VISUAL PHYSICS

SHORT NOTES

CHAPTER

Motion in 2 & 3 Dimensions

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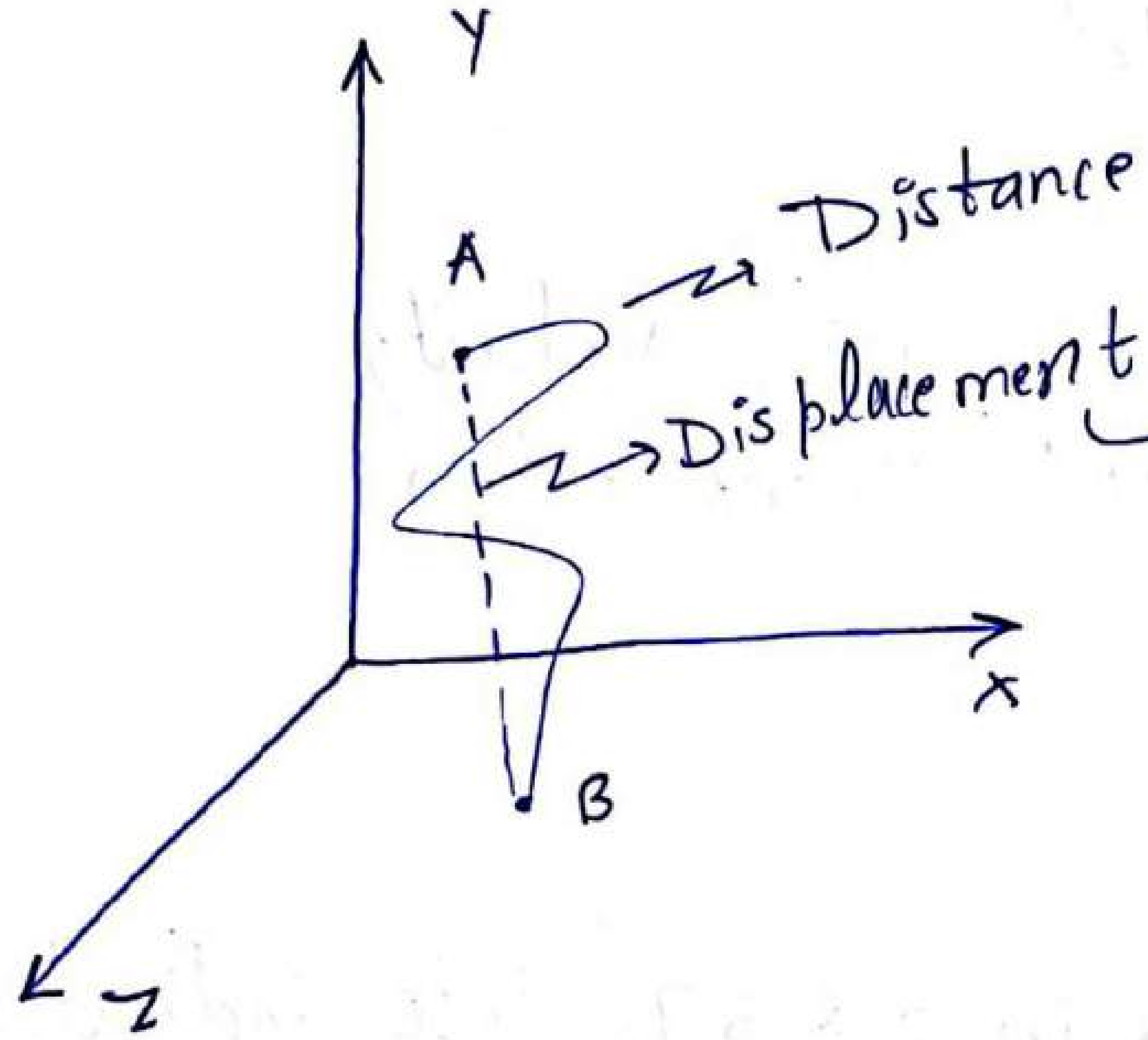
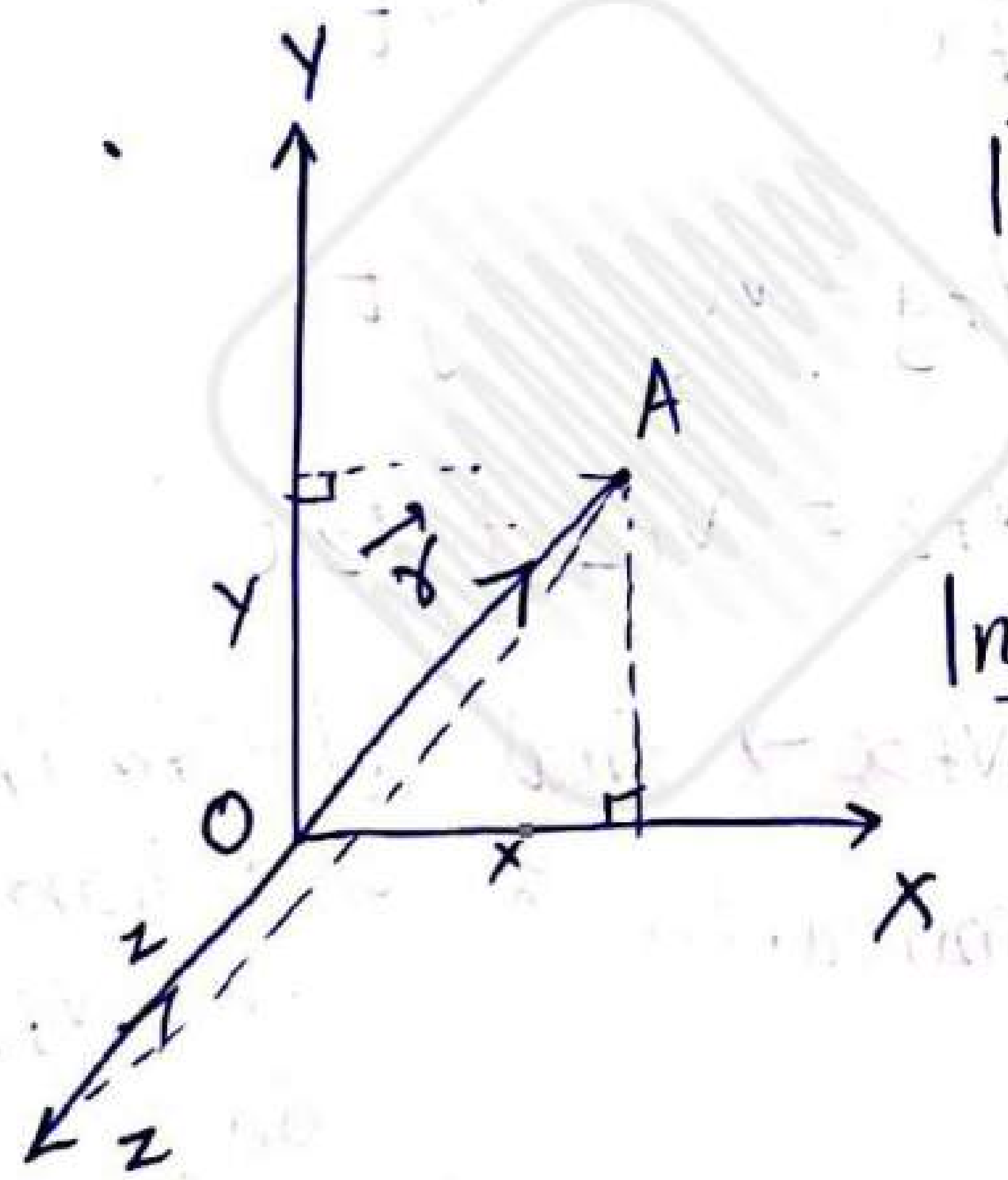
Motion in 2 & 3 D



Displacement → smallest distance between initial and final position.

