



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Simple Harmonic Motion

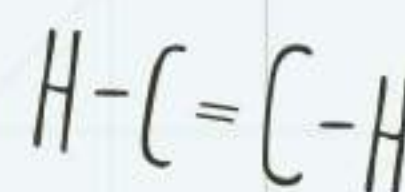
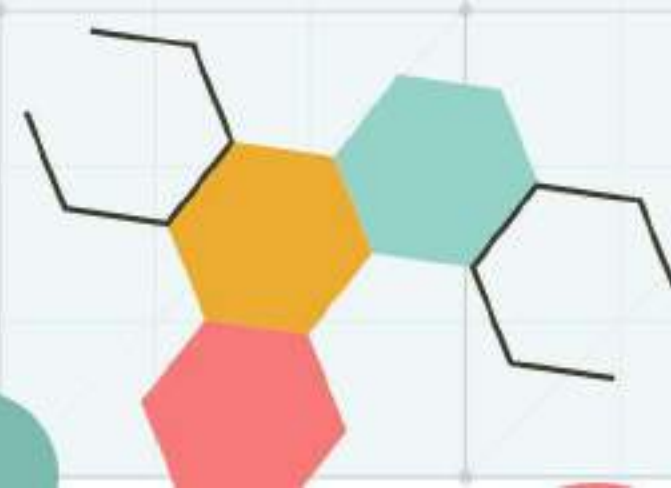
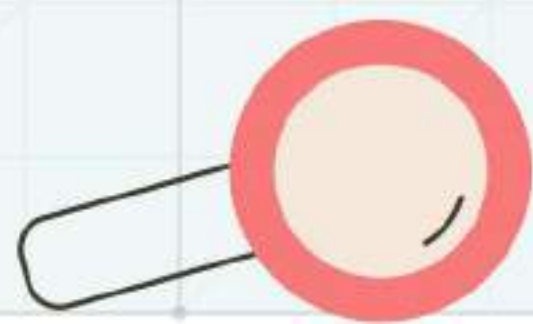
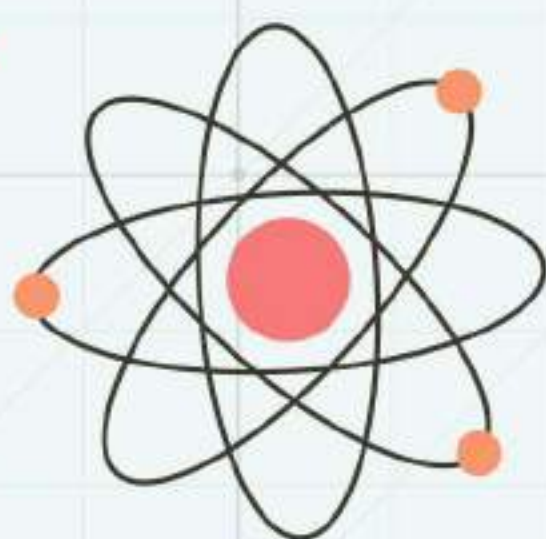
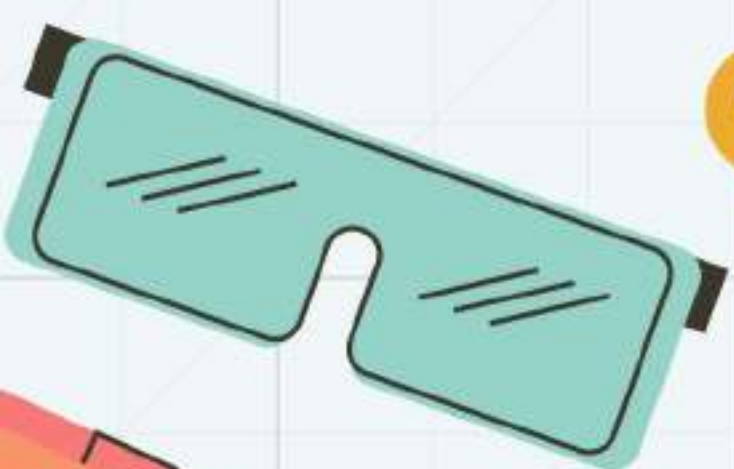
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SIMPLE HARMONIC MOTION

Periodic motion: Motion of an object that regularly returns to given position after a fixed time interval.

Oscillations $\rightarrow$ A special type of periodic motion in which particles moves to and fro about a fixed point.

SHM (Simple Harmonic Motion): Special case of oscillation in which particle move in straight line and acceleration $\propto$ displacement

$$a = -\omega^2 x$$

displacement

angular frequency

direction of 'a' is opposite to displacement direction

mean position

amplitude

amplitude

a

$-a$

The diagram illustrates the relationship between displacement and acceleration in Simple Harmonic Motion. It features a horizontal dashed line representing the mean position. A vertical line marks the mean position. To the left of the mean position, a point is labeled $-x$ and a double-headed arrow indicates the displacement. To the right, a point is labeled x and a double-headed arrow indicates the displacement. Below the mean position line, a point is labeled a and a double-headed arrow indicates the acceleration. To the right, a point is labeled $-a$ and a double-headed arrow indicates the acceleration. The distance from the mean position to the point x is labeled 'amplitude'. The distance from the mean position to the point $-a$ is also labeled 'amplitude'. The equation $a = -\omega^2 x$ is shown above the diagram, with arrows pointing from the terms to their respective parts: 'a' points to the acceleration arrows, ' ω^2 ' points to the angular frequency label, and 'x' points to the displacement arrows.

Amplitude $\rightarrow$ maximum displacement

$\rightarrow$ Time period $\rightarrow$ Time to cover one cycle

$\rightarrow$ frequency $\rightarrow \frac{1}{T}$, $\omega = \text{angular frequency} = \frac{2\pi}{T}$

EQUATION OF SHM :

$$a = -\omega^2 x$$

Necessary Condition for motion to be SHM

$F \propto x$ force constant
restoring force $F = -kx$ restoring force
direction is opposite to displacement

$$ma = -kx$$

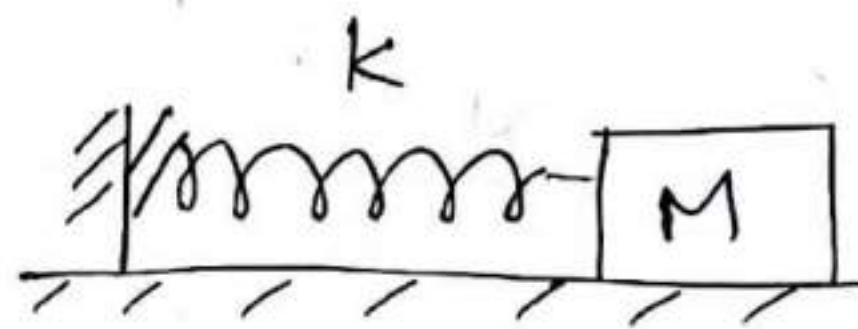
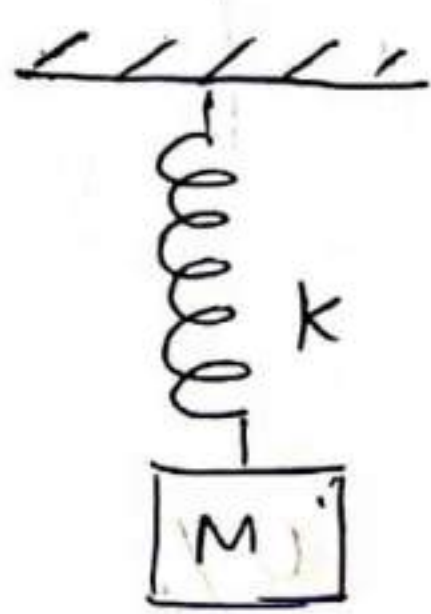
$$a = -\frac{k}{m} x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x = -\omega^2 x$$

solving this we get

$$x = A \sin(\omega t + \phi)$$

$\phi \rightarrow$ Initial phase $\omega \rightarrow$ angular frequency
 $A \rightarrow$ Amplitude

