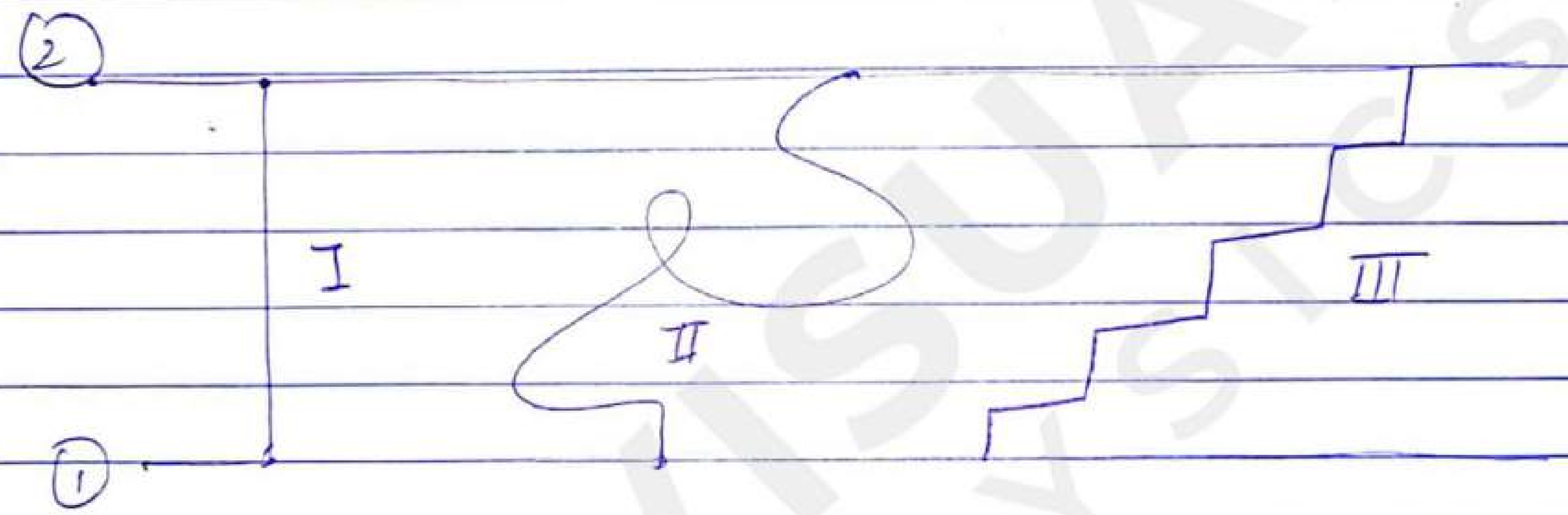


In all cases work is same

e.g. gravity force, spring force.

Non - Conservative force :

Work depends on path taken by the body.



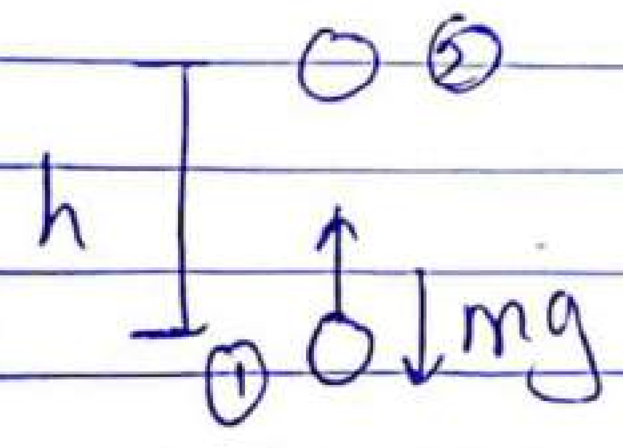
In all three cases work are different.

→ Work depend on path

e.g. friction, drag force.

$\boxed{\text{Theory section Q8}}$

Work in taking object from ground to height h



$$W = -mgh$$

Potential Energy:

Energy associated because of the configuration of one body with respect to other body.

In simple term object will have different potential energy with different frame of point of observation because of the change in configuration.

