VISUAL PHYSICS

SHORT NOTES

CHAPTER

Rotational Dynamics

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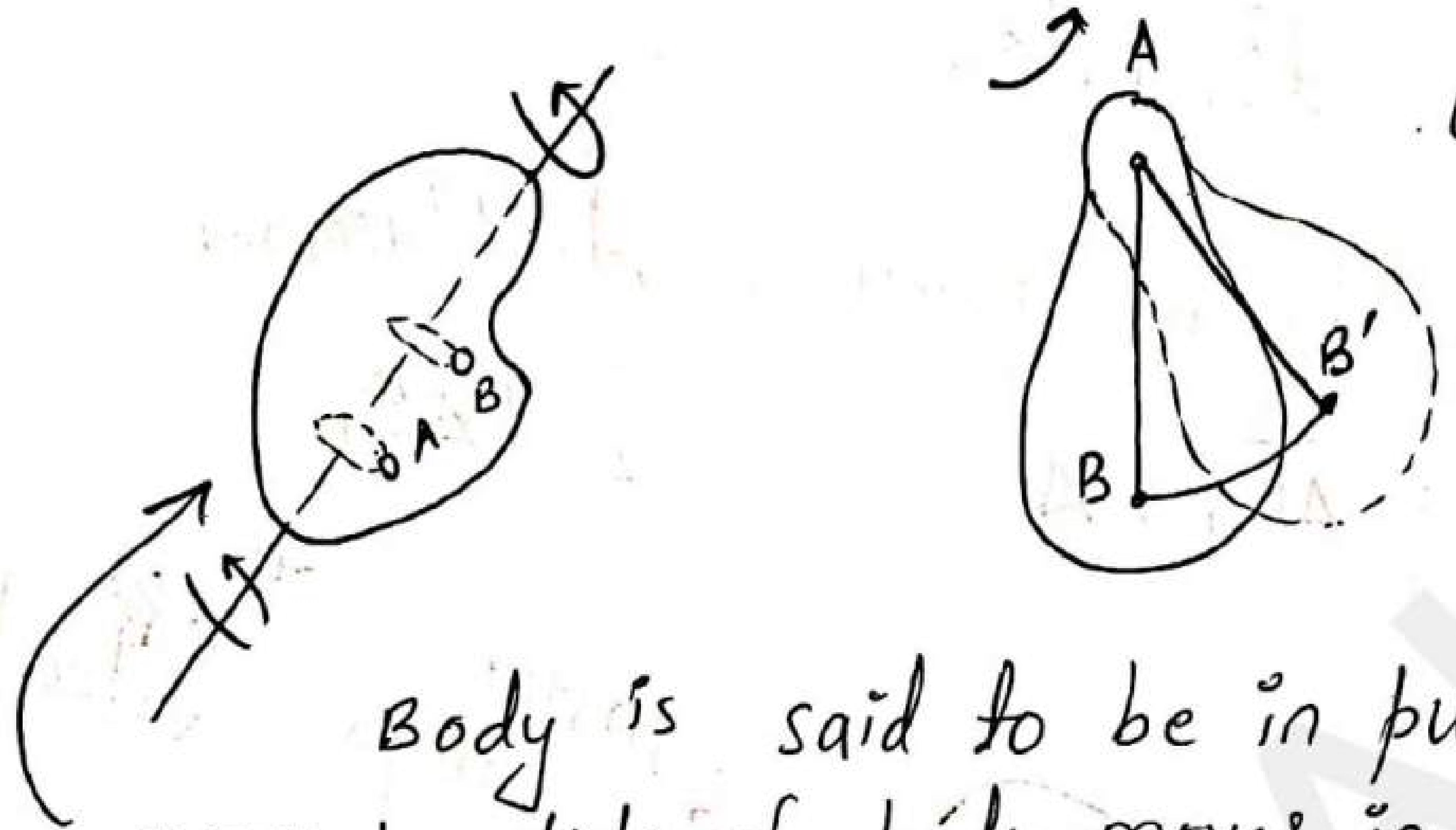
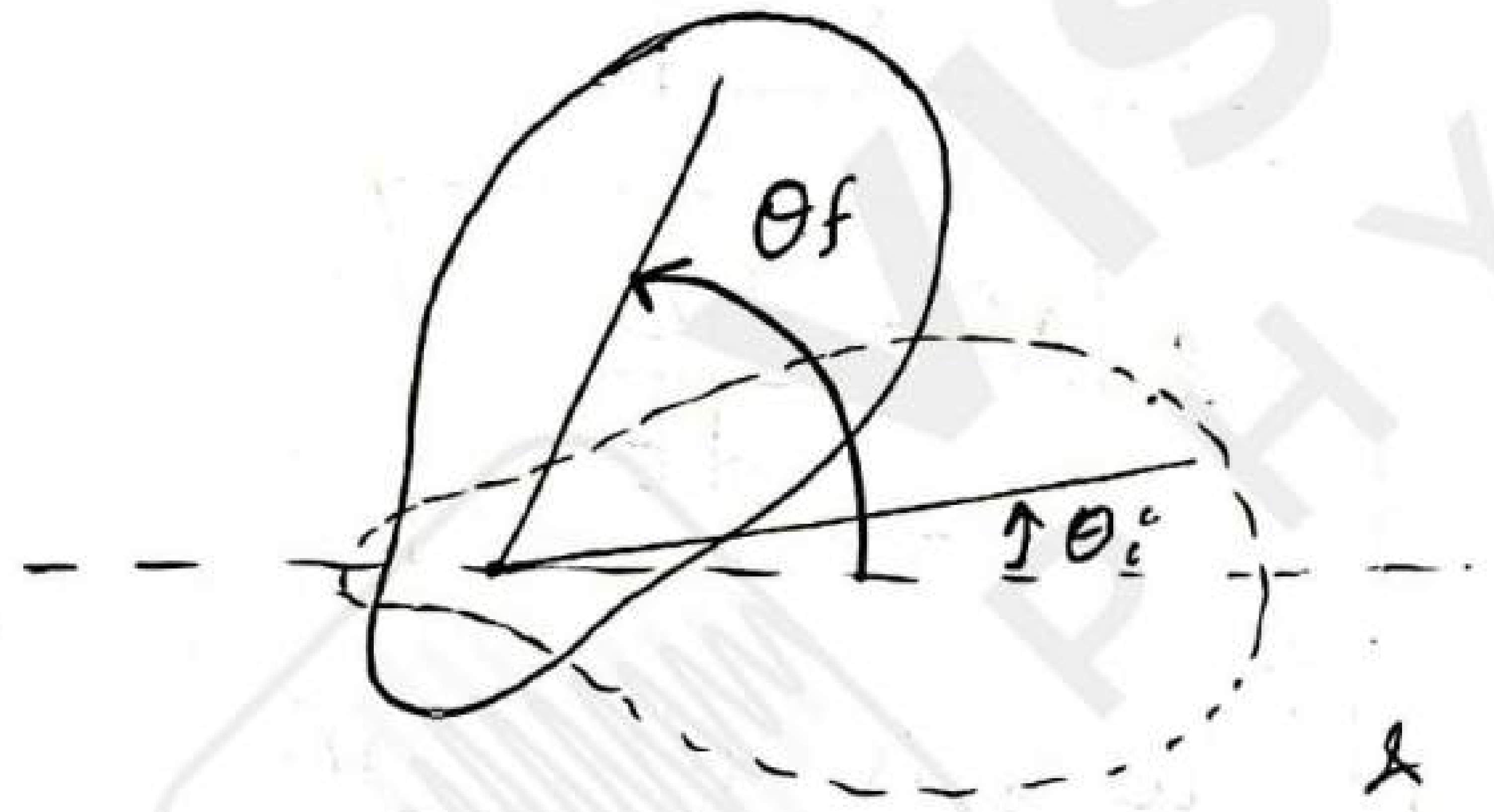
ROTATIONAL DYNAMICS



each particle move through same angle

Body is said to be in pure rotation if every particle of body moves in a circle & centre of all circles lie on a straight line.
→ called Axis of Rotation