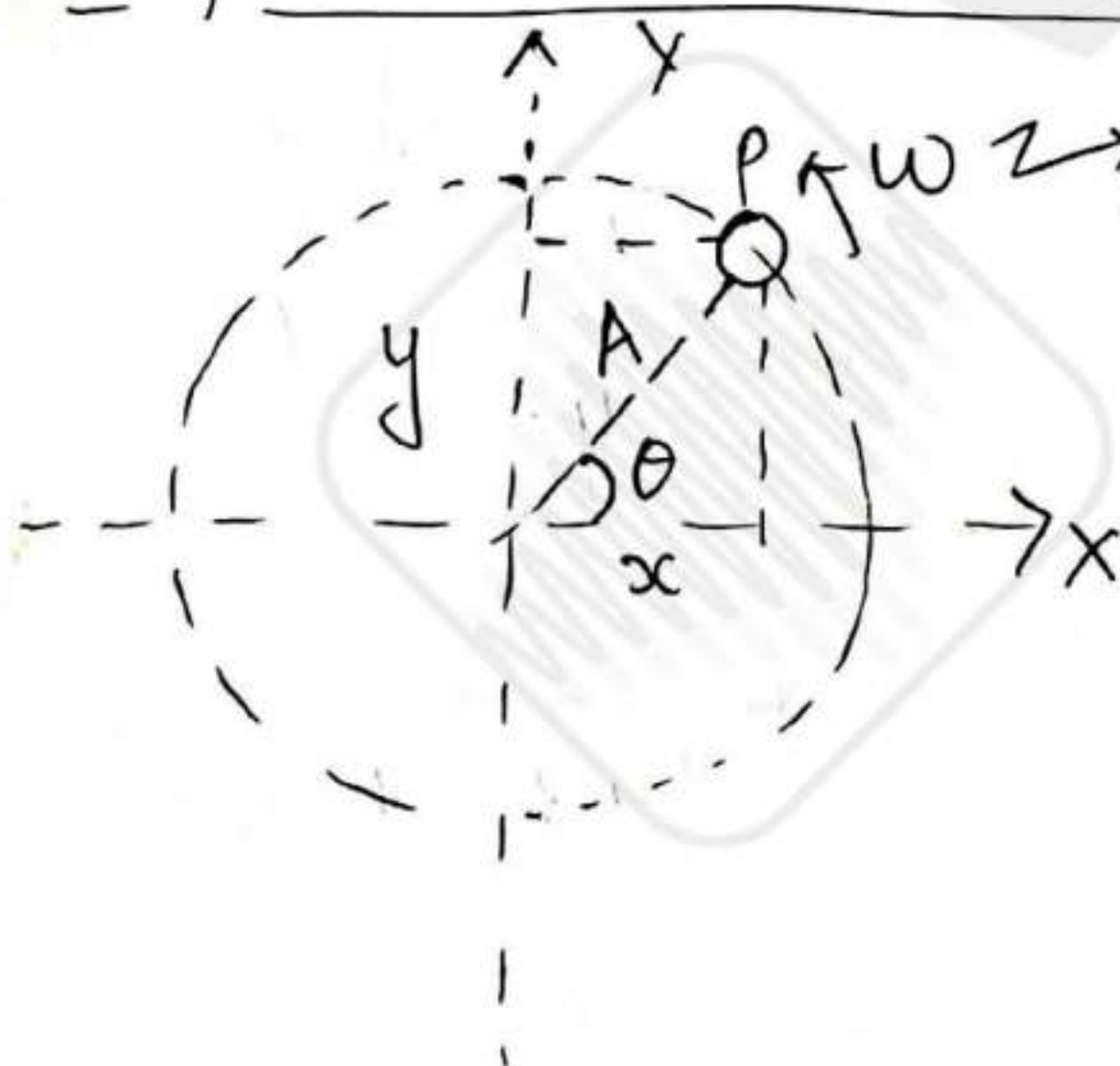


In both cases, $x = A \sin(\omega t + \phi)$

do not depend on orientation $\rightarrow$

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Harmonic Motion with Uniform Circular Motion:



$\omega \rightarrow$ angular speed

$$x = A \cos \theta \quad \& \quad y = A \sin \theta$$

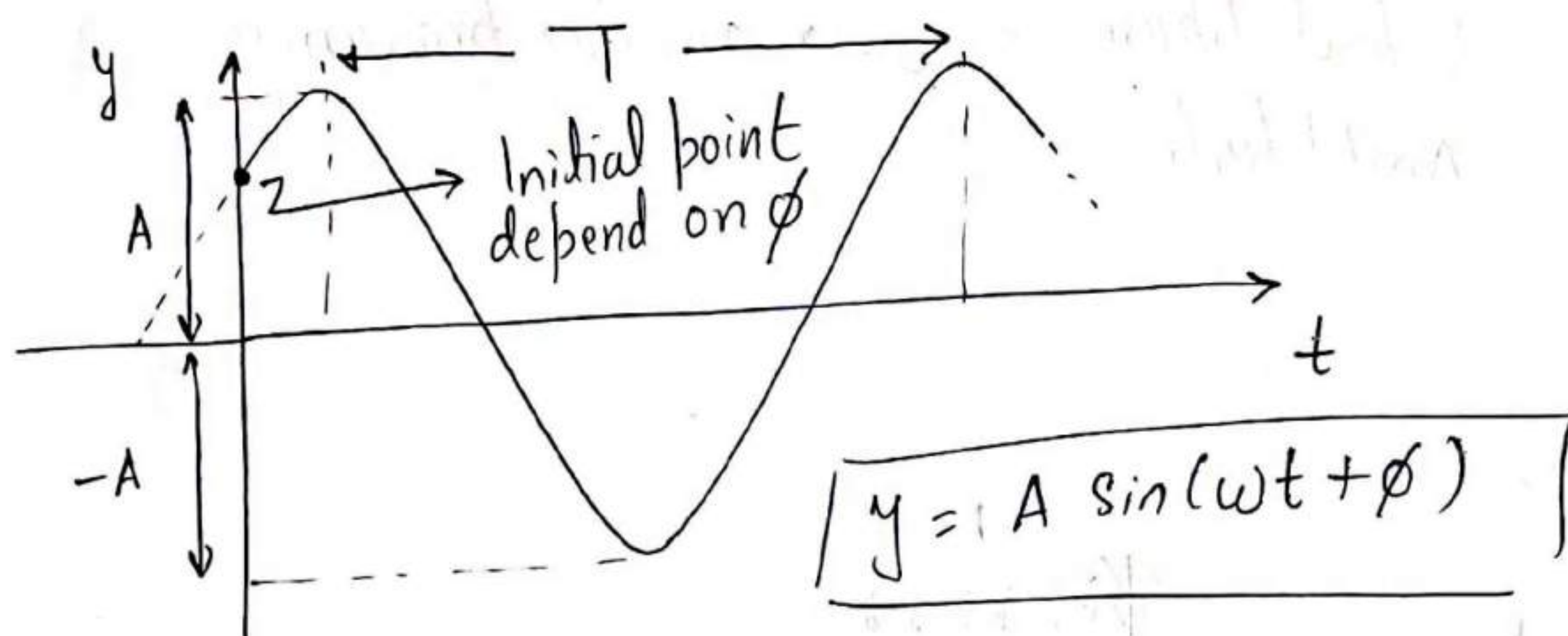
we can say,

$$\theta = \omega t + \phi$$

$\phi \rightarrow$ Initial position

$$x = A \cos(\omega t + \phi) \quad , \quad y = A \sin(\omega t + \phi)$$

Hence projection of uniform circular motion give or behave as SHM.



so, $y = A \sin(\omega t + \phi)$, $\boxed{\sin(\omega t + \phi) = y/A}$

so, $v = \frac{dy}{dt} = \frac{d}{dt} [A \sin(\omega t + \phi)]$

$v = \omega A \cos(\omega t + \phi)$

$\cos(\omega t + \phi) = \sqrt{1 - \sin^2(\omega t + \phi)} = \sqrt{1 - \frac{y^2}{A^2}}$

$\cos(\omega t + \phi) = \frac{1}{A} \sqrt{A^2 - y^2}$

$\boxed{v = \omega A \left[\frac{1}{A} \sqrt{A^2 - y^2} \right] = \omega \sqrt{A^2 - y^2}}$

so, $\boxed{a = \frac{dv}{dt} = -A \omega^2 \sin \omega t = -A \omega^2 y}$

Now, $y = A \sin(\omega t + \phi)$

phase

phase constant $\rightarrow$ when $t=0$, phase = phase constant

$\Rightarrow t=0$

$\Rightarrow$ phase constant $= \omega(0) + \phi$

$\Rightarrow$ $\text{phase constant} = \phi$

for a given oscillation

phase, $\phi_1 = \omega t_1 + \phi$

& $\phi_2 = \omega t_2 + \phi$

$\Delta \phi = \phi_2 - \phi_1 = \omega(t_2 - t_1)$

$\omega = \frac{2\pi}{T} \rightarrow$ Time constant (Time period)

$$\Delta\phi \rightarrow 2\pi \quad (\text{for e.g.})$$

$$\Rightarrow \omega \Delta t = 2\pi$$

$$\frac{2\pi}{T} \Delta t = 2\pi$$

$$\text{or } \boxed{\Delta t = T}$$

$\Rightarrow$ for 2π phase difference time difference is 'T'