We can always find change in potential energy

$$\boxed{W_{conc} = -\Delta U}$$

ΔU → change in p.E

W_{conc} → Work by Conservative force

OR

$$F_c\, dr = -du$$

$$\boxed{F_c = -\frac{du}{dr}}$$

So, taking ground as reference:

ground p.E

$$\Delta U_g = -W$$
$$= -(-mgh)$$
$$U = mgh.$$

$$\Delta U = -\left(\frac{-1}{2}kx^2\right)$$

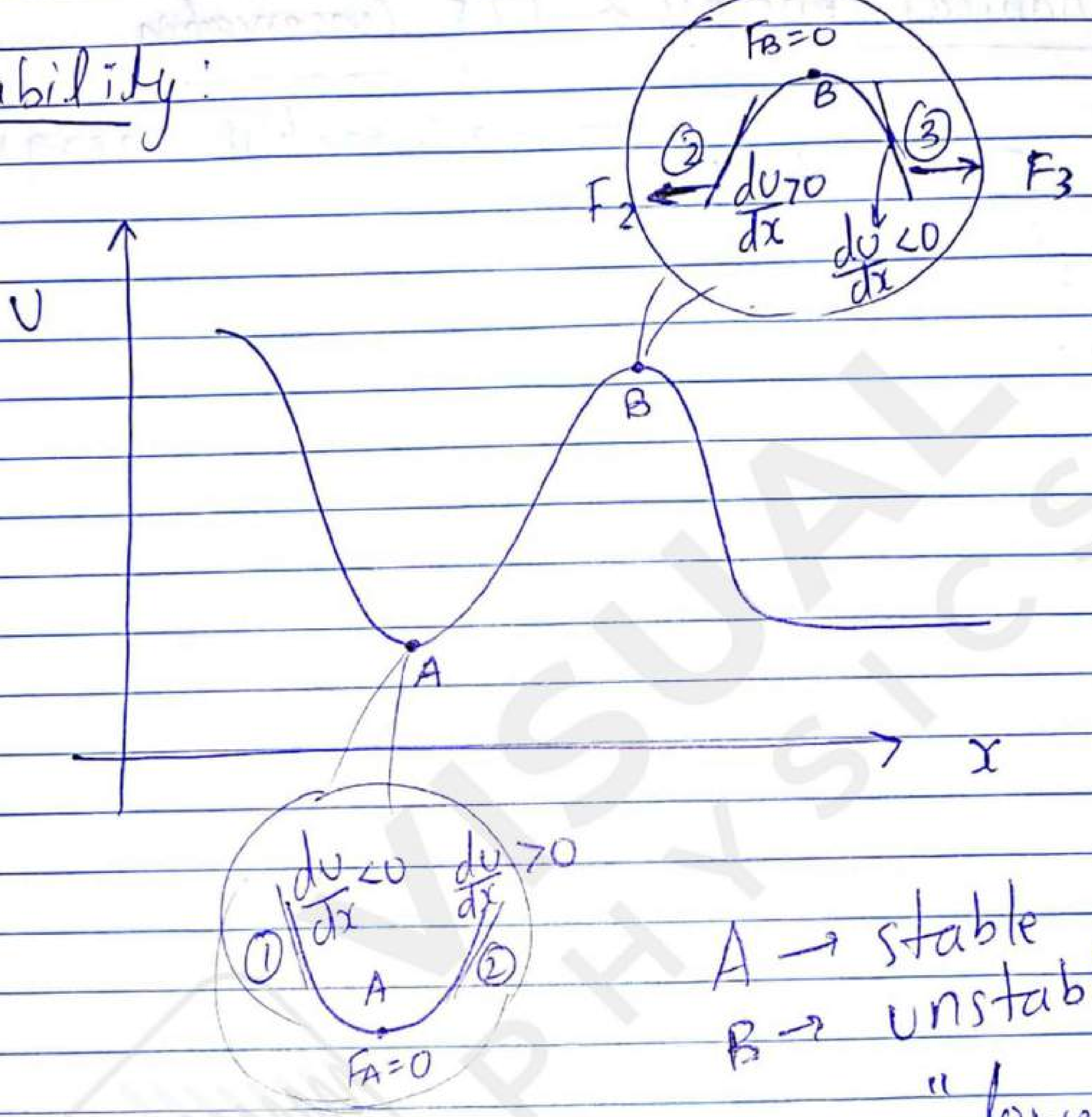
Spring P.E

$$\Delta U_s = 1/2\, kx^2$$

We take $U_{ground} = 0$ (for reference) so at height 'h'

$$\Delta U = U - 0 = mgh.$$

stability:



A → stable
B → unstable

"lowest potent Stable point"

for A, when we move along x

at ① $\frac{dU}{dx} < 0$, $F_1 = -\frac{dU}{dx}$

& ② $\frac{dU}{dx} > 0$, $F_2 = -\frac{dU}{dx}$

F_1 & F_2 always toward A.

but for (B), case is opposite, always opposite to B.

