VISUAL PHYSICS

SHORT NOTES

CHAPTER

Inductance

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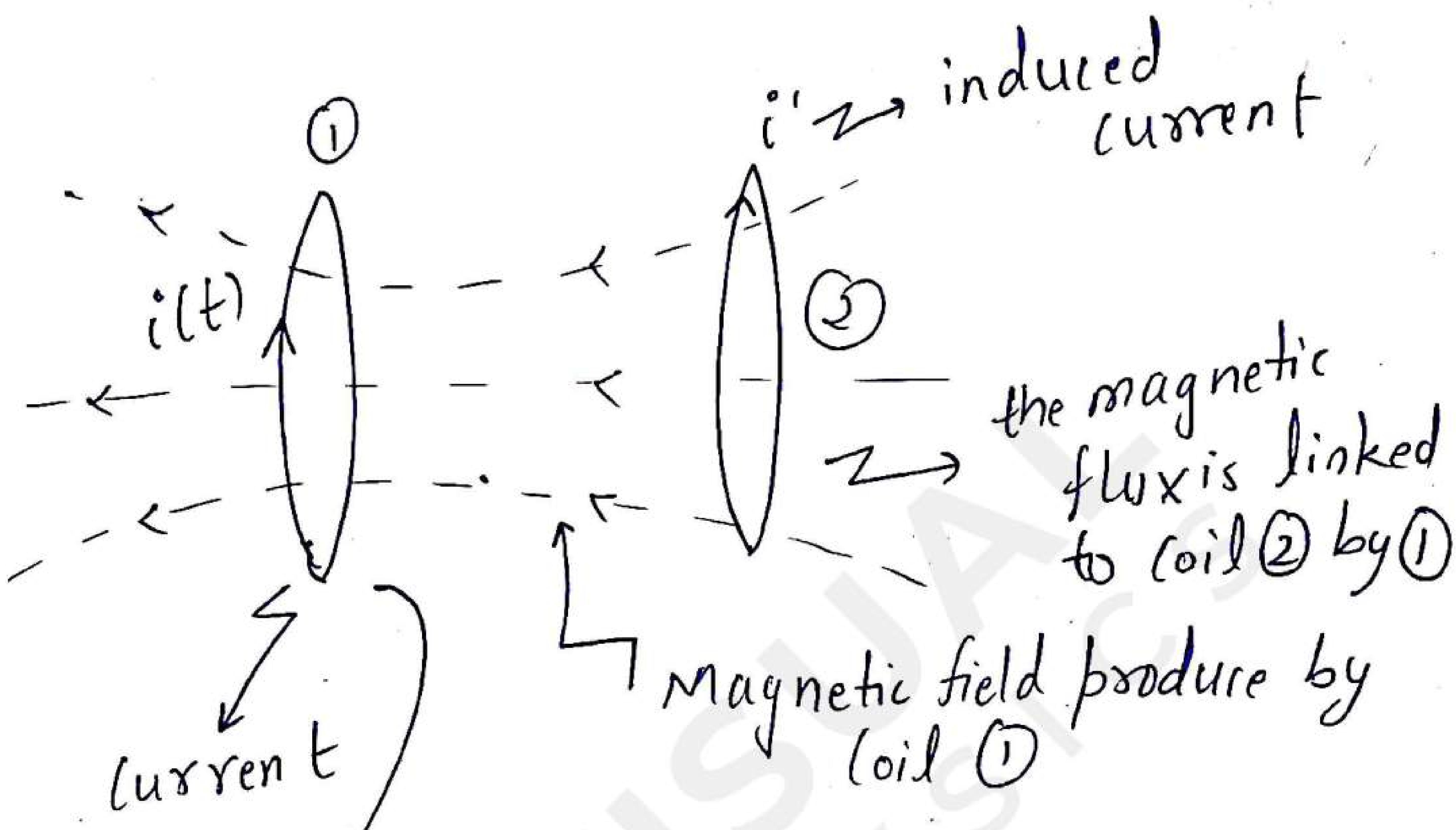
INDUCTANCE



Now changing i(t) in coil ①, changes the magnetic flux linked with ②, Hence induced current i'

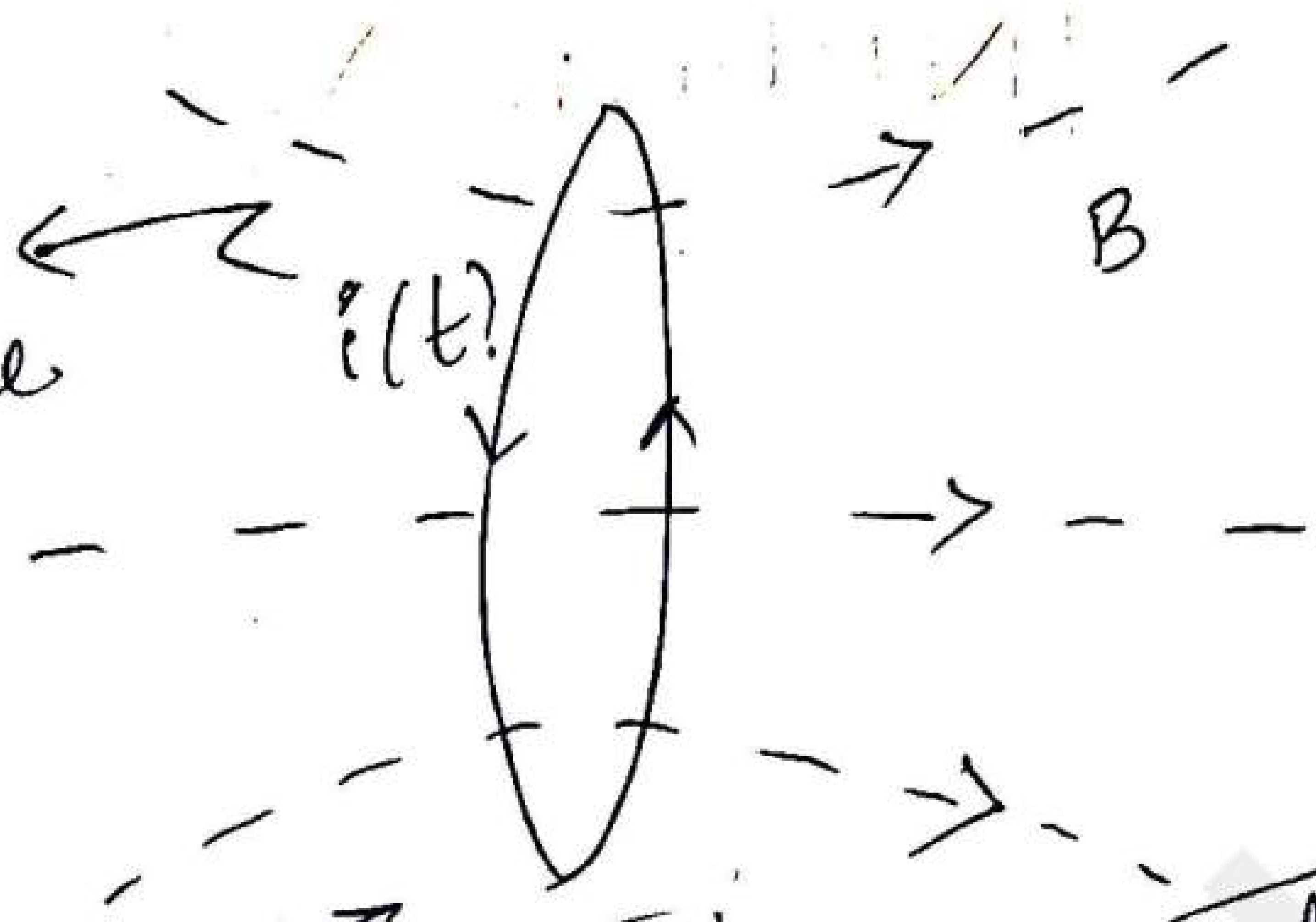
as, $\boxed{\begin{array}{l} B \propto i \\ \Rightarrow \phi \propto B \end{array}}$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$

this is called Mutal inductance

★ Current induce in coil ②, when curent in other coil changes.

current changing with time



* Due to current in coil there is flux linked with the coil itself

So if i(t) changed in coil, there is still change in flux

Hence, as $\varepsilon = -\dfrac{d\phi}{dt}$

back emf is induced that opposes the change in current i(t)

emf induced in itself

known as self induction

Now, for Mutal inductance.

$\phi \propto i$

Net flux linked.

$\Rightarrow \boxed{\phi = M i}$ ⟶ Mutual inductance

Now for N turns,

$\phi = N \phi'$ ⟶ flux linked with one coil

$\therefore \boxed{M = \frac{N\phi'}{i}}$

for Self Inductance

$\phi \propto i$

$\Rightarrow \boxed{\phi = L i}$ ⟶ self Inductance.

independent of current

$\phi = N\phi'$ ⟶ linked with one Coil.

$\boxed{L = \frac{N\phi'}{I} = \frac{N\phi'}{i}}$

current

Property of coil.

L ⟶ depends on geometrical feature only

unit of Self Inductance

$$= \frac{T\,m^2}{A} \Rightarrow \text{Henry } (H)$$

symbol

Net ϕ

Now, $L = \frac{N\phi'}{i}$

$$\Rightarrow \boxed{\varepsilon = -\frac{d\phi}{dt} = -N\frac{d\phi'}{dt}}$$

Now $N\phi' = Li$

$$\Rightarrow \boxed{\varepsilon = -L\frac{di}{dt}} \quad \boxed{= - L\frac{di}{dt}}$$

opposes change

back emf

do not depend on magnitude of i

depents on rate of change of i

$\Rightarrow$ Current $\uparrow \rightarrow \varepsilon$ such that i decrease and vice-versa

induced emf

polarity of $\varepsilon \rightarrow$ oppossite to i
$\hookrightarrow$ if $i \uparrow$

polarity of $\varepsilon \rightarrow$ Same direction of i
$\hookrightarrow$ if $i \downarrow$

$\rightarrow$ ☆ $\boxed{\text{Inductor tries to maintain the same current through it.}}$

Inductance of solenoid

$B = \mu_0 n i$ $\rightarrow$ Number of turns per unit length

So, $\boxed{\phi' = BA = \mu_0 n i A}$

flux per coil

Now net flux

$$\phi = N\phi'$$

$$\phi = N(\mu_0 n i)A$$

$n l$

$$\rightarrow \boxed{\phi = (n^2 \mu_0 i l)A}$$

Now, $L = \dfrac{\text{Shared / Net flux}}{i} = \dfrac{\phi}{i}$

$$L = \frac{n^2 l \mu_0 i A}{i}$$

$$\boxed{L = n^2 l \mu_0 A}$$

depends on geometry

Now if there other medium $\boxed{\mu = \mu_0 \mu_r}$

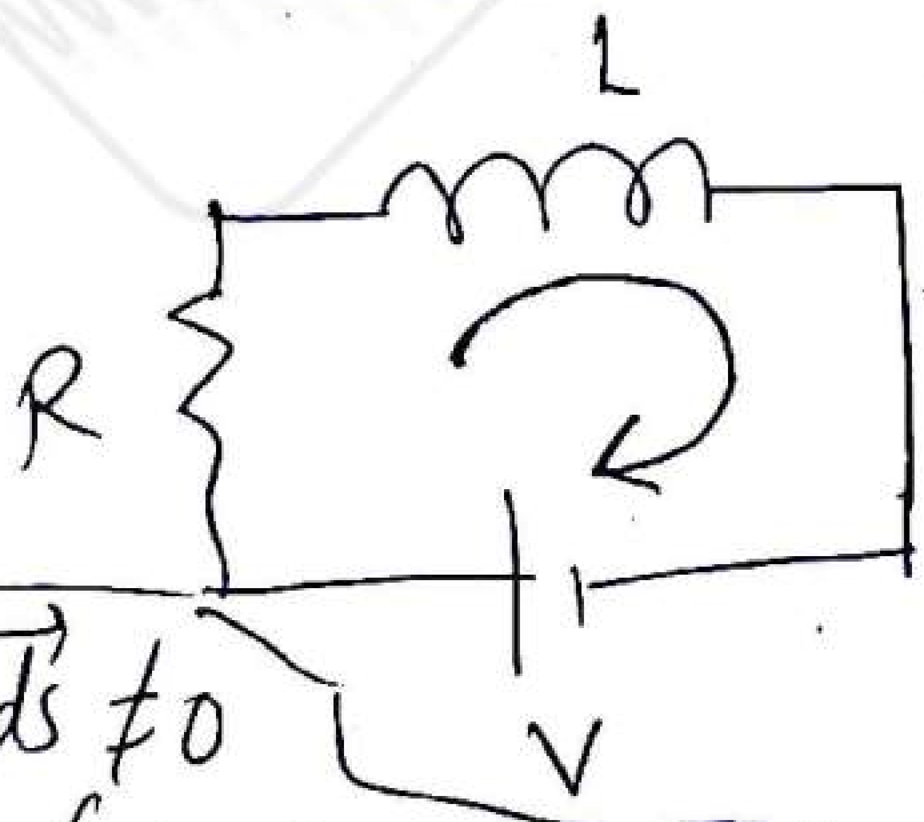
so $\boxed{L = n^2 l \mu_r \mu_0 A}$

for krichhoff's law

Now, $\boxed{\mathcal{E} = -L\dfrac{di}{dt}}$ also considered as p.d across Inductor

techincally

krichhoff's rule cannot be applied



as $\oint \vec{E}\cdot\vec{ds} \neq 0$ for Non conservative field

as induced '$\mathcal{E}$' is not conservative field

The actual Rule is faraday's path

$$\boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi}{dt} \rightarrow \varepsilon_{\text{induced}}}$$

$$V - iR = \varepsilon$$

$$\Rightarrow \boxed{V - iR - \varepsilon = 0}$$

* Same as of krichhoff's loop law $\oint \vec{E}\cdot d\vec{s} = 0$

Considering the correct polarity of ε

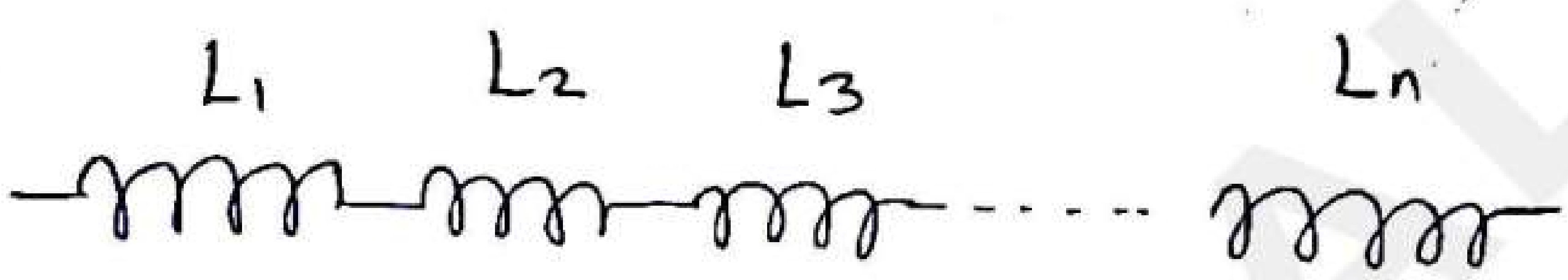
so for getting equation we use the same method as krichhoff's law

$$\boxed{V - iR - \varepsilon = 0}$$

but technically it is farady's path method.

* by considering ε as p.d across inductor

Series parallel combination :



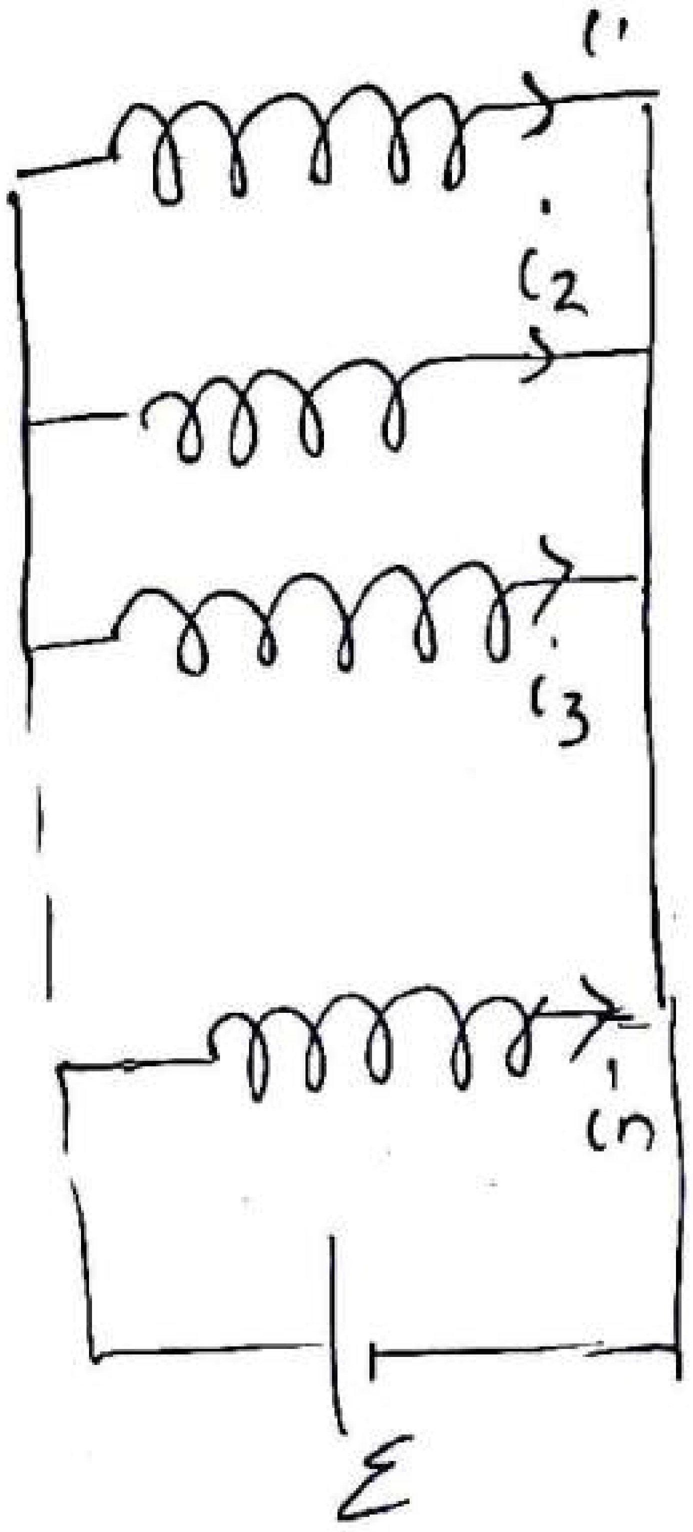
$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \cdots \mathcal{E}_n$$

$$\mathcal{E} = -L_1 \frac{di}{dt} + \left(-L_2 \frac{di}{dt}\right) + \cdots \cdots \left(-L_n \frac{di}{dt}\right)$$

$$\Rightarrow \quad \mathcal{E} = -(L_1 + L_2 + \cdots L_n)\frac{di}{dt}$$

$$\Rightarrow \quad \boxed{L_{net} = L_1 + L_2 + L_3 + \cdots L_n}$$

$$\Rightarrow \quad \boxed{\mathcal{E} = -L_{net}\frac{di}{dt}}$$



$$\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2 = \ldots \mathcal{E}_n$$

$$\Rightarrow i = i_1 + i_2 + \ldots i_n$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \ldots \frac{di_n}{dt}$$

$$\Rightarrow -\frac{\mathcal{E}}{L_{eq}} = -\left[\frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2} + \ldots \frac{\mathcal{E}}{L_n}\right]$$

$$\Rightarrow \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$

Mutual Inductance :

$\phi_{21} \rightarrow$ flux in coil 2 due to ~~ks~~ current in coil 1

$$\phi_{21} \propto i_1$$

$$\phi_{21} = \boxed{M_{2,1}}\, i_1$$

Mutual Inductance of the second coil wrt to first coil

Now if second coil have 'N' turns

$\Rightarrow$ $$\boxed{N_2 \phi_{2,1} = M_{2,1} i_1}$$

$$N_2 \frac{d\phi_{2,1}}{dt} = M_{2,1} \frac{di_1}{dt}$$

$$\Rightarrow \quad \mathcal{E}_2 = -N_2 \frac{d\phi_{2,1}}{dt}$$

$$\mathcal{E}_2 = -M_{2,1} \frac{di}{dt} \quad \Rightarrow \quad \boxed{M_{2,1} = \frac{N_2 \phi_{2,1}}{i_1}}$$

Similary if coil 2 is carrying current

$\phi_{1,2} \rightarrow$ flux in coil 1 due to current in coil 2

always $\quad N_1 \phi_{1,2} \propto i_2$

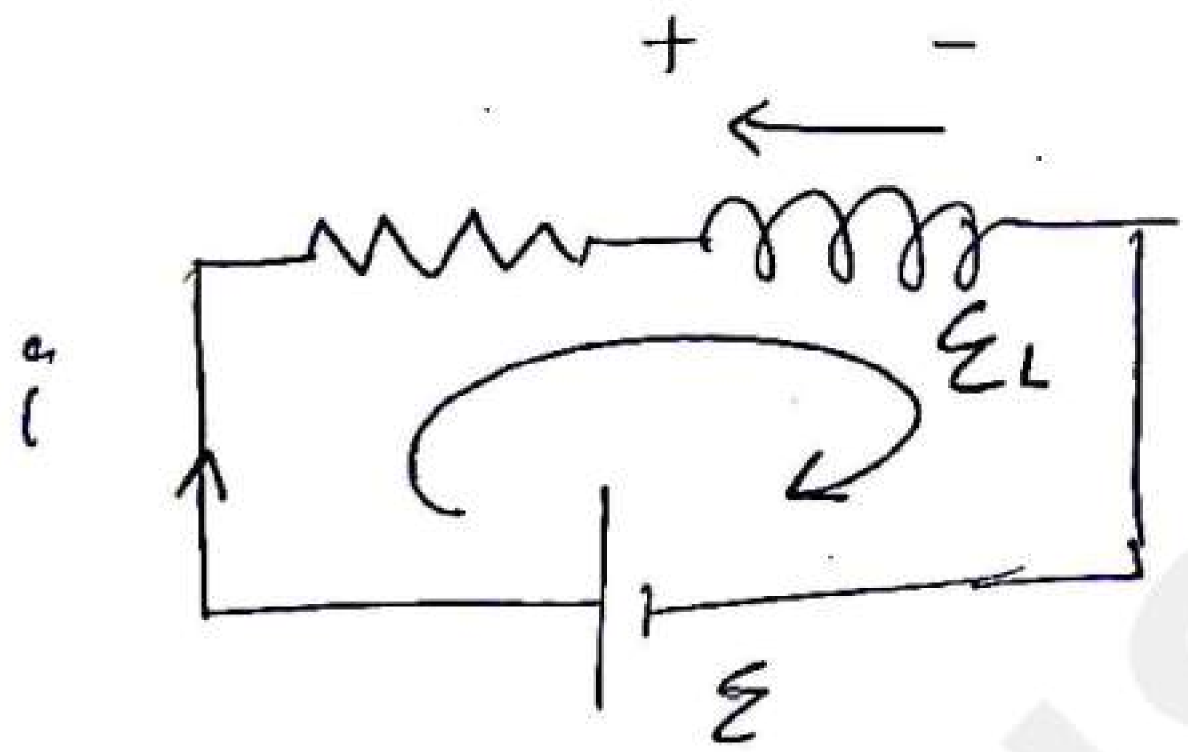
$$\Rightarrow \quad N_1 \phi_{1,2} = M_{1,2} i_2$$

Mutual Inductance, of the first coil wrt second coil

$$* \boxed{M_{2,1} = M_{1,2} = M}$$

Mutual Inductance ~~of~~ between coil

$$\boxed{\mathcal{E}_1 = -M\frac{di_2}{dt}}, \quad \boxed{\mathcal{E}_2 = -M\frac{di_1}{dt}}$$



Inductor does not allow a sudden change of current

$i\uparrow \quad \mathcal{E}_L = -L\frac{di}{dt}$

Series R-L circuit

$$\mathcal{E} - iR - \mathcal{E}_L = 0$$

So, $$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

$$\Rightarrow L\frac{di}{dt} = \mathcal{E} - iR$$

$$\int_0^i \frac{di}{\mathcal{E} - iR} = \int_0^t \frac{dt}{L}$$

$$\left.\frac{\ln(\mathcal{E} - iR)}{-R}\right|_0^i = \frac{t}{L}$$

$$\ln\left(\frac{\varepsilon - iR}{\varepsilon}\right) = \frac{-R}{L}t$$

$$1 - \frac{R}{\varepsilon}i = e^{-(R/L)t}$$

$$\Rightarrow \frac{R}{\varepsilon}i = 1 - e^{-(R/L)t}$$

$$\boxed{i = \frac{\varepsilon}{R}\left[1 - e^{-(R/L)t}\right]}$$

$t = 0$

$$i_0 = \frac{\varepsilon}{R}(1 - e^0)$$

$$\Rightarrow \boxed{i_0 = 0}$$

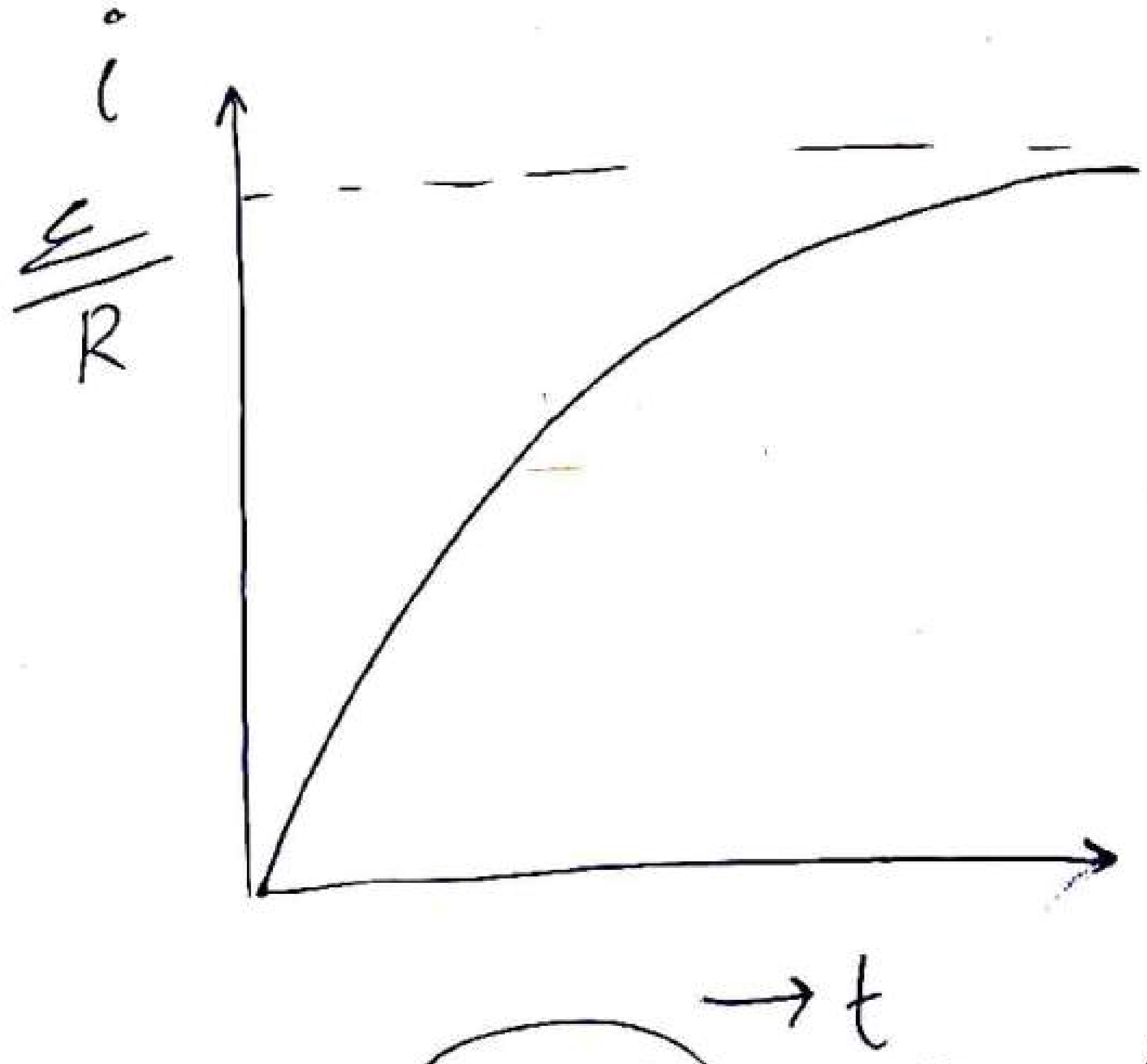
Just when battery is connected there is no current in circuit

$t \to \infty$

when, there is no Inductor

$$\Rightarrow i_\infty = \frac{\varepsilon}{R}(1 - e^{-\infty})$$

$$\Rightarrow \boxed{i_\infty = \frac{\varepsilon}{R}}$$



if $i_0 \to 0$

⇒ p.d across Inductor $= -\varepsilon$

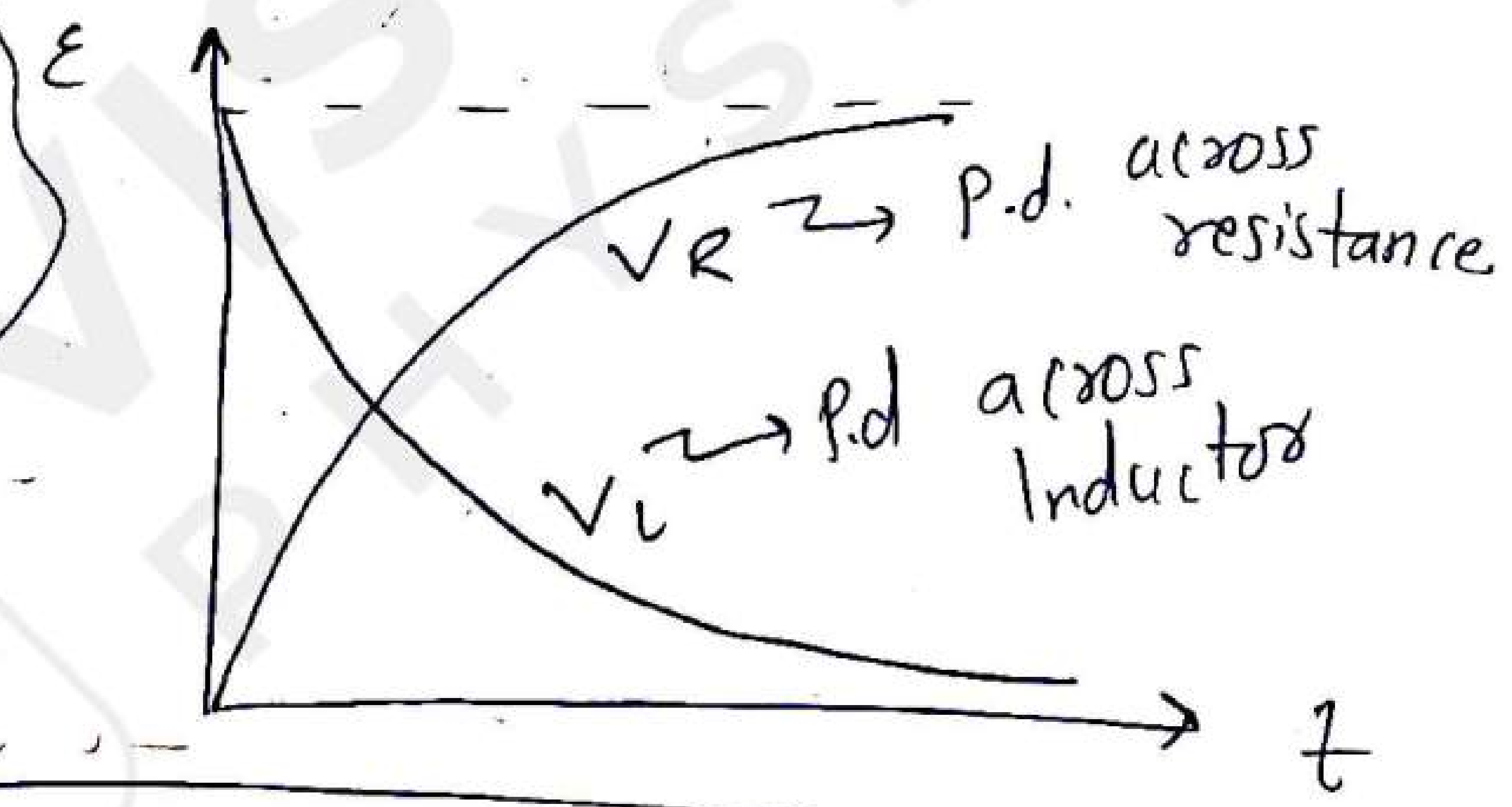
when $i_0 \to 0$

★ means induce Inductor is zero

means $\frac{di}{dt} = 0$

★ so, when Inductor is connected it acts as open circuit

after τ, $i \to$ 63% of i_{max}



$$\boxed{i = \frac{\varepsilon}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)}$$

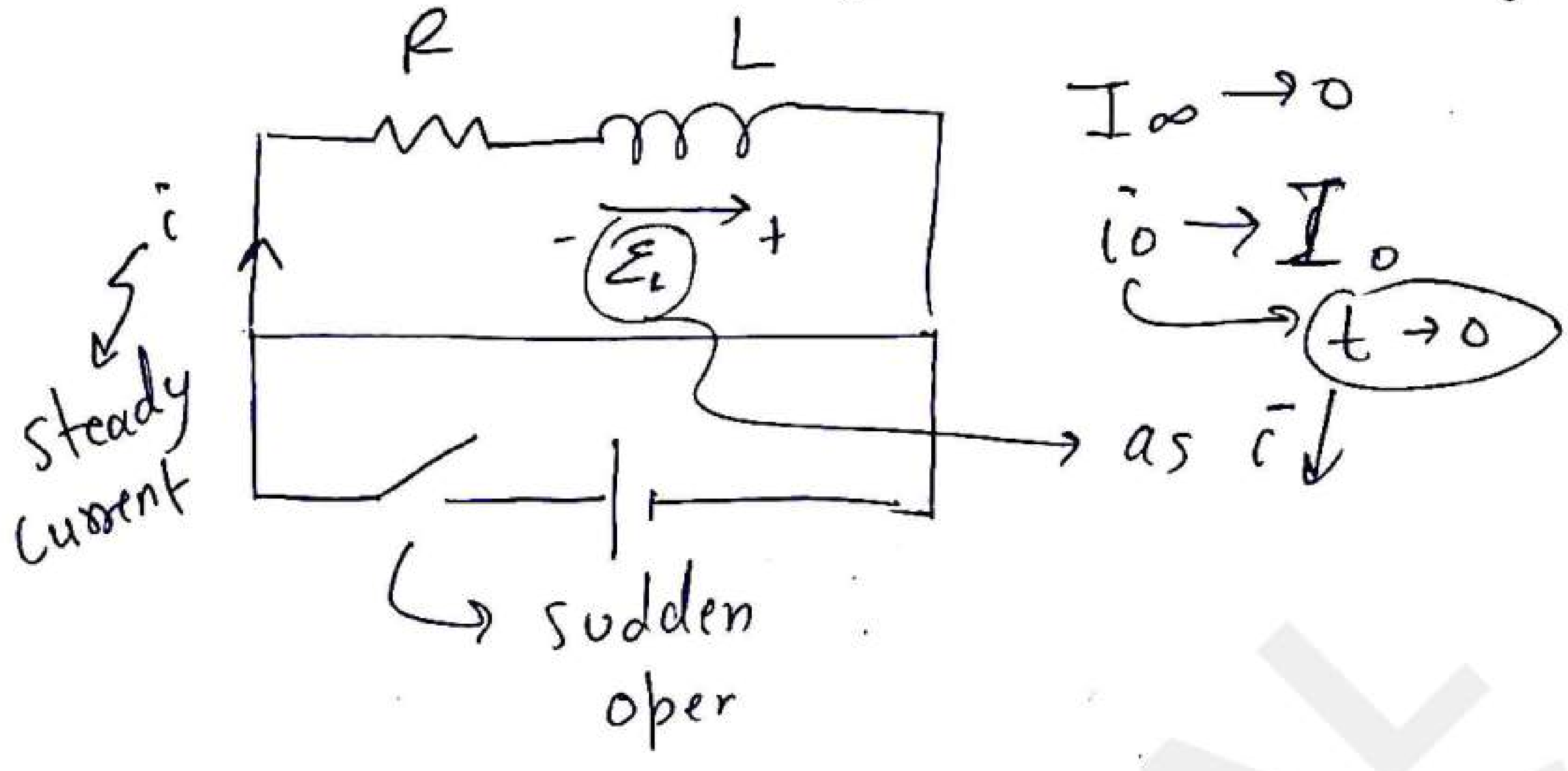
Now, $\frac{L}{R} \to \tau$ → Inductive time constant

unit [T]

$$\boxed{q = Q_0\left(1 - e^{-\frac{t}{(RC)}}\right)}$$

charge on capacitor → for RC circuit

opposses the sudden change



$$\Rightarrow \mathcal{E}_L = -L\frac{di}{dt}$$

$$\Rightarrow -Ri + \left(-L\frac{di}{dt}\right) = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}\,dt$$

$$\Rightarrow \boxed{i = I_0 e^{-(R/L)t}}$$

$t \to \tau$

$i \to 37\%$ of I_0

$$\boxed{\frac{L}{R} \to \tau_L} \to \text{Time Constant}$$

p.d of R $\to Ri$

$$\boxed{\mathcal{E}_R = \left(I_0 e^{-(R/L)t}\right)R}$$

$$\mathcal{E}_L = -L\frac{di}{dt}, \quad \boxed{\mathcal{E}_L = (I_0 R)e^{-(R/L)\tau}}$$

if we suddenly disconnect from circuit

$\rightarrow \frac{di}{dt} \rightarrow \infty$