ENERGY OF A BODY IN SIMPLE HARMONIC MOTION :

for SHM

$$F = -kx$$

$$dw = F dx = -kx dx$$

$$W = \int_0^x (-kx) dx = -\frac{1}{2} kx^2$$

$U(x) \rightarrow$ potential energy when displacement 'x'.

$$\text{Now } \Delta U = U(x) - U(0) = -W$$

$$\boxed{U(x) = \frac{1}{2} kx^2}$$

→ When at mean position, $x=0$, $U(x)=0$

$$U(x) = \frac{1}{2} kx^2$$

$$x = A \sin(\omega t), \quad v = \frac{dx}{dt} = A\omega \cos(\omega t)$$

$$U = \frac{1}{2} k A^2 \sin^2(\omega t)$$

$$\text{Now, } k = m\omega^2, \quad \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow U = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t)$$

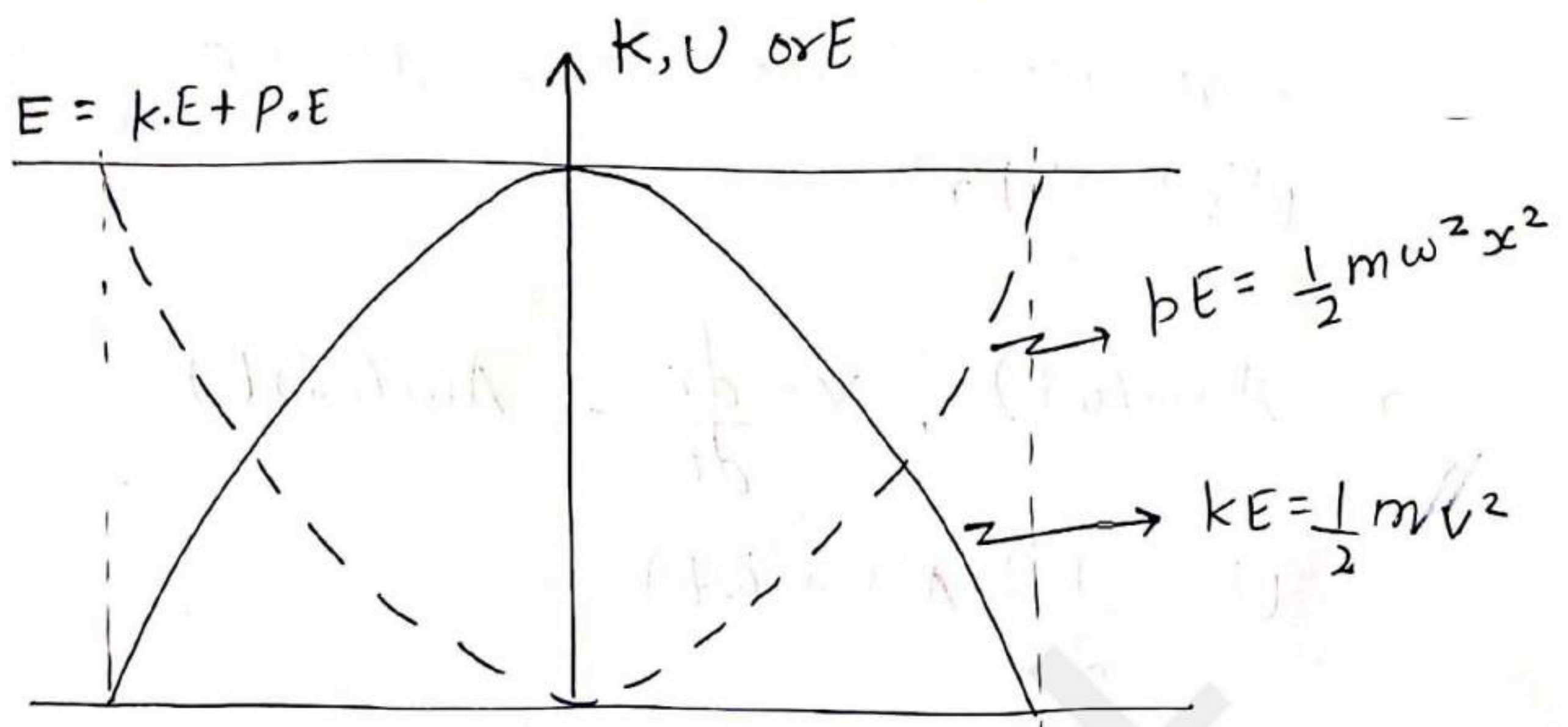
$$U = \frac{m\omega^2 A^2}{4} [1 - \cos 2\omega t]$$

$$\Delta \quad k = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t)$$

$$k = \frac{m\omega^2 A^2}{4} [1 + \sin 2\omega t]$$

$$E = U + k = \frac{1}{2} m\omega^2 A^2 [\sin^2(\omega t) + \cos^2(\omega t)] \quad \nearrow 1$$

$$\boxed{E = \frac{1}{2} m\omega^2 A^2}$$



Hence energy at time 't' is independent of 't'.
Thus the mechanical energy remains constant as expected.

Average PE & KE

$$U = \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

$$U_{\text{average}} = \frac{1}{T} \int_0^T U dt = \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi) dt$$

$$U_{\text{average}} = \frac{1}{4} m \omega^2 A^2$$

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2$$

$$KE = \frac{1}{2} m \left[\frac{d}{dt} [A \sin(\omega t + \phi)] \right]^2$$

$$= \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi)$$

$$KE_{\text{average}} = \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi) dt$$

$$= \frac{m \omega^2 A^2}{4T} \int_0^T \{1 + \cos 2(\omega t + \phi)\} dt$$

$$\boxed{KE_{\text{average}} = \frac{1}{4} m \omega^2 A^2}$$

SUPERPOSITION OF TWO SIMPLE HARMONIC MOTION

In same direction and of same frequency:

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

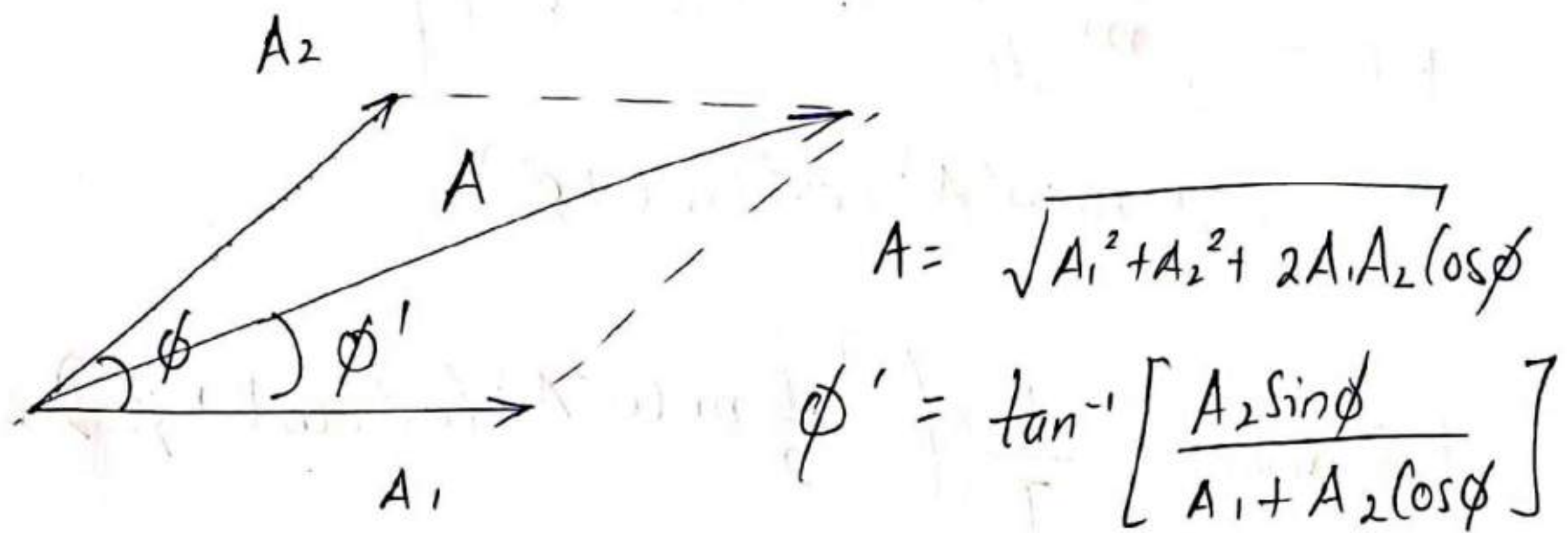
$$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$