Position vector in terms of components we can write as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



$|\vec{r}|$ = magnitude of position vector

$$= \sqrt{x^2+y^2+z^2}$$

Instantaneous velocity:

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

components in x, y, z direction

$$\text{Instantaneous speed} = |\vec{v}| = \sqrt{v_x^2+v_y^2+v_z^2}$$

$\vec{a}$ = instantaneous acceleration = $\frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Equation of motion:

Always deal motion in 2 & 3 D into individualy in each components of its motion.
Like for a motion with velocity $\vec{v}$, acceleration $\vec{a}$, displacement $\vec{d}$ & time t.

$$\vec{x} = v_{ix}t + \frac{1}{2}a_x t^2, \quad v_{fx} = v_{ix} + a_x t$$

$$y = v_{iy}t + \frac{1}{2}a_y t^2, \quad v_{fy} = v_{iy} + a_y t$$

$$z = v_{iz}t + \frac{1}{2}a_z t^2, \quad v_{fz} = v_{iz} + a_z t$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$v_{fx} \rightarrow$ final velocity in x-direction
similarly for v_{fy} & v_{fz} in y & z direction

$v_{ix}, v_{iy}, v_{iz} \rightarrow$ initial velocity in x, y, z direction

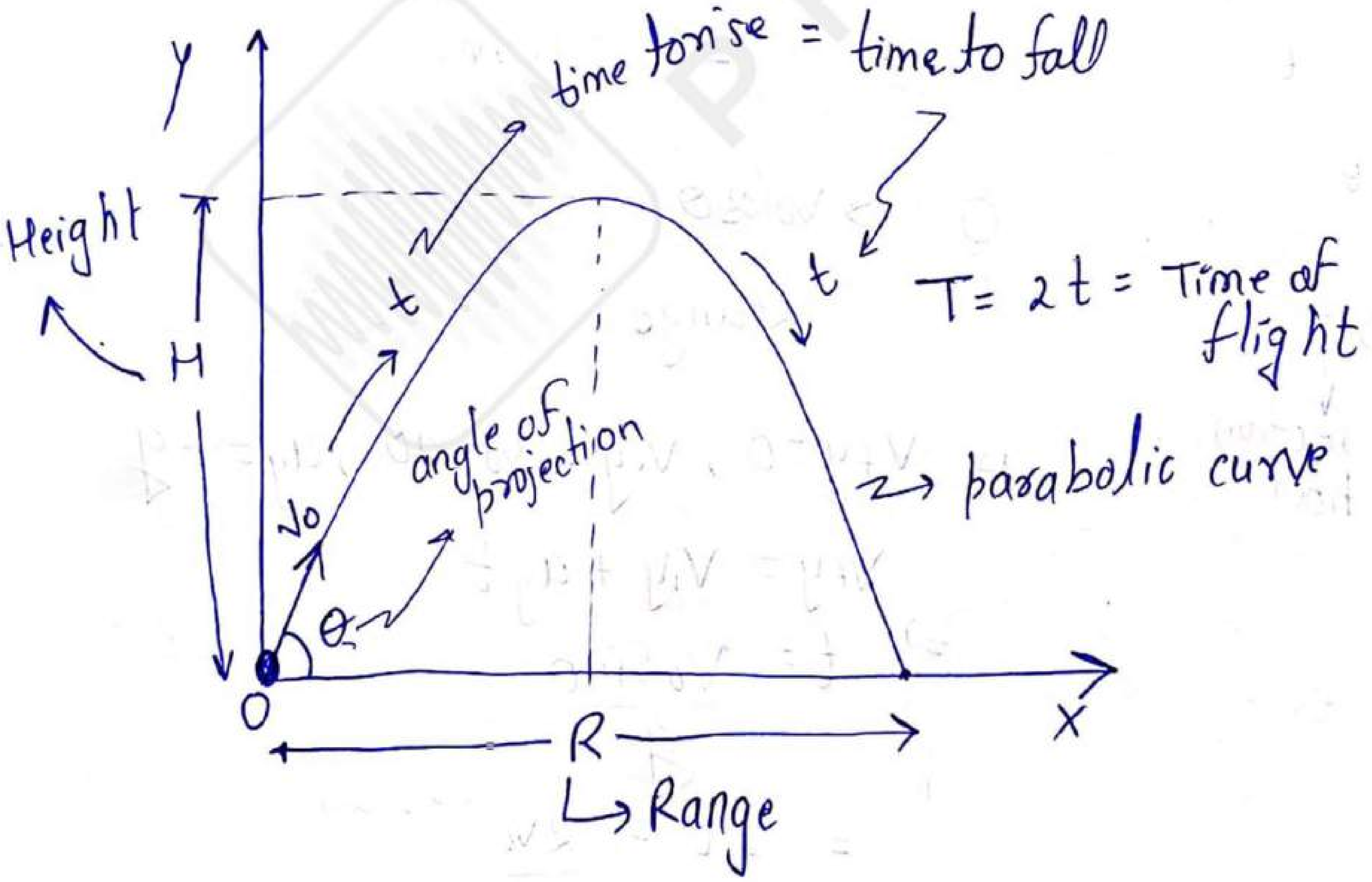
&

$$V_{fx}^2 = V_{ix}^2 + 2a_x x$$
$$V_{fy}^2 = V_{iy}^2 + 2a_y y$$
$$V_{fz}^2 = V_{iz}^2 + 2a_z z$$

x, y, z, displacement in x, y, z direction respectively.

Projectile motion:

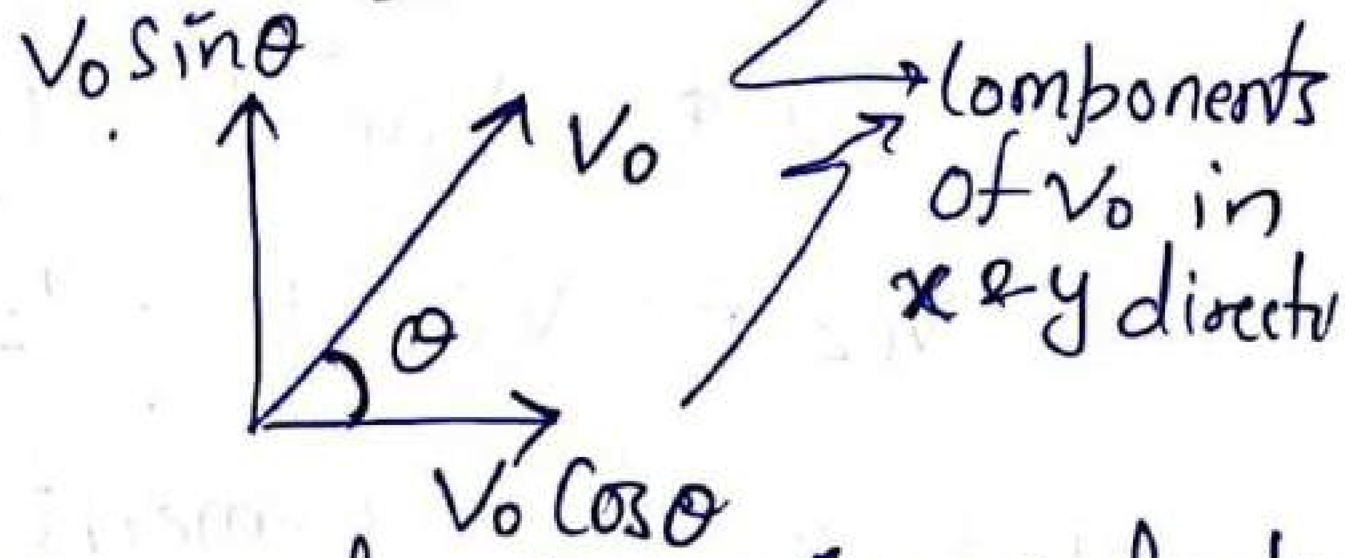
Any body that is moving only under the influence of gravity and no other source of energy providing any sense of motion to it.



dealing motion in x & y direction seperately.

$V_x = V_0 \cos\theta$

$V_y = V_0 \sin\theta$



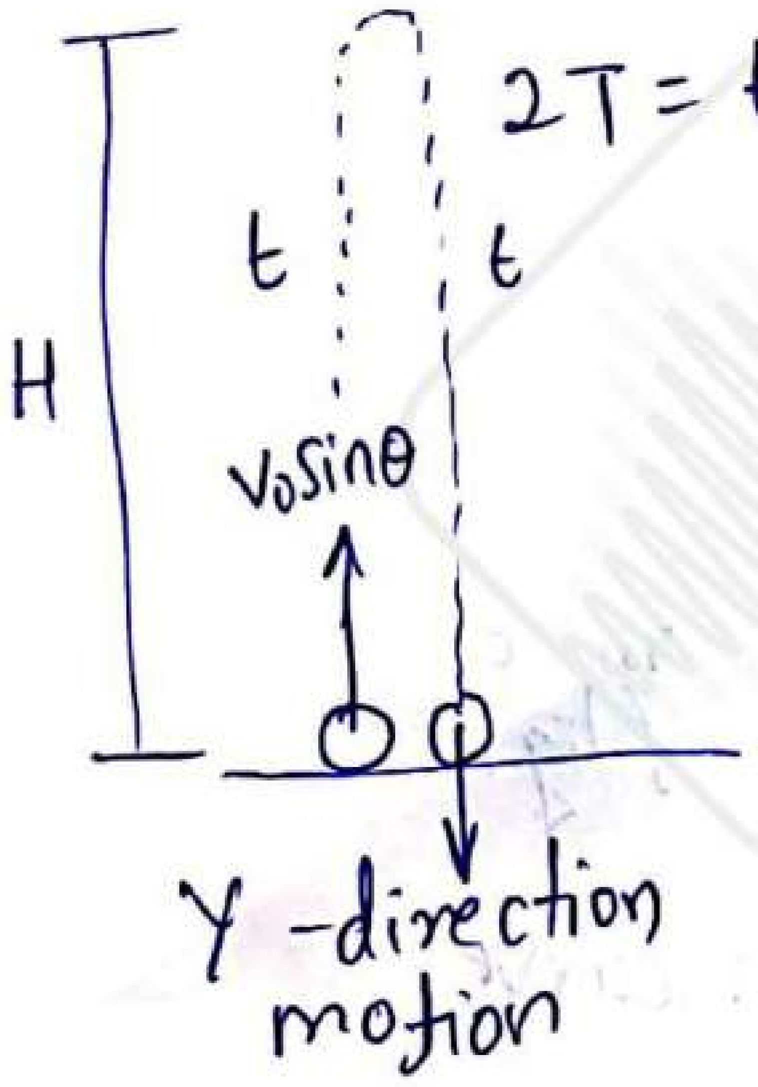
neglecting air drag so no acceleration in x-direction (Horizontal)

