VISUAL PHYSICS

# SHORT NOTES

CHAPTER

# Magnetic Field

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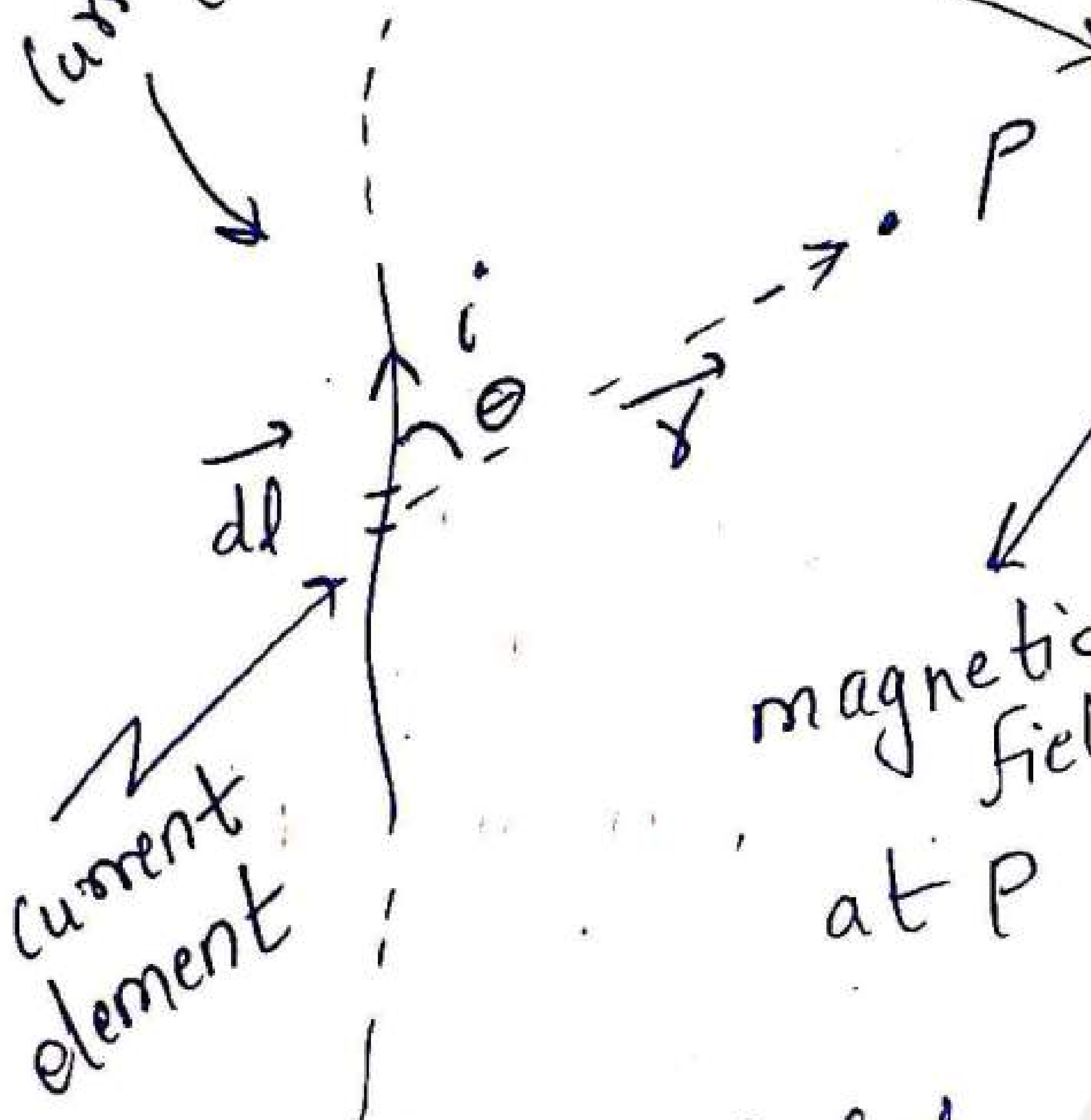
# Magnetic Field

current carrying wire

Biot'savart law




permeability of air

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dl} \times \vec{r}}{r^2}}$$

magnetic field at P

$4\pi \times 10^{-7}\ N s^2 c^{-2}$

current element

$\mu \rightarrow$ permeability $\rightarrow$ resistance of material agaist the formation of magnetic field.

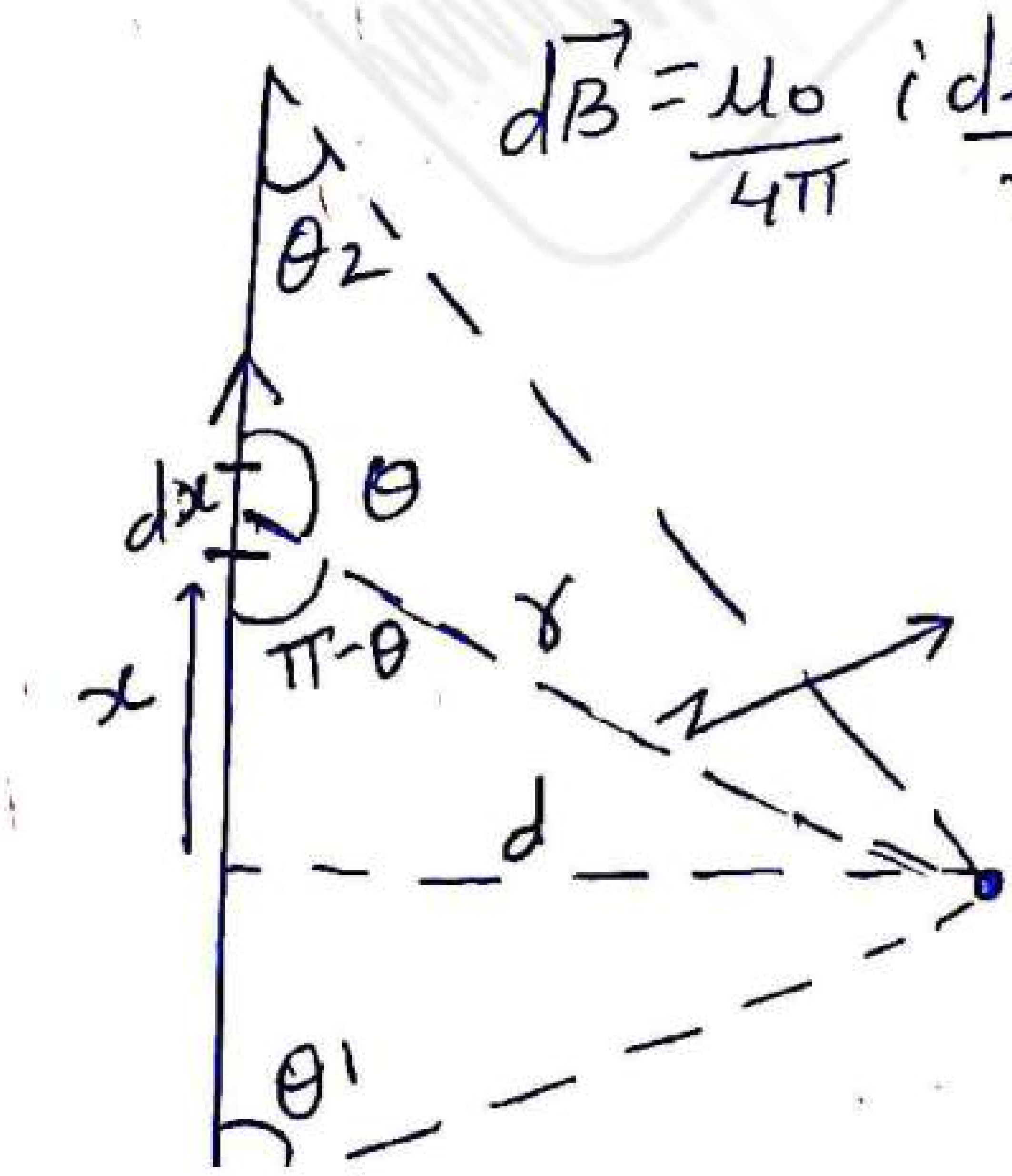
if air

$\mu_0 = 4\pi \times 10^{-7}\ N s^2 c^{-2}$

$$\boxed{\vec{B} = \frac{\mu_0 \mu_r i}{4\pi} \frac{\vec{dl} \times \vec{r}}{r^2}}$$

if not air

$\mu_0 \mu_r \rightarrow$ relative permeability of medium




$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dx} \times \hat{r}}{r^2} \qquad |d\vec{B}| = \frac{\mu_0}{4\pi} \frac{i\,dx \sin\theta}{r^2}$$

$x = -d\cot\theta$

$\Rightarrow dx = -d\,(-\mathrm{cosec}^2\theta\, d\theta)$

$\boxed{dx = d\,\mathrm{cosec}^2\theta\, d\theta}$

$\sin\theta = \sin(\pi-\theta) = d/r$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dx}{r^2} \frac{d}{r}$$