Resultant

$$x = A \sin(\omega t + \phi')$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi'}$$

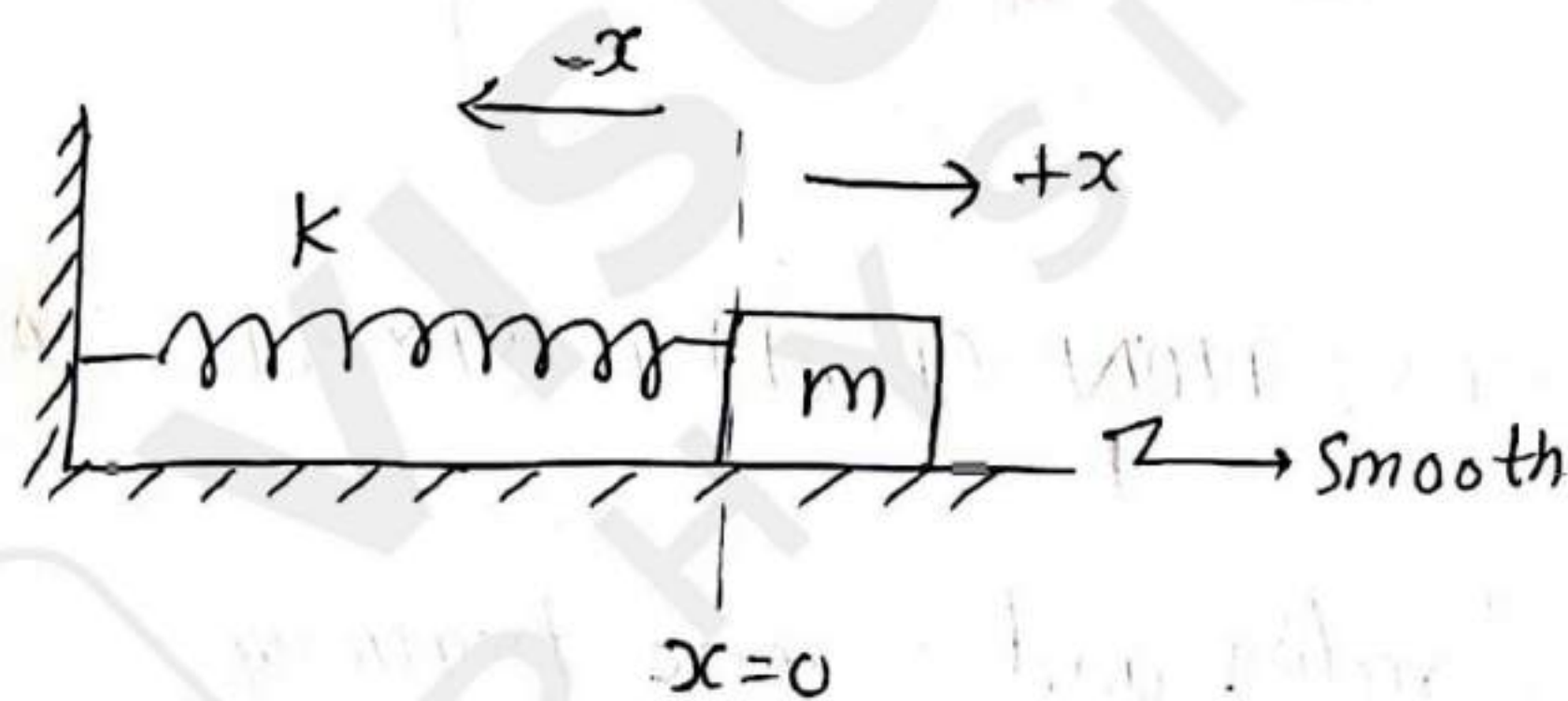
$$\phi' = \tan^{-1} \left[\frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi} \right]$$



THE SPRING-MASS SYSTEM

Case 1: In Horizontal motion

Method 1:



$$F = m \frac{d^2x}{dt^2} = -kx, \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m} \right) x = 0$$

$$\Rightarrow \omega^2 = \frac{k}{m}, \quad \omega = \sqrt{\frac{k}{m}}$$

$$x = A \sin(\omega t + \phi)$$

Method 2: Energy method:

Total mechanical Energy is constant [friction=0]

$$\Rightarrow \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \text{constant}$$

differentiating w.r.t time

$$\frac{1}{2} m (2v \frac{dv}{dt}) + \frac{1}{2} k (2x \frac{dx}{dt}) = 0$$

$$\frac{1}{2} m (2v a) + \frac{1}{2} k (2x v) = 0$$

$$a = -\left(\frac{k}{m}\right)x, \Rightarrow \omega^2 = \frac{k}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \quad \Rightarrow \quad \boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

Case II: Vertical motion

Method 1: Force

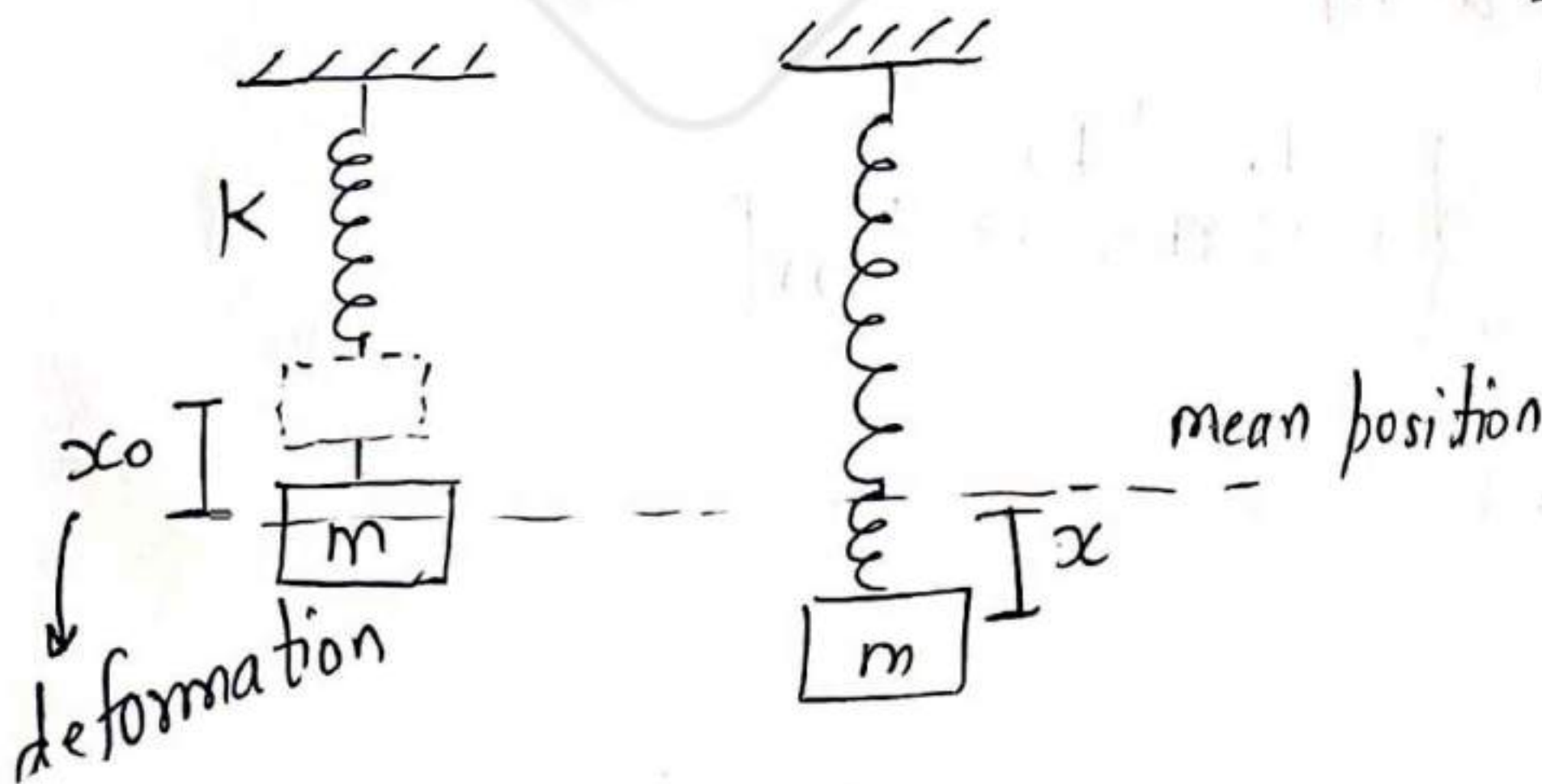
$$ma = -kx$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\omega^2 = \frac{k}{m}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$