$a_x = 0$

as for y-direction (vertical) gravity is acting

Considering $\uparrow \Rightarrow$ +ve $\downarrow$ -ve

so, $a_y = -g$ → as gravity is downwards



as $V_{fy} = 0$, $V_{iy} = V_0 \sin\theta$, $a_y = -g$

$$V_{fy} = V_{iy} + a_y t$$

$$\Rightarrow t = \frac{V_0 \sin\theta}{g}$$

so $\boxed{T = 2t = \frac{2V_0 \sin\theta}{g}}$

And, $y = H$

$$y = V_{iy} t + \frac{1}{2} a_y t^2$$

$$H = V_0 \sin\theta \times \frac{V_0 \sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{V_0 \sin\theta}{g}\right)^2$$

$$\boxed{H = \frac{V_0^2 \sin^2\theta}{2g}}$$

Now Range, $R = V_x T = V_0 \cos\theta \times \left(\frac{2 V_0 \sin\theta}{g}\right)$

$$R = \frac{2 V_0^2 \sin\theta \cos\theta}{g}$$

As $2 \sin\theta \cos\theta = \sin 2\theta$

$$\Rightarrow \boxed{R = \frac{V_0^2 \sin 2\theta}{g}}$$

For maximum Range

$$R \rightarrow R_{max} = \frac{V_0^2 \overset{\rightarrow max}{\sin 2\theta}}{g} = \boxed{\frac{V_0^2}{g} = R_{max}}$$

$\Rightarrow$ $\sin 2\theta \rightarrow max = 1$

$\sin 2\theta = 1 \Rightarrow 2\theta = \sin^{-1}(1)$

$2\theta = 90^\circ \longrightarrow$ as $\sin 90 = 1$.

$\Rightarrow \boxed{\theta = 45^\circ}$

★ if angle $\theta \rightarrow 90 - \theta = \theta'$

Range remains same

$$R' = \frac{V_0^2 \sin 2\theta'}{g} = \frac{V_0^2 \sin 2(90-\theta)}{g} = \frac{V_0^2 \sin(180 - 2\theta)}{g}$$

$$\boxed{R' = R = \frac{V_0^2 \sin 2\theta}{g}}$$

when $R \rightarrow R_{max} \rightarrow \theta = 45^\circ$

$$H = \frac{V_0^2 \sin^2(45^\circ)}{2g}$$

$$H = \frac{V_0^2 (1/\sqrt{2})^2}{2g} = \frac{V_0^2}{4g}.$$

$$\boxed{H = \frac{R_{max}}{4}}$$

if $R = H$, then $\theta = \tan^{-1}(4)$

if $R = 4H$, then $\theta = \tan^{-1}(1)$

$$R = nH \Rightarrow \frac{u_0^2 \sin 2\theta}{g} = n\left(\frac{u_0^2 \sin^2\theta}{2g}\right)$$

$$\tan\theta = [4/n] \text{ or } \boxed{\theta = \tan^{-1}(4/n)}$$

Equation of trajectory:

as $x = v_0 \cos\theta \, t$

$$t = \frac{x}{v_0 \cos\theta}$$

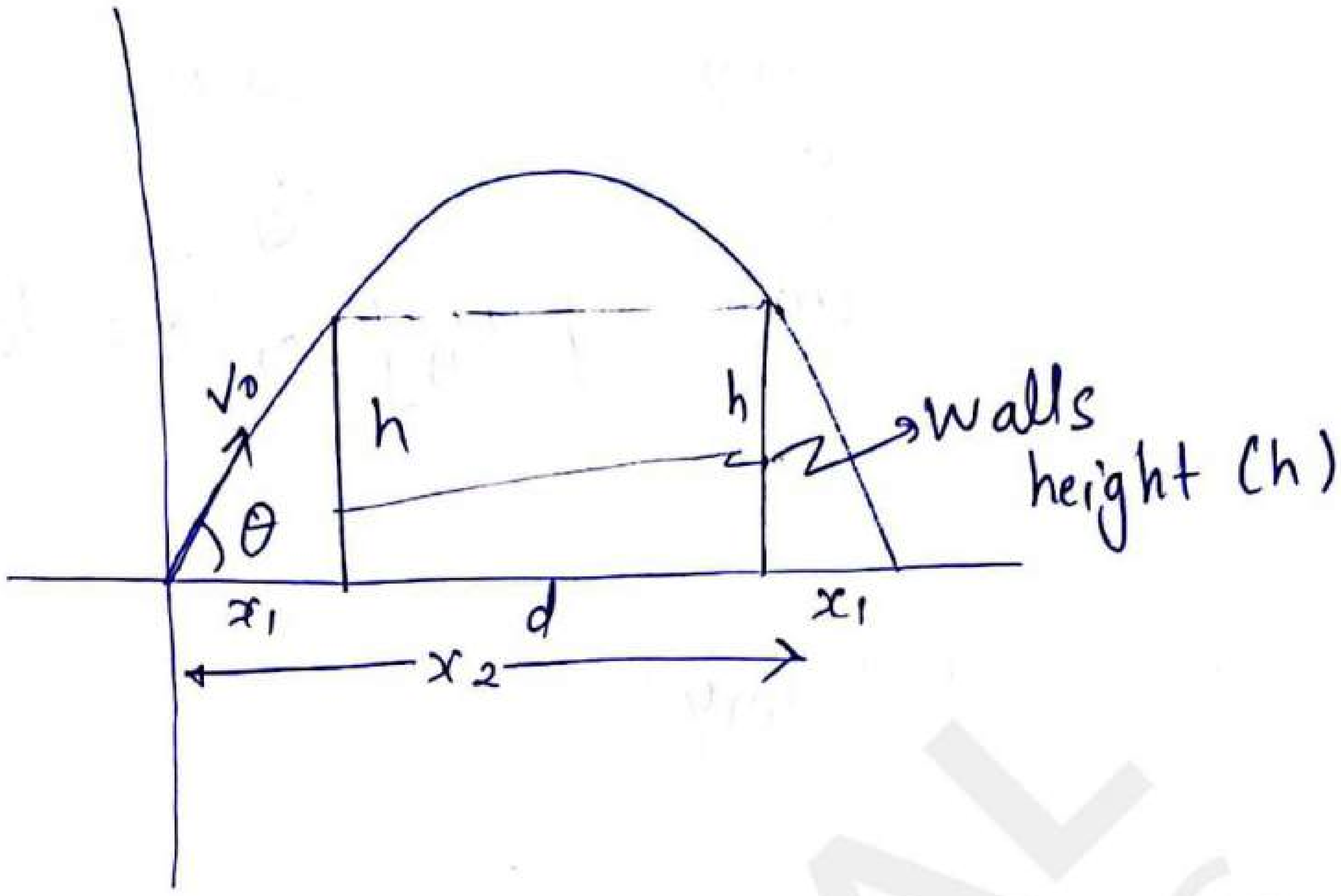
as $y = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$$y = u_0 \sin\theta \left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{1}{2} \frac{g x^2}{u_0^2 \cos^2\theta}}$$

$$\Rightarrow y \propto ax - bx^2$$

projectile of shape parabola.

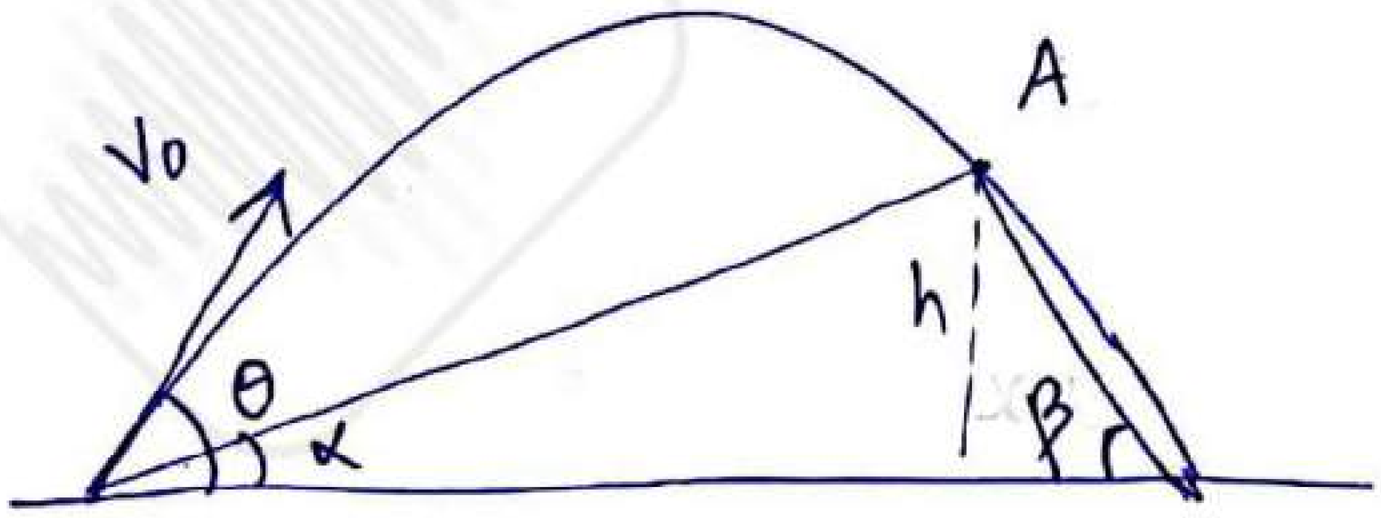


$$y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$$

x_1 & x_2 are roots of this equations.

