VISUAL PHYSICS

SHORT NOTES

CHAPTER

Mechanical Waves

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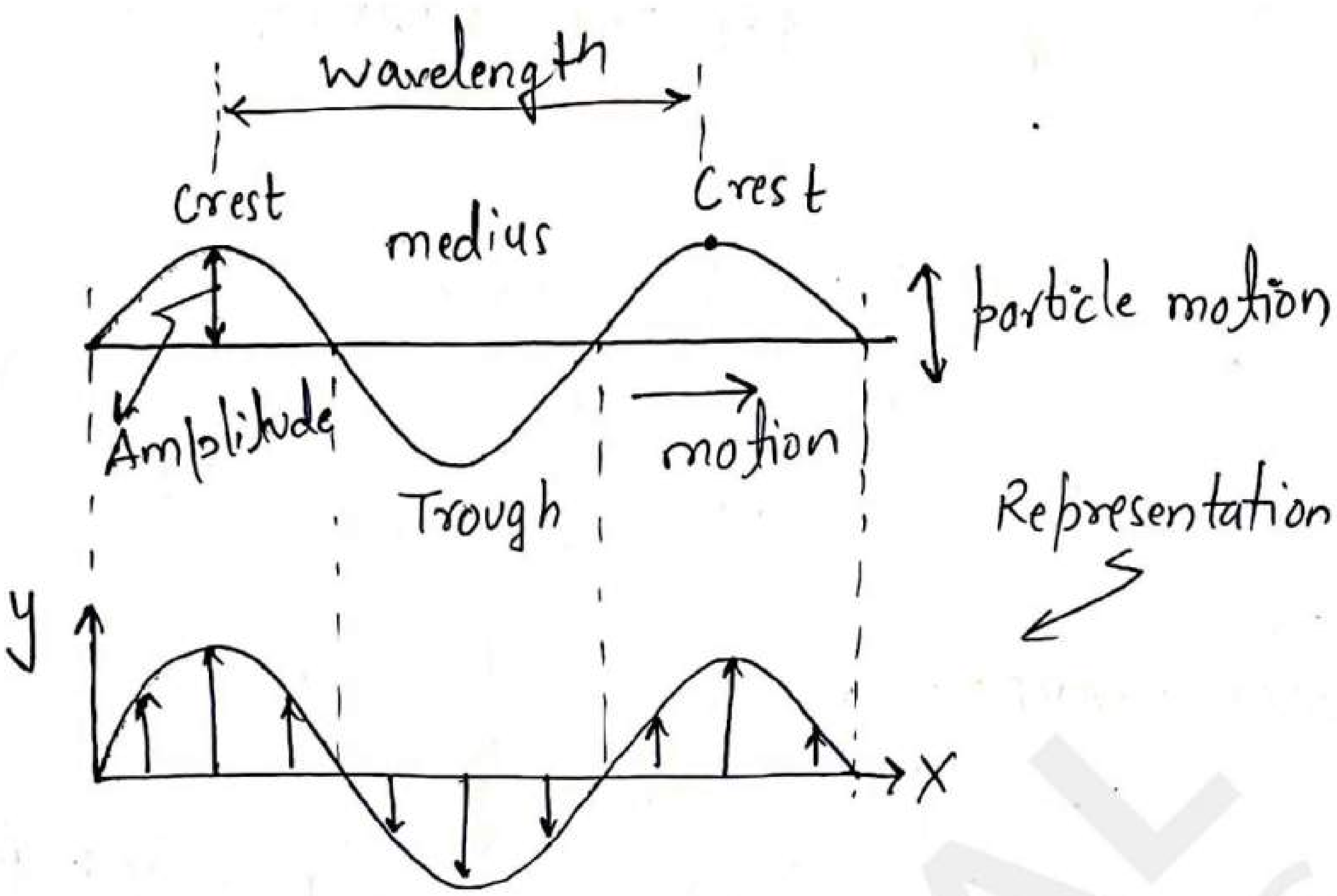
MECHANICAL WAVES

Wave motion → The transfer of energy through space without the accompanying transfer of matter. (Waves travell through and using medium)

MECHANICAL WAVES: A mechanical wave can be produced and propagated only in those material media which possess elasticity & inertia.

Type of waves:

→ **Transvere Waves**: Elements of the disturbed medium to move perpendicular to the direction of propagation is called a transverse wave.

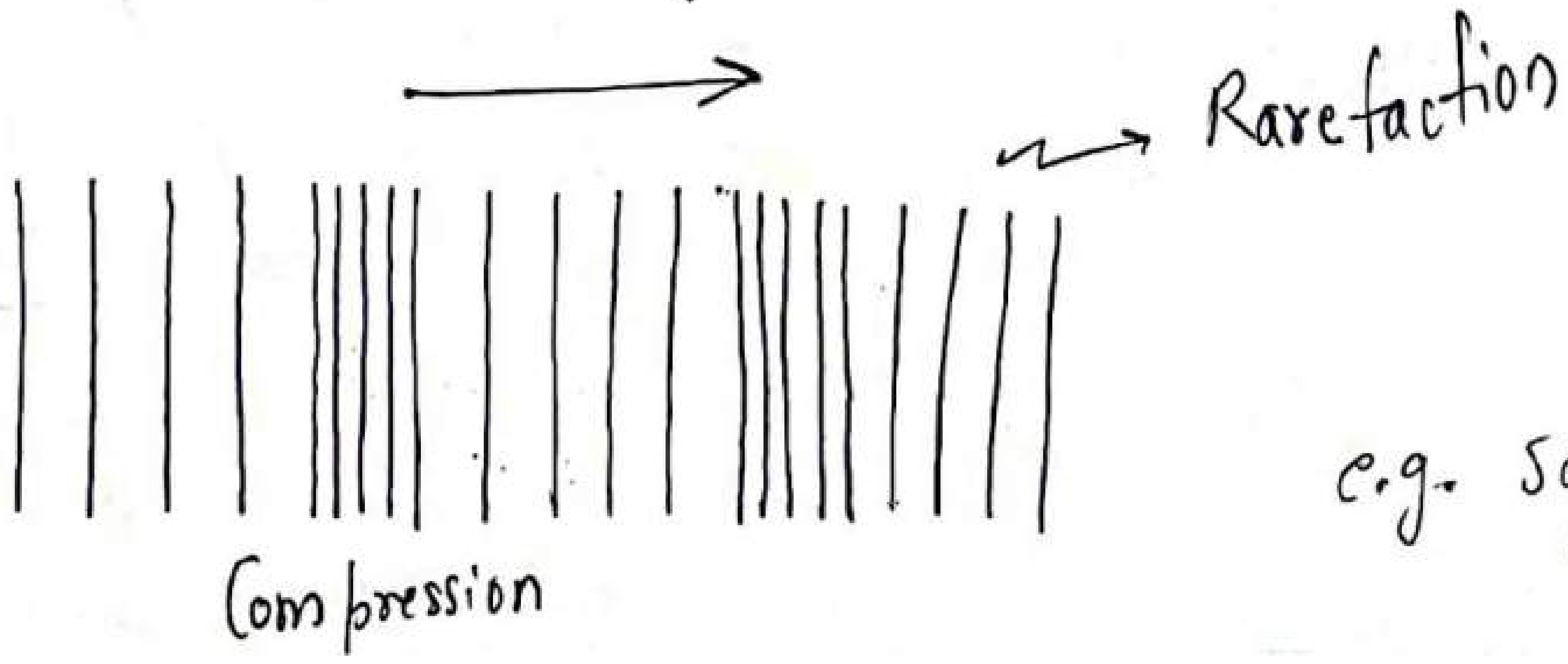


→ A particle at the crest or the trough has zero velocity. and distance of particle from mean position is termed as amplitude of wave.

→ Distance between two consecutive crests/trough is wavelength.

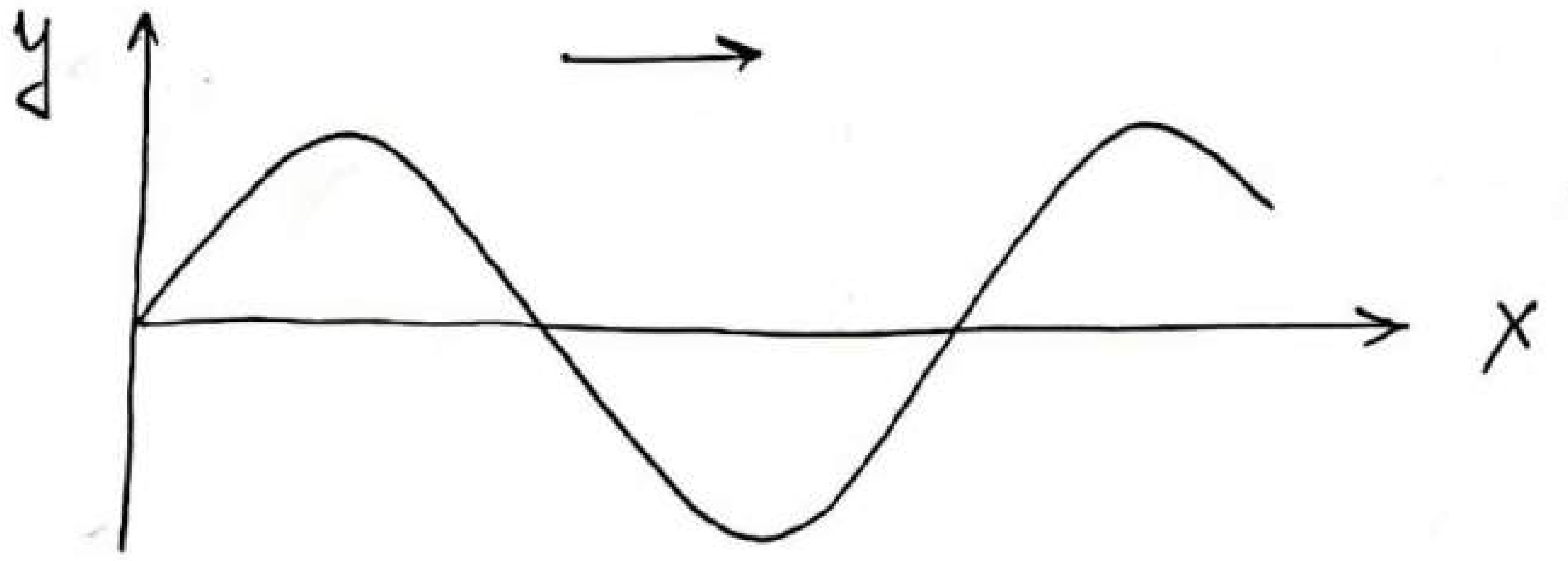
Longitudinal waves

→ kind of wave motion in which individual particles of a medium execute periodic motion about their mean position along the direction of wave motion.