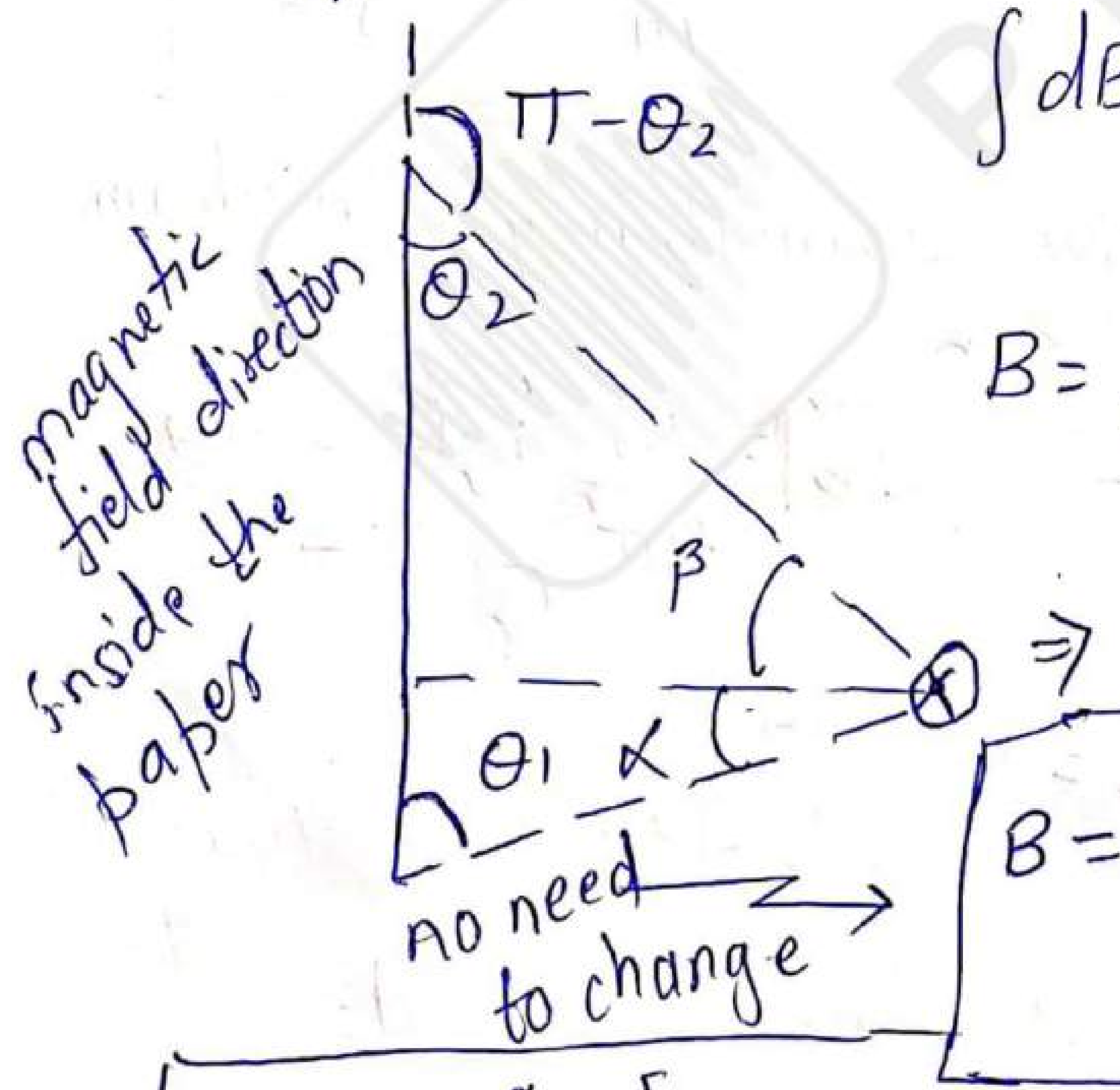
$$dB = \frac{\mu_0}{4\pi} \frac{i\,d(\mathrm{cosec}^2\theta\, d\theta)\, d}{d^3\, \mathrm{cosec}^3\theta}$$

$$\boxed{dB = \frac{\mu_0 i}{4\pi d} \sin\theta\, d\theta} \Rightarrow$$ direction using right hand thumb rule

Thumb in current direction, the fingers give magnetic field direction

angle always in between current direction & position vector

$$\int dB = \frac{\mu_0 i}{4\pi d} \int_{\theta_1}^{\pi-\theta_2} \sin\theta\, d\theta$$

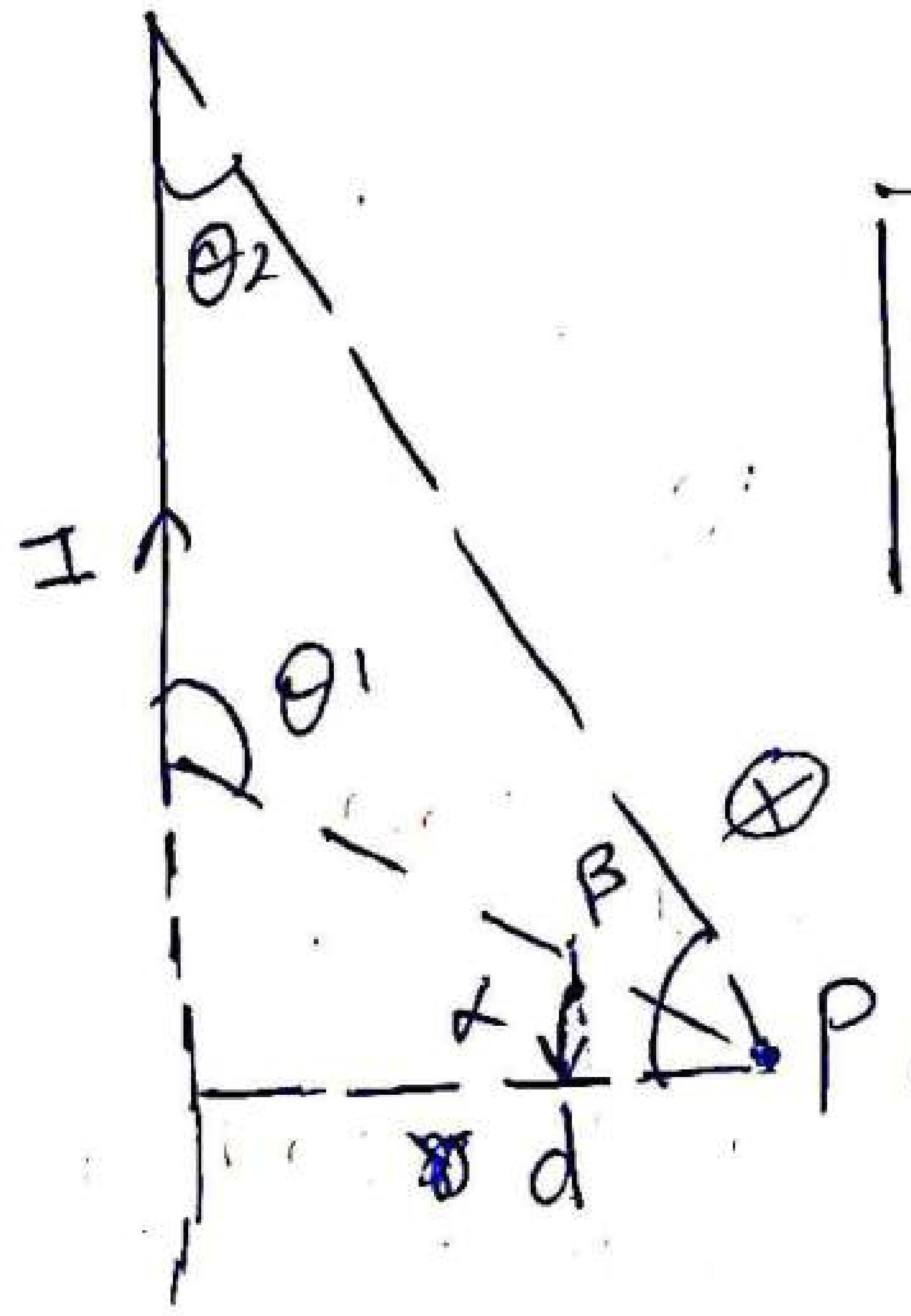



$$B = \frac{\mu_0 i}{4\pi d} \left[-\cos\theta\right]_{\theta_1}^{\pi-\theta_2}$$