So, $\boxed{d = x_2 - x_1}$

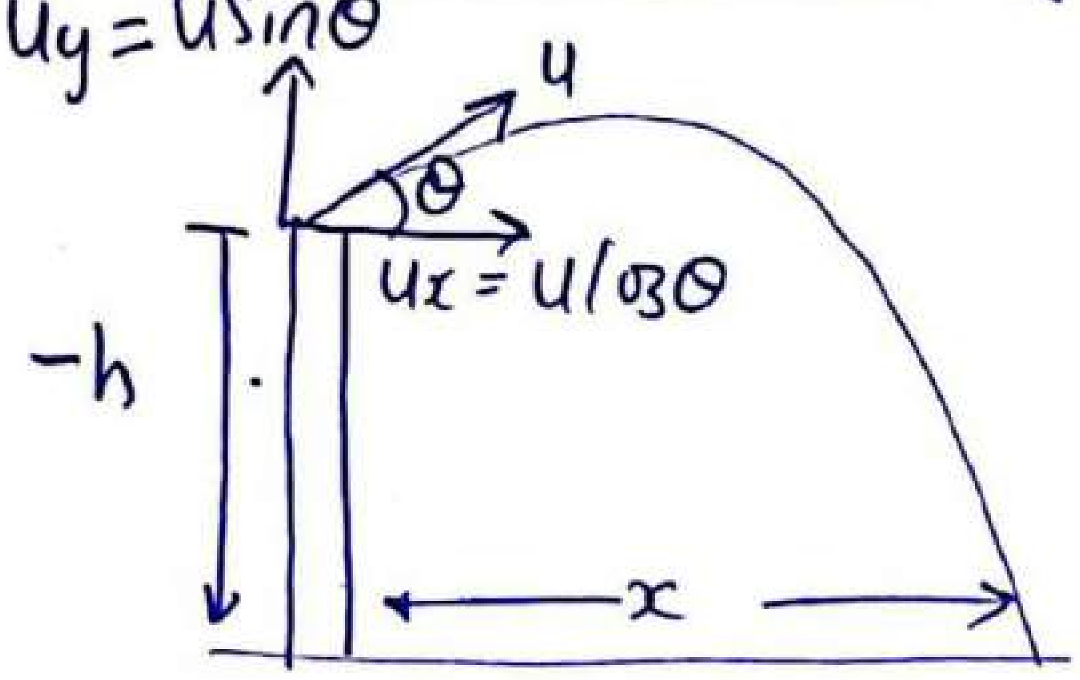
$$\boxed{h = x_1\tan\theta - \frac{g\,x_1^2}{2v_0^2\cos^2\theta}}$$



$$\boxed{\tan\theta = \tan\alpha + \tan\beta}$$

At any point net $\boxed{V = \sqrt{V_x^2 + V_y^2}}$

Projectile from Height at Certain Angle with Horizontal



$S_y = -h,\ u_y = u\sin\theta$

$a_y = -g,\ t = T \rightarrow$ Time of flight T

$S_y = u_y t + \frac{1}{2} a_y t^2$

$-h = (u\sin\theta)T - \frac{1}{2} g T^2$

$R = u_x T = u\cos\theta\, T$

$v_y^2 = u_y^2 + 2 a_y S_y$

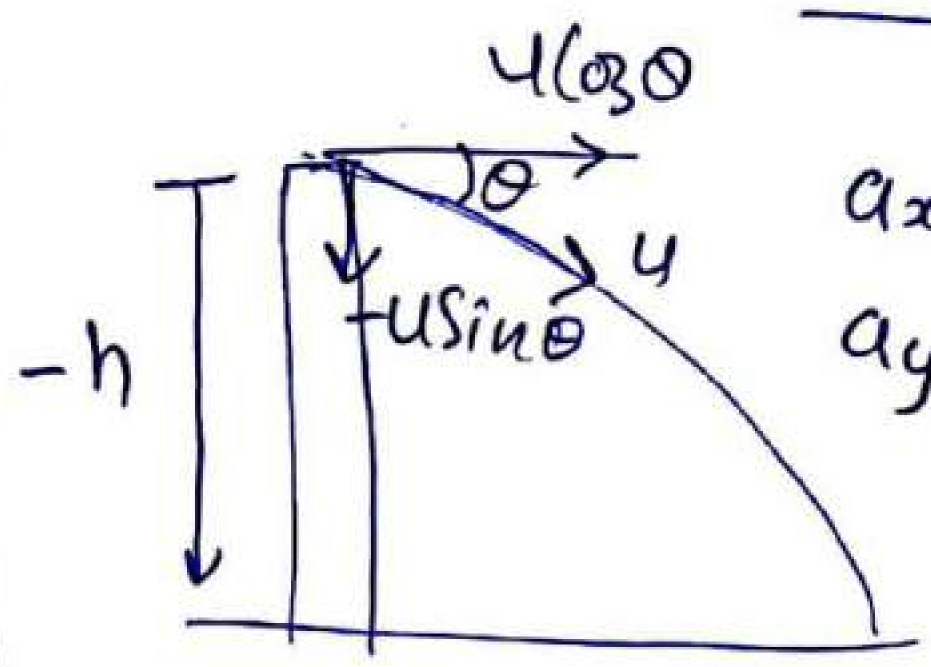
$= u^2\sin^2\theta + 2gh$

$V_x = u\cos\theta$

$V_B = \sqrt{V_y^2 + V_x^2}$

$V_B = \sqrt{u^2 + 2gh}$

as $V_x = u_x = u\cos\theta$



$a_x = 0$

$a_y = -g$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$S_y = -h,\ u_y = -u\sin\theta$