Mechanical Energy & ITS Conservation

$E = K + U$

$U \rightarrow$ potential energy

$K \rightarrow$ kinetic energy

$E \rightarrow$ Mechanical energy

if only Conservative forces acts while changing state from one state to other state.

$E_f = E_i$

$E_i \rightarrow$ Mechanical energy initial

$E_f \rightarrow$ Mechanical energy Final

$K_f + U_f = K_i + U_i$

But for work energy theorem

$$\boxed{W_{\text{Conservative}} + W_{\text{non-conservative}} = \Delta K}$$

Power:

↳ rate of doing work

$$P = \frac{dW}{dt}$$

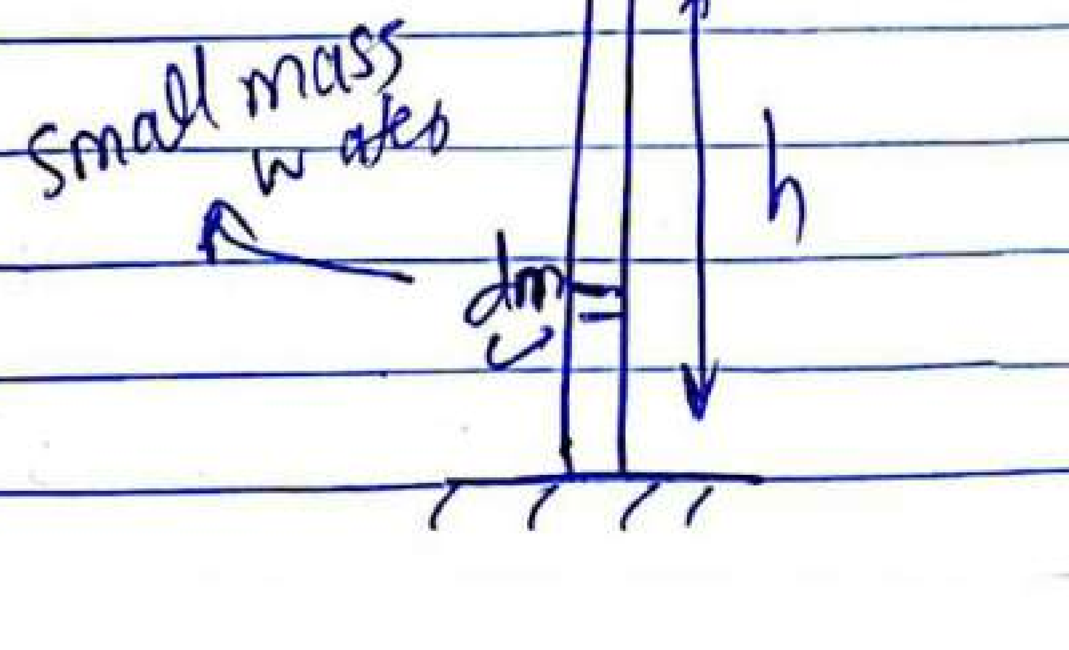
$$P = \frac{d}{dt}(\vec{F} \cdot \vec{s})$$

if force is constant

$$= \vec{F} \cdot \frac{d\vec{s}}{dt}$$

$$\boxed{P = \vec{F} \cdot \vec{V}}$$

Power of water-drawing pump.