Method 2: Energy Method

$$\frac{1}{2} m v^2 + \frac{1}{2} k (x_0 + x)^2 - mgx = \text{Constant}$$

$\hookrightarrow$ Gravitation p.e

differentiating w.r.t 't'

$$\frac{1}{2} m (2va) + \frac{1}{2} k [2(x_0 + x)v] - mgv = 0$$

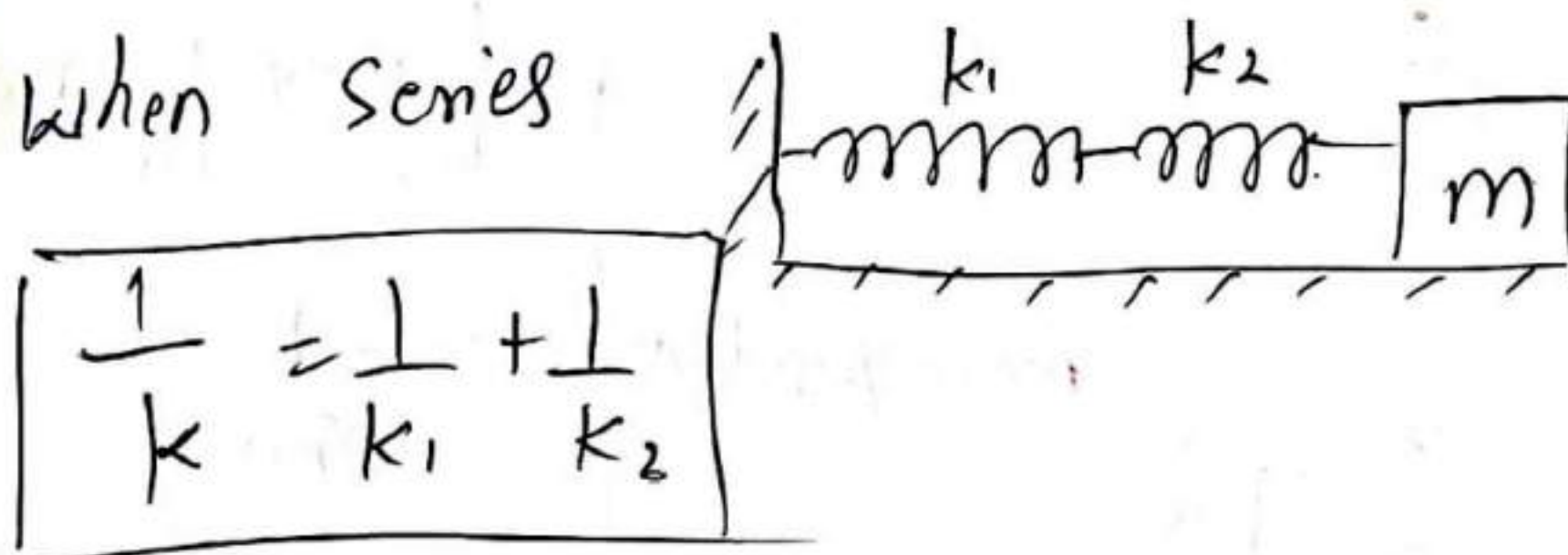
$$a = -\left(\frac{k}{m}\right)x \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

★ Time period of block - spring system depends only on mass of block & force constant of spring. (not positioning).

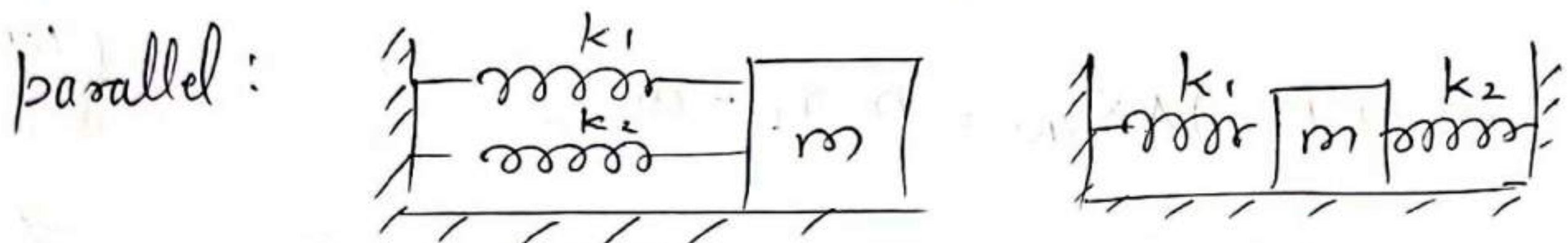
So, $\omega = \sqrt{\frac{k}{m}}$

$k \rightarrow$ equivalent

So, when series

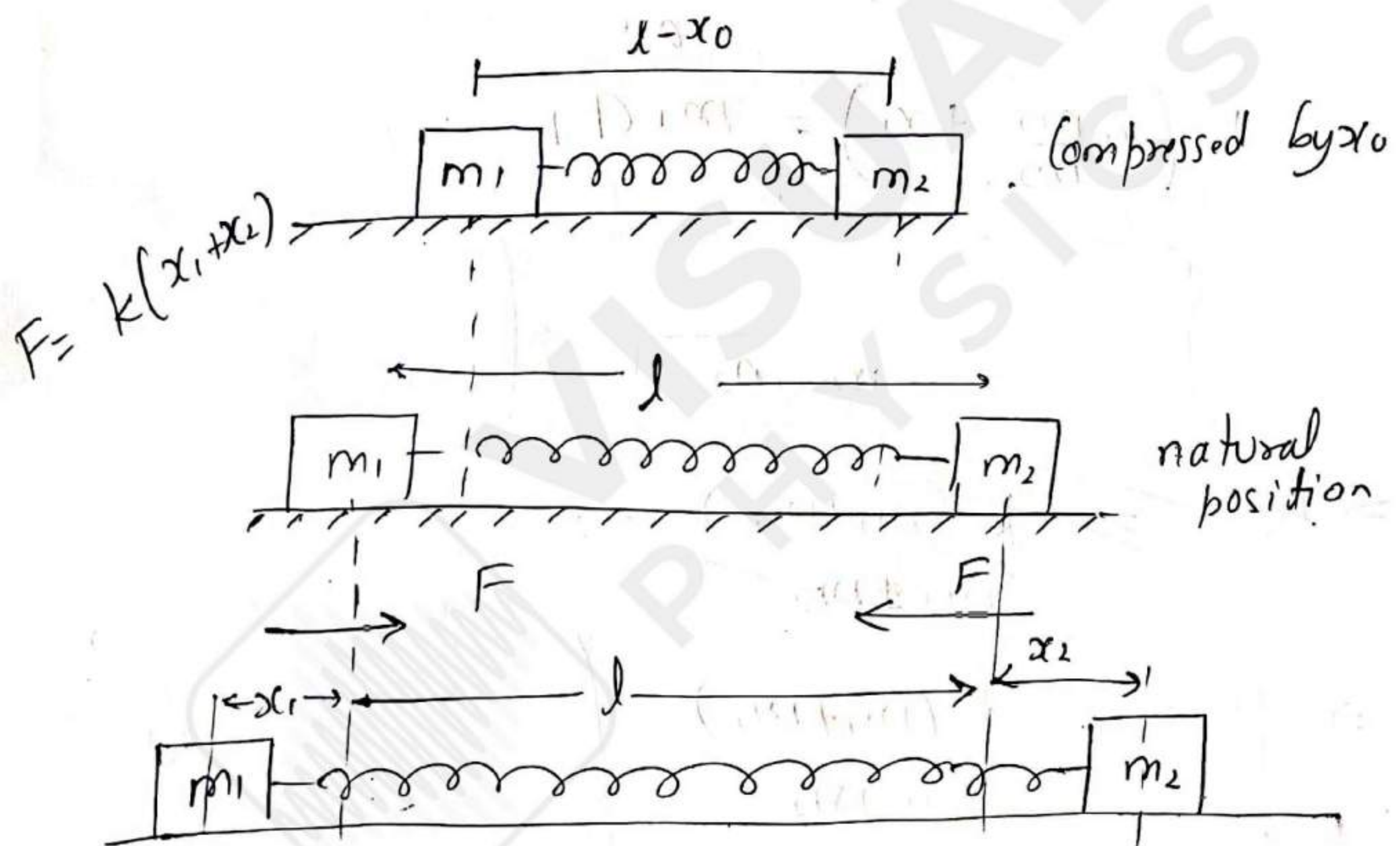


$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$



$$k = k_1 + k_2$$

Oscillation of a two-particle system two-body problem:



No external force, so, $\Delta X_{cm} = 0$

change in centre of mass position

$$\Rightarrow (m_1 + m_2) \Delta \dot{X}_{cm} = m_1 \dot{x}_1 - m_2 \dot{x}_2$$

$$\Rightarrow \boxed{m_1 x_1 = m_2 x_2} \quad \text{as } \boxed{\Delta X_{cm} = 0}$$

for on first particle. $k(x_1 + x_2)$

$$k(x_1 + x_2) = m_1 \frac{d^2 x_1}{dt^2}$$

$$K \left(x_1 \frac{m_1}{m_2} + x_1 \right) = m_1 a_1$$

$$\boxed{a_1 = \frac{k(m_1 + m_2) x_1}{m_1 m_2}}$$

$$\Rightarrow \boxed{\omega^2 = \frac{k(m_1 + m_2)}{m_1 m_2}}$$

$$T = 2\pi \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$

or for such system $\mu = \text{reduce mass}$

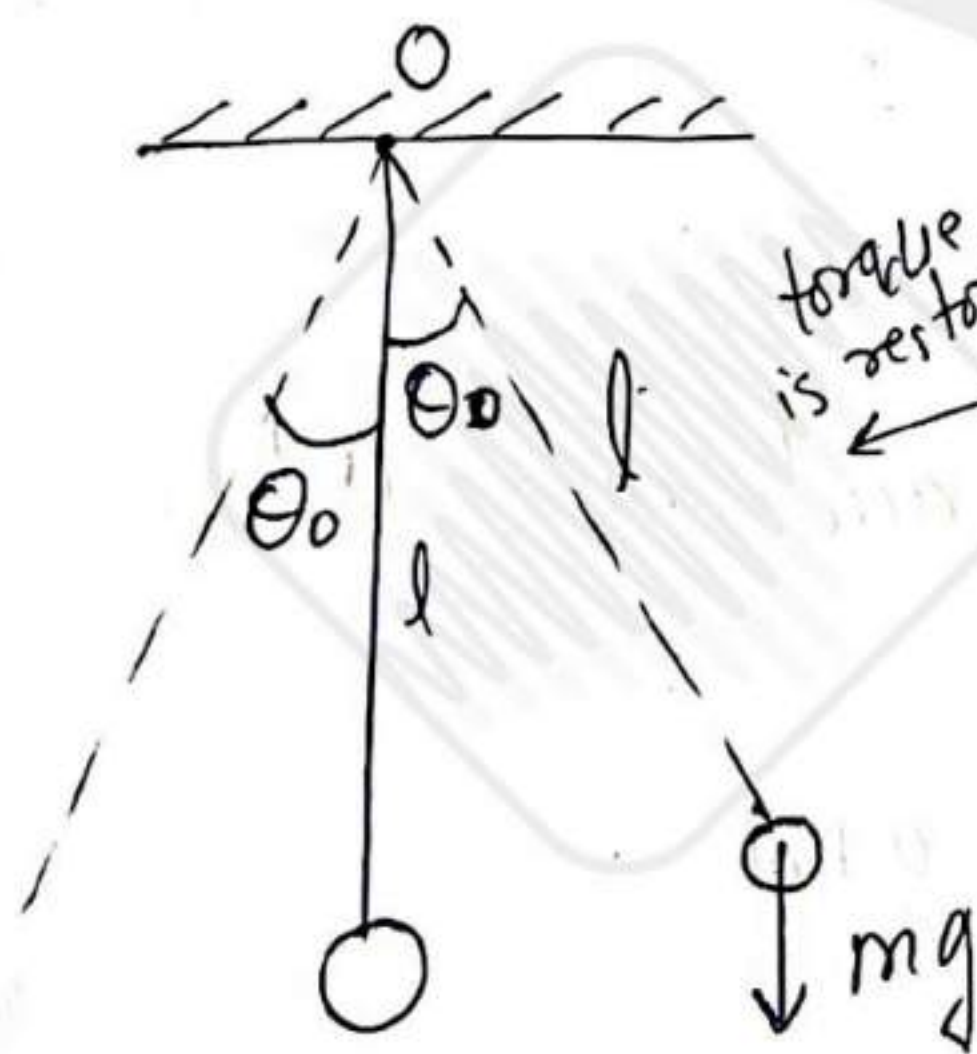
$$\boxed{\mu = \frac{m_1 m_2}{(m_1 + m_2)}}$$

Hence for connected system,

$$T = 2\pi \sqrt{\frac{\mu}{k}}, \quad \omega^2 = \frac{k}{\mu}$$

we use reduce mass system.

SIMPLE PENDULUM:



torque about 'O':

$$\tau = -mgl \sin \theta \quad (\text{at any position } \theta)$$

torque is restoring

for small angle ' θ '

$$\sin \theta \sim \theta$$

$$\Rightarrow \tau = -mgl \theta$$

$$\tau = I \alpha = ml^2 \left(\frac{d^2 \theta}{dt^2} \right)$$

$$\Rightarrow ml^2 \frac{d^2 \theta}{dt^2} = -mgl \theta$$

$$\rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta}$$

similar to $\frac{d^2x}{dt^2} = -\frac{k}{m}x$

$$\rightarrow \boxed{\omega^2 = \frac{g}{l}}$$

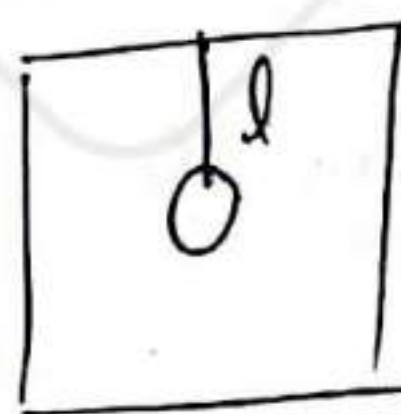
$$\text{So, } \boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

Independent on mass of the bob.