+ anticlockwise → positive

& − (clockwise angle direction negative)

$$\boxed{\Delta\theta = \theta_f - \theta_i}$$

→ Units is radian

→ angular displacement → both → magnitude, direction

* (for small angular displacement, it is considered as vector)

* but large angular displacement is not vector

As for small $d\theta$, it follows cumulative law of angular displacements

i.e. $d\bar{\theta} = d\bar{\theta}_1 + d\bar{\theta}_2 = d\bar{\theta}_2 + d\bar{\theta}_1$

but for large angular displacements.

$\overline{\Delta\theta} = \overline{\Delta\theta}_1 + \overline{\Delta\theta}_2 \neq \Delta\theta_2 + \Delta\theta_1$

Hence not considered vector.

$\boxed{\omega_{avg} = \frac{\Delta\theta}{\Delta t}}$ angular velocity, $\boxed{\omega = \frac{d\theta}{dt}}$ → Vector

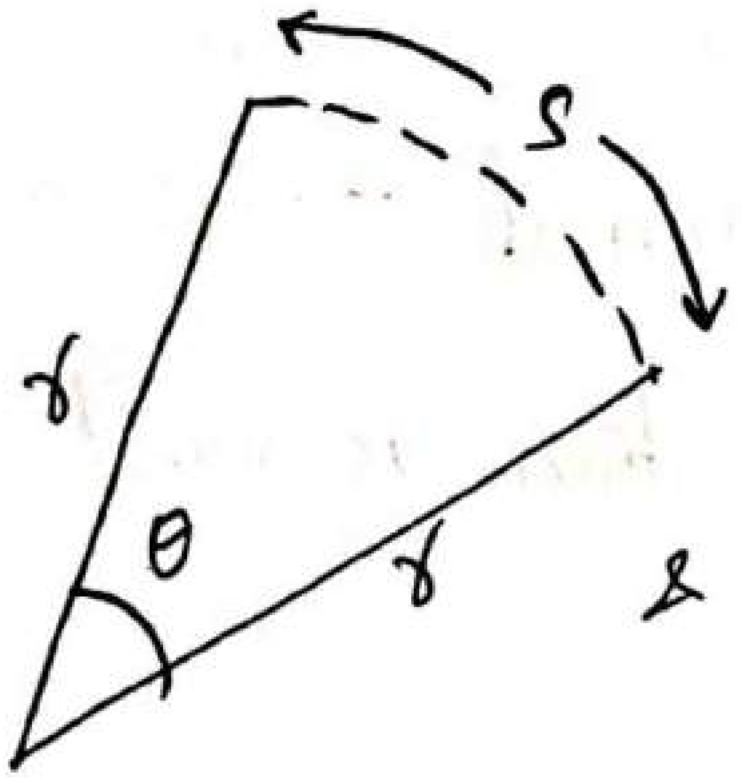
instantaneous angular velocity.

angular acceleration $\boxed{\alpha_{avg} = \frac{\Delta\omega}{\Delta t}}$ & $\boxed{\alpha = \frac{d\omega}{dt}}$ → Vector

★ direction of ω, is using right hand thumb rule.

circular motion	translation motion
i) $\omega_f = \omega_i + \alpha t$	i) $v = u + at$
ii) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$	ii) $v^2 - u^2 = 2as$
iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$	iii) $s = ut + \frac{1}{2}at^2$

Now,



$$s = r\theta$$

& $$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\Rightarrow \boxed{v = r\omega}$$

v → tangential velocity

ω → angular velocity

units

$\omega \rightarrow rad/s$

$\alpha \rightarrow rad/s^2$

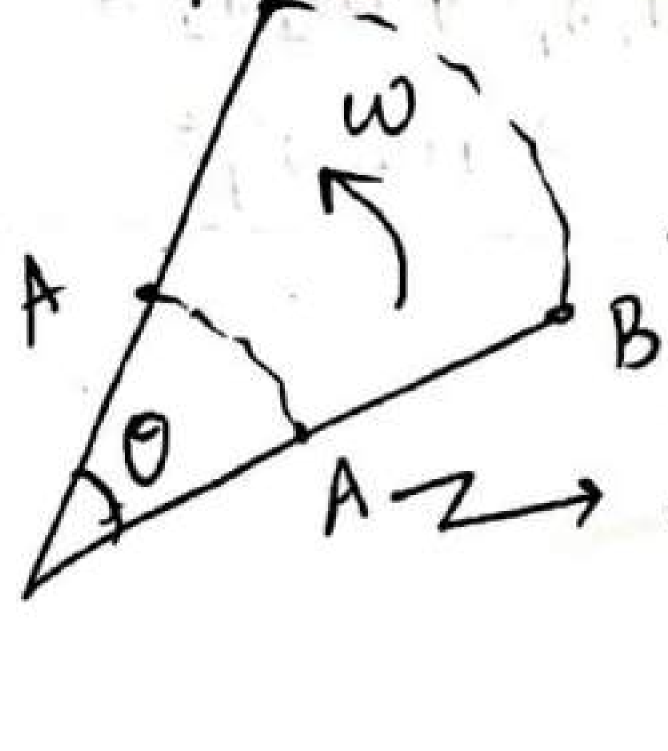
now $$a = \frac{dv}{dt} = \frac{d}{dt}(r\omega)$$ (for $r \rightarrow$ same)

$$\Rightarrow \boxed{a = r\alpha}$$

a → tangential acceleration

α → angular acceleration

$\Rightarrow$ for same ω, and α if r increases v & a increases.