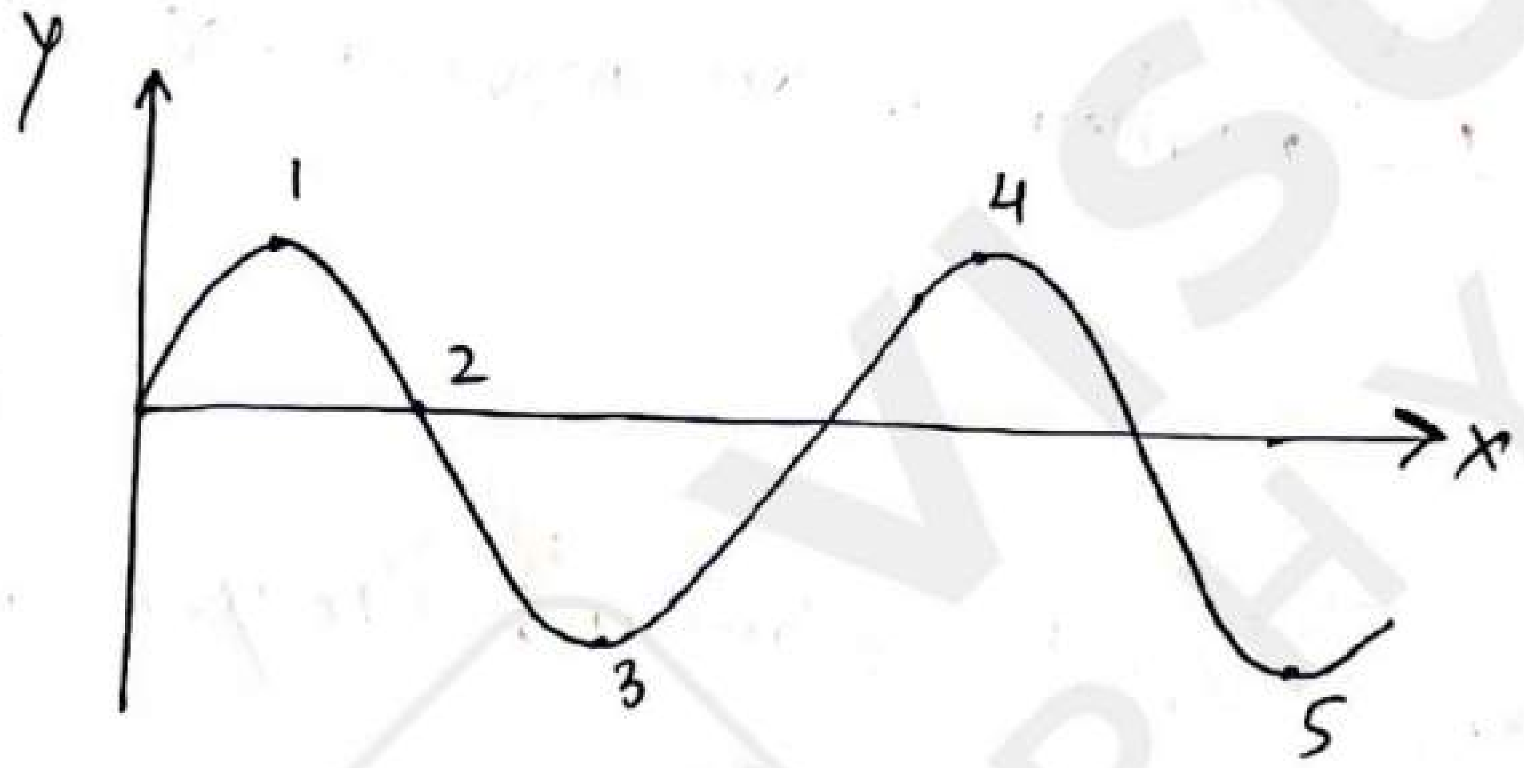


e.g. Sound waves, Spring wave

Representation



phase → Defines the position (in terms of distance from mean position). and velocity of a particle oscillating under the Influence of a wave.



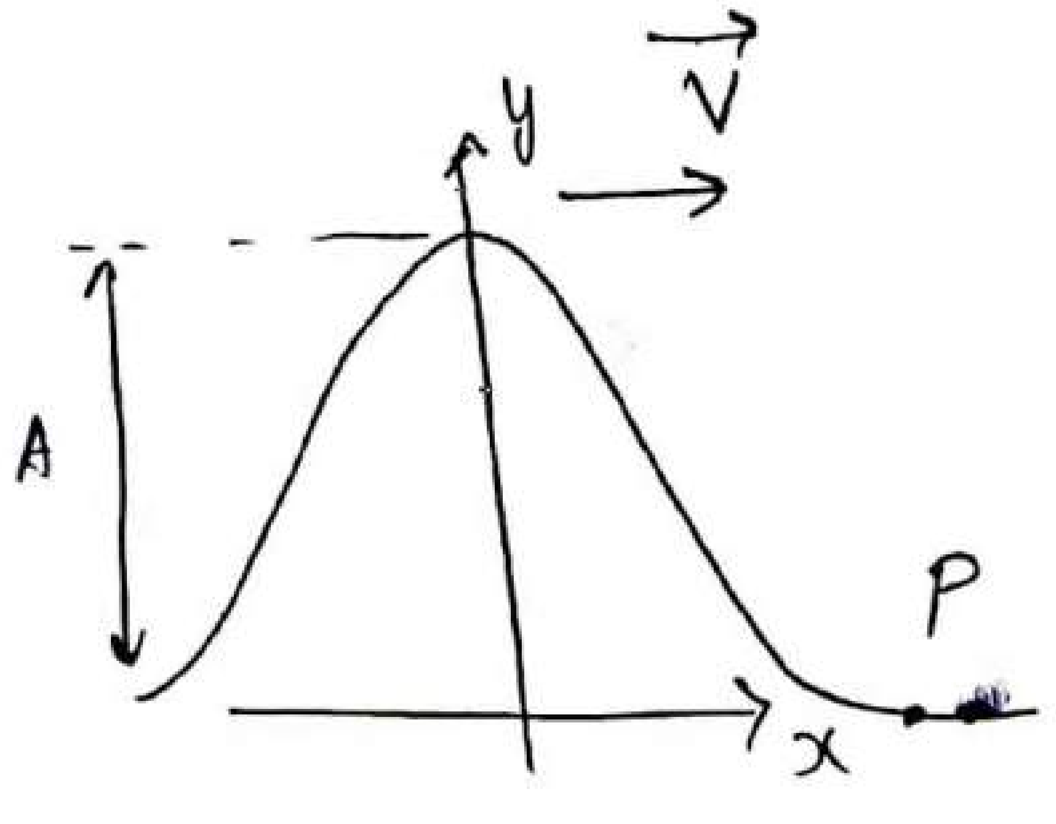
1, 4 → in phase
3, 5 → in phase
4, 5 → out of phase

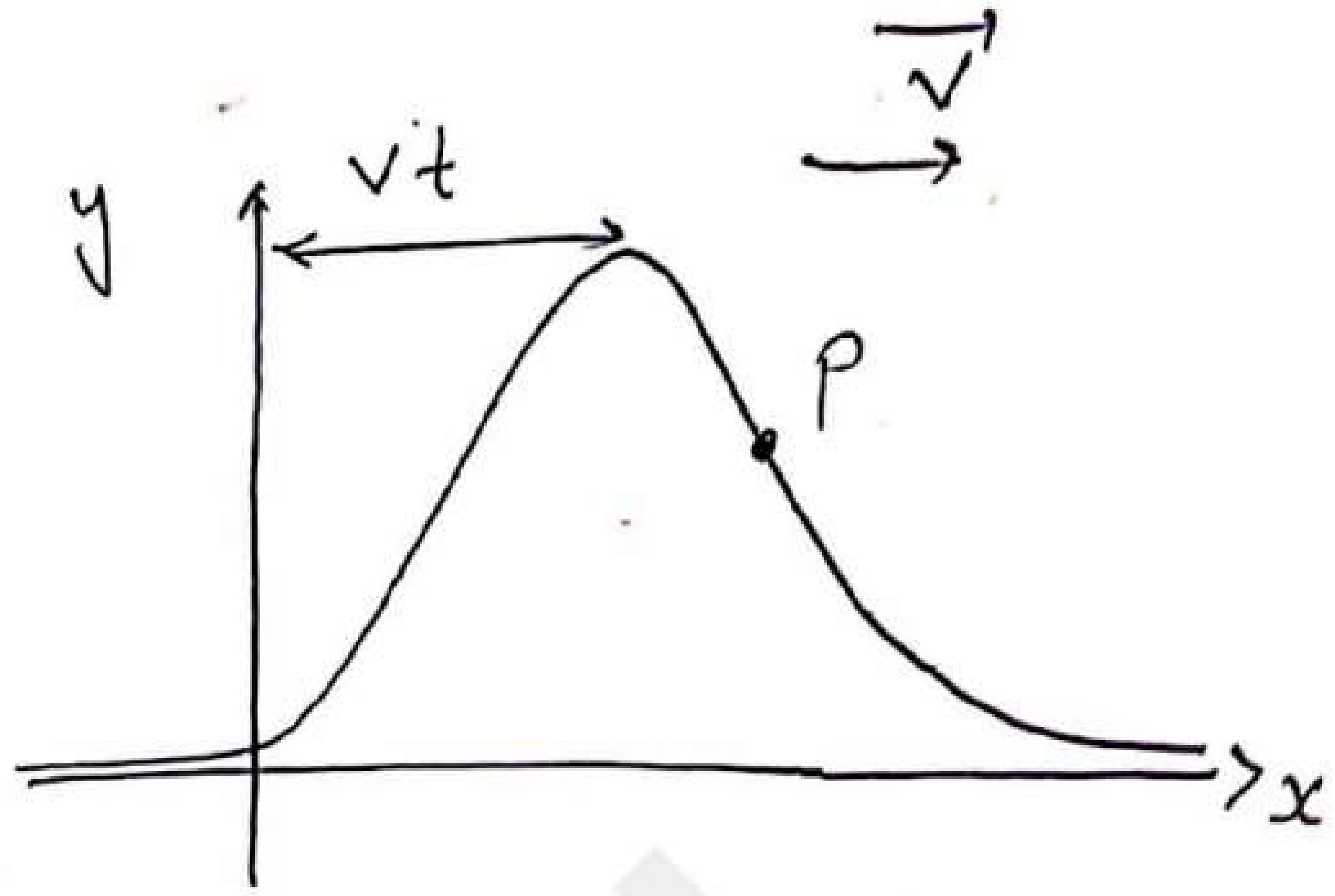
Wave speed → Distance travelled by the wave in unit time.

Time period → Time taken by particle from mean position to one extreme to other extreme position and back to mean position.

Intensity of wave → Intensity of the wave is the energy transmitted per unit area per second in the form of the wave in the direction of the propogation of the wave by the source.