$$\boxed{T = 2\pi \sqrt{\frac{l}{g_{\text{eq}}}}}$$

equivalent acceleration

1. When pendulum going up by acceleration 'a'

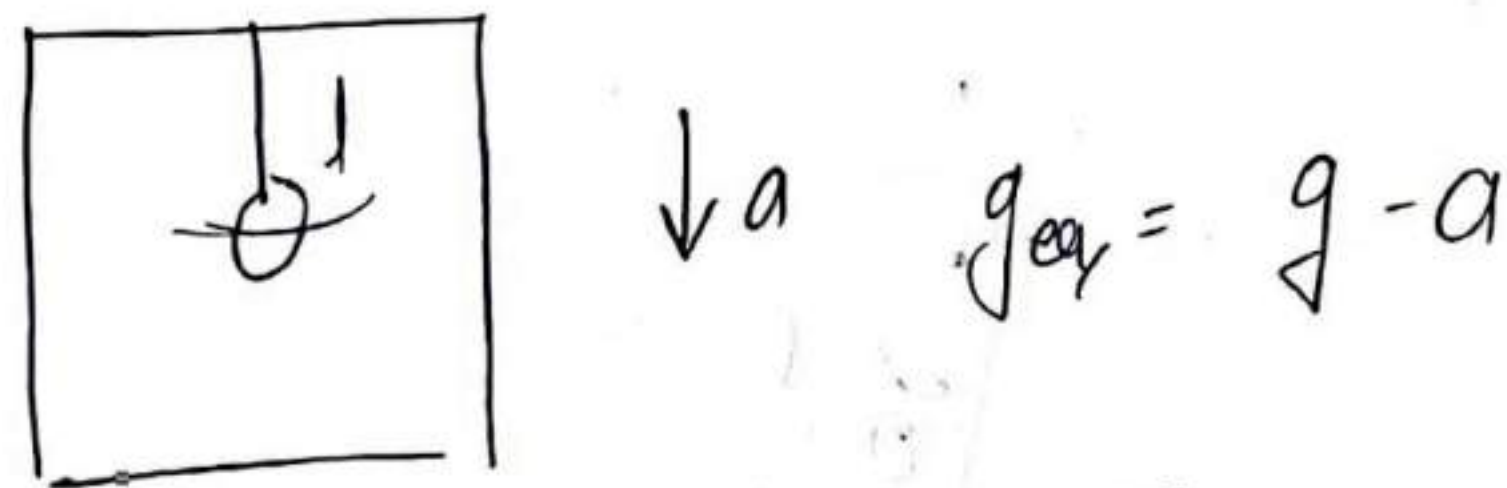


$\uparrow a$

$$g_{\text{eq}} = g + a$$

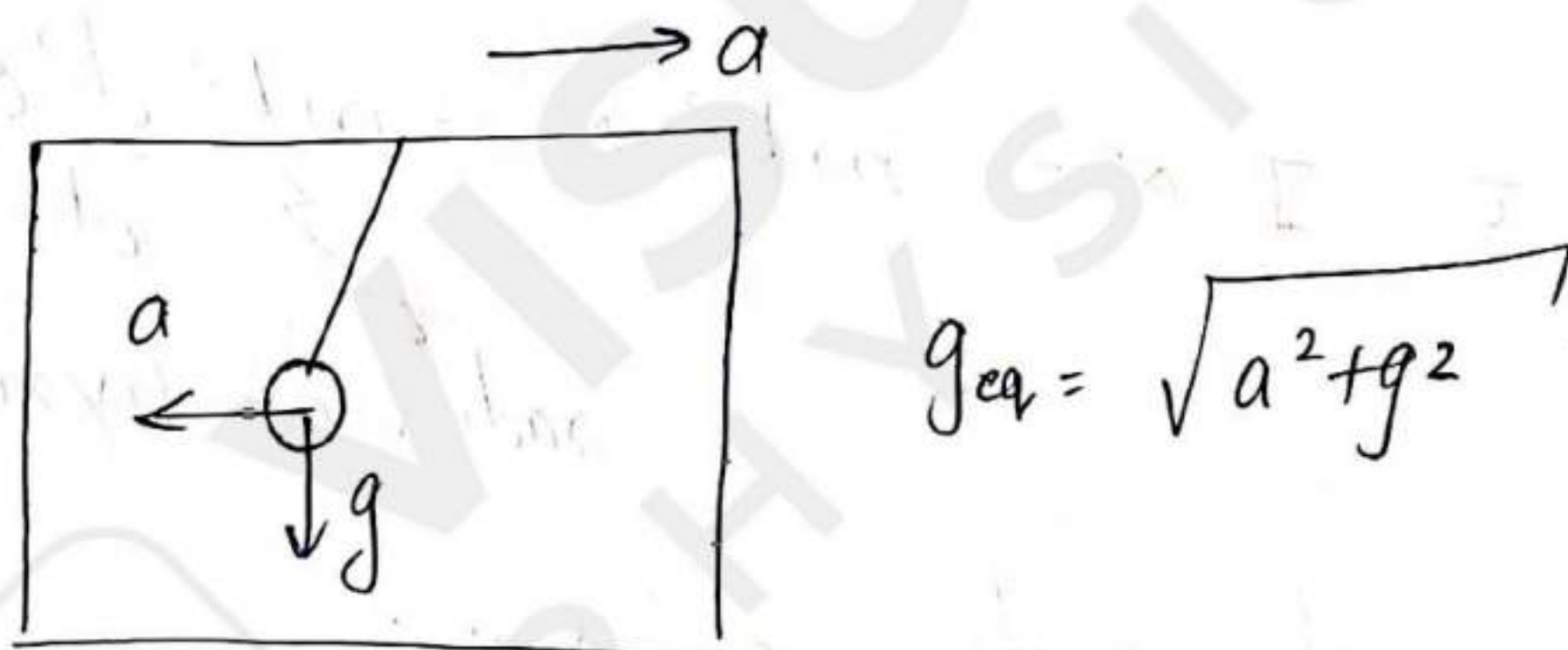
$$\boxed{T = 2\pi \sqrt{\frac{l}{g+a}}}$$

2. When moving down with acceleration 'a'



$$\text{So, } T = 2\pi \sqrt{\frac{l}{g-a}}$$

3. for horizontal acceleration:

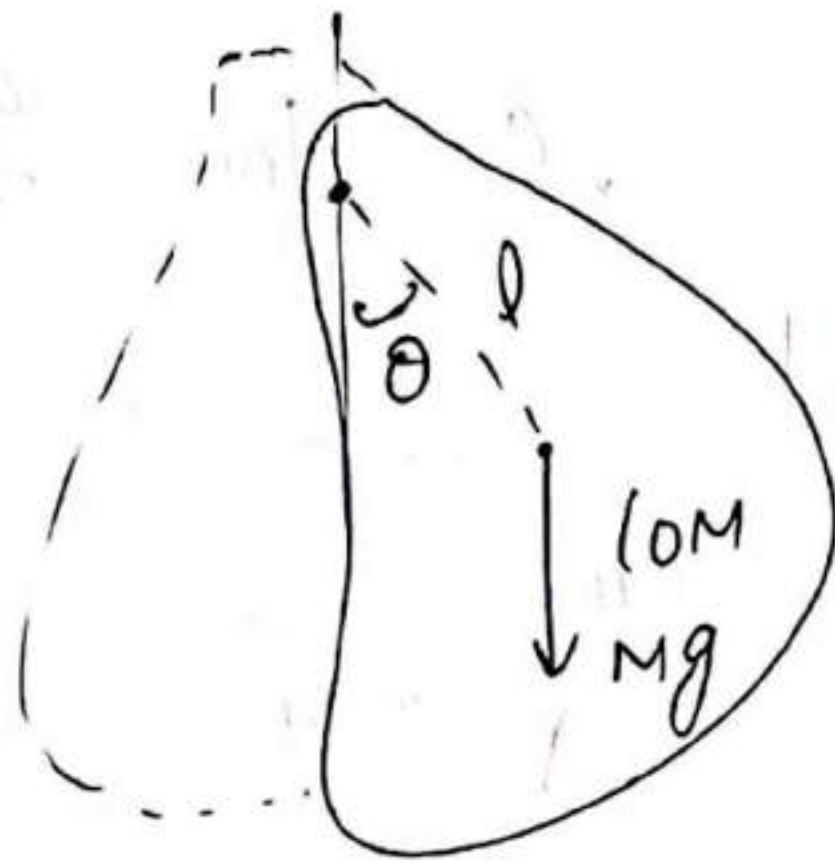


$$T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

PHYSICAL PENDULUM (COMPOUND PENDULUM)

Any rigid body suspended from a fixed support constitutes a physical pendulum.

$l \rightarrow$ distance b/w point of suspension & center of mass



$$\tau = -mgl \sin \theta$$

$$\tau = I \alpha = mk^2 \alpha = mk^2 \frac{d^2 \theta}{dt^2}$$

radius of Gyration

for small angle, $\theta \sim \sin \theta$

$$\Rightarrow mk^2 \frac{d^2 \theta}{dt^2} = -mgl \theta$$

$$\Rightarrow \frac{d^2 \theta}{dt^2} = -\frac{gl}{k^2} \theta$$

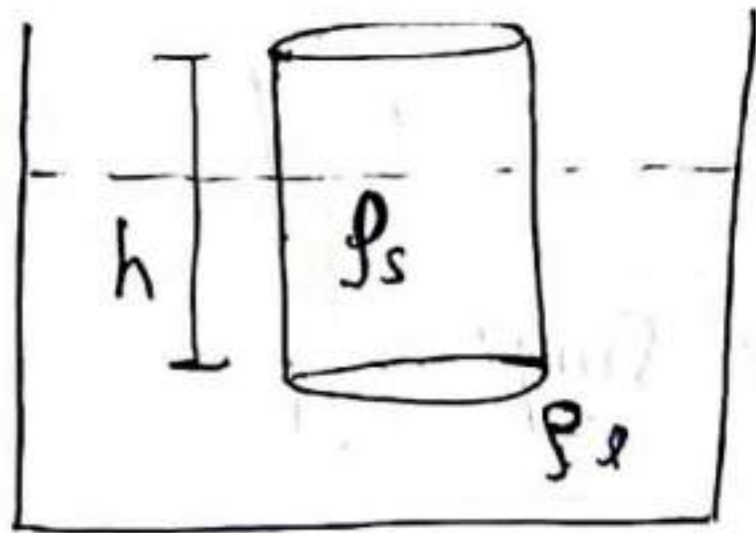
$$\omega^2 = \frac{gl}{k^2}$$

$$T = 2\pi \sqrt{\frac{k^2}{gl}}$$

$$\text{or } T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$I = mk^2$$

OSCILLATION OF FLOATING BODY:



$$\rho_l > \rho_s$$

now at equilibrium, $W = B$
Weight $\swarrow$ Buoyancy.