$t = T,\ a_y = -g$

$-h = -(u\sin\theta)T - \frac{1}{2} g T^2$

$h = u\sin\theta\, T + \frac{1}{2} g T^2$

$R = u_x T = u\cos\theta\, T$

$V_x = u\cos\theta$

$V_y^2 = u_y^2 + 2a_y S_y$

$V_y^2 = u^2\sin^2\theta + 2(-g)(-h)$

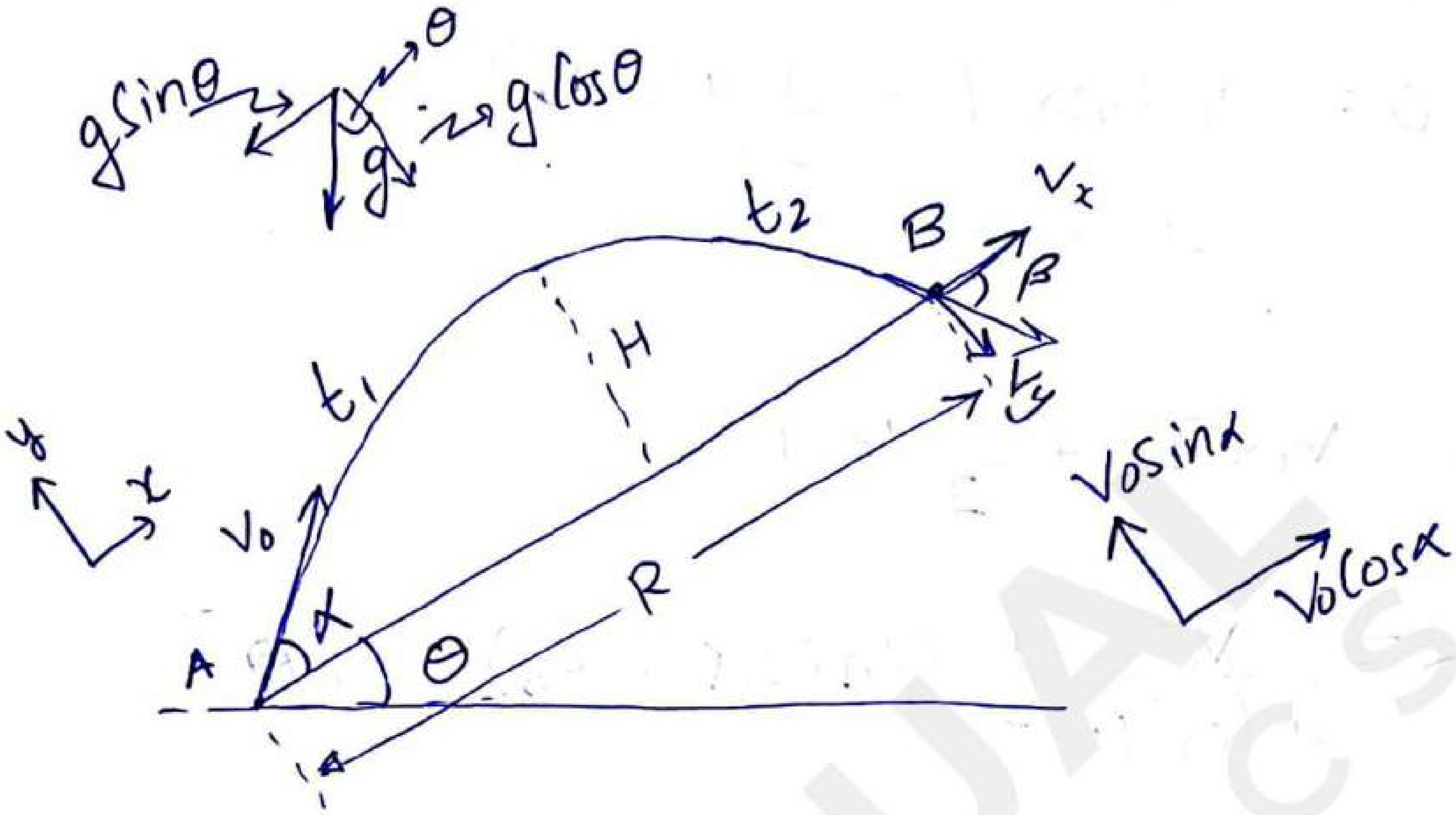
$V_{yB}^2 = u^2\sin^2\theta + 2gh$

$V_B = \sqrt{V_x^2 + V_y^2}$

as $V_x = u_x = u\cos\theta$

$$\boxed{V_B = \sqrt{u^2 + 2gh}}$$

projectile motion on an Inclined plane:



$a_x = -g\sin\theta$, $a_y = -g\cos\theta$, $V_{ix} = V_0\cos\alpha$
$V_{iy} = V_0\sin\alpha$

$y = 0 \Rightarrow$ displacement net in y-direction

T = Time of flight

$a_y = -g\cos\theta$, $V_{Fy} = 0$

$V_{Fy} = V_{iy} + a_y t$

$t_1 = \dfrac{V_0\sin\alpha}{g\cos\theta}$

$$\boxed{T = 2t_1 = \frac{2V_0\sin\alpha}{g\cos\theta}}$$ as $t_1 = t_2$

So

$$y = V_{iy} T + \frac{1}{2} a_y T^2$$

$$0 = V \sin\alpha\, T - \frac{1}{2} g \cos\theta_0 T^2$$

$$\Rightarrow T$$

$$R = V_{ix} T + \frac{1}{2} a_x T^2$$

$$\boxed{R = \frac{V_0^2}{g\cos^2\theta}\left[\sin(2\alpha + \theta_0) - \sin\theta\right]}$$

R_{max}

$$2\alpha + \theta = \pi/2$$

$$\alpha = \frac{\pi}{4} - \frac{\theta}{2}$$

$$\boxed{R_{max} = \frac{V_0^2}{g(1+\sin\theta)}}$$

<u>Velocity at B</u>:

$$v = u + at$$

$$V_{fx} = V_0 \cos\alpha - g \sin\theta\, T$$

$$= V_0\cos\alpha - g\sin\theta\left(\frac{2V_0\sin\alpha}{g\cos\theta}\right)$$

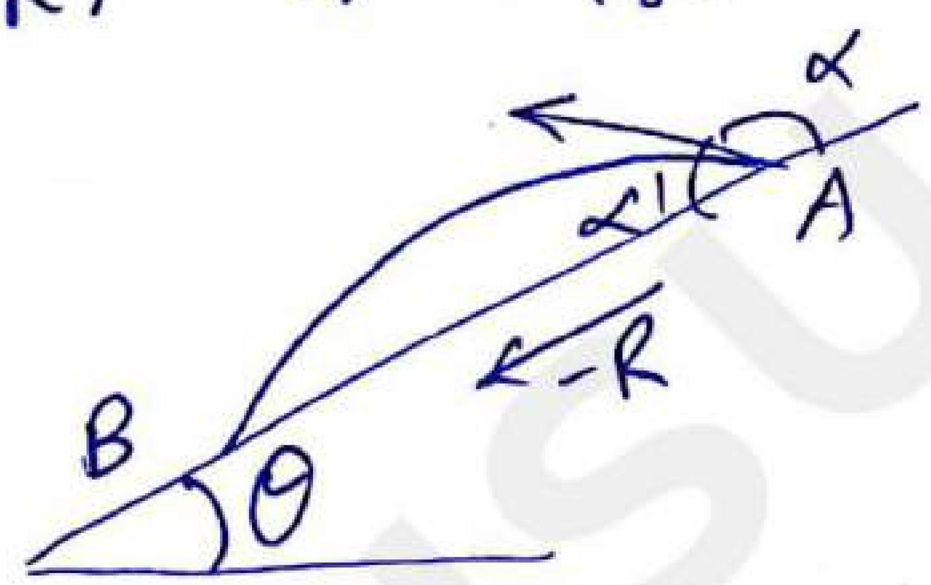
$$\boxed{V_{fx} = V_0\cos\alpha\,(1 - 2\tan\alpha\tan\theta)}$$

$$V_y = u_y + a_y t$$

$$\boxed{V_y = V_0 \sin\alpha - g\cos\theta\left(\frac{2V_0 \sin\alpha}{g\cos\theta}\right)}$$

If the projectile were projected down the inclined plane:

$R \to (-R)$ $\alpha \to 180^\circ - \alpha'$



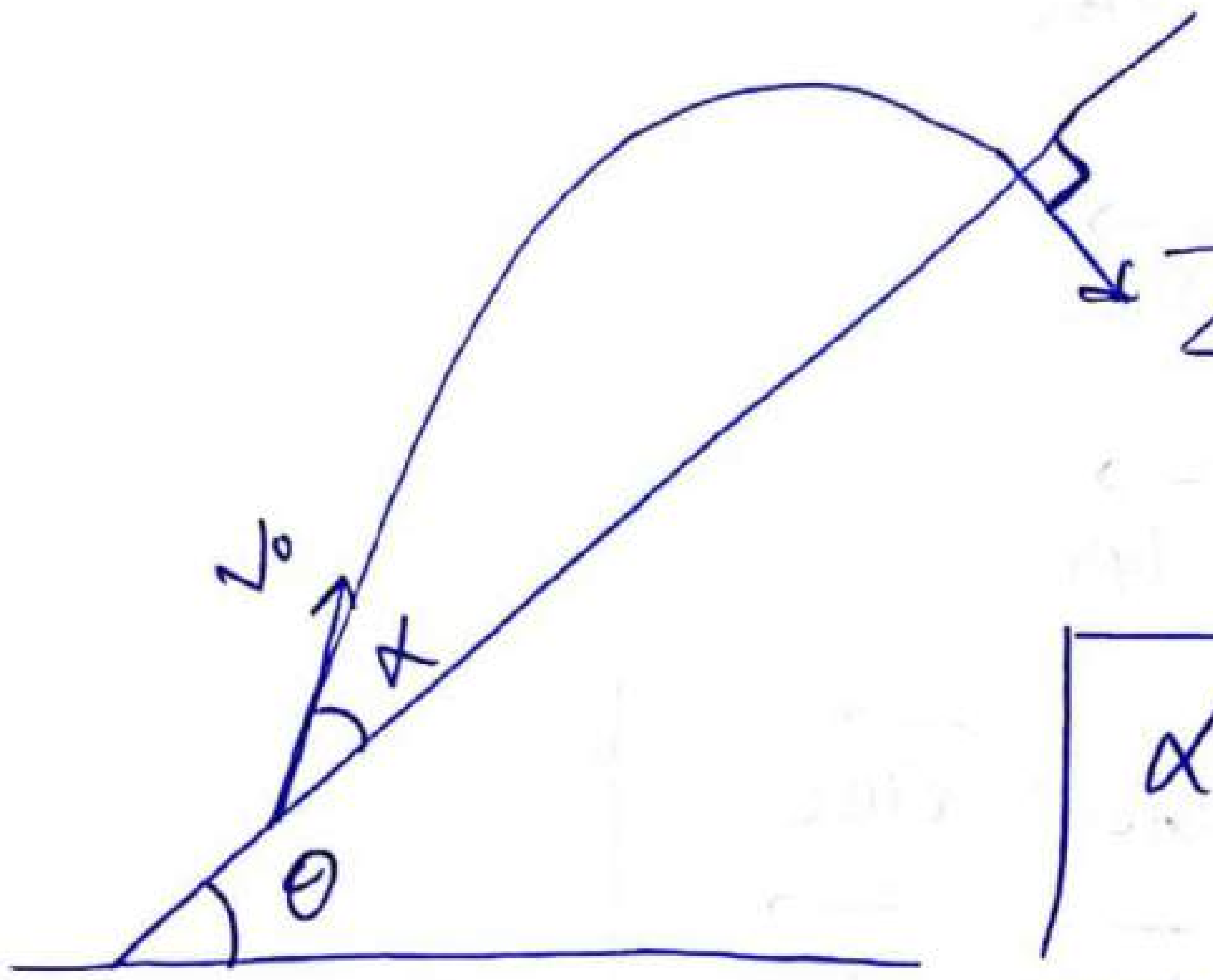
$$-R = \frac{V_0^2}{g\cos^2\theta}\left[\sin\{2(180-\alpha')+\theta\} - \sin\theta\right]$$