Wave function:





$y(x,t) = f(x-vt)$ → when wave move in +x direction

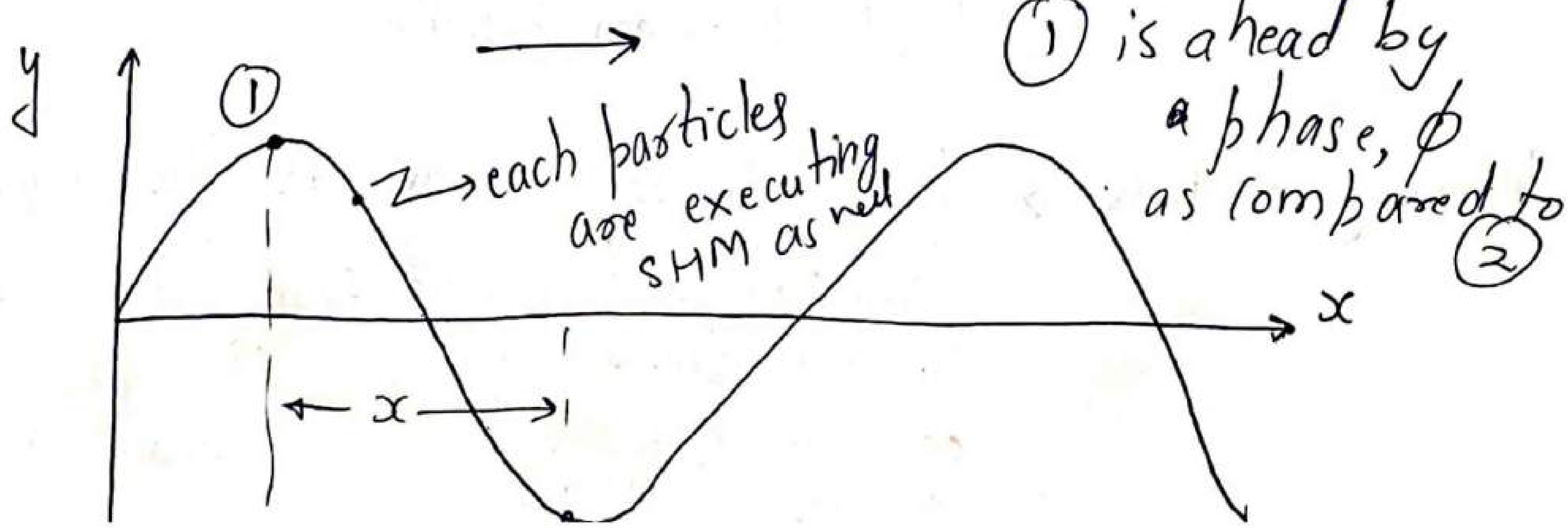
$y(x,t) = f(x+vt)$ → when move move in −x-direction

→ wave function

The wave function, $y(x,t)$ represents the y-coordinates

→ if we fix 't', $y(x)$ called waveform.

Sinusoidal travelling wave:



for λ (wavelength) $\rightarrow$ 2π (phase difference)

$\Rightarrow$ $\Delta\phi \rightarrow$ phase difference between two particles seperated by Δx

$$\boxed{k = \frac{\Delta\phi}{\Delta x} = \frac{2\pi}{\lambda}} \quad ; \quad \boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$

k → angular wave number.

$$\boxed{y_1 = A \sin \omega t}$$

ω → angular frequency for each particle (Hence wave)

↳ SHM equation of any particle

now a particle at 'x' distance from this particle will have

$$\phi = \frac{2\pi}{\lambda} x$$, phase difference.

or $\phi = kx$

so for second particle

$$\boxed{y_2 = A \sin(\omega t - kx)}$$

↳ equation giving relation in terms of position and time 't'.

Hence the equation of travelling Sinusoidal wave

$$y = A\sin(\omega t - kx)$$

→ wave moving in +x direction

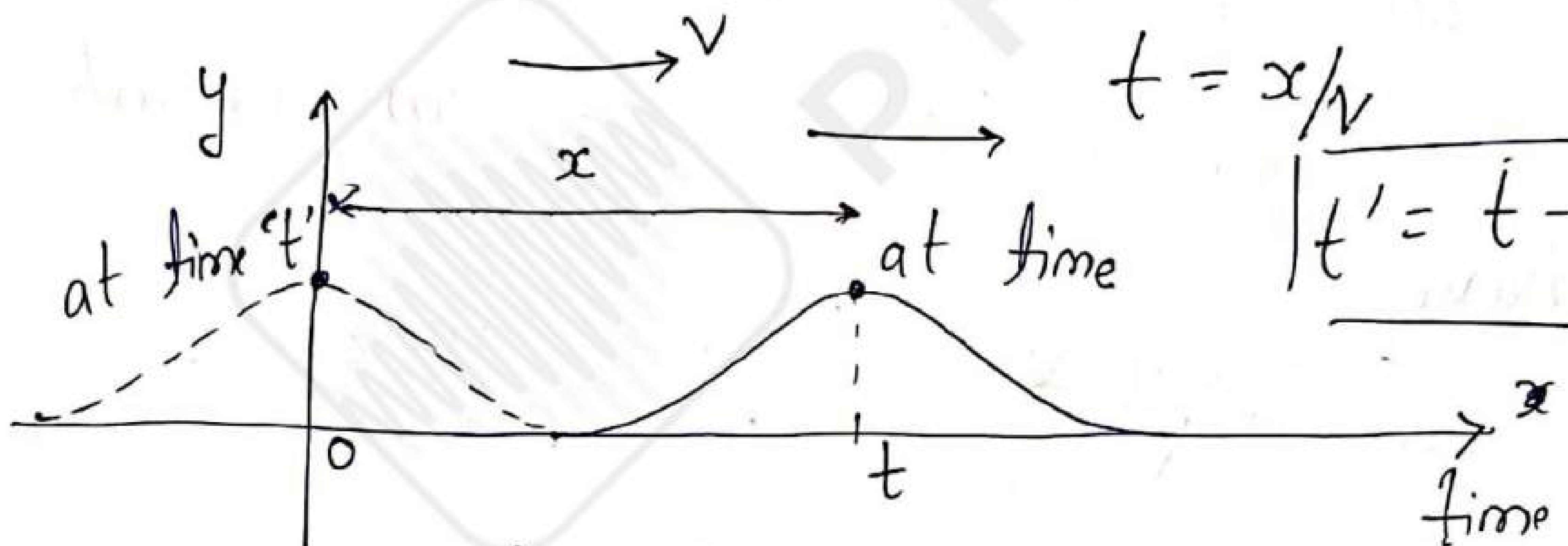
And in general case,

$$y_1 = A\sin(\omega t + \phi)$$

→ have some initial condition

$$\Rightarrow \boxed{y = A\sin(\omega t + \phi - kx)}$$

General wave equation:



$t = x/v$

$$\boxed{t' = t - x/v}$$

$y(x,t) = f(x,t')$

$y(0,t') = f(t')$

position of particle at 'x' is same as equation of particle at $x=0$, at time

$$\Rightarrow y(x,t) = y\left(0, t - \frac{x}{v}\right) \qquad \boxed{t' = t - x/v}$$

Hence, $\boxed{f\left(t-\frac{x}{v}\right) = y(x,t)}$

↳ general equation

Differential wave equation

$$y = f\left(t - \frac{x}{v}\right)$$

for t = constant = c

$$y = f\left(c - \frac{x}{v}\right) = g(x)$$

$$\left.\frac{dy}{dx}\right|_{t=\text{constant}} = \frac{\partial y}{\partial x} = f'\left(t - \frac{x}{v}\right)\left(-\frac{1}{v}\right).$$

(t → constant)

for x = constant

$$y = f\left(t - \frac{x}{v}\right)$$

($\frac{x}{v}$ → constant)

$$\Rightarrow \boxed{\frac{\partial y}{\partial x} = \frac{\partial y}{\partial t}\left(-\frac{1}{v}\right)}$$

$$\frac{\partial y}{\partial t} = u = f'\left(t - \frac{x}{v}\right)$$

u → particle speed

v → wave speed

Or

$$\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$$

$\frac{\partial y}{\partial t}$ → Transverse speed of particle

v → wave speed

$\left(\frac{\partial y}{\partial x}\right)$ → slope of y-x graph

$$\frac{\partial y}{\partial x} = f'\left(t-\frac{x}{v}\right)\left(-\frac{1}{v}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = f''\left(t-\frac{x}{v}\right)\left(-\frac{1}{v}\right)\left(-\frac{1}{v}\right)$$

& $$\frac{\partial y}{\partial t} = f'\left(t-\frac{x}{v}\right)$$

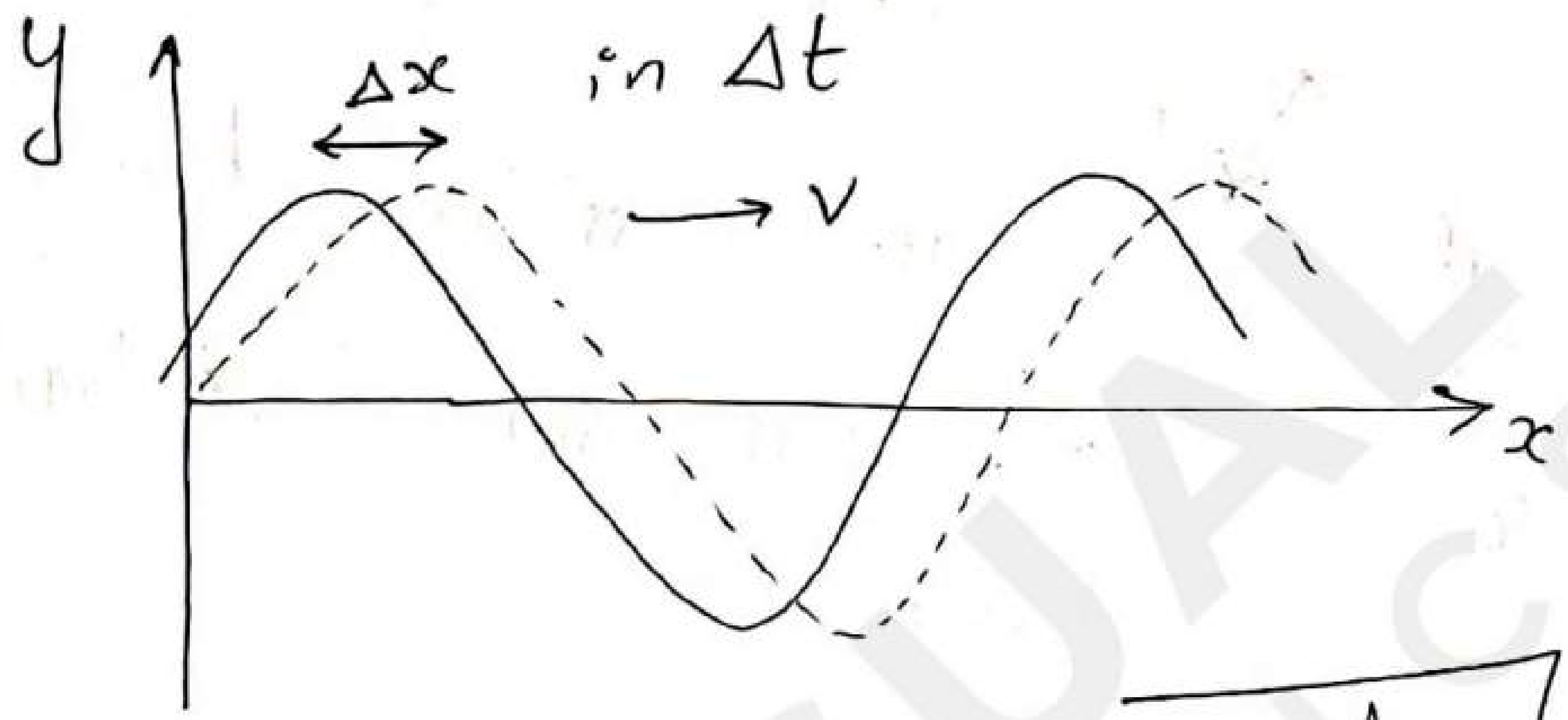
$$\frac{\partial^2 y}{\partial t^2} = f''\left(t-\frac{x}{v}\right)$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