now when displaced by x from mean position
net Buoyant force act upwards,

$$\text{Net force} = -\rho_l A x g$$

$\swarrow$
displacement from mean position

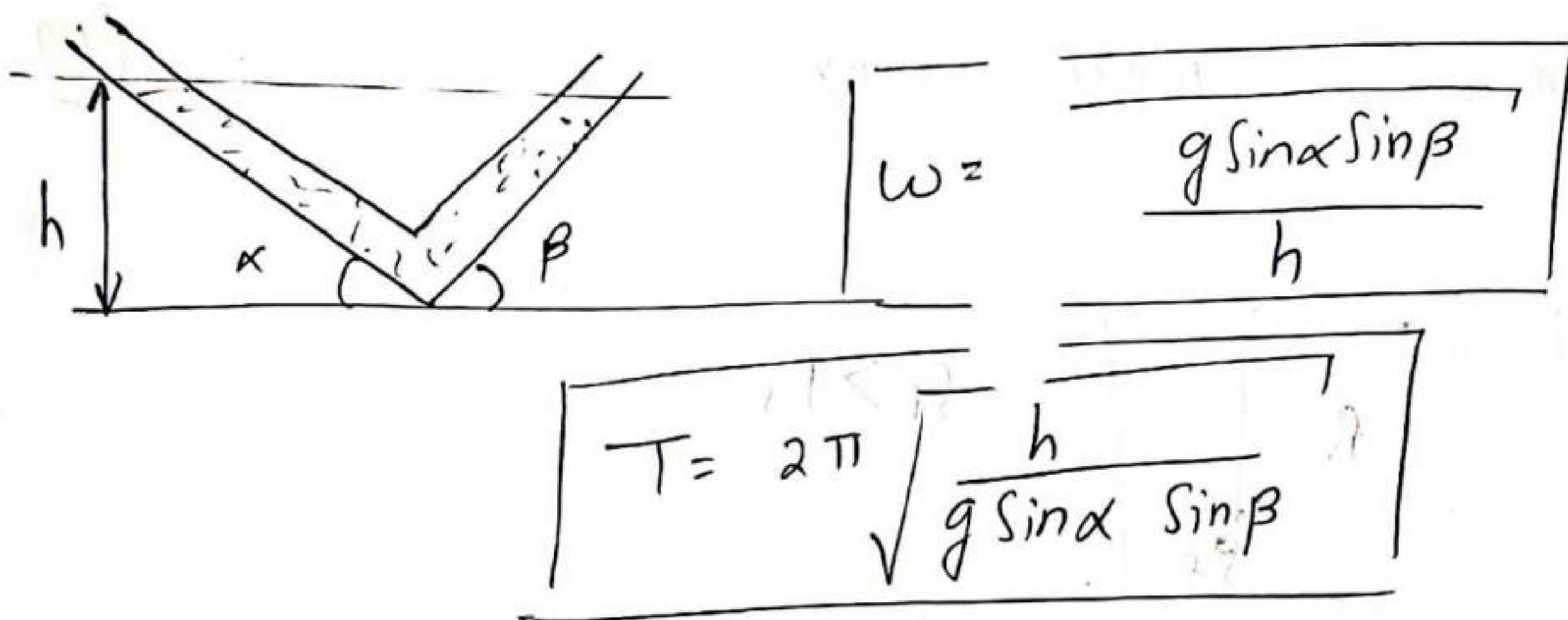
$$\Rightarrow ma = -\rho_l A x g$$

$$\rho_s A h \frac{d^2 x}{dt^2} = -\rho_l A g x$$

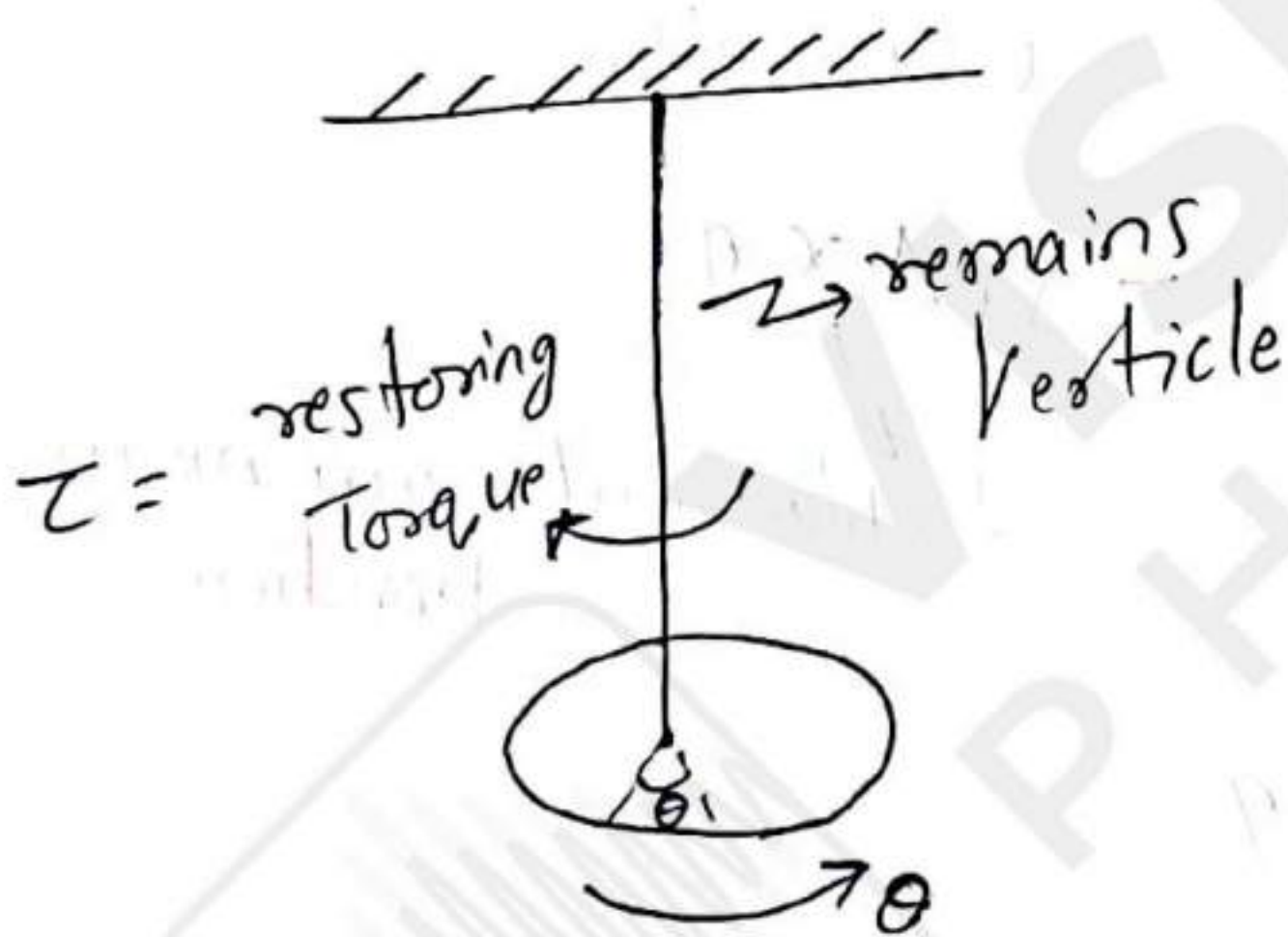
$$\Rightarrow \frac{d^2 x}{dt^2} = -\frac{\rho_l g}{\rho_s h} x$$

$$\omega^2 = \frac{\rho_l g}{\rho_s h}$$

$$T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi \sqrt{\frac{\rho_s h}{\rho_l g}}$$



Torsional pendulum:



$$\tau = -k\theta$$

Torsional Constant of thread or wire.

$$\Rightarrow I\alpha = -k\theta$$

$$\Rightarrow \alpha = -\frac{k}{I}\theta$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{I}{k}}$$