$$\boxed{-R = \frac{V_0^2}{g\cos^2\theta}\left[\sin(2\alpha'-\theta) + \sin\theta\right]}$$

for max Range $2\alpha' - \theta = \pi/2$

$$\alpha' = \pi/4 + \frac{\theta}{2}$$

$$\boxed{R_{max} = \frac{V_0^2}{g(1-\sin\theta)}}$$



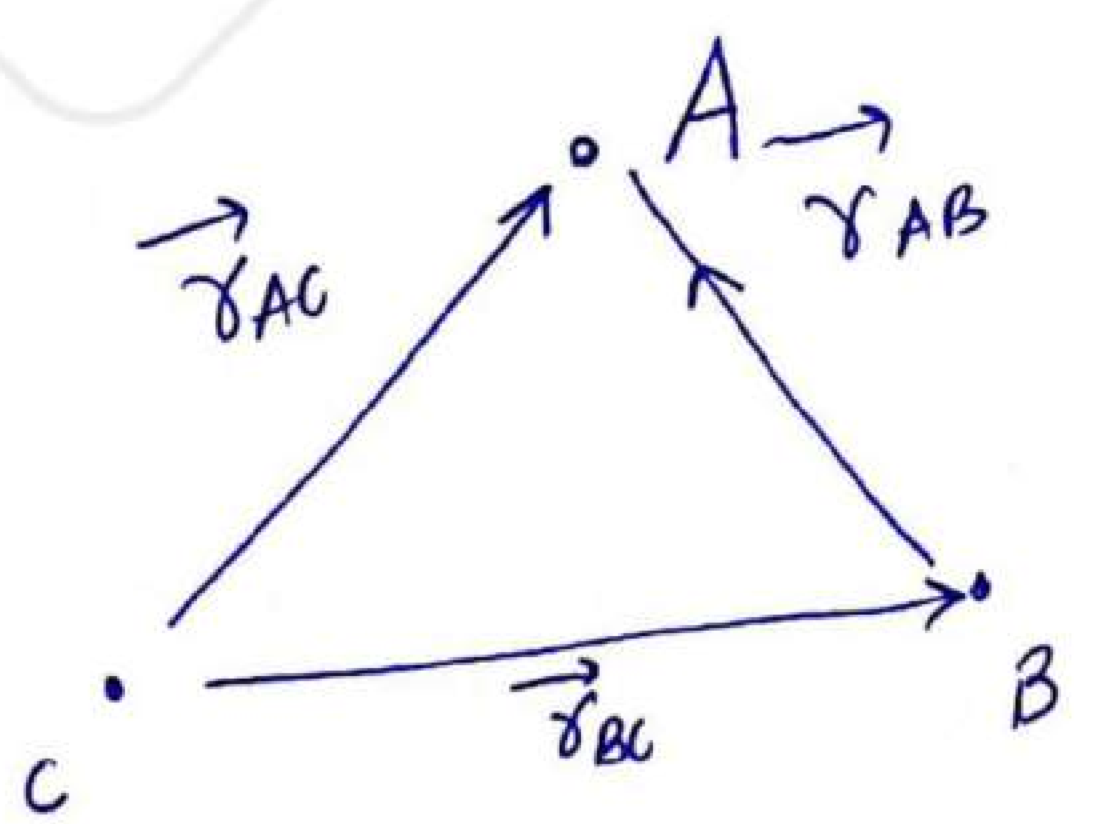
$\rightarrow$ for final angle to be 90°

$$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\cot\theta\right)}$$

And in this case

$$\boxed{T = \frac{2V_0}{g\sqrt{1+3\sin^2\theta}}}$$

Relative Motion (General)



$\vec{r}_{AC} \rightarrow$ position of A wrt C

$\vec{r}_{BC} \rightarrow$ position of B wrt C

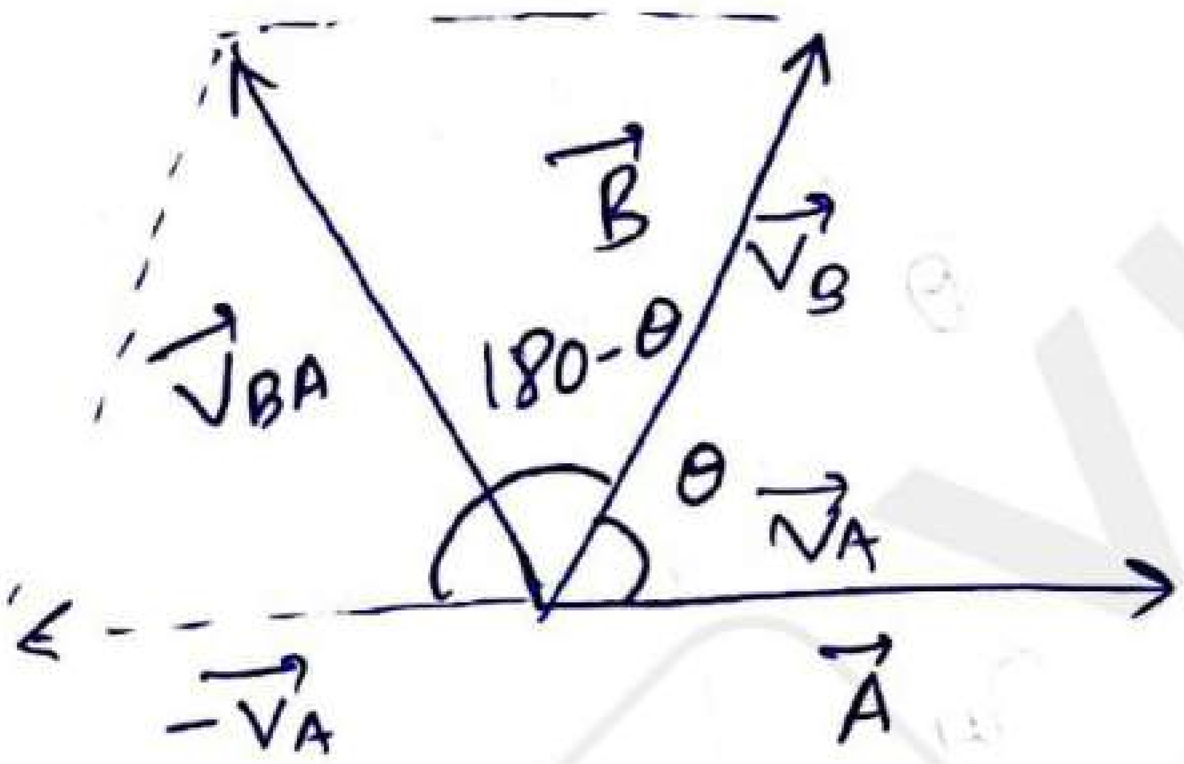
$\vec{r}_{AB} \rightarrow$ position of A wr B

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC}$$

$$\Rightarrow \vec{V}_{AC} = \vec{V}_{AB} + \vec{V}_{BC}$$

$$\vec{a}_{AC} = \vec{a}_{AB} + \vec{a}_{BC}$$

so
$$\boxed{\begin{aligned} \vec{a}_{AB} &= \vec{a}_{AC} - \vec{a}_{BC} \\ \& \quad \vec{V}_{AB} &= \vec{V}_{AC} - \vec{V}_{BC} \end{aligned}}$$