↳ Differential eq. of wave motion

* Any function satisfying this represents travelling waves.

speed of sinusoidal wave:



$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \Rightarrow \boxed{V = \frac{dx}{dt}}$$

y constant for Δx

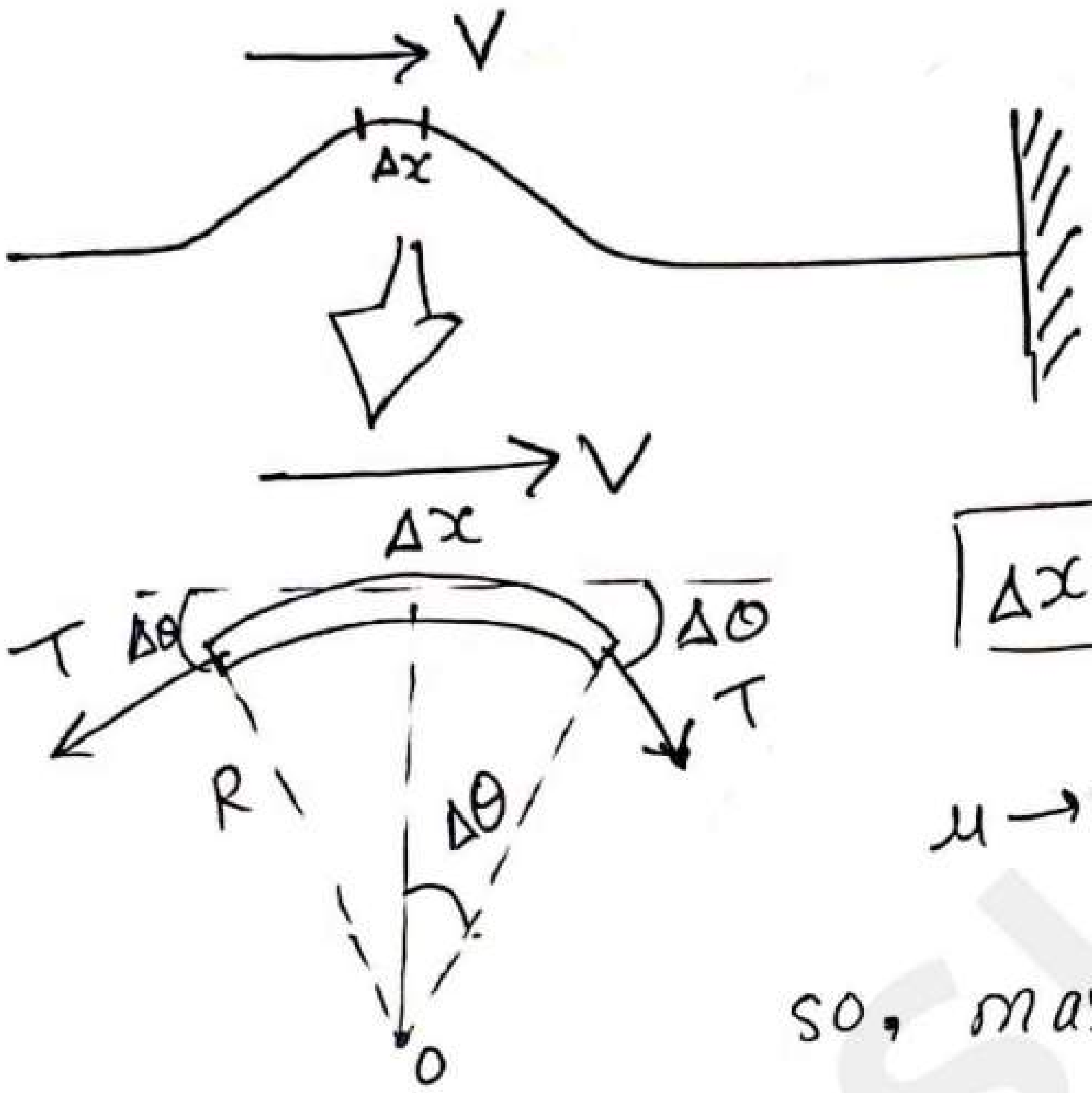
$\Rightarrow \sin(kx - \omega t + \phi) = \text{constant}$

$(kx - \omega t + \phi) = \text{constant}$

$\Rightarrow k\frac{dx}{dt} - \omega + 0 = 0$

$$\Rightarrow \boxed{V = \frac{\omega}{k}}$$

Speed of waves on string:



$\boxed{\Delta x = 2R\Delta\theta}$

$\mu \rightarrow$ mass per unit length.

so, mass, Δm of Δx part

$$\Delta m = \mu 2R\Delta\theta$$

force towards center O, $= T\sin\Delta\theta + T\sin\Delta\theta$

$$F_c = 2T\sin\Delta\theta$$

if $\Delta\theta$ is small

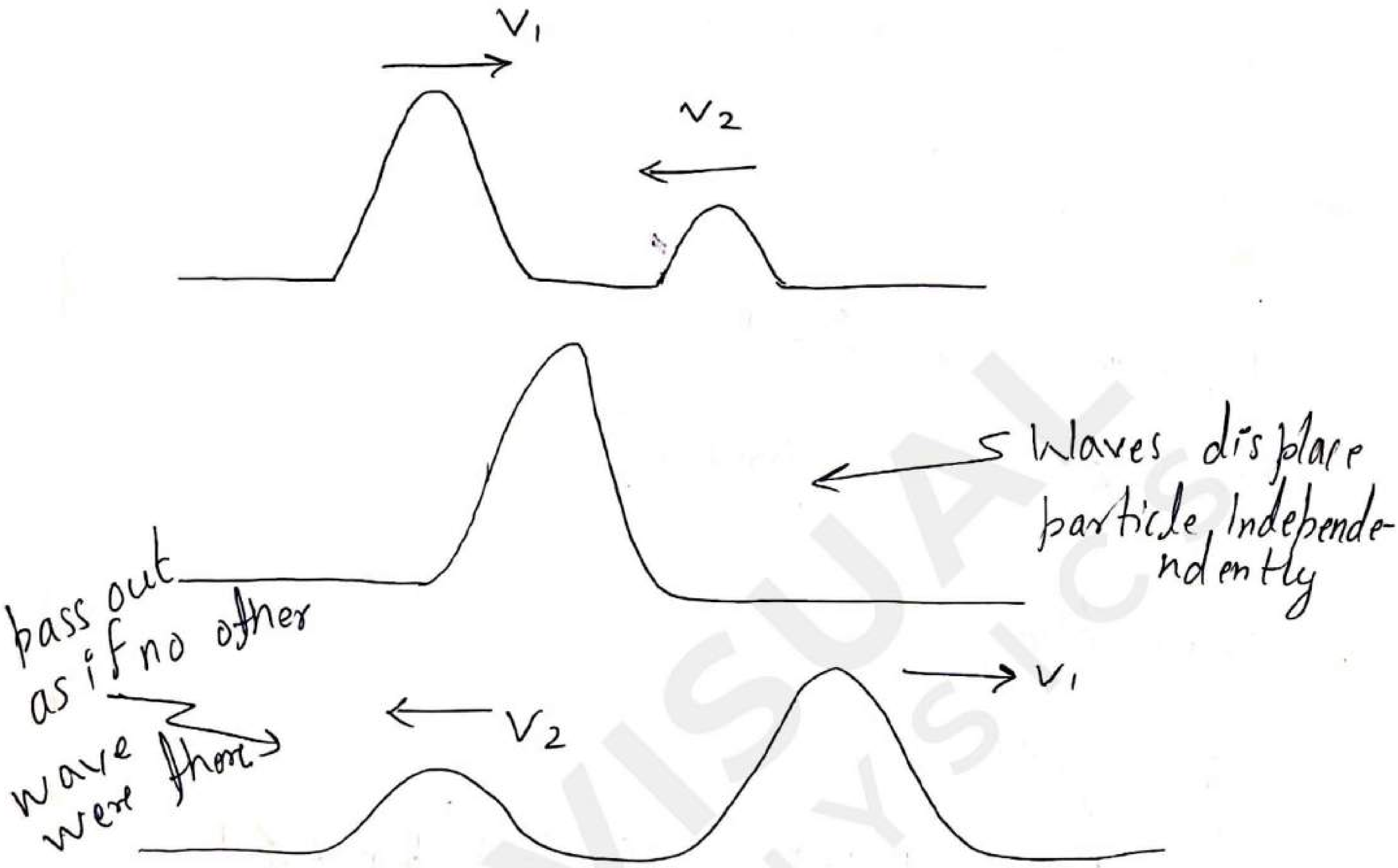
$$\Rightarrow \sin\Delta\theta \rightarrow \Delta\theta$$

$$\Rightarrow 2T\Delta\theta = F_c = \frac{mV^2}{R}$$

$$\Rightarrow 2T\Delta\theta = \frac{\mu 2R\Delta\theta V^2}{R}$$

$$\Rightarrow \boxed{V = \sqrt{\frac{T}{\mu}}}$$

SUPERPOSITION :



not effected in presence of other wave.

principle of superposition:

$\vec{y} = \vec{y_1} + \vec{y_2}$ net displacement is vector sum of individual waves

$\vec{u} = \vec{u_1} + \vec{u_2}$ similar for particle velocities

($\vec{u}$: particle velocities)

Assuming Interfering waves having same frequency and wavelength.

$$\Rightarrow \vec{y} = \vec{y_1} + \vec{y_2}$$

$$y_1 = A_1 \sin(\omega t - kx)$$
$$y_2 = A_2 \sin(\omega t - kx + \phi)$$

In same plane:

$$y = y_1 + y_2$$
$$y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$$
$$y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx)\cos\phi + A_2 \cos(\omega t - kx)\sin\phi$$

$$y = \underbrace{(A_1 + A_2 \cos\phi)}_{P} \sin(\omega t - kx) + \underbrace{A_2 \sin\phi}_{Q} \cos(\omega t - kx)$$

$$y = P\sin(\omega t - kx) + Q\cos(\omega t - kx)$$

$$y = \sqrt{P^2 + Q^2}\left[\frac{P}{\sqrt{P^2+Q^2}} \sin\omega t + \frac{Q}{\sqrt{P^2+Q^2}} \cos\omega t\right]$$

$$\cos\alpha = \frac{P}{\sqrt{P^2+Q^2}}, \quad \sin\alpha = \frac{Q}{\sqrt{P^2+Q^2}}$$

$$y = A'\left[\sin(\omega t - kx)\cos\alpha + \cos(\omega t - kx)\sin\alpha\right]$$

$$\boxed{y = A'\sin(\omega t - kx + \alpha)}$$ ⇒ frequency remain Same.