B → more v & a

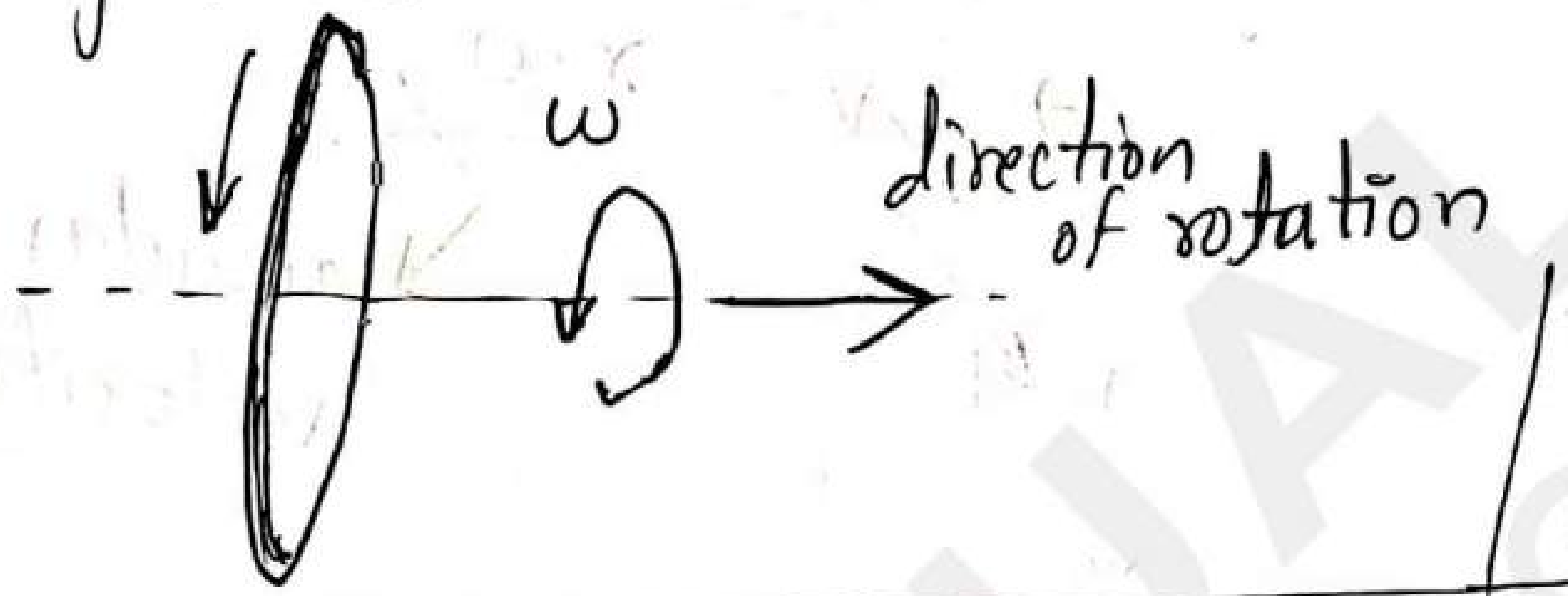
A → less v & a

ω is same.

MOMENT OF INERTIA:

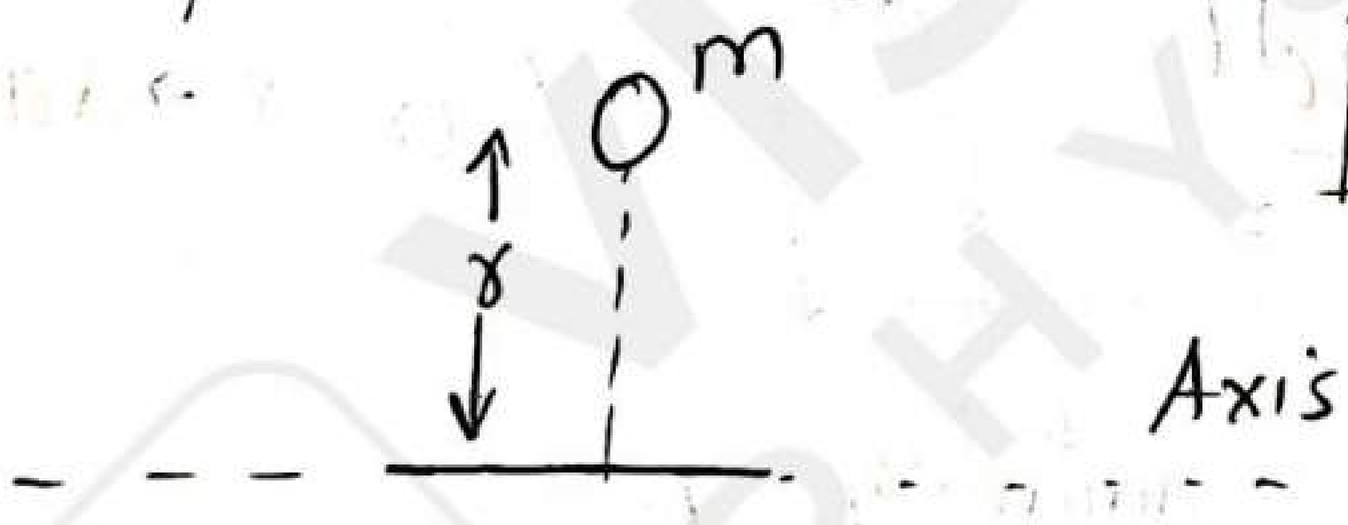
↳ Ability of the body to resist its change in rotational state of motion.

★ Rotation have both direction as well as magnitude.

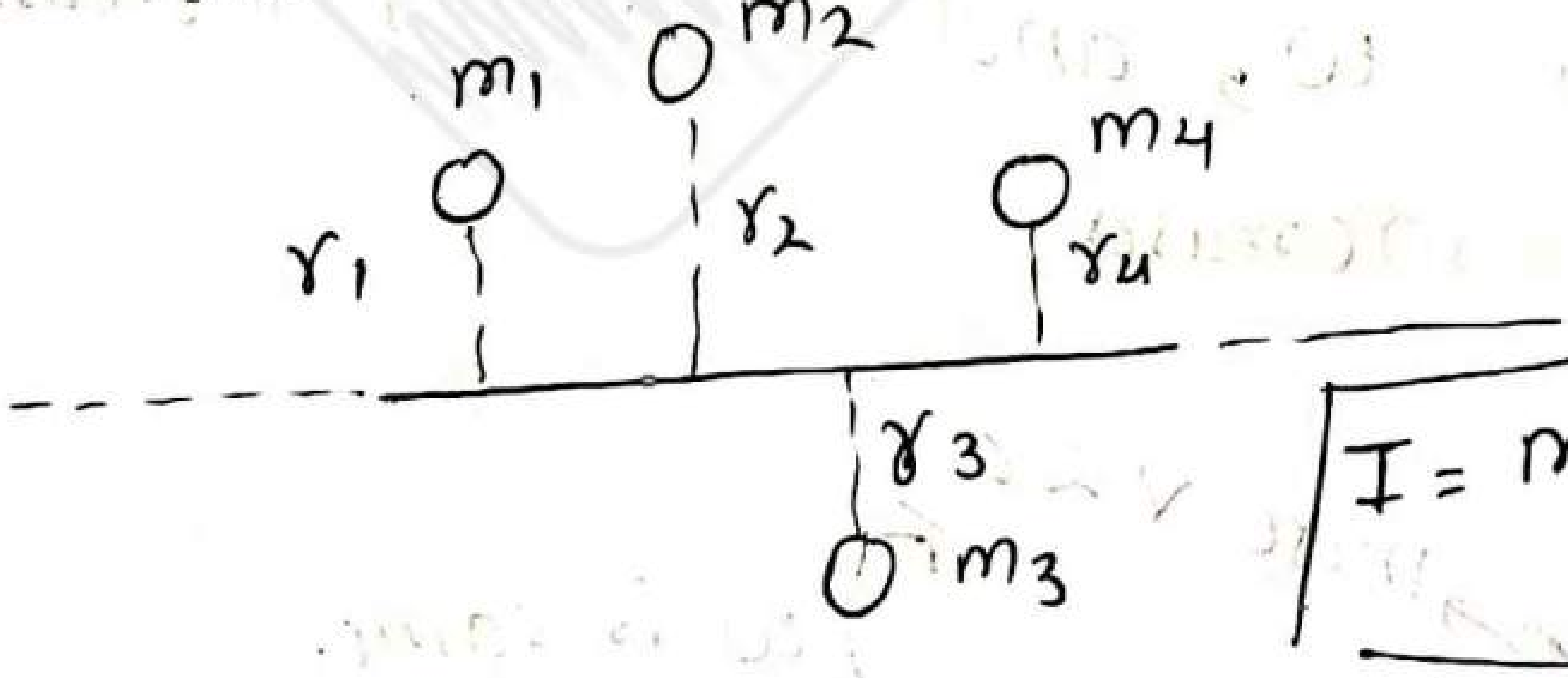


for a particle of mass 'm' ↗ moment of inertia

$$\boxed{I = mr^2}$$



so for $m_1, m_2, \ldots$
the net moment of inertia of the system



$$\boxed{I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2}$$

for discrete system of particles ↗

so, for continuous body

$$\boxed{I = \int dm\, r^2 = \int r^2\, dm}$$

here, 'r' is perpendicular distance of a particle of mass 'dm' of rigid body from axis of rotation.

→ 'I' depends on axis of rotation.

→ 'I' depends on mass of body.

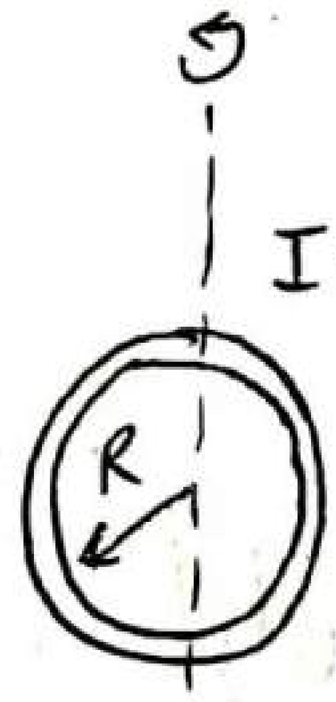
→ 'I' depends on the distribution of mass about an axis. The farther the mass is distributed from the axis, higher will be 'I'.

$I = \frac{MR^2}{2}$

DISC

$I = MR^2$

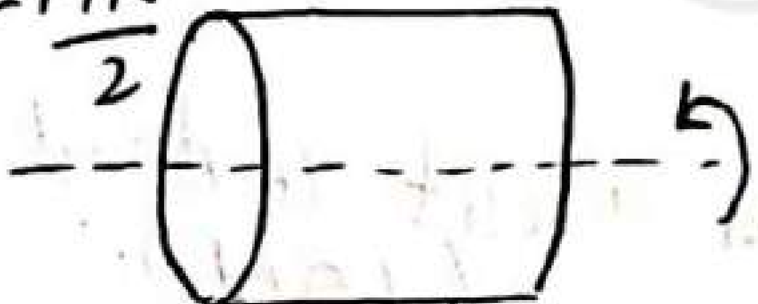
RING



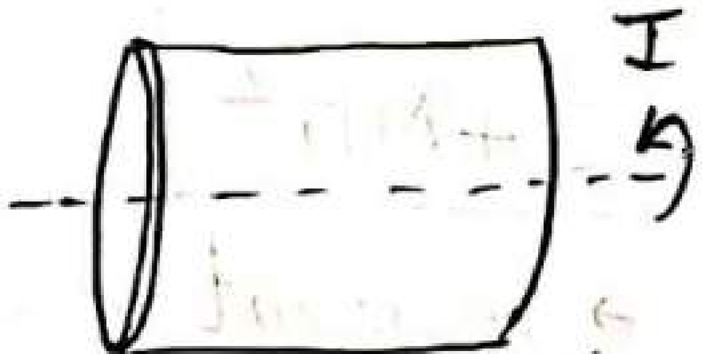
$I = \frac{MR^2}{2}$

RING

$I = \frac{MR^2}{2}$

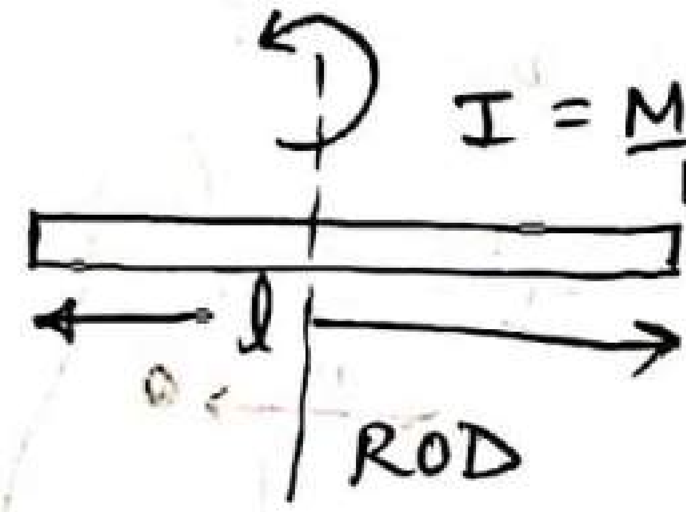


Cylinder



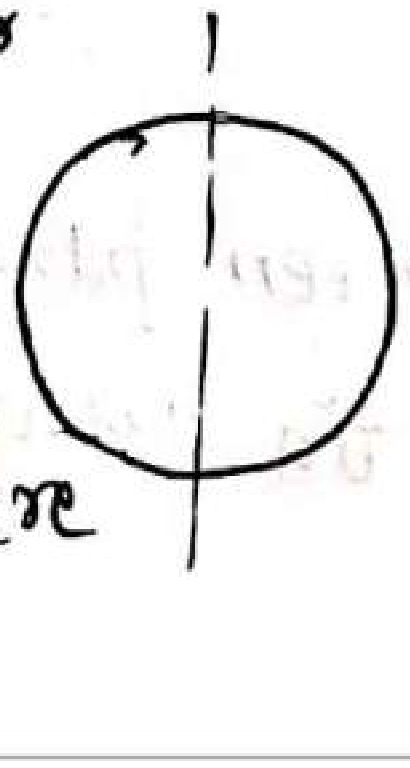
$I = MR^2$

Hollow RING



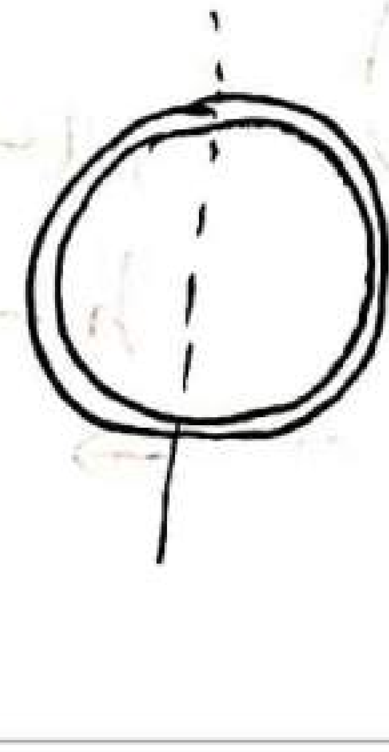
$I = \frac{Ml^2}{12}$

ROD



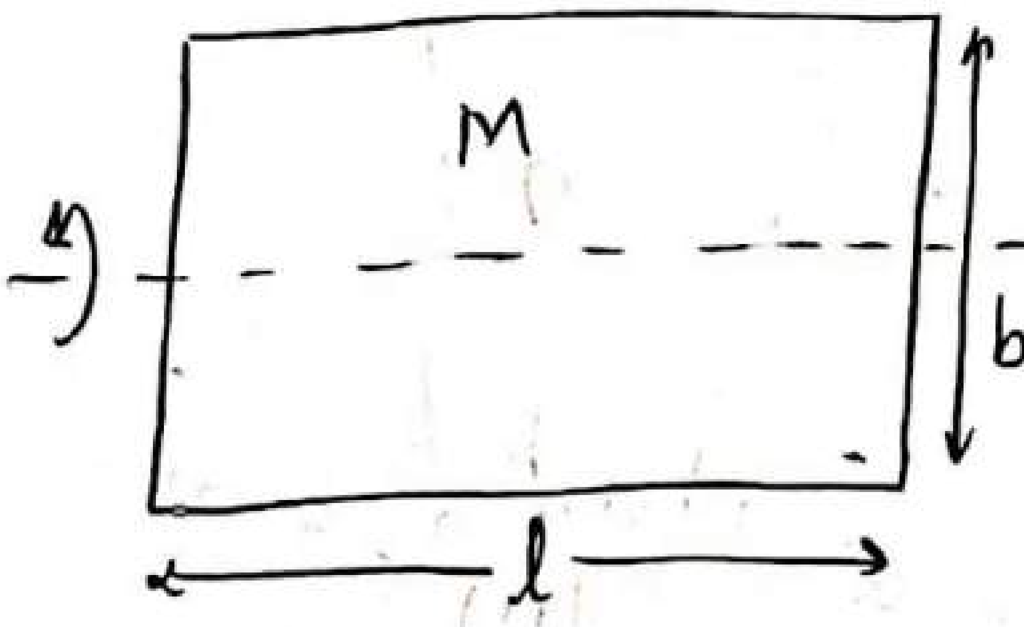
$I = \frac{2}{5} MR^2$

solid sphere

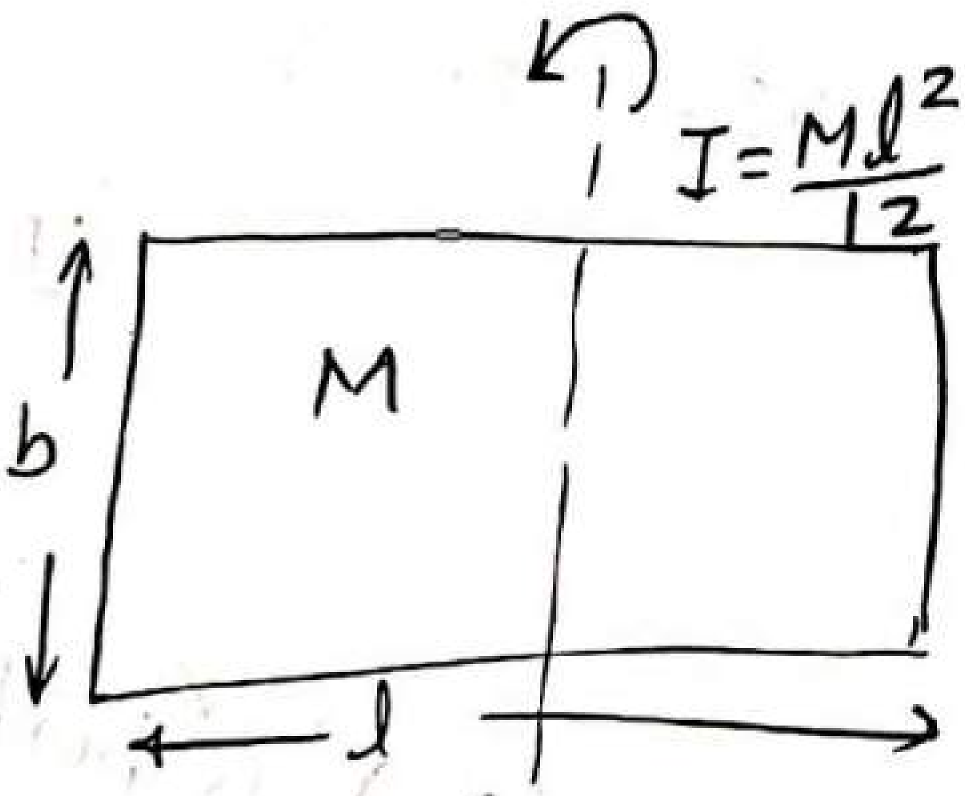


Hollow sphere

$I = \frac{2}{3} MR^2$

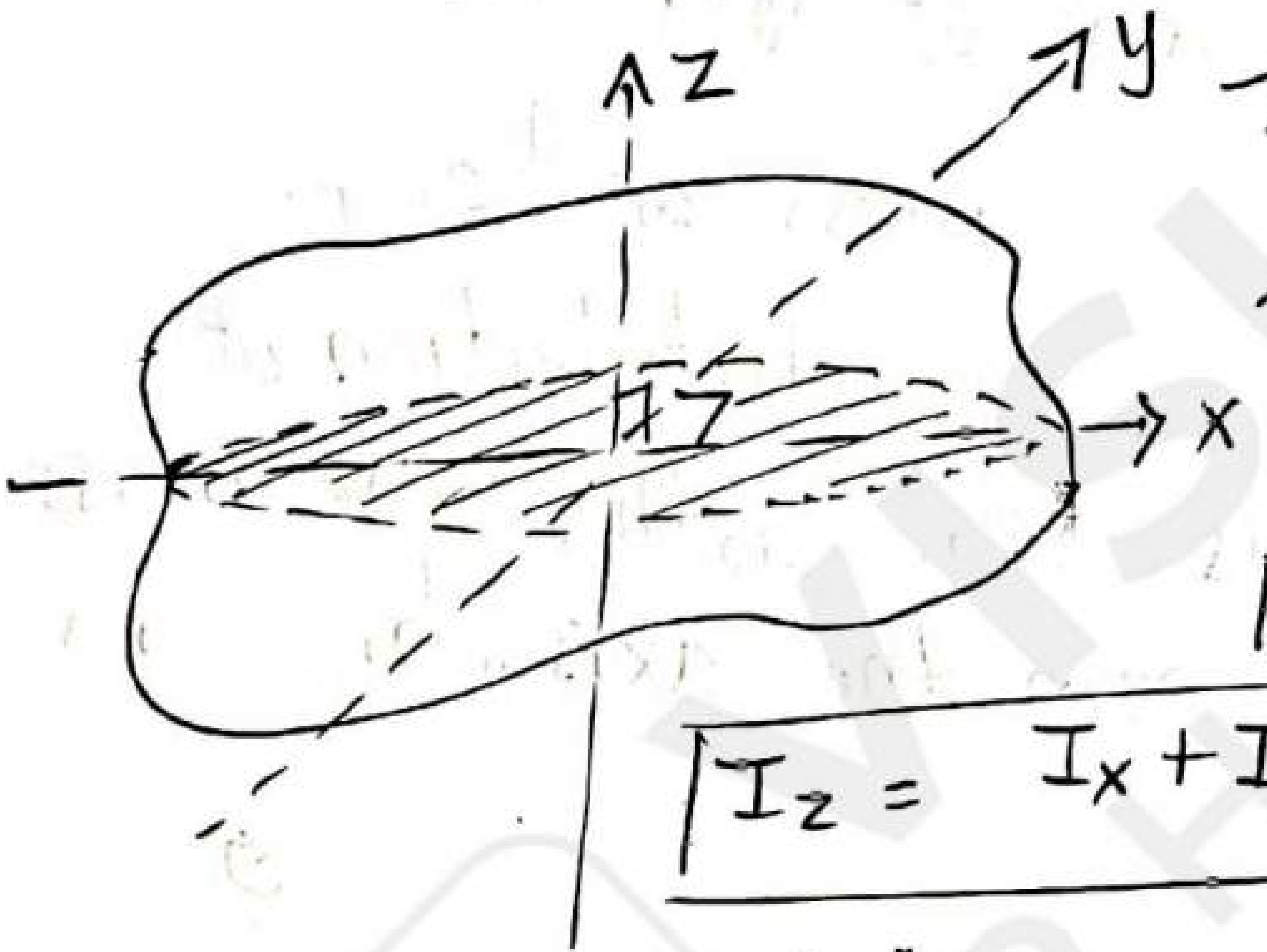


$$I = \frac{Mb^2}{12}$$



$$I = \frac{Ml^2}{12}$$

Perpendicular - Axis Theorem

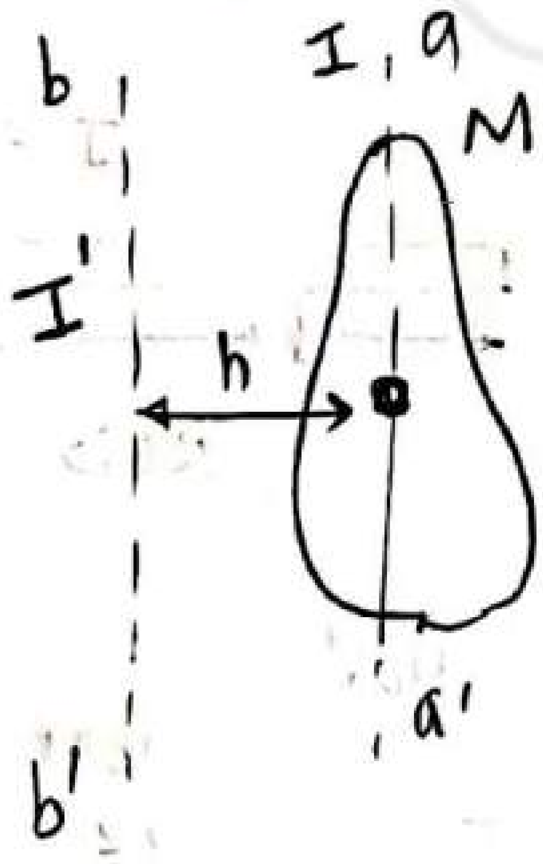


Are on same plane

$$\boxed{I_z = I_x + I_y}$$

Sum of moments of inertia of laminar object about two mutually perpendicular axes lying in the plane of lamina is equal to moment of inertia about axis normal to plane of lamina and passing through the two perpendicular axis.

parallel - Axis Theorem



$$\boxed{I' = I + Mh^2}$$

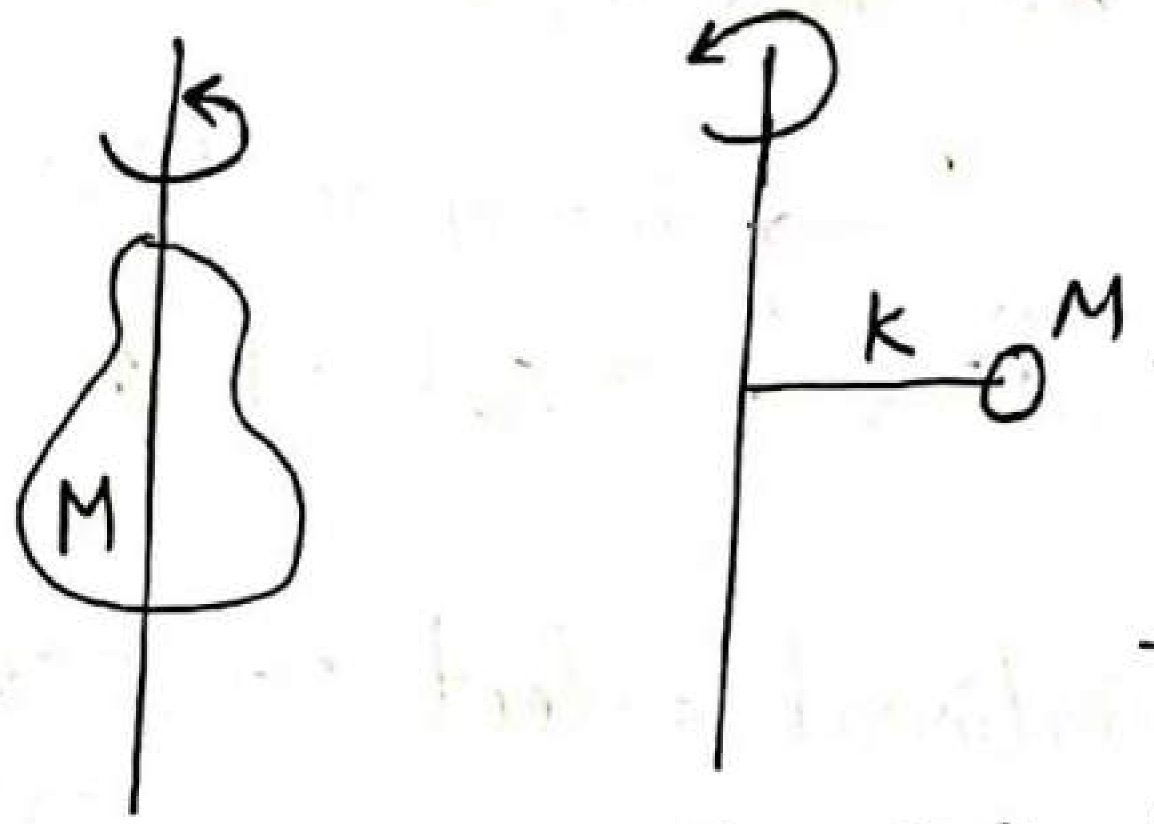
$I \rightarrow$ moment of inertia about centroid (COM). usually

$M \rightarrow$ net mass

$h \rightarrow$ distance between parallel axis

$I' \rightarrow$ moment of inertia about parallel axis

RADIUS OF GYRATION:



$k \rightarrow$ radius of gyration.

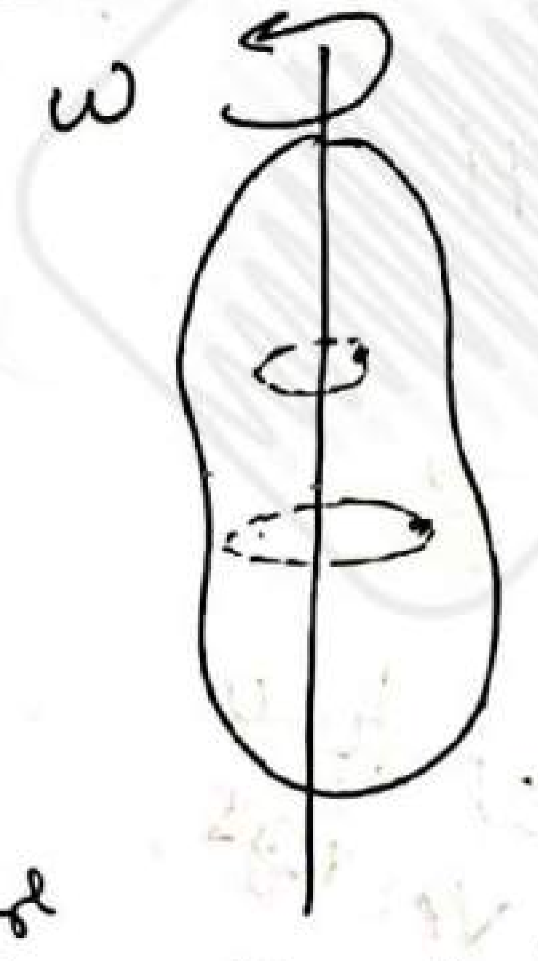
$$\boxed{I = Mk^2 = \int r^2 dm}$$

→ radius of gyration (k) of a body about an axis is effective distance from axis where whole mass can be assumed to be concentrated so that moment of inertia remains the same.

i.e. for solid sphere, $I = \frac{2}{5}MR^2 = Mk^2$

$$\Rightarrow \boxed{k = \sqrt{\frac{2}{5}}\,R}$$

Rotational KE:



as per particle

$$d(k.E) = \frac{1}{2}(dm)v^2$$

as $\boxed{v = \omega r}$

$$\Rightarrow \int d(k.E) = k.E = \int \frac{1}{2}\, dm(\omega r)^2$$

$$\Rightarrow k.E = \frac{1}{2}\int dm\,(\omega^2 r^2)$$

$$= \frac{1}{2}\,\omega^2 \int dm\, r^2$$

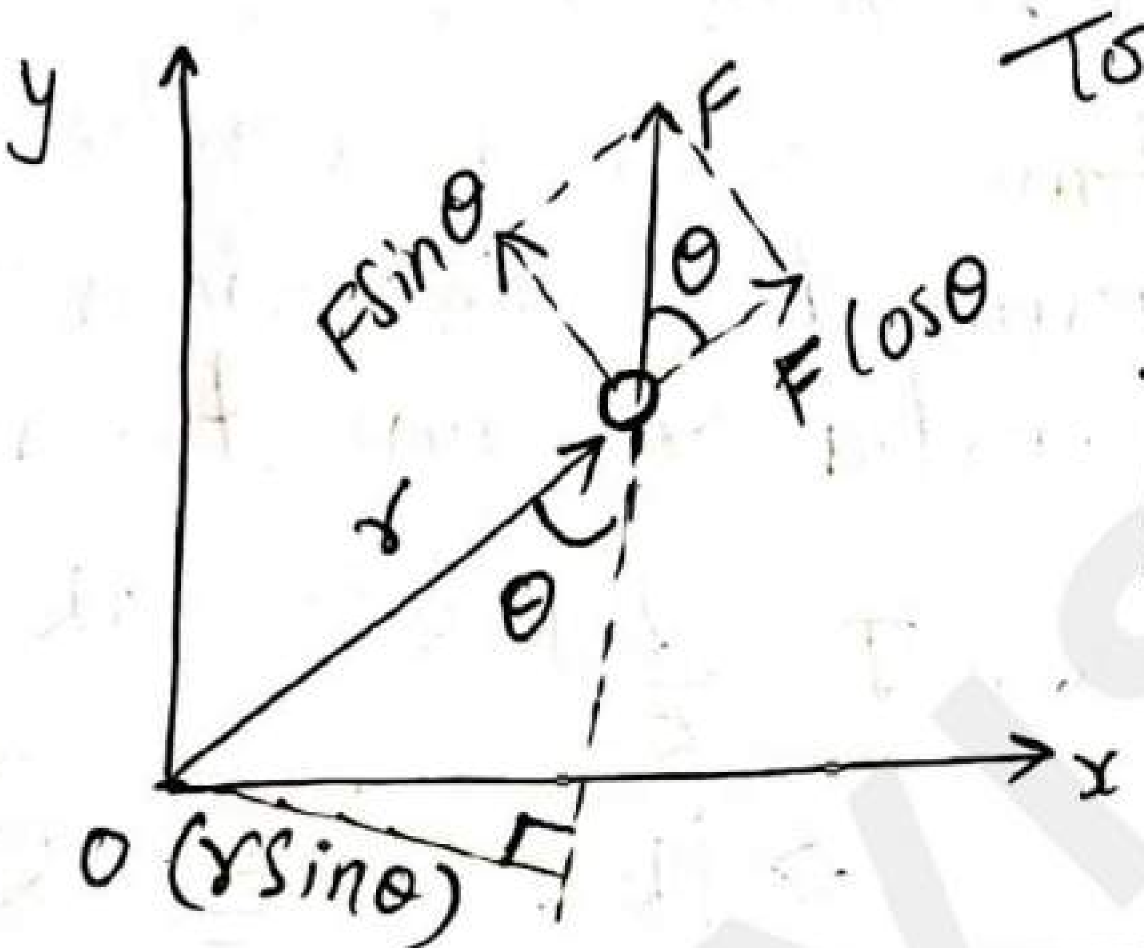
moment of inertia $\rightarrow I$

for pure rotation kinetic energy

$$\boxed{k.E = \frac{1}{2} I \omega^2}$$

TORQUE : $\tau \longrightarrow$ Rotational effect of force is torque.

torque cause angular acceleration



Torque about 'O'

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

$\vec{r}$ from position (axis) of rotation

$$\Rightarrow \boxed{|\vec{\tau}| = \tau = r(F\sin\theta) = rF_{\perp}}$$

$F_{\perp}$ (Force component perpendicular to $\vec{r}$)

$$\boxed{\tau = (r\sin\theta)F = r_{\perp}F}$$

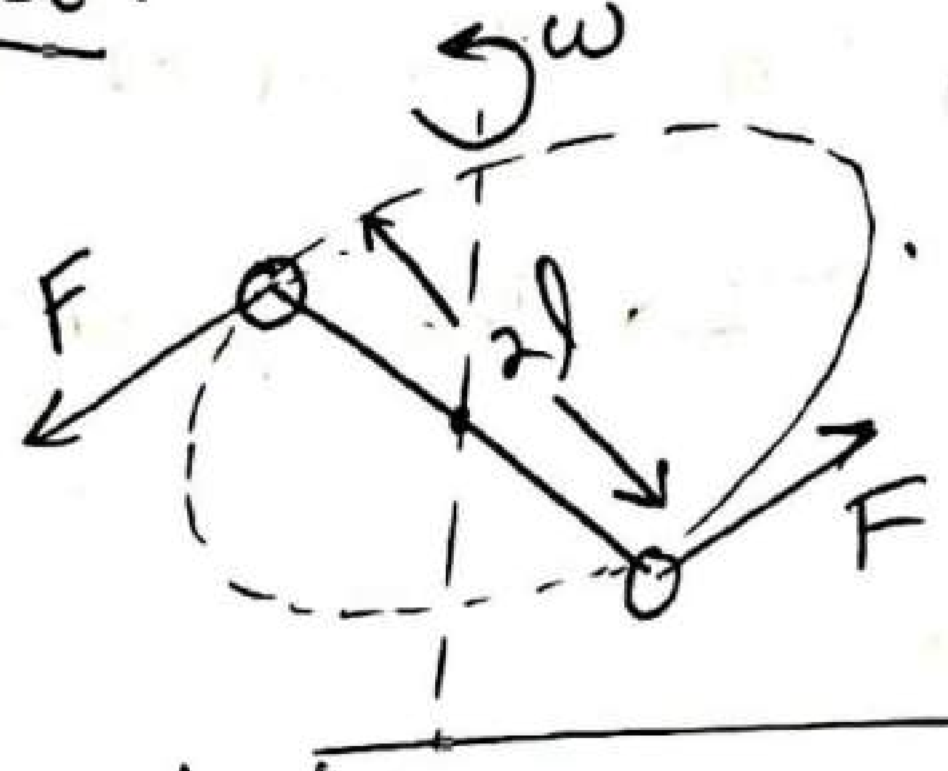
$r_{\perp}$ means from 'r' $\perp$ to F

For direction of torque $\vec{\tau} = \vec{r} \times \vec{F}$

cross product of two vectors

'Right hand rule'

COUPLE:



Two equal and opposite forces whose line of action do not coincide make a couple.

And $\boxed{|\vec{\tau}| = \tau = 2lF}$

LAWS OF ROTATION:

$\tau_{\text{abot the axis of rotation}} = I_{\text{about axis of rotation}} \times \alpha$

$\alpha \rightarrow$ angular acceleration about axis of rotation

$\Rightarrow \boxed{\tau = I\alpha} \quad \Rightarrow \boxed{\vec{\tau} = I\vec{\alpha}}$

$|\vec{\tau}| = \tau = rF = r(ma) \times \frac{r}{r}$

$= r^2 m \left(\frac{a}{r}\right)$

external torque

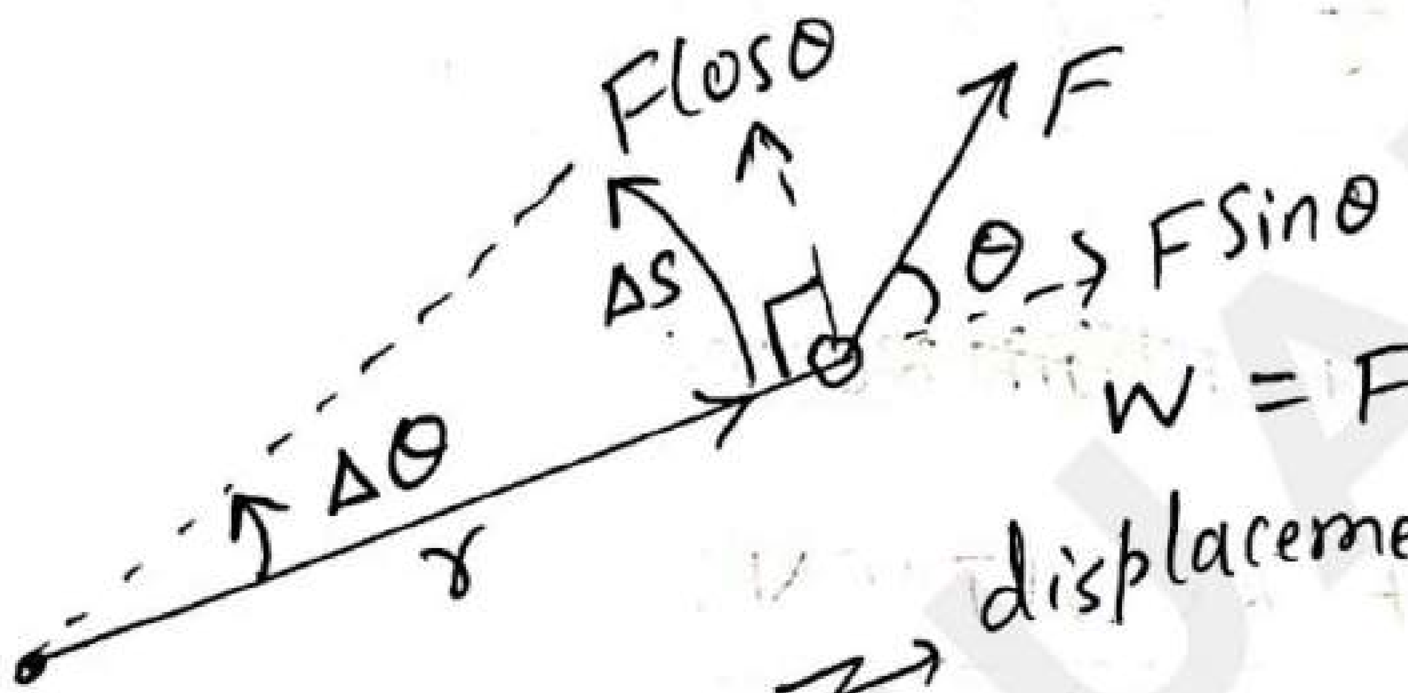
$\boxed{\tau = I\alpha}$

Valid for any body

Similar to $\boxed{F = ma}$

(derived for particle)

Work - Energy Theorem !



$W = F\cos\theta \times s$

displacement

Work done $= (F_\perp)\Delta s = (F_\perp) r \Delta\theta$

now $F_\perp r = \tau =$ Torque.

$\Rightarrow \quad dw = F_\perp r \, d\theta = \tau \, d\theta$

$$\Rightarrow \boxed{\int dw = \underbrace{W}_{\text{Net work}} = \int \tau \, d\theta}$$

external Torque

$$\underbrace{\Delta K.E}_{\text{change in kinetic Energy}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

for single particle

$$W = \Delta K.E = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\int \tau \, d\theta = \frac{1}{2} m(\omega r_f)^2 - \frac{1}{2} m(\omega r_i)^2$$

$$\int \tau \, d\theta = \frac{1}{2} m r^2 (\omega_f^2 - \omega_i^2)$$

$$\boxed{\int \tau \, d\theta = \frac{1}{2} I (\omega_f^2 - \omega_i^2)}$$

As for single particle $I = mr^2$

but applicable for any body.

Hence in general, for rotational dynamics.

scalar * $\boxed{W = \int \tau \, d\theta = \frac{1}{2} I (\omega_f^2 - \omega_i^2) = \Delta K.E}$

As Power $= P = \frac{dW}{dt}$

Now $dW = \tau \, d\theta$

$$\Rightarrow P = \frac{d}{dt} (\tau \, d\theta) = \tau \frac{d\theta}{dt}$$

$\Rightarrow$ $\boxed{P = \tau \omega z}$ $\rightarrow$ rotational speed.

scalar

similar to $\boxed{P = FV}$

Angular Momentum:

for a moving particle about a point is defined as, $\boxed{\vec{L} = \vec{r} \times \vec{p}}$

As Torque $= \vec{r} \times \vec{F} = \vec{\tau}$

$\Rightarrow$ if $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \vec{r} \times \dfrac{d}{dt}\vec{p}$

external Torque

$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \boxed{\vec{\tau} = \dfrac{d\vec{L}}{dt}}$

applicable for any system of particles & rigid body

now, $\vec{L} = \vec{r} \times m\vec{v}$

as $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$

$\Rightarrow \vec{L} = \vec{r} \times (m\vec{\omega} \times \vec{r})$

for system of particles.

$$\vec{L} = \Sigma (m_i r_i^2) \vec{\omega}$$

Vector quantity

$$\boxed{\vec{L} = I\vec{\omega}}$$

$$\Rightarrow \boxed{|\vec{L}| = I\omega}$$

As,

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

if $\vec{\tau} = 0 = \frac{d\vec{L}}{dt}$

Means change in angular momentum is zero

if external Torque is zero than, angular moment of the system is not zero.

$$\boxed{d\vec{L} = 0}$$

change in angular momentum is zero.

$\Rightarrow \cancel{f_o}$

$$\Rightarrow \boxed{L_f - L_i = 0}$$

$L_f \rightarrow$ final angular momentum

$L_i \longrightarrow$ Initial angular momentum

$$\boxed{L_f = L_i}$$

angular momentum of rotational system remains same.

or $\boxed{I_f \omega_f = I_i \omega_i}$

$I_f \rightarrow$ final ~~an~~ moment of Inertia

$I_i \rightarrow$ initial moment of Inertia

$\omega_i \rightarrow$ ~~a~~ initial angular velocity

$\omega_f \rightarrow$ final angular velocity.