$\Rightarrow$

$$\boxed{B = \frac{\mu_0 i}{4\pi d} \left[\cos\theta_1 + \cos\theta_2\right]}$$

$$\boxed{B = \frac{\mu_0 i}{4\pi d} \left[\sin\alpha + \sin\beta\right]}$$

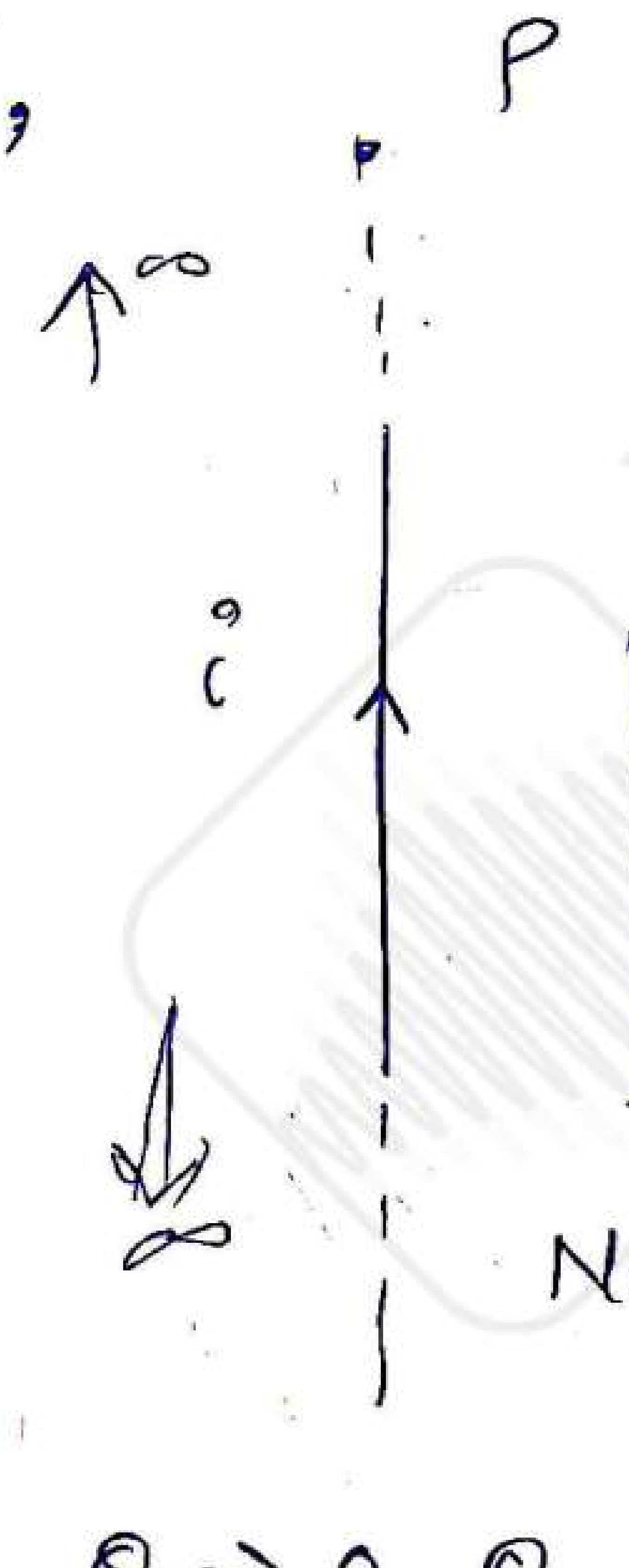



$$\boxed{B = \frac{\mu_0 I}{4\pi d}\left[\sin\beta - \sin\alpha\right]}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi d}\left[\cos\theta_1 + \cos\theta_2\right]}$$

still remain same

if,




$$B = \frac{\mu_0 i}{4\pi d}\left[\cos 0 + \cos 180\right]$$

$$\boxed{B = 0}$$

magnetic field on axis of wire and along the wire is always zero.

Now if wire length goes to infinite

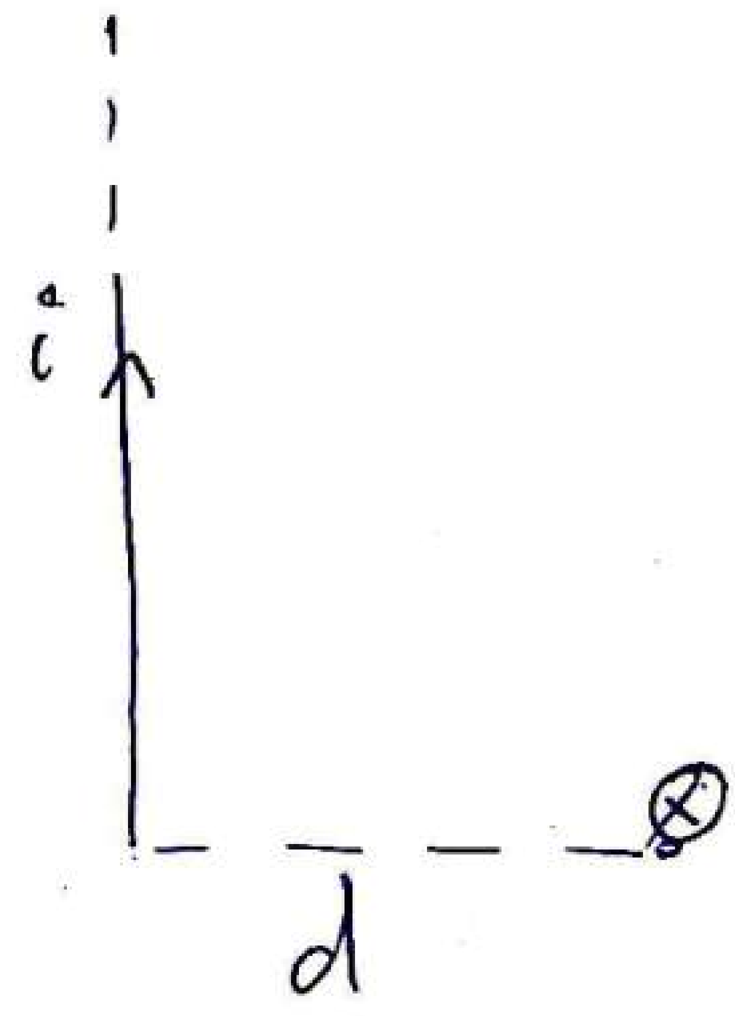
$\theta_1 \to 0,\ \theta_2 \to 0$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d}\left[\cos 0 + \cos 0\right]$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

if the wire is semi-infinite.




$\theta_1 = 90, \theta_2 = 0$

$\cancel{B = \frac{\mu_0 i}{4\pi d}[\sin 0 + \sin}$

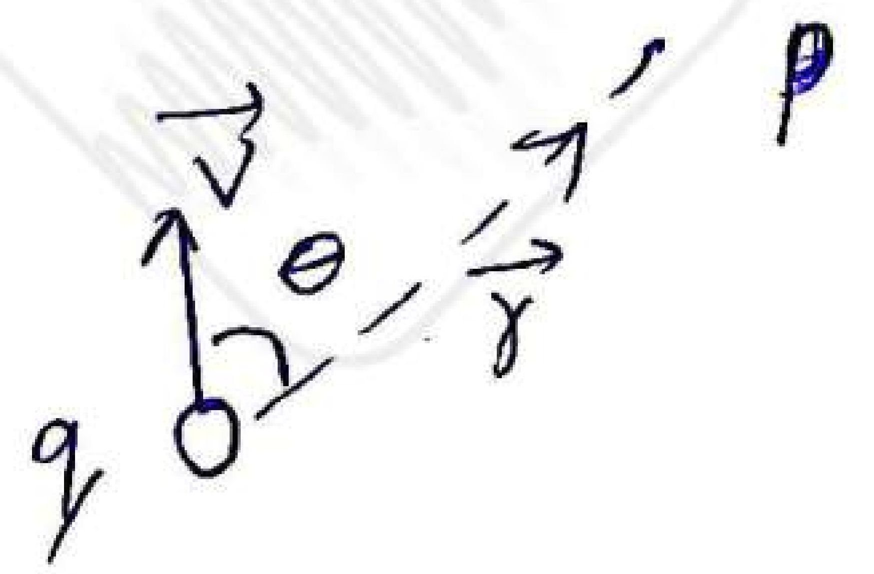
$B = \frac{\mu_0 i}{4\pi d}[\cos 0 + \cos 90]$

$\Rightarrow \boxed{B = \frac{\mu_0 i}{4\pi d}[1] = \frac{\mu_0 i}{4\pi d}}$

<u>magnetic field due to a moving charge</u>:

experimentally:

$\vec{B}$ at $p$ is:



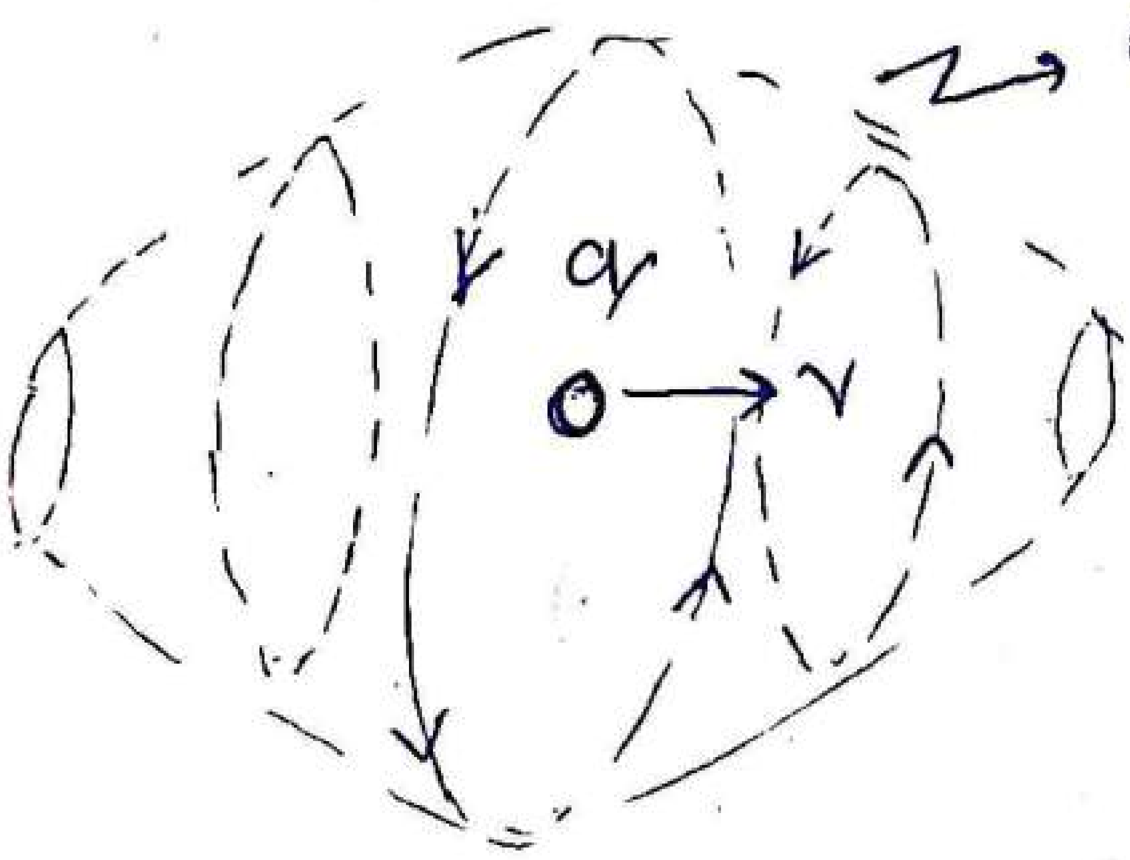

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}}$$

so, when $\theta \to 0$, $B = 0$

★ $\boxed{\vec{B} \text{ along the line of motion of charge} = 0}$

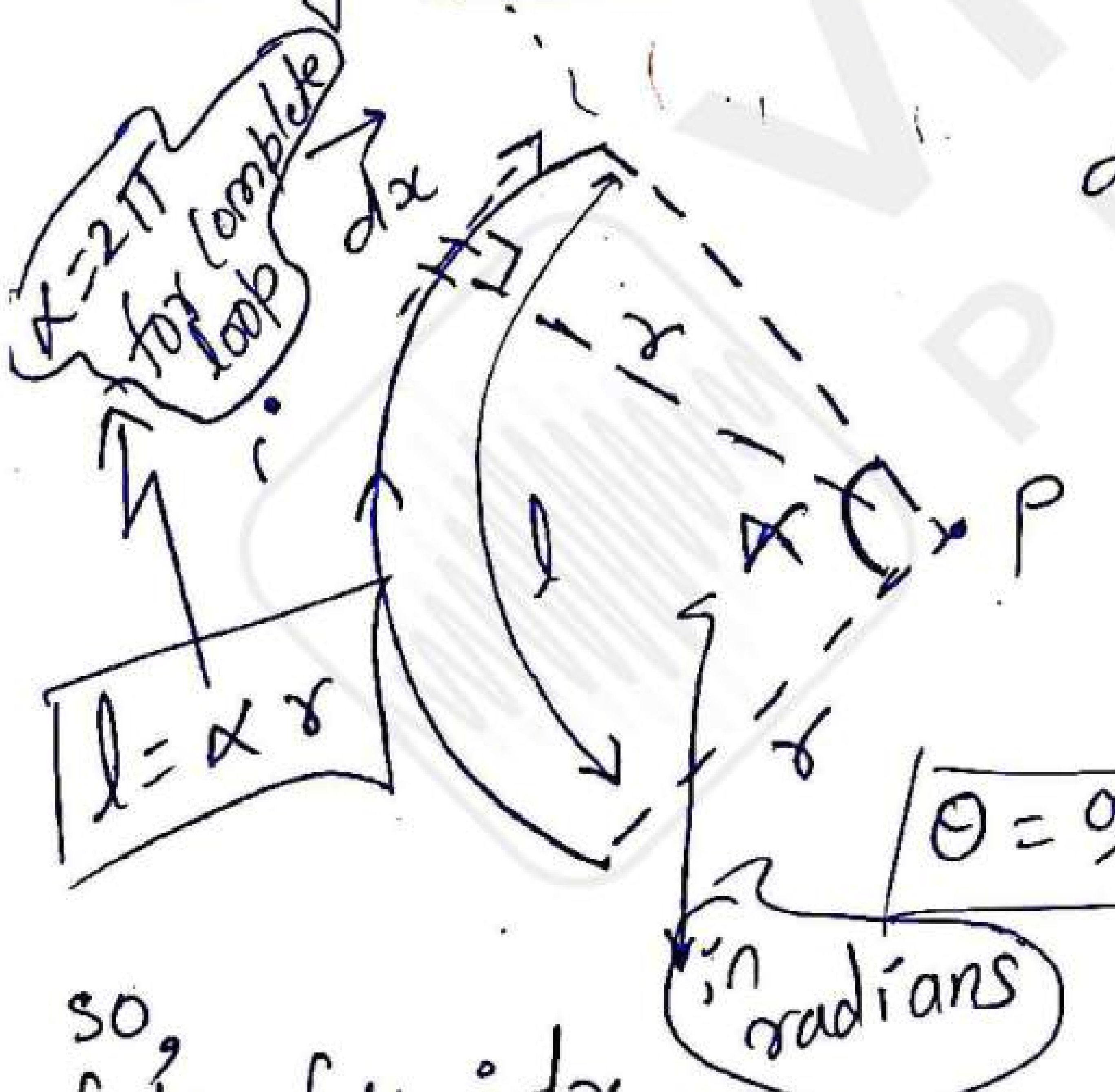
so, for a constant 'r,'

$\vec{B}_{max}$ at $\theta = 90$, $\vec{B}_{min}$ at $\theta = 0$




magnitude of B around a moving charge q.

## Magnetic field at the center of Arc




$$dB = \frac{\mu_0 i \, dx \sin 90}{4\pi r^2}$$

magnetic field at P due to current element dx

$\theta = 90°$, as angle between current direction & $\vec{r}$ is 90°

tangent

so,

$$\int dB = \int \frac{\mu_0 i \, dx}{4\pi r^2}$$

$$\boxed{B = \frac{\mu_0 i}{4\pi r^2} l}$$

so, $\boxed{B = \frac{\mu_0 i \alpha}{4\pi r^2}}$

for circular loop, $\alpha = 2\pi$

'B' at centre of loop

$$= \frac{\mu_0 i (2\pi r)}{4\pi r^2}$$

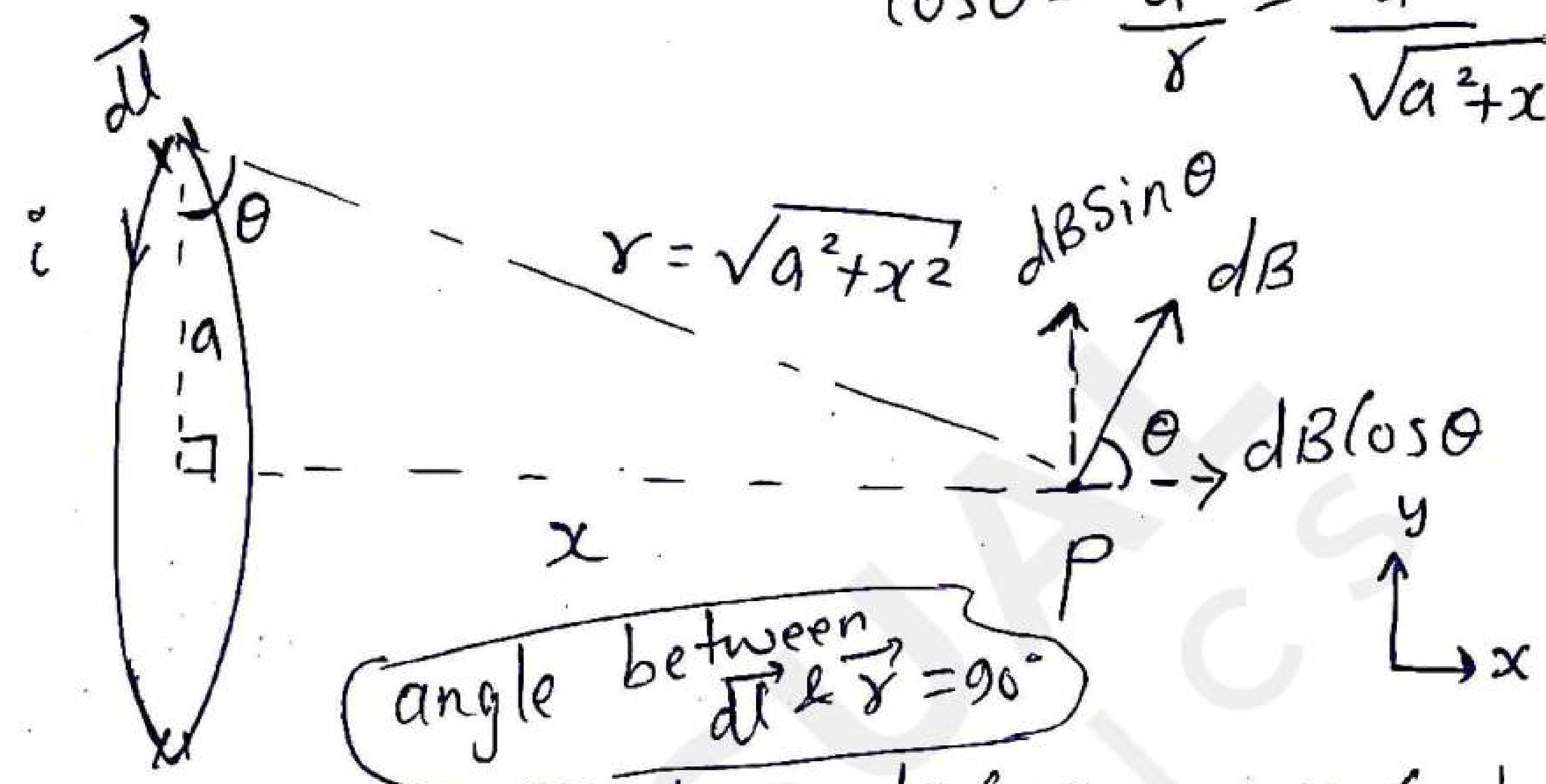
$$\boxed{B = \frac{\mu_0 i}{2r}}$$

for 'n' loop, $\alpha = n(2\pi)$

so $$\boxed{B = \frac{n\mu_0 i}{2r}}$$

# Magnetic field of a circular current loop at axis :

$$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$$




* as, we only consider $dB\cos\theta$, as y component for any current element will be cancel out by opposite current element
  
  diametrical

so, $$dB_{net} = dB\cos\theta = \frac{\mu_0 i \, dl \sin 90 \cos\theta}{4\pi r^2}$$

$$\Rightarrow \quad B_{net} = \frac{\mu_0 i a}{4\pi r^3}\int dl$$

$$\boxed{B_{net} = \frac{\mu_0 i a^2}{2(x^2+a^2)^{3/2}}}$$ in x-direction

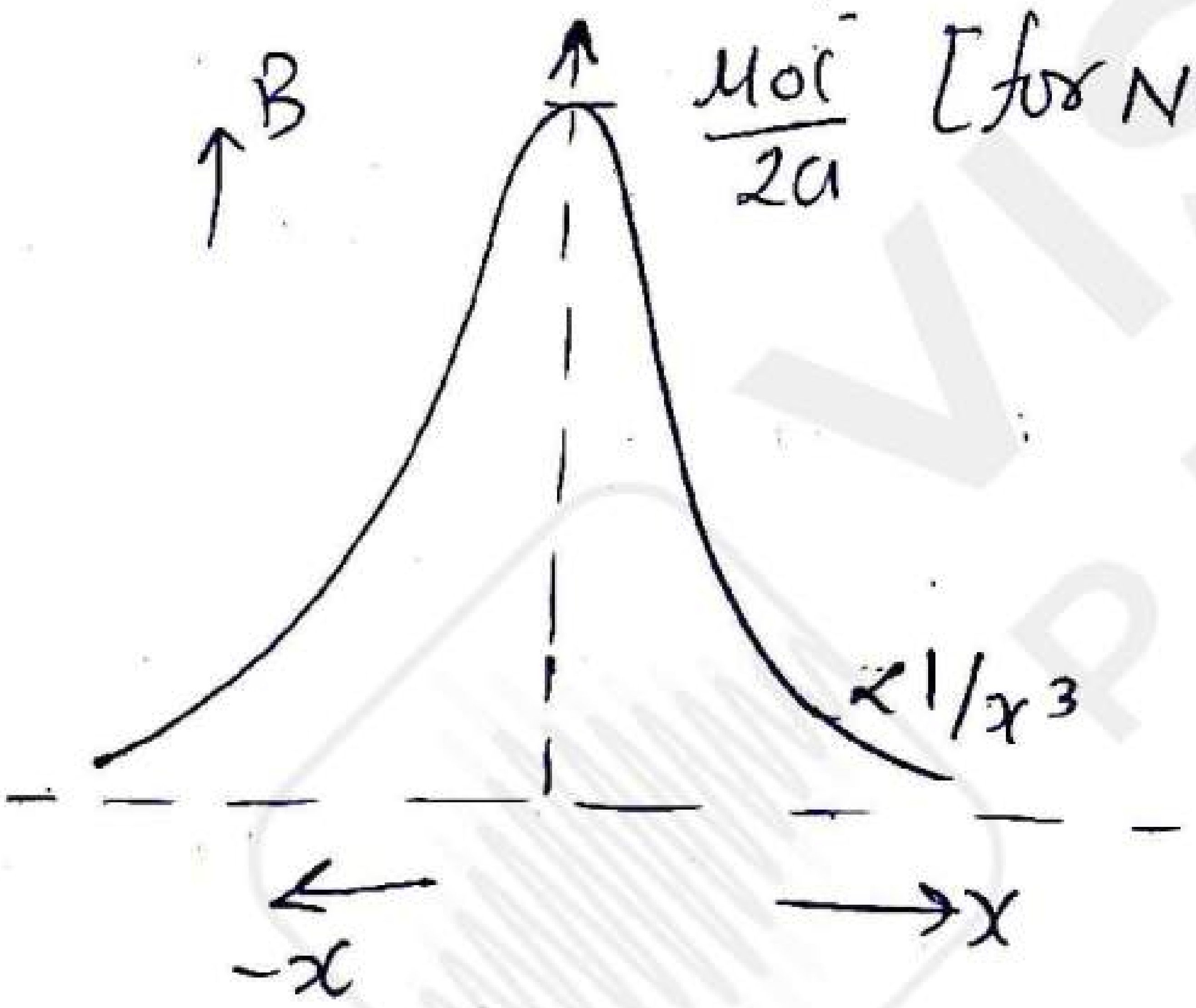
When $x >> a$

$$\Rightarrow \boxed{B_{net} = \frac{\mu_0 I a^2}{2x^3}} \Rightarrow B \propto \frac{1}{x^3}$$

at $x = 0$

$$\boxed{B_{net} = \frac{\mu_0 i}{2a}} = B_{max}$$

if N turns $B_{net} = \frac{N \mu_0 i a^2}{2(x^2+a^2)^{3/2}}$




As (M = magnetic dipole moment)

$M = i$ Area

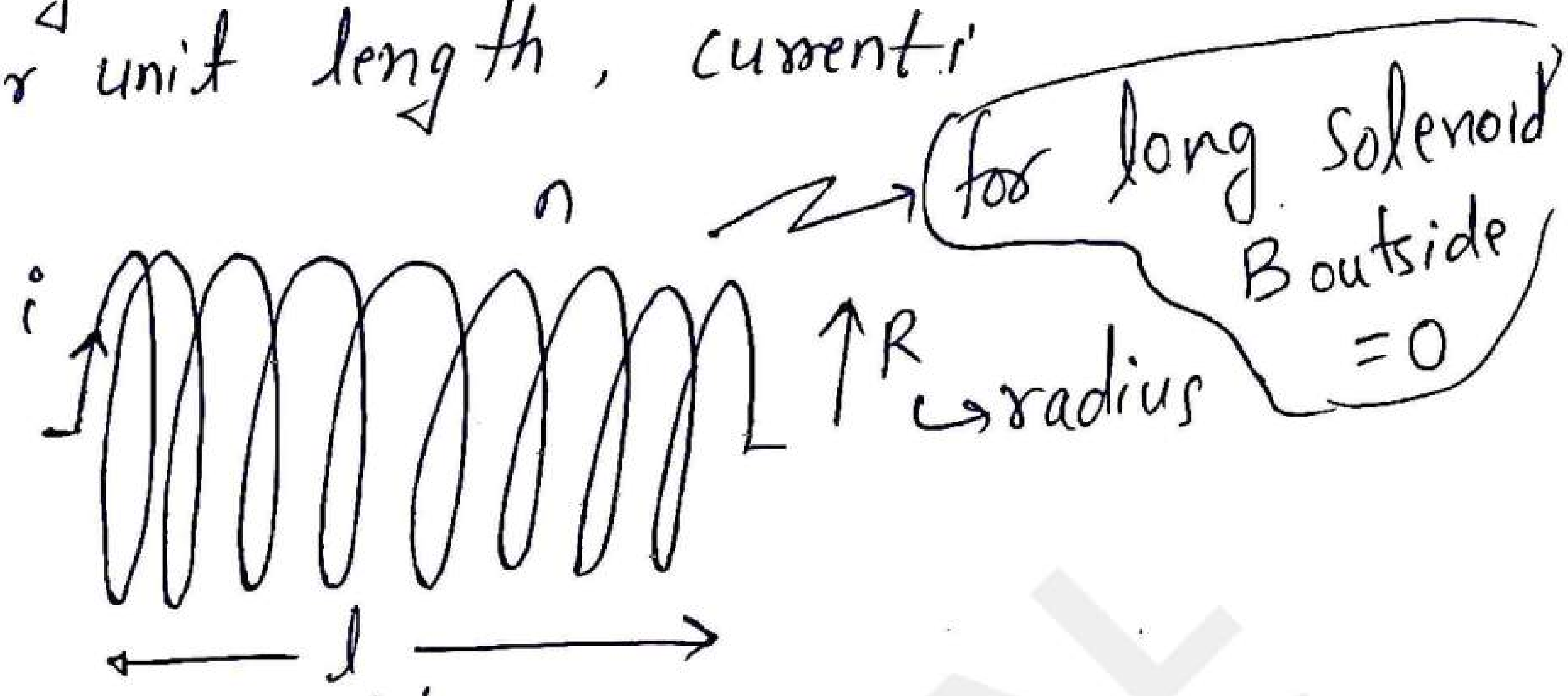
$= i \times \pi a^2$

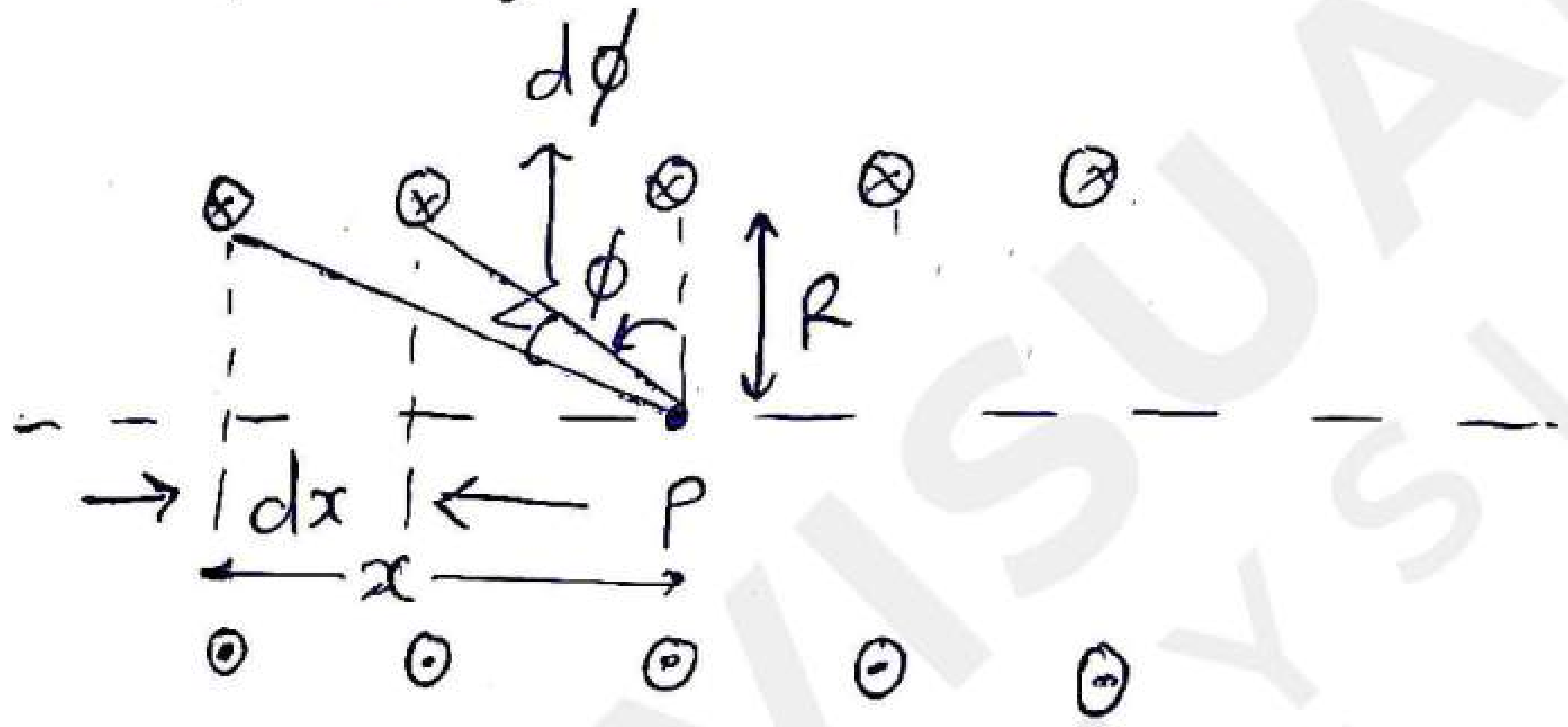
$B_{net} = \frac{B_{net} \times 2\pi}{2\pi}$

$$\Rightarrow B_{net} = \frac{N \mu_0 i a^2 \times 2\pi}{4\pi (x^2+a^2)^{3/2}}$$

so $$\boxed{B_{net} = \frac{\mu_0}{4\pi} \frac{2M}{(x^2+a^2)^{3/2}}}$$

Magnetic field due to solenoid, N turns per unit length, current i




n = number of turns per unit length

$N = n\,dx \;,\quad x = R\tan\phi \;,\quad dx = R\sec^2\phi\, d\phi$

turns in dx length

$$dB = \frac{\mu_0 n\,dx \times i R^2}{2\left(R^2 + R^2\tan^2\phi\right)^{3/2}}$$

for loop

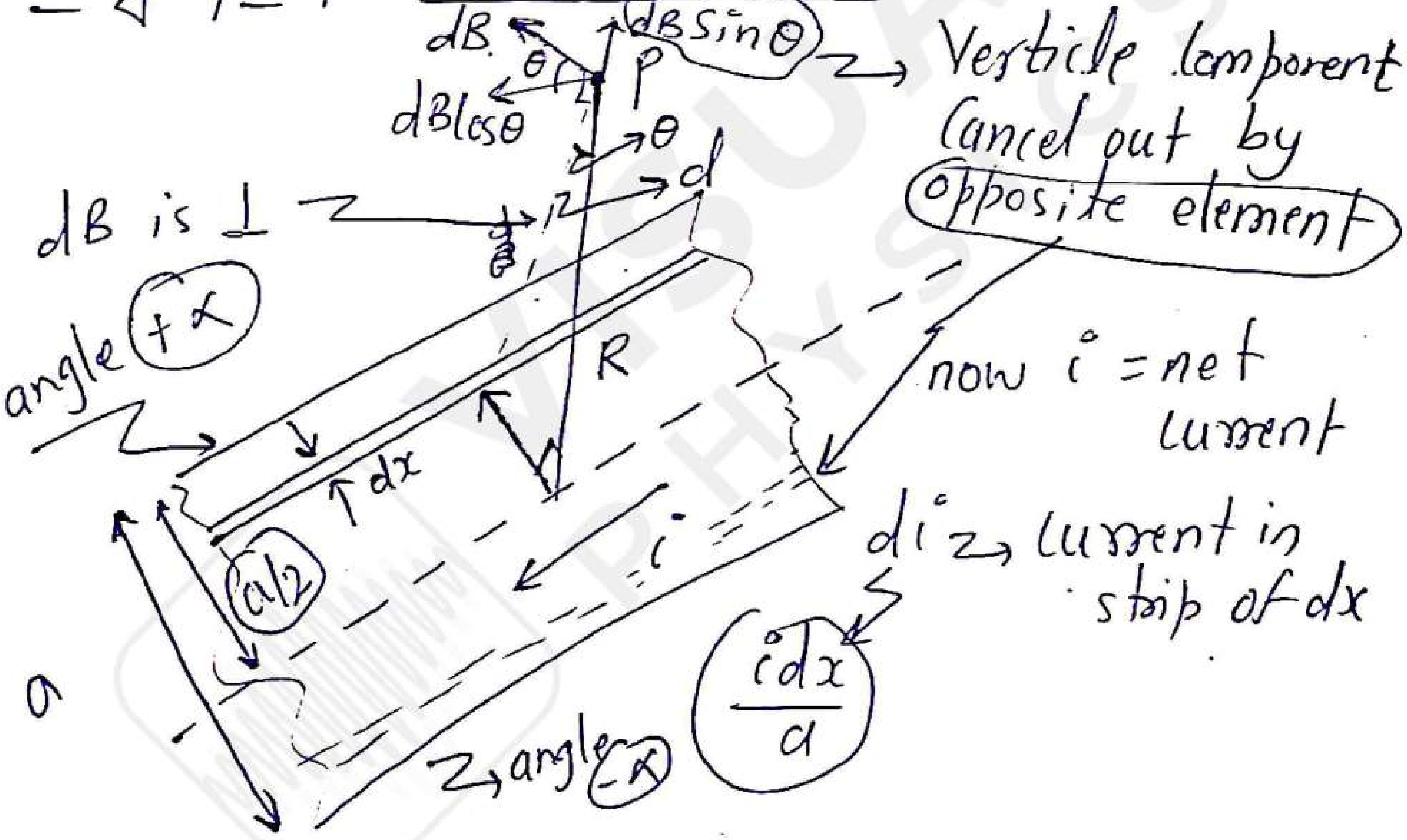
$$B = \int dB = \int_{\alpha}^{\beta} \frac{\mu_0 n\,dx \times i R^2}{2\left(R^2 + R^2\tan^2\phi\right)^{3/2}}$$

$$\boxed{B = \frac{\mu_0 n i}{2}\left[\sin\alpha + \sin\beta\right]}$$

for long, $\alpha = 90^\circ = \beta = 90^\circ$

$\Rightarrow \boxed{B_{net} = \mu_0 n i}$

Magnetic field $\vec{B}$ at point P, at a distance R from centre of a Flat strip, width 'a', along perpendicular Bisector:




$$dB = \frac{\mu_0 di}{2\pi d} = \frac{\mu_0}{2\pi} \frac{i(dx/a)}{R\sec\theta}$$

act as current wire

$$dB_{net} = \int dB \cos\theta$$

$$B_{net} = \int \frac{\mu_0 i (dx/a)}{2\pi R \sec\theta} \cos\theta$$

$$= \frac{\mu_0 i}{2\pi a R} \int \frac{dx}{\sec^2\theta}$$

$x = R\tan\theta$, $dx = R\sec^2\theta \, d\theta$

$$B_{net} = \frac{\mu_0 i}{2\pi a R} \int_{-\alpha}^{+\alpha} d\theta$$

$$\boxed{B_{net} = \frac{\mu_0 i}{2\pi a} \alpha = \frac{\mu_0 i}{\pi a} \tan^{-1}\left(\frac{a}{2R}\right)}$$

## Ampere's law

$$\boxed{\oint_{\text{closed loop}} \vec{B} \cdot \vec{dl} = \mu_0 I_{enclosed}}$$

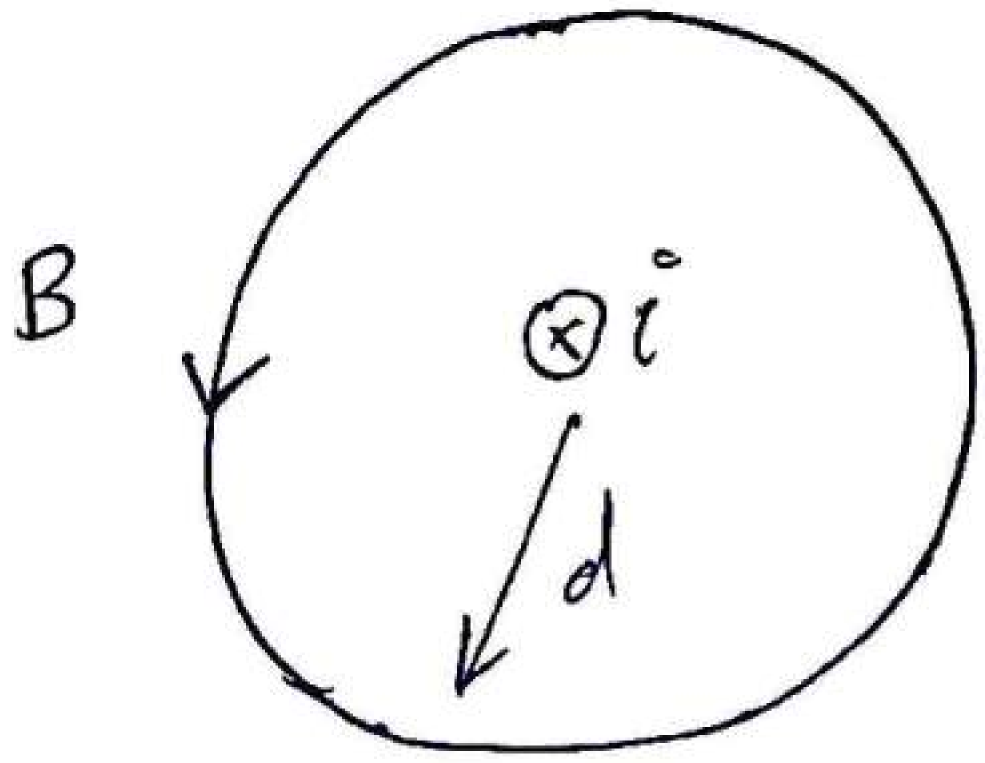
closed loop integeral of magnetic field = $\mu_0$ time the net current enclosed inside the loop

→ Current into paper plane → −ve

→ Current out of paper → +ve

→ Counterclockwise → positive

## Field of a long, straight current carrying Conductor:




from ampere's law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i$$

$$\Rightarrow B \times 2\pi d = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

## Magnetic field Inside and outside a Cylinder wire:


$$\pi R^2 \longrightarrow i.$$

$$\Rightarrow \pi r^2 \rightarrow i_1 = \frac{i \times \pi r^2}{\pi R^2}$$

$$\Rightarrow \boxed{i_1 = i\left(\frac{r^2}{R^2}\right)}$$

for $r<R$

inside

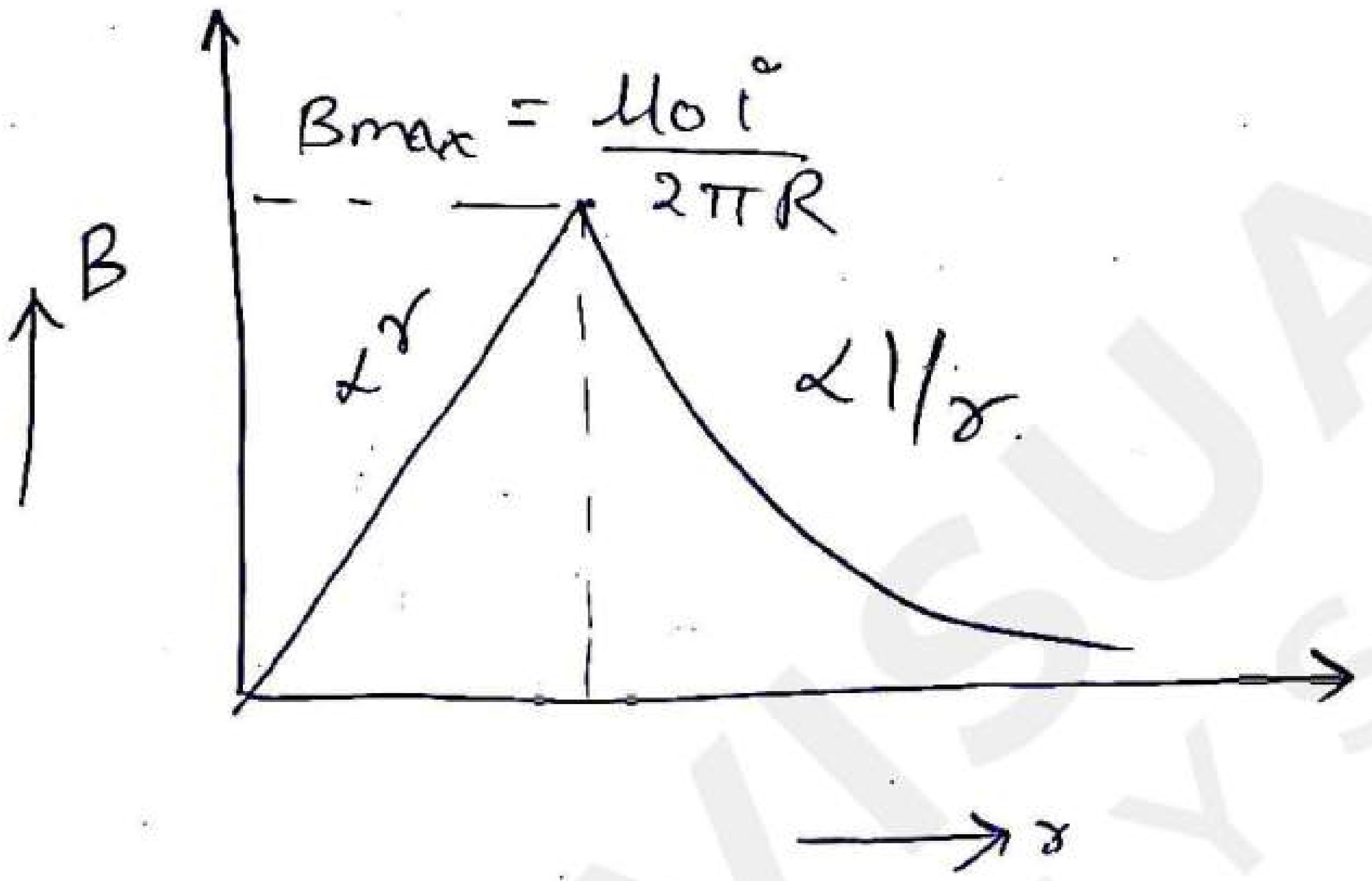
$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \mu_0 i_1$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i r^2}{R^2 \times 2\pi r} = \frac{\mu_0 i r}{2\pi R^2}} \quad \boxed{B \propto r}$$

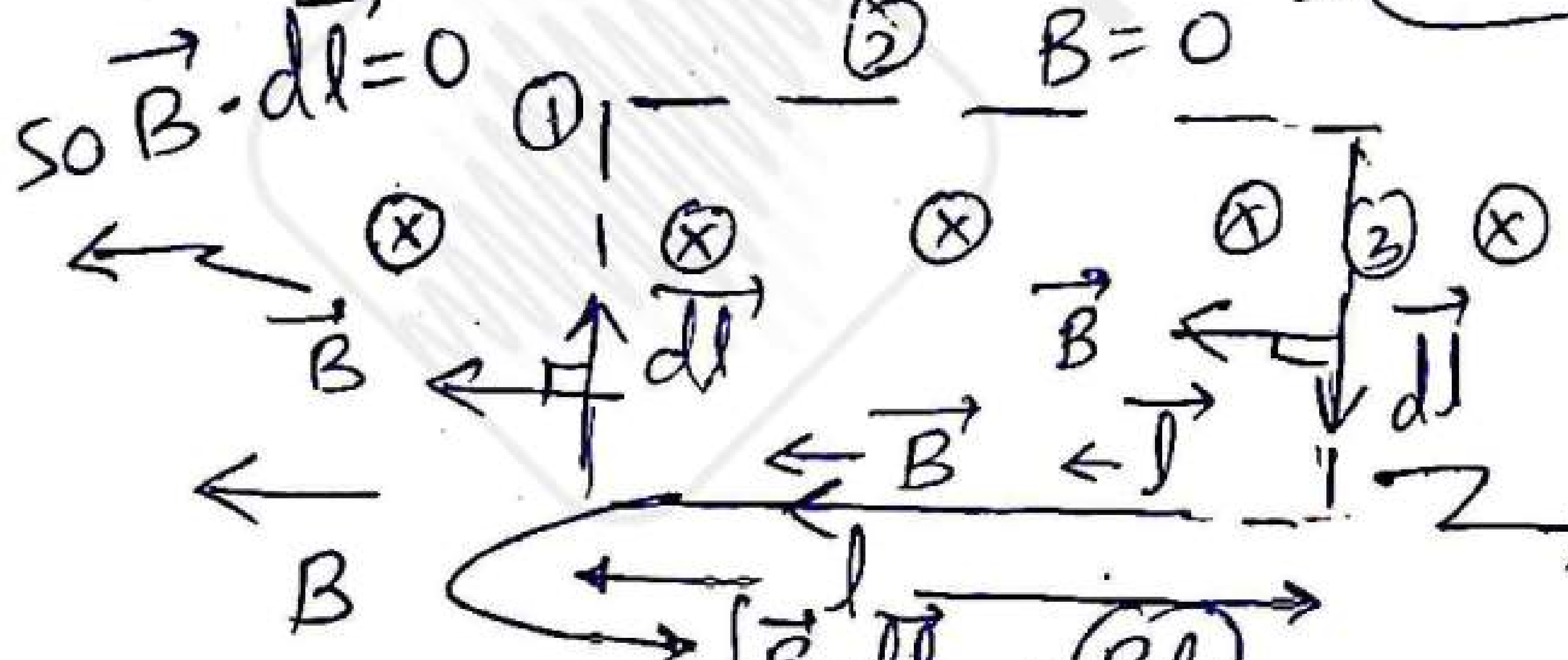
outside, means $r \geq R$

$i_{enclosed} = i$

$$\Rightarrow \quad B_{net} = B = \frac{\mu_0 i}{2\pi r} \quad , B \propto \frac{1}{r}$$




## Field for Solenoid




$2 \to \int \vec{B}\cdot dl = 0$

$n$ = number of turns per unit lengt

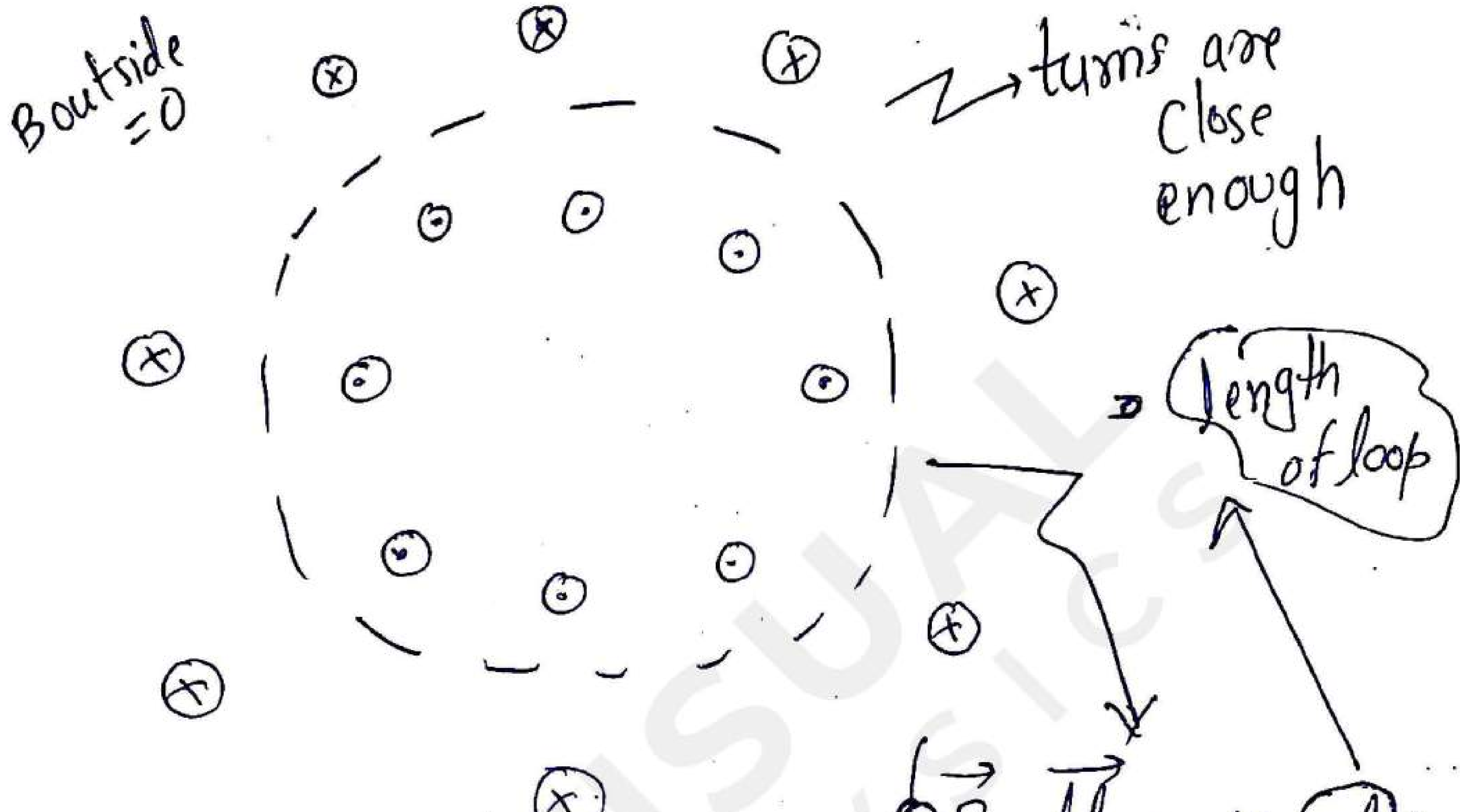
net current $= (nl)\,i$

as $\int_1 \vec{B}\,\vec{dl} = \int_2 \vec{B}\,\vec{dl} = \int_3 \vec{B}\cdot\vec{dl} = 0$

So, $\oint \vec{B}\cdot\vec{dl} = Bl = \mu_0 (nl)\,i$

$$\boxed{B = \mu_0 n i}$$

## Similarly for toroid.




$$\oint \vec{B}\cdot\vec{dl} = \mu_0 n l i$$

if turns are close enough

length of Ampere loop = $l$

$nl = N$ = net number of turns in toroid

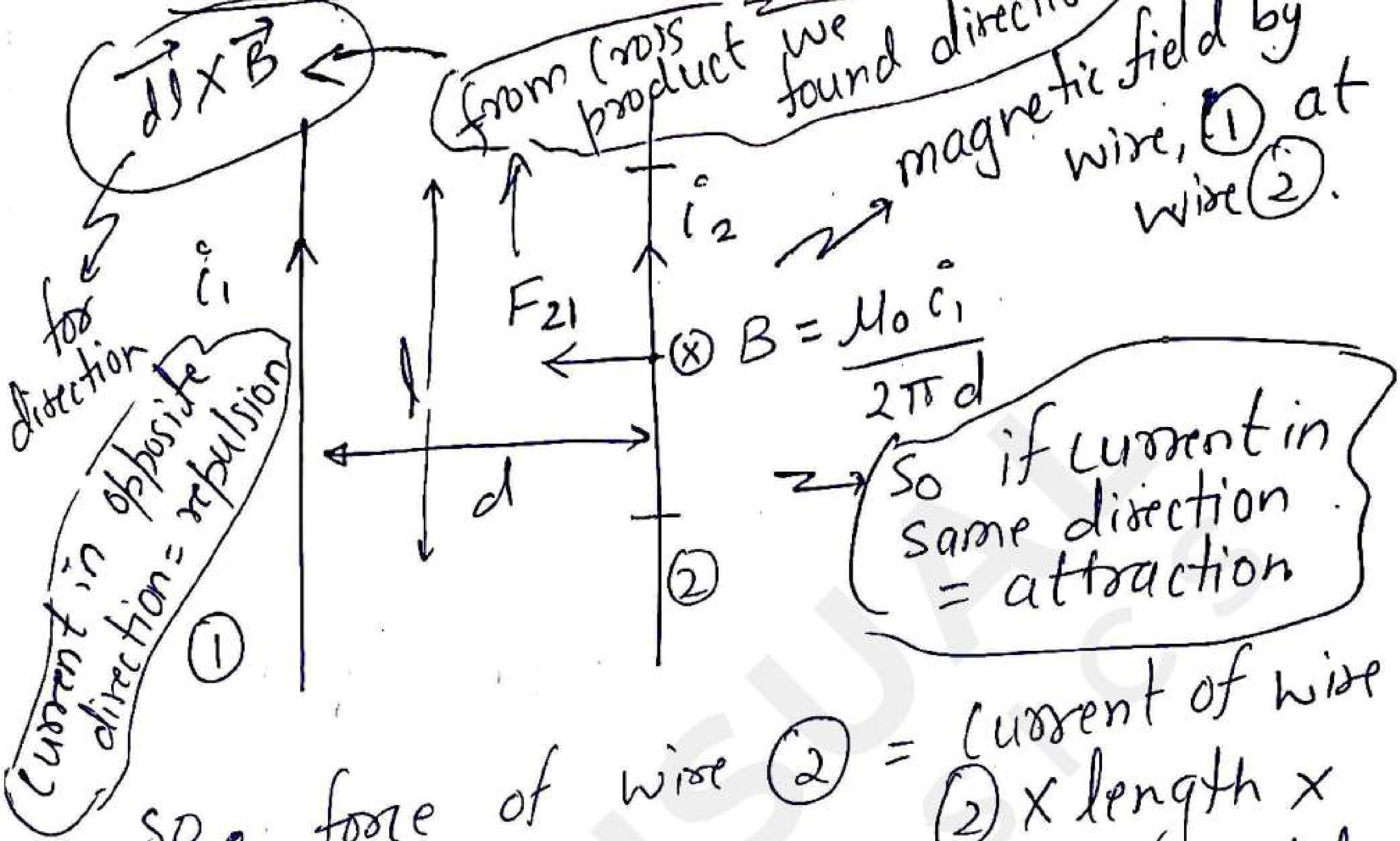
no. of turns per unit length

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = Bl = \mu_0 n l i$$

$$\Rightarrow \boxed{B = \mu_0 n i}$$

Same as of solenoid

# Force between parallel wires:




so, force of wire ② = current of wire ② × length × magnetic field of wire ①.

$\Rightarrow$ $F_{21}$ = force on ② by ① = $i_2 \times l \times \frac{\mu_0 i_1}{2\pi d}$

So force per unit length

$$\Rightarrow \boxed{f_{21} = \frac{F_{21}}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}}$$

similarly

$f_{12}$ = force per unit length on ① by ②

$$\boxed{f_{12} = \frac{\mu_0 i_1 i_2}{2\pi d}}$$

## Magnetic pressure:

↳ if we have a surface current distribution

$$\boxed{\text{surface current density} = \sigma}$$

So, magnetic pressure = $P_B$

$$P_B = \sigma\left[\frac{1}{2}\left(B_{above} + B_{below}\right)\right]$$

$P_B$ → magnetic pressure

$\sigma$ → surface current density

$\frac{1}{2}\left(B_{above} + B_{below}\right)$ → Average of magnetic field on either side