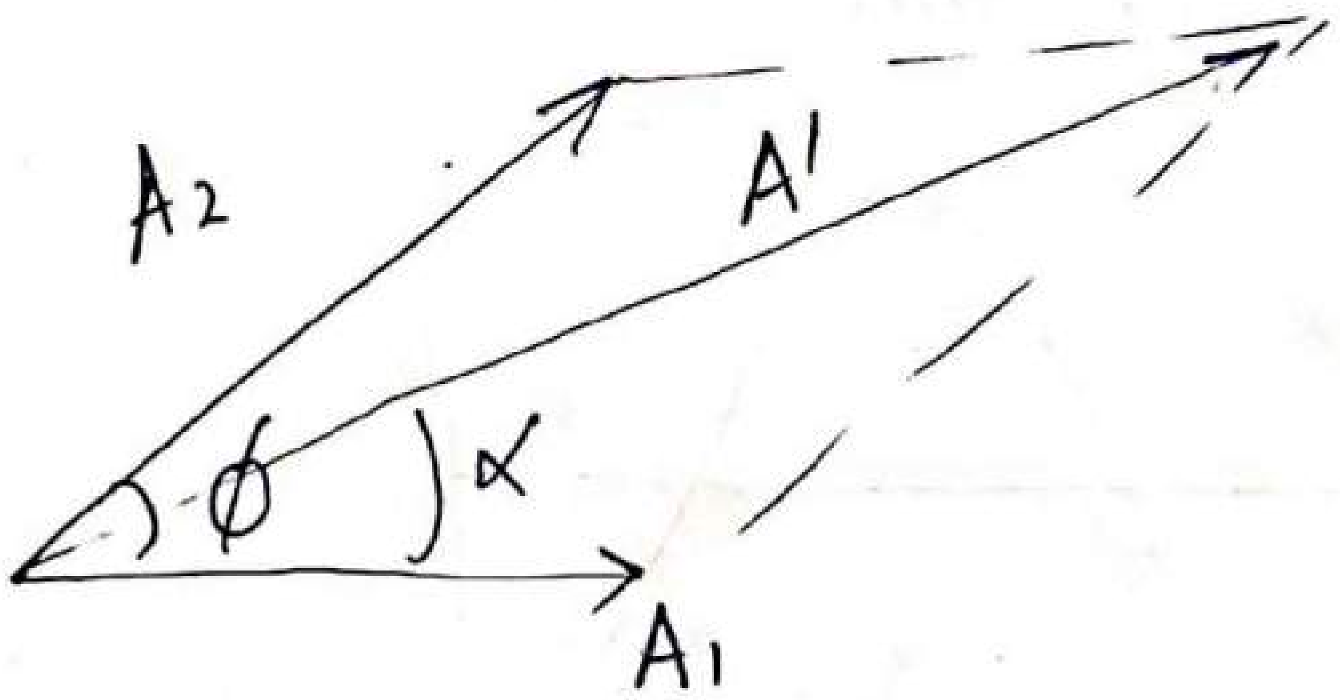
↳ $\sqrt{P^2+Q^2}$

$$A' = \sqrt{P^2+Q^2}$$

$$\boxed{A' = \sqrt{A_1^2 + 2A_1A_2\cos\phi + A_2^2}}$$

$$\boxed{\tan\alpha = \frac{Q}{P} = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}}.$$

We can also use vector method



if $A_1 = A_2$

$$A' = \sqrt{A^2 + 2AA\cos\phi + A^2}$$

$\Rightarrow$ for $\phi = 0, 2\pi, 4\pi \ldots$

$A' = 2A$ fully Constructive Interference.

$\alpha = 0$

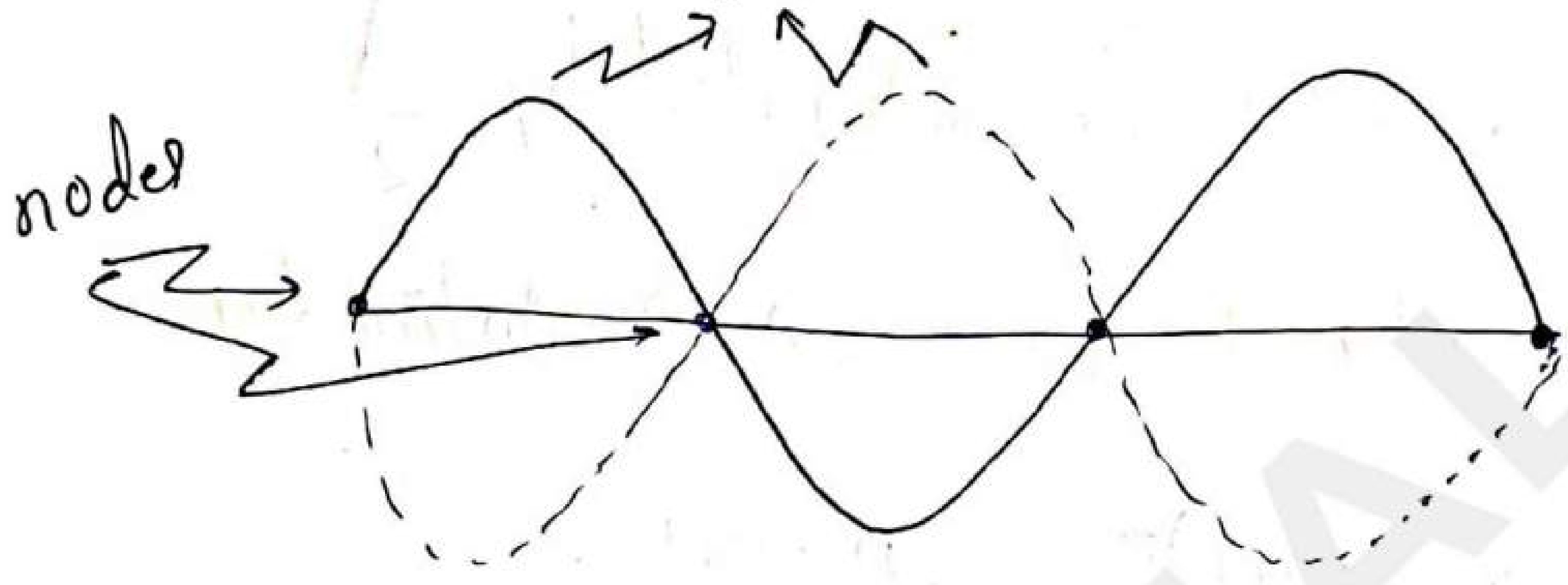
$\phi = \pi, 3\pi, 5\pi \ldots \quad (2n+1)\pi$

$A' = 0$ fully destructive Interference

STANDING WAVES :

→ Two similar waves superposition in opposite direction

→ pattern formed is stationary, hence standing waves



nodes → particles are stationary

antinodes → particle executing motion with maximum amplitude.

→ In standing waves, each particles have different amplitudes of motion

$$y = y_1 + y_2$$

$$y = A\sin(\omega t + kx) + A\sin(kx - \omega t)$$

$$y = A\left[2\sin\left(\frac{2kx}{2}\right)\cos\left(\frac{-2\omega t}{2}\right)\right]$$