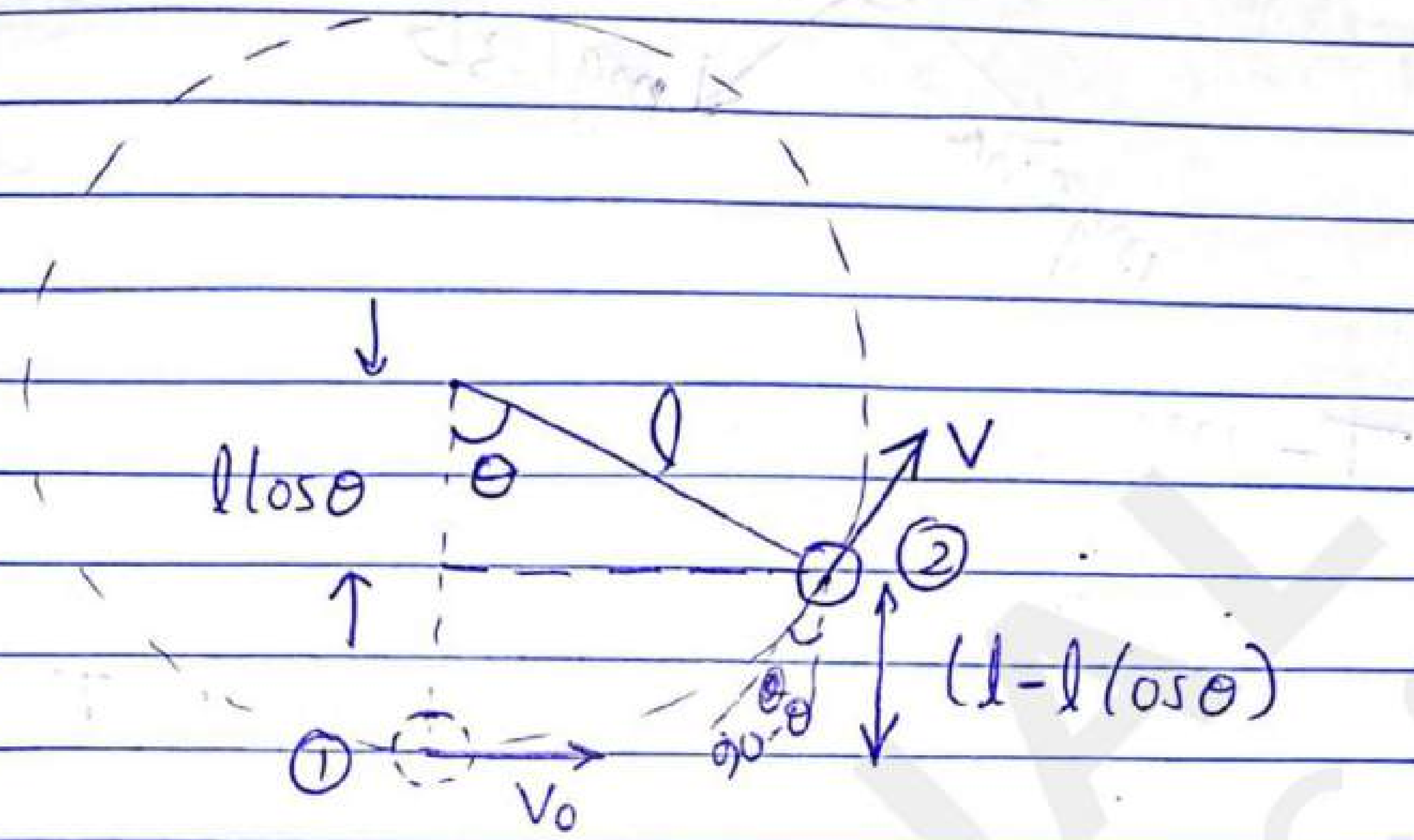


$$dW = (dm)gh + \frac{1}{2}(dm)v^2$$

P.E K.E

$$P = \frac{dW}{dt} = \frac{dm}{dt}\left[gh + \frac{v^2}{2}\right]$$

motion in vertical circle :-



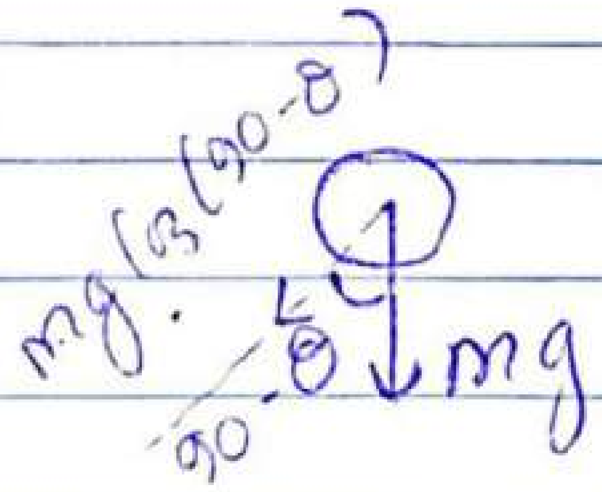
M.E Conservation.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg(l - l\cos\theta)$$

$$\boxed{v = \sqrt{v_0^2 - 2gl(1-\cos\theta)}} \quad -(1)$$

tangential acceleration :