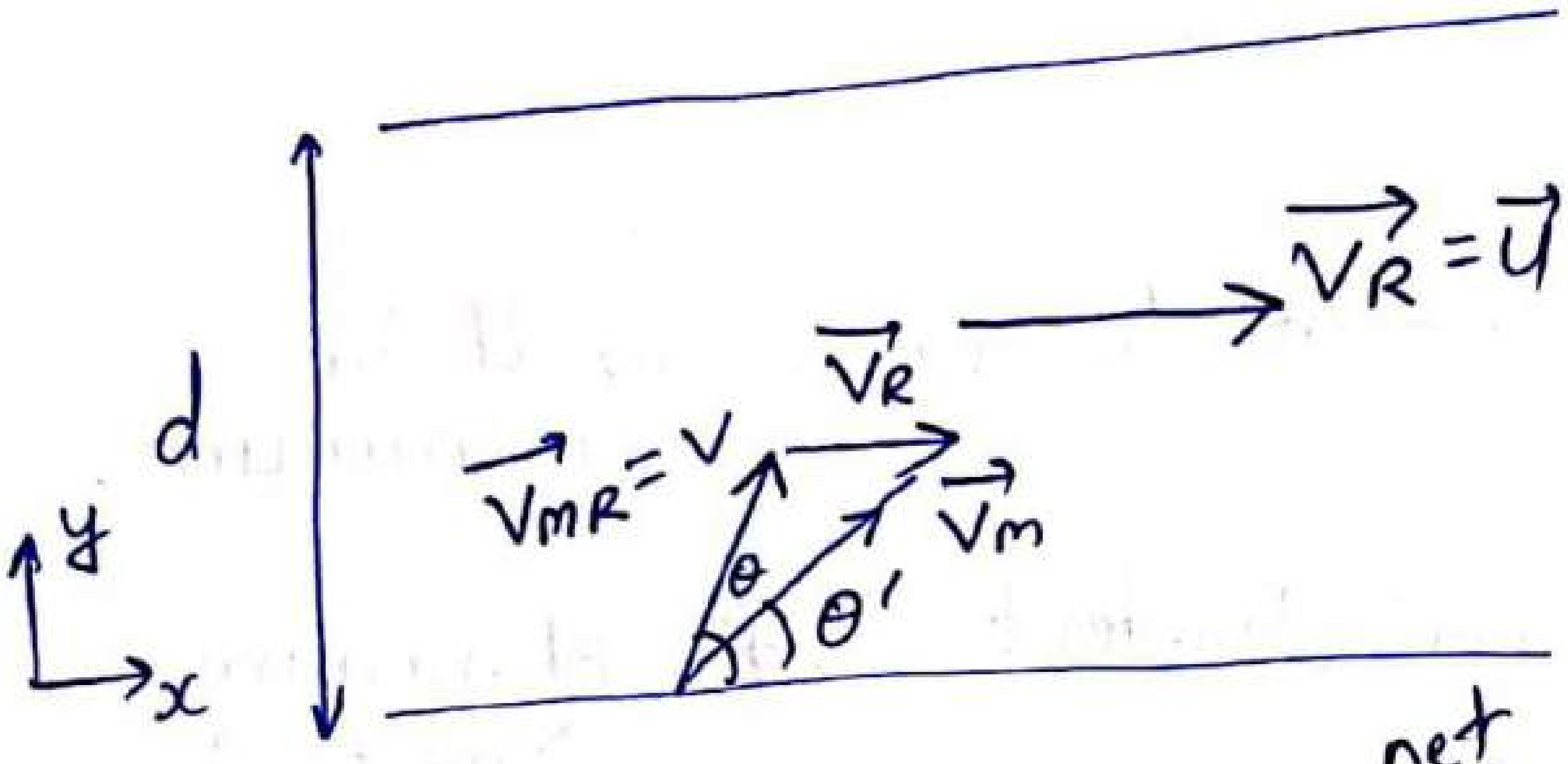


$\vec{V}_{BA} \rightarrow$ Velocity of B wrt A

$$= \vec{V}_B - \vec{V}_A$$

$$\boxed{\tan\alpha = \frac{V_B \sin\theta}{V_A - V_B \cos\theta}}$$

Motion of man Swimming in a River :



when river is not moving

$\vec{V}_{mR}$ = Velocity of man with respect to River

net velocity $\vec{V}_m$ = Velocity of man

$\vec{V}_R$ = velocity of river

so, $\vec{V}_m = \vec{V}_{m,R} + \vec{V}_R$

$$\vec{V}_{m,R} = (V\cos\theta\hat{i} + V\sin\theta\hat{j})$$

$$\vec{V}_m = (V\cos\theta\hat{i} + V\sin\theta\hat{j}) + u\hat{i}$$

$$= (V\cos\theta + u)\hat{i} + V\sin\theta\hat{j}$$

now $\boxed{t = \dfrac{d}{V_y} = \dfrac{d}{V\sin\theta}}$

Drift $= V_x \times t$

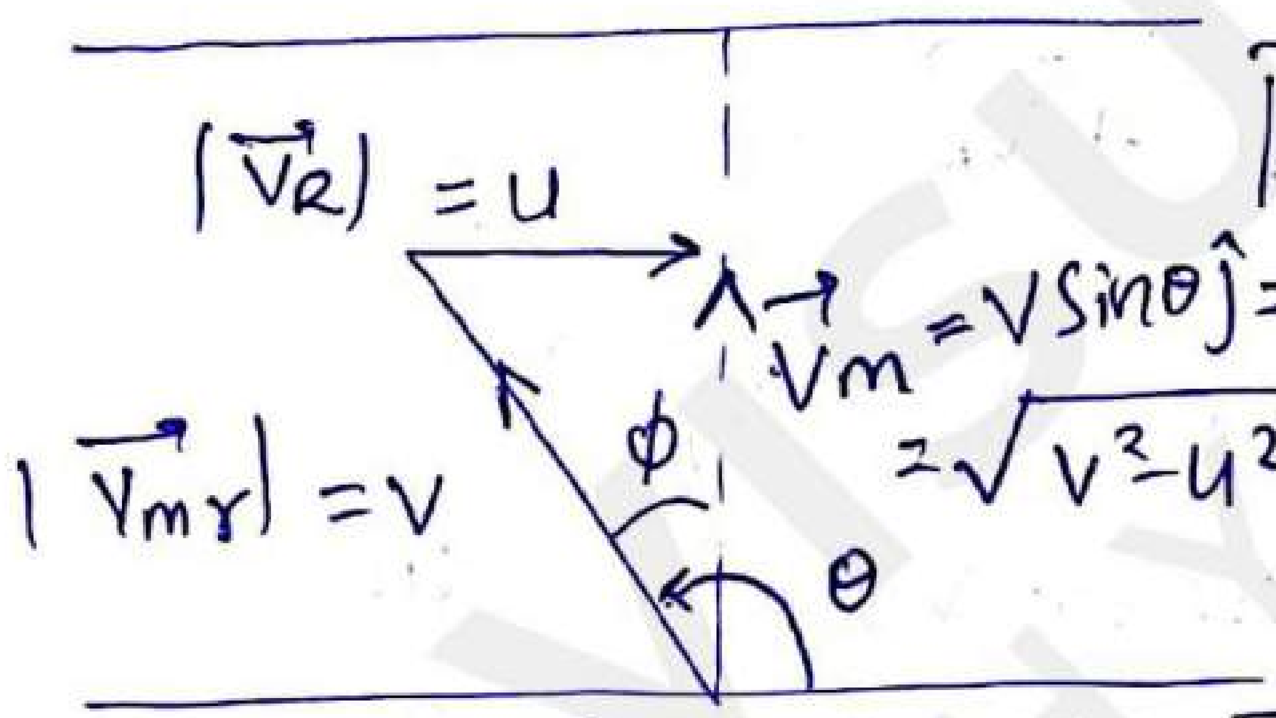
$$\boxed{x = (V\cos\theta + u) \times \frac{d}{V\sin\theta}}$$

for crossing the river in shortest Time:

$$t = \frac{d}{v\sin\theta}$$

so when $\theta = 90^\circ$ $\longrightarrow$ time to cross will be minimum.

Crossing the river in shortest Path, Minimum Drift :



$\boxed{x_{min} = 0}$

$\vec{V_m} = v\sin\theta\,\hat{j} \Rightarrow (v\cos\theta + u) = 0$

$= \sqrt{v^2 - u^2} \Rightarrow \boxed{\cos\theta = \frac{-u}{v}}$

$\Rightarrow \boxed{u \le v}$

$$\text{so, } \boxed{t = \frac{d}{v\sin\theta} = \frac{d}{\sqrt{v^2 - u^2}}}$$

Rain - Man problem

$\vec{V_r}$ → velocity of rain

$\vec{V_m}$ → Velocity of man

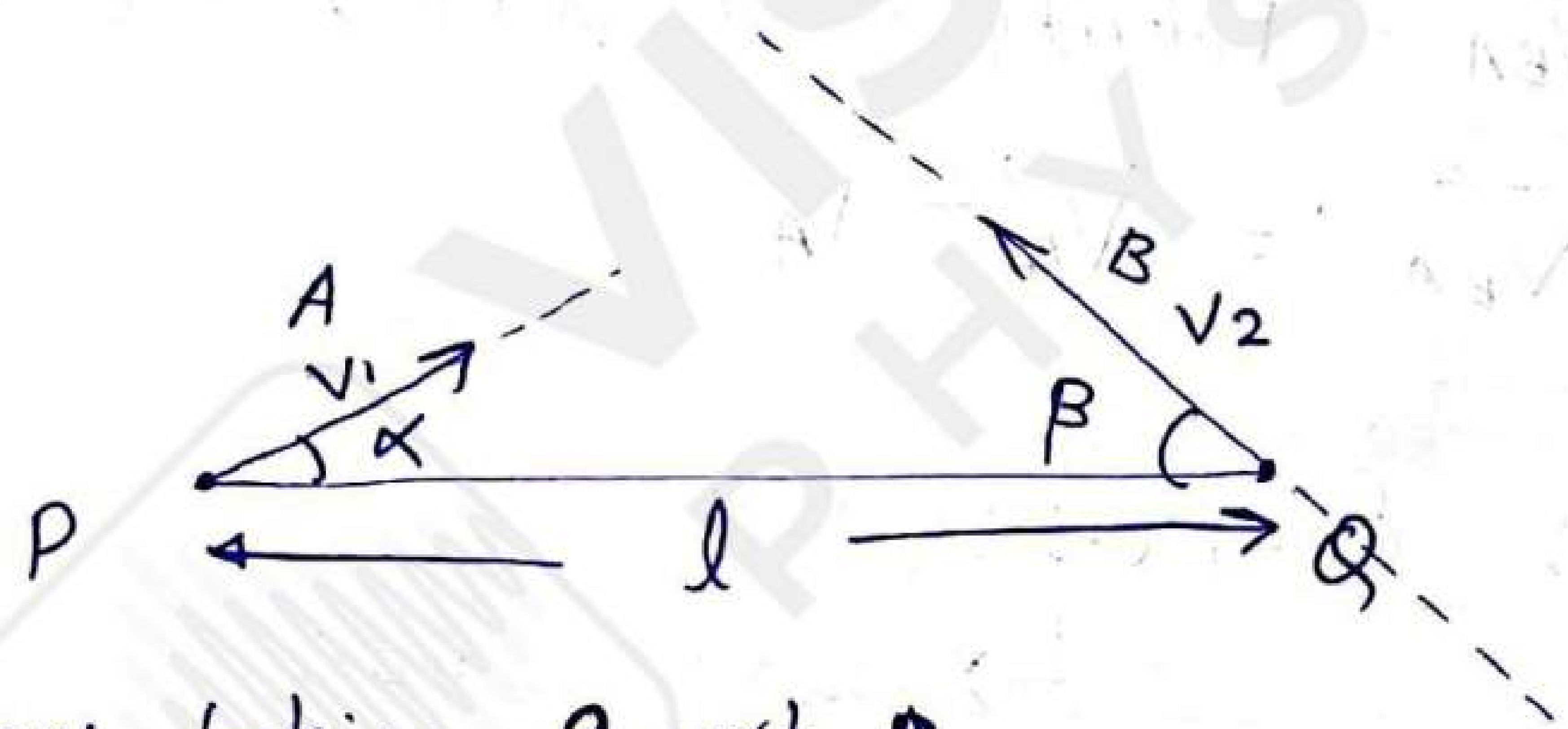
$\vec{V_{rm}}$ → velocity of rain wrt man