$$\boxed{y = 2A\sin kx\cos\omega t}$$

→ standing as $y \neq f\left(t - \frac{x}{v}\right)$

for $\sin kx = 0$, $y = 0 \Rightarrow$ nodes

$kx = n\pi \quad [n = 0, 1 \ldots]$

$$\Rightarrow \boxed{x = \frac{n\pi}{k} = \frac{n\lambda}{2}}$$

$\hookrightarrow$ nodes are seperated by $\frac{\lambda}{2}$

for $\sin kx = 1$, $y = 2A \Rightarrow$ Antinodes

$kx = \frac{(2n+1)\pi}{2}$, $n = 0, 1 \ldots$

$$\boxed{x = (2n+1)\frac{\lambda}{4}}$$

Hence between two nodes there is one anti-node and vice-versa

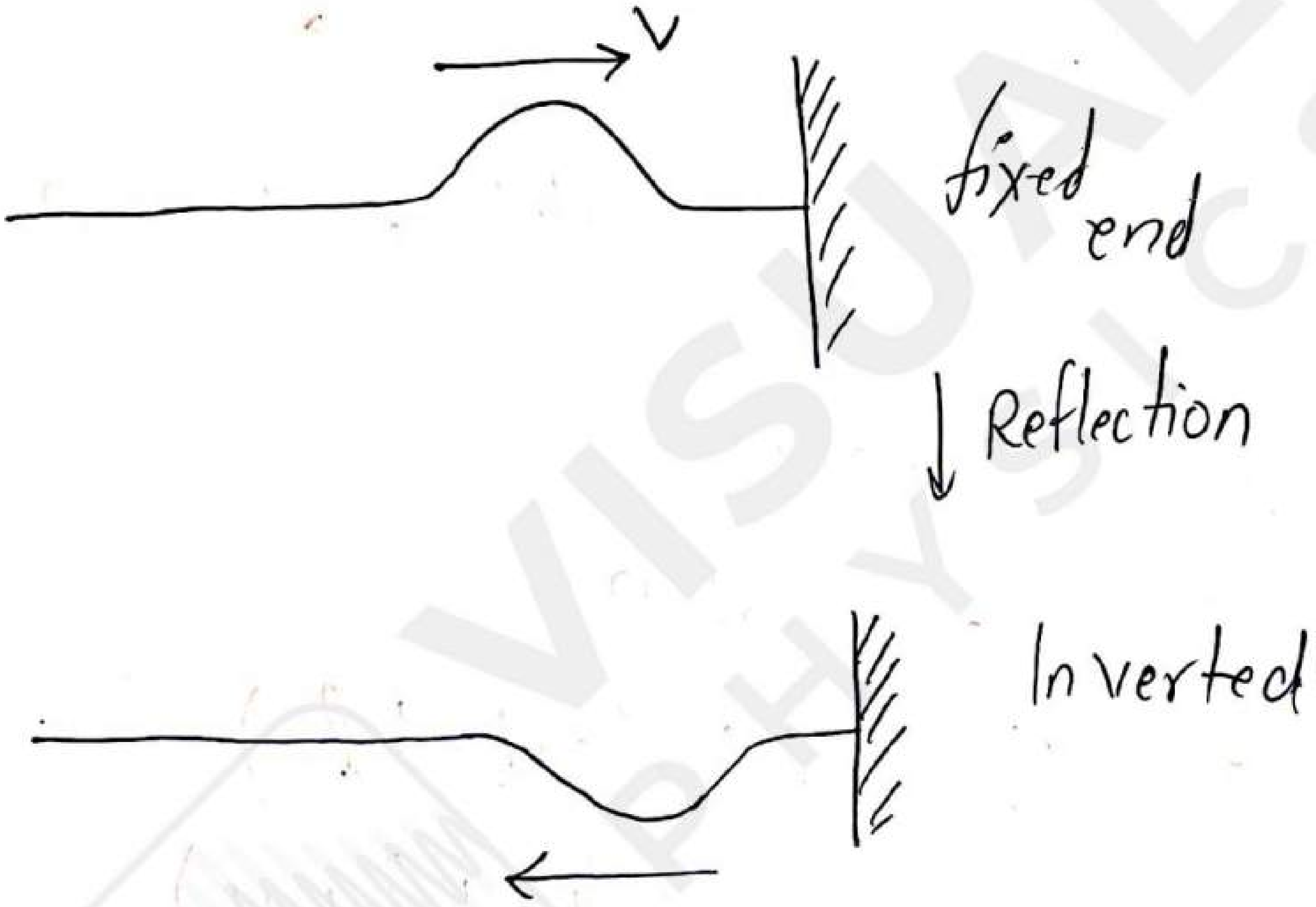
$$\boxed{y = 2A \sin kx \cos \omega t}$$

$2A \sin kx$ → amplitudes at different 'x'

$\cos \omega t$ → Variation of pattern

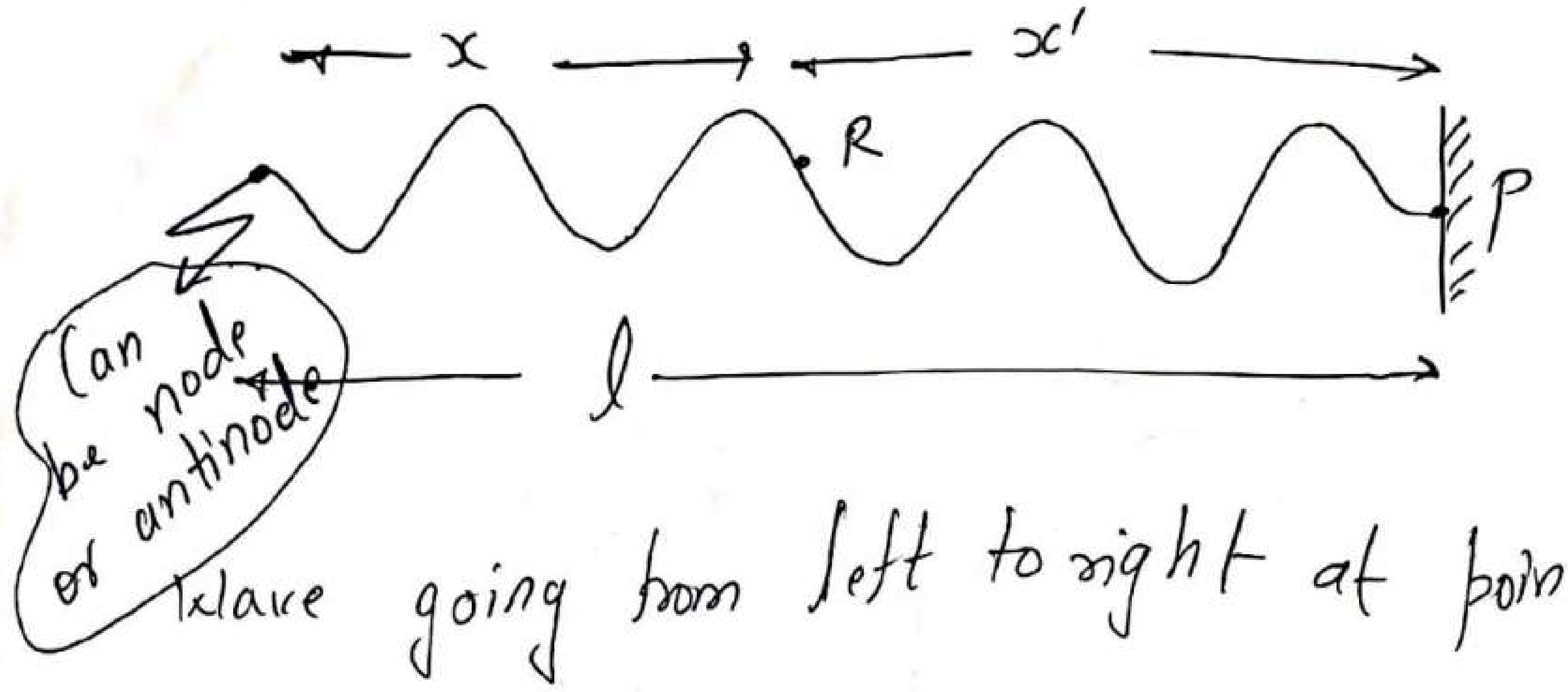
→ Standing wave equation

REFLECTION:



→ Same amplitude, frequency, (no loss of energy is assumed)

$y = A\sin(\omega t - kx)$



Wave going from left to right at point R

$$y_1 = A\sin(\omega t - kx)$$

& from reflected at point P

as $y'_{r_p} = -A\sin(\omega t - kl)$

$$y'_{r_p} = A\sin(\omega t - kl + \pi)$$

Now wave reflected reaching at 'R'

$$y_2 = A\sin(\omega t - kl + \pi - kx')$$

$$y_r = y_2 = A\sin[\omega t - kl + \pi - k(l - x)]$$

⟶ equation of reflected wave

so, $y = y_1 + y_2$

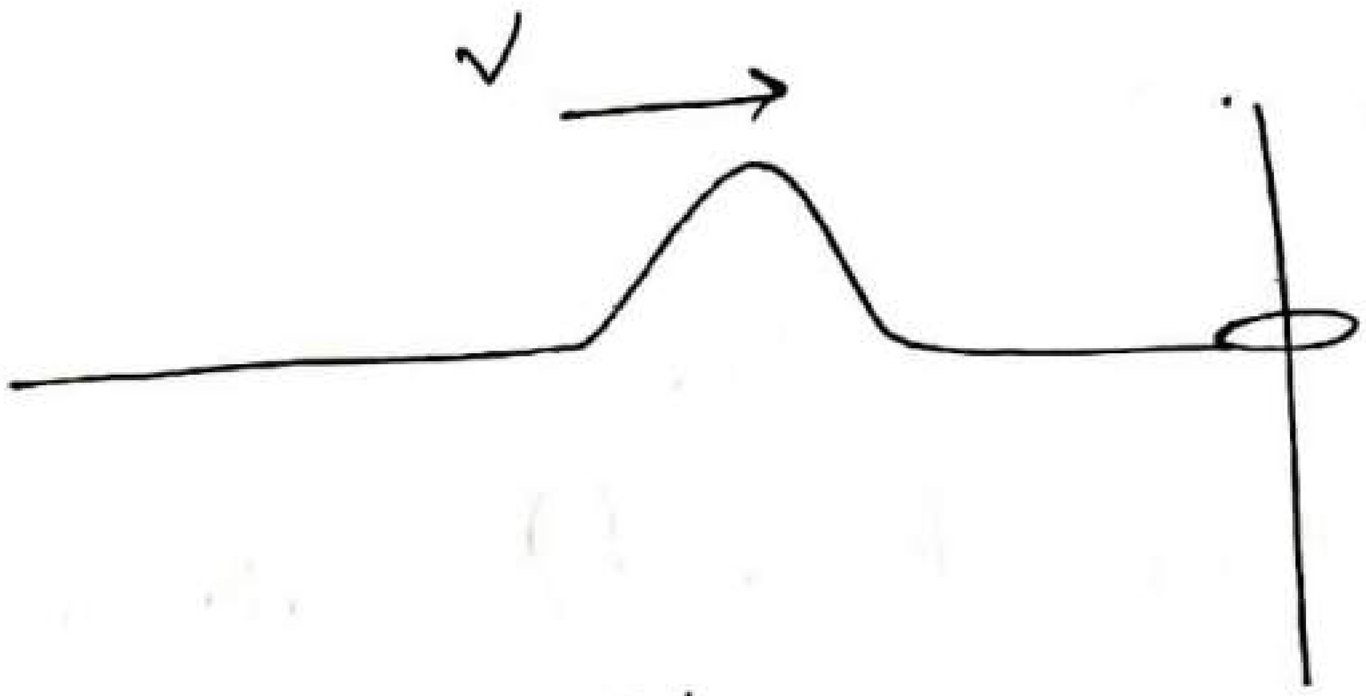
for $x = l$, $A' = 0$ (nodes)

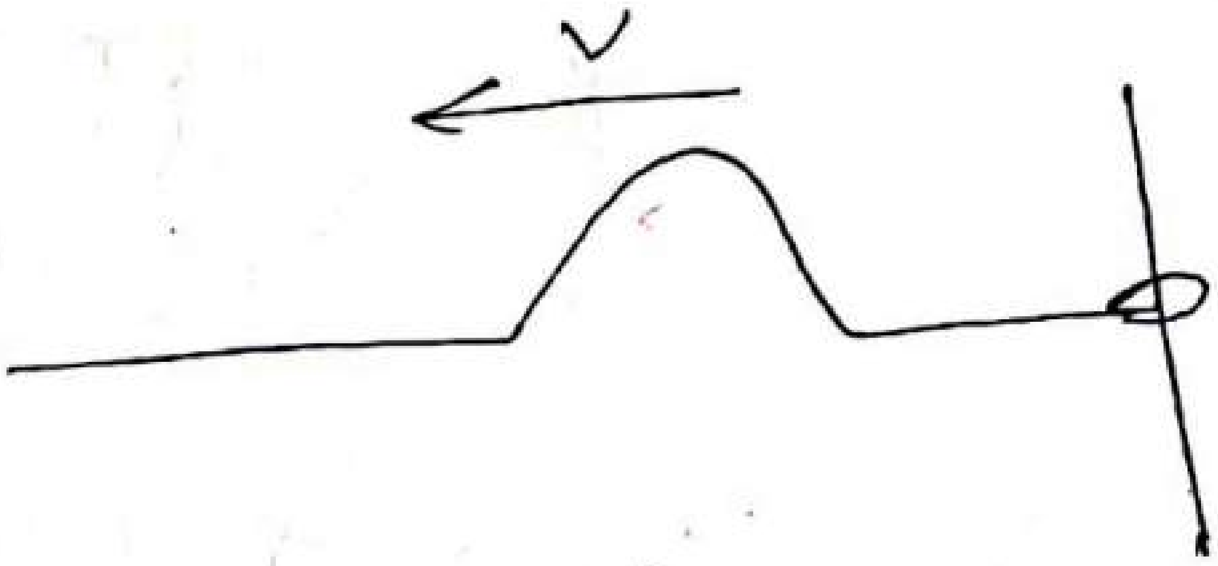
$x = 0$, $A' = 0$ for $l = n(\lambda/2)$