$a_t =$

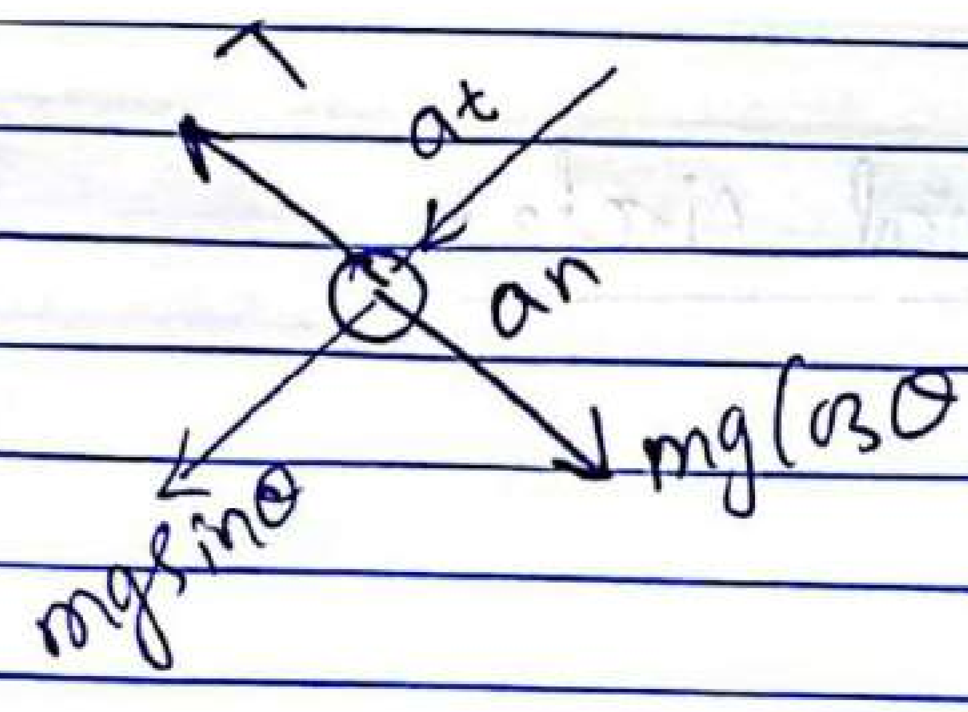


$$F = mg\cos(90-\theta) = ma_t$$

centripital

$$\boxed{a_t = g\sin\theta} \quad \text{and} \quad \boxed{a_n = \frac{v^2}{l}}$$

tangential



$T - mg\cos\theta = ma_r$

$$\Rightarrow \boxed{T = \frac{mv^2}{l} + mg\cos\theta} \quad \text{— V}$$

from (I), $T \to T_{min}$, at top as $\theta \to 180$

so, $T_{min} = \frac{mv^2}{l} + mg\cos 180$

$T_{min} = \frac{mv^2}{l} - mg$

if string slag at Top $T_{min} = 0$

$$\Rightarrow \frac{mv^2}{l} = mg$$

$$\boxed{V = \sqrt{gl}}$$

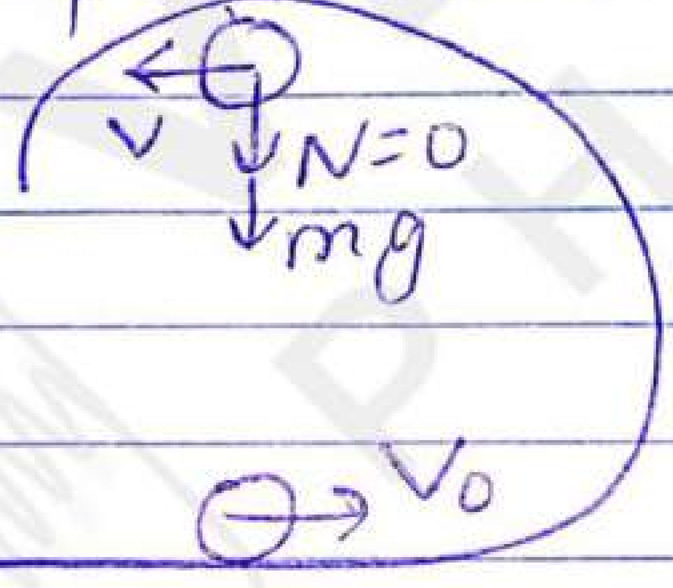
Now for $V_{top} = \sqrt{gl} = V_{lowest}$

from (1)

$$\sqrt{gl} = \sqrt{V_0^2 - 2gl(1-\cos\theta)} \Rightarrow \boxed{V_0 \geq \sqrt{5gl}}$$

that is the minimum velocity for just looping the loop for radius 'l' is $\sqrt{5gl}$.

⇒ Same condition applies for moving a car through a ~~ver~~ vertical circular track, in that case at top Normal $= 0$



$v_0 = \sqrt{5gR}$

$R \to$ radius

minimum speed to cover a loop.