$$\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$$

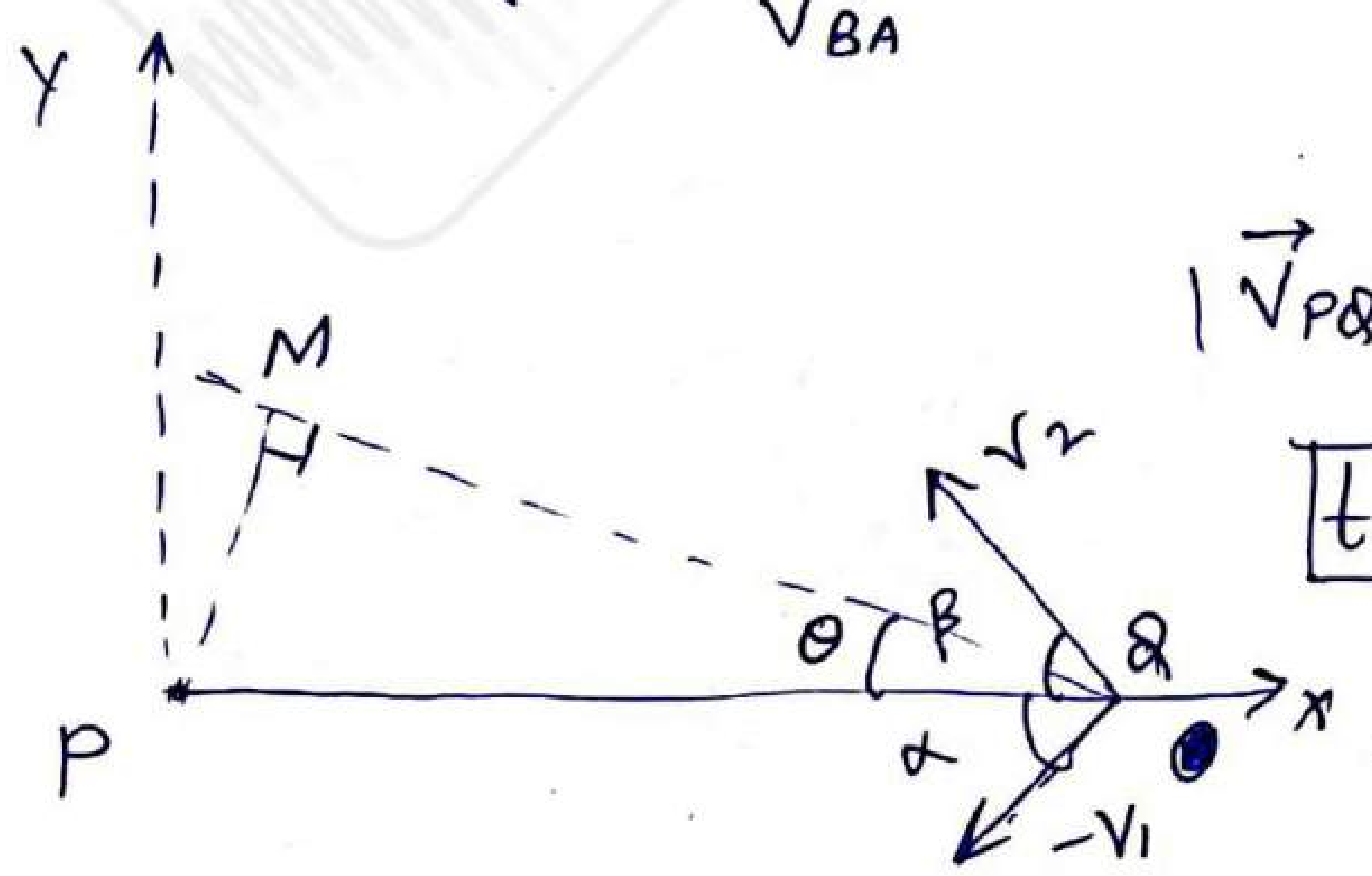
$$|\vec{V}_{rm}| = \sqrt{V_r^2 + V_m^2}$$

and $\boxed{\theta = \tan^{-1}\left(\frac{V_m}{V_r}\right)}$

shortest distance between two moving particle



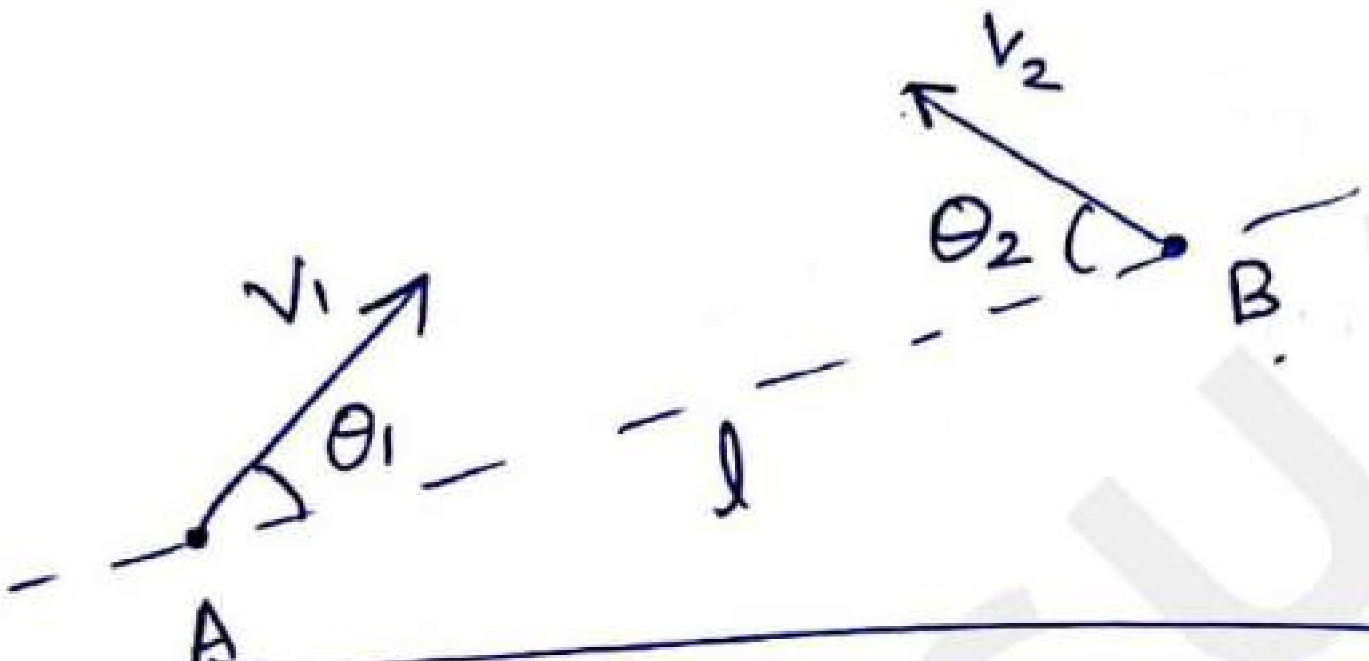
Now taking Q wrt P
$\vec{V}_{BA}$



$$|\vec{V}_{PQ}| = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\alpha+\beta)}$$

$$\boxed{t_{min} = \frac{PM}{\sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\alpha+\beta)}}}$$

Velocity of Approach



$$\boxed{V = \underbrace{v_1\cos\theta_1 + v_2\cos\theta_2}_{\text{velocity of approach}} = -\frac{dl}{dt}}$$

So, in terms of vector

$\vec{V}_{BA}$ = Velocity of B wrt A = velocity of approach

$$\boxed{\vec{V}_{BA} = \vec{V}_B - \vec{V}_A}$$