$A = 2A$ for $l = (2n+1)(\lambda/4)$

standing wave

$$y = 2A\cos\left(kx - \frac{\pi}{2} - kl\right)\sin\left(\omega t + \frac{\pi}{2} - kl\right)$$





$y_1 = A\sin(\omega t - kx)$

$y(l, t) = A\sin(\omega t - kl)$

so, from slider, $y_r = A\sin(\omega t - kl - kx')$

$$y_r = A\sin[\omega t - kl - k(l - x)]$$

↳ Reflected equation

$y_1 = A\sin(\omega t - kx)$

$y_2 = A\sin(\omega t + kx - 2kl)$

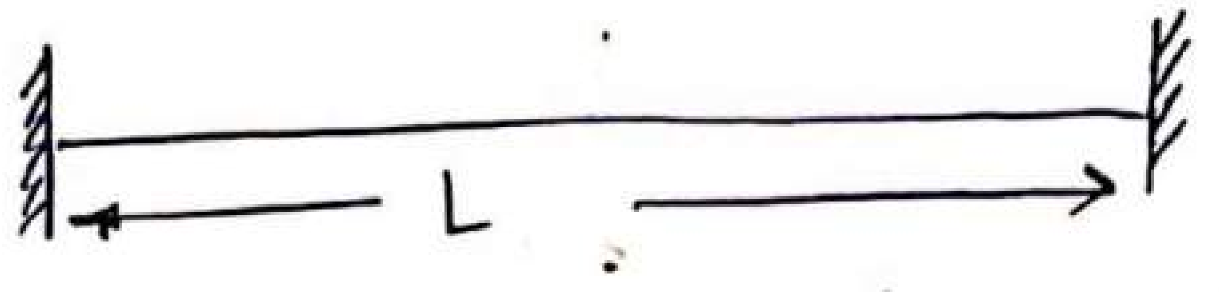
$y = y_1 + y_2$ → for $x = l$, $A' = 2A$ → antinode as expected

$$y = 2A\sin(\omega t - kl)\cos(kl - kx)$$

↳ standing wave

MODES OF VIBRATION:

In standing waves consideration
(both end fixed)



$2\sin kx \cos\omega t = y$

$\sin kx = 0$

$\sin kL = 0$

$$L = \frac{n\lambda}{2}$$

The fundamental mode: [two nodes, 1 antinode) (one loop)

$\lambda = 2L$

fundamental frequency → $$\nu = \frac{V}{2L}$$

first harmonic OR fundamental mode

similarly for 3 nodes (two loop)

$\lambda = L$

$$\nu_1 = \frac{V}{L} = 2\left(\frac{V}{2L}\right)$$

second harmonic
OR
first overtone

similarly in general (n loop)

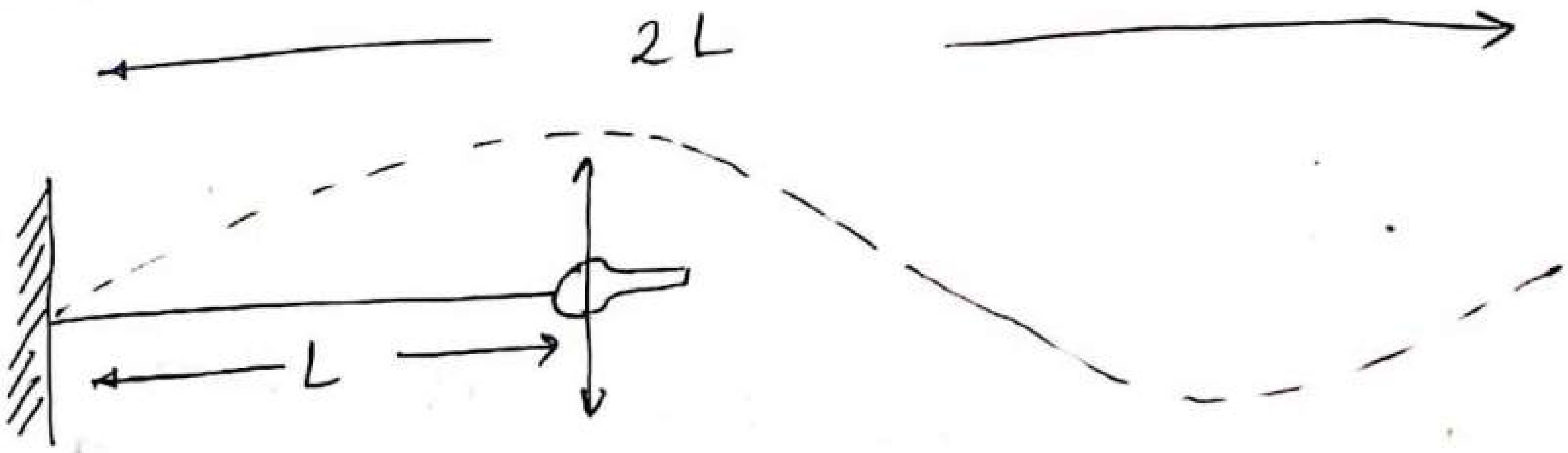
$$\lambda = 2\frac{L}{n}$$

$$\nu_n = n\nu$$

n^{th} harmonic
OR
$(n-1)^{th}$ overtone

(ν → fundamental frequency)

One end fix & other end is antinode.



⇒ for first harmonic or fundamental mode

$4L = \lambda \Rightarrow \boxed{L = \frac{\lambda}{4}}$

$\boxed{\nu = \frac{V}{4L}}$ → fundamental frequency

∴ in general

$\boxed{\lambda = \frac{4L}{(2n+1)}}$ $n = 0, 1, 2 \ldots$

$\nu = (2n+1)\left(\frac{V}{4L}\right)$ n^{th} harmonic OR $(n-1)^{th}$ overtone.

As, OR

$y = 2 \sin kx \cos \omega t$

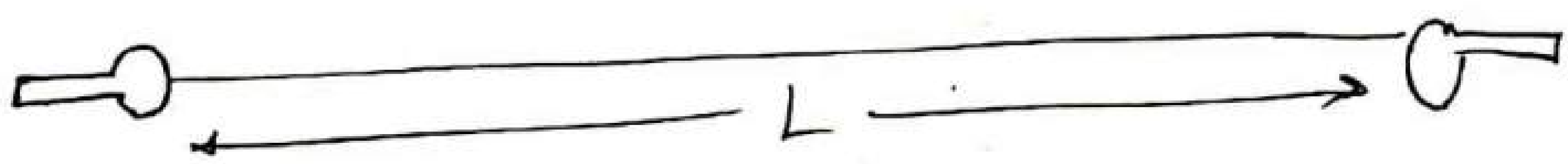
$\sin kL = 1$

$kL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\left(\frac{2\pi}{\lambda}\right) L = \frac{\pi}{2}, \frac{3\pi}{2} \ldots (2n+1)\frac{\pi}{2}$

$\Rightarrow \boxed{\lambda = 4L/(2n+1)}$

for both ends antinodes



It will behave same as when both ends were fix & hence harmonic will be same.

$$L = n\frac{\lambda}{2}$$

$$\nu_n = n\left(\frac{v}{2L}\right)$$

n^{th} harmonic
OR
$(n-1)^{th}$ overtone.

Sonometer:

As fundamental frequency for both end fix:

$$\nu = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

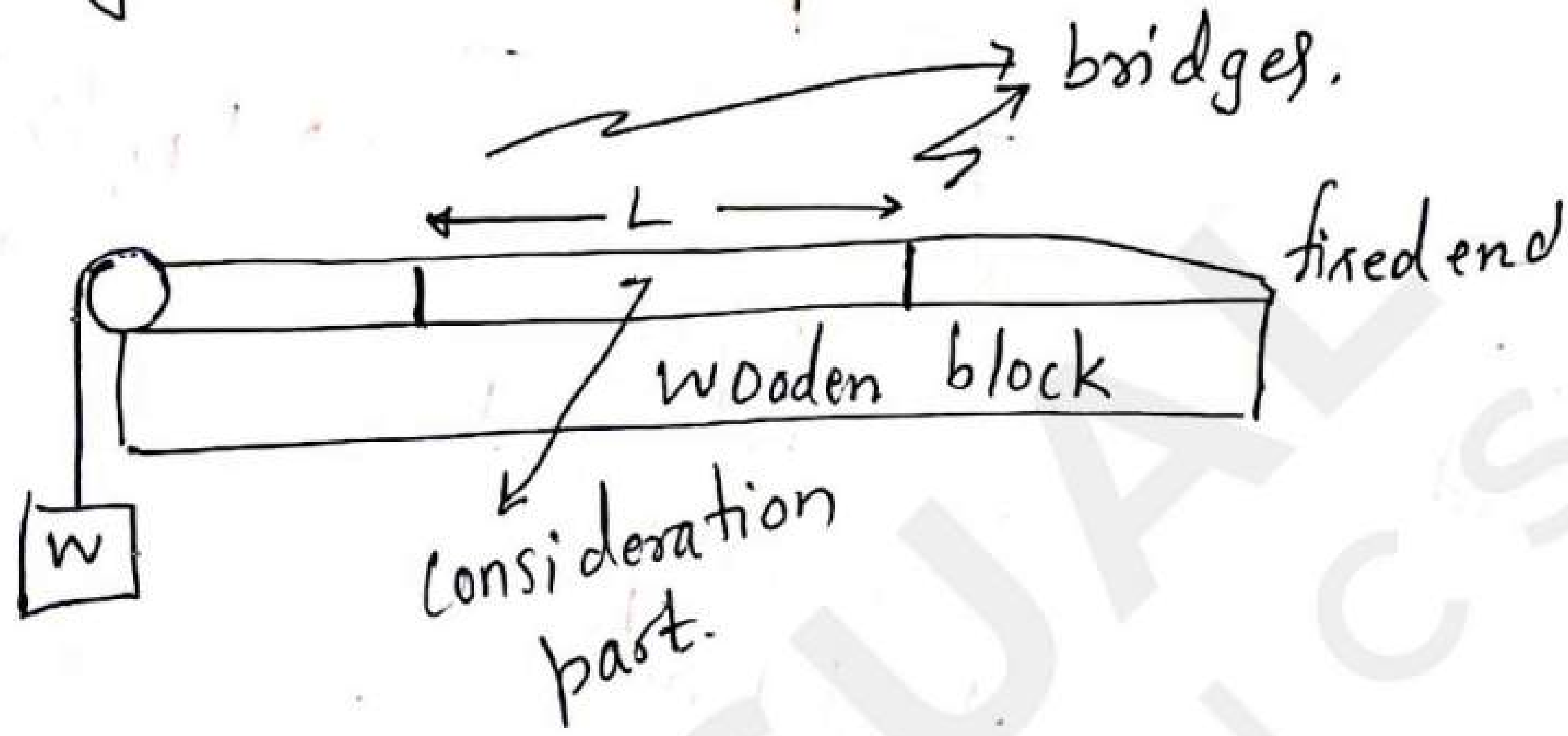
Hence ⇒

$$\boxed{\nu \propto \frac{1}{L}}$$

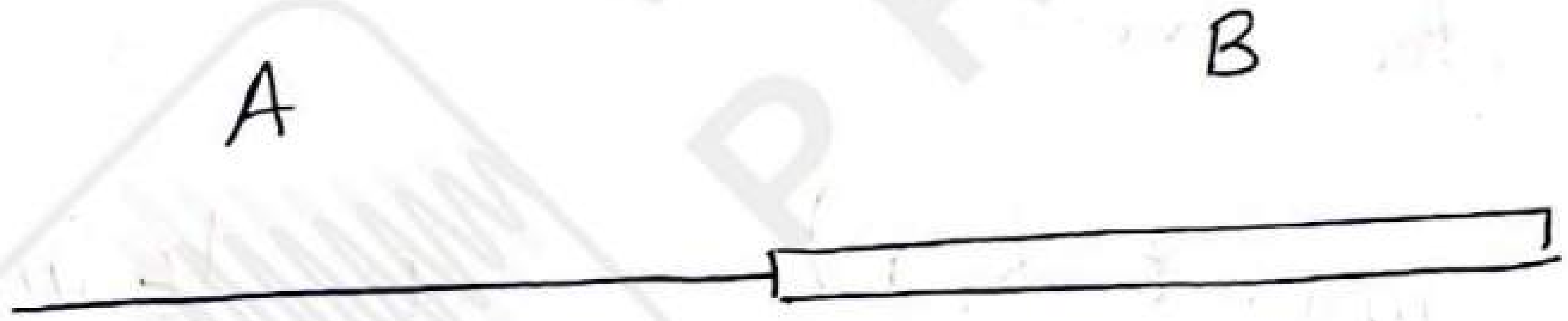
$$\boxed{\nu \propto \sqrt{T}}$$

$$\boxed{\nu \propto \frac{1}{\sqrt{\mu}}}$$

Sonometer is a device with wooden box and moving bridges, over which wire is passed ~~over~~ having one end fixed & other attached with weights. To verify the previous results.



Transmission :

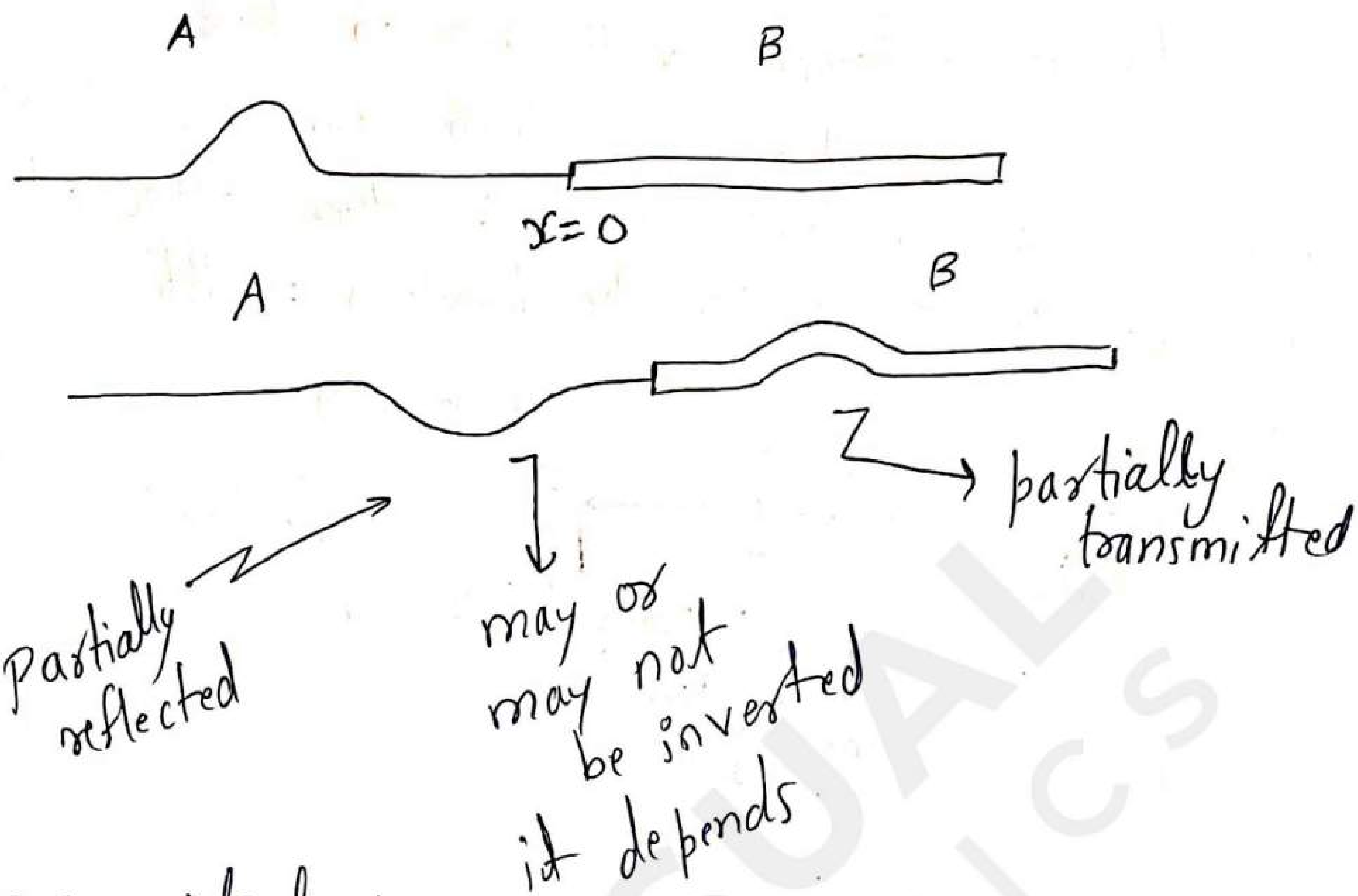


$$\boxed{\mu_A < \mu_B}$$

$$V_A = \sqrt{\frac{T}{\mu_A}} > V_B = \sqrt{\frac{T}{\mu_B}}$$

$$\Rightarrow \quad \lambda_A > \lambda_B$$

frequency decided by oscillator & hence remains constant.



k_i = initial wave
k_t = transmitted wave
k_r = reflected wave

} angular wave number

$y_i = A_i \sin(\omega t - k_i x)$ $\qquad$ $y_t = A_t \sin(\omega t - k_t x)$

$y_r = A_r \sin(\omega t + k_r x)$

As, $\boxed{A_i + A_r = A_t}$ —(i)

and also, slope of string on either side of $x=0$, must be same

$\therefore$ due to continuity

$$\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x}$$

$$-A_i k_i \cos(\omega t - k_i x) + A_r k_r \cos(\omega t + k_r x) = -A_t k_t \cos(\omega t - k_t x)$$

at $x = 0$,

$$-A_i k_i \cos\omega t + A_r k_r \cos\omega t = -A_t k_t$$

as $k_i = k_r$

$$-k_i(A_i - A_r) = -A_t k_t$$

$$\boxed{\frac{k_i}{k_t}(A_i - A_r) = A_t} \quad \text{(ii)}$$

from (i) & (ii) Putting $k = \frac{\omega}{v}$

We get:

$$\boxed{\begin{aligned} A_r &= A_i\left[\frac{v_t - v_i}{v_t + v_i}\right] \\ A_t &= A_i\left[\frac{2v_t}{v_t + v_i}\right] \end{aligned}}$$

if $v_t < v_i$

$A_r \rightarrow -ve$

(reflected is Inverted)

As $-ve$ means π phase shift hence Inverted.

if $V_t > V_i$

$A_r \rightarrow +ve$ (not inverted)

hence reflect wave is not inverted.

Rate of energy transfer:

As energy associate with the particles will be kinetic energy as well as potential energy.

so for 'dm' mass

$$dK = \text{Kinetic Energy} = \frac{1}{2}(dm)v^2$$

v = speed of particle [not wave speed)

as $y = A\sin(kx - \omega t)$

so, $v = \frac{dy}{dt} = -\omega A\cos(kx - \omega t)$

hence

$$dk = \frac{1}{2}(\mu dx)(-\omega A\cos(kx - \omega t))^2 dx$$

$\mu dx = dm$ [mass of dx part]

for t=0,

$$\boxed{dk_{t=0} = \frac{1}{2}\mu\omega^2 A^2\cos^2(kx)\,dx}$$

K_λ = kinetic energy Involved with one wavelength

$$K_\lambda = \int dK = \int_0^\lambda \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx)\, dx$$

$$K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

Similarly we get same amount of potential energy associated with one full wave length

$$U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

Total Energy for 1 wavelength

$$\Rightarrow E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

Now average power P, for one time period

$$P = \frac{E_\lambda}{T} = \frac{1/2\, \mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \left(\frac{\lambda}{T}\right)$$

(T: time period of oscillation)

$$P = \frac{1}{2}\mu\omega^2 A^2 v'$$

$v' = \frac{\lambda}{T}$ = wave speed

$$\Rightarrow \boxed{\begin{aligned} P &\propto \omega^2 \\ P &\propto A^2 \\ P &\propto v' \end{aligned}}$$

v' → wave speed

Intensity:

→ Energy transfered per unit Area of Cross-Section of string per unit time is known as Intensity.

$$I = \frac{\text{Power}}{\text{area of Cross-Section}} = \frac{\left(\frac{1}{2}\right)\mu\omega^2 A^2 v'}{S}$$

$\mu = \rho S$ → S: Cross-section area

ρ → mass density $= \frac{\text{mass}}{\text{volume}}$

$$\Rightarrow \boxed{I = \frac{1}{2}\rho\omega^2 A^2 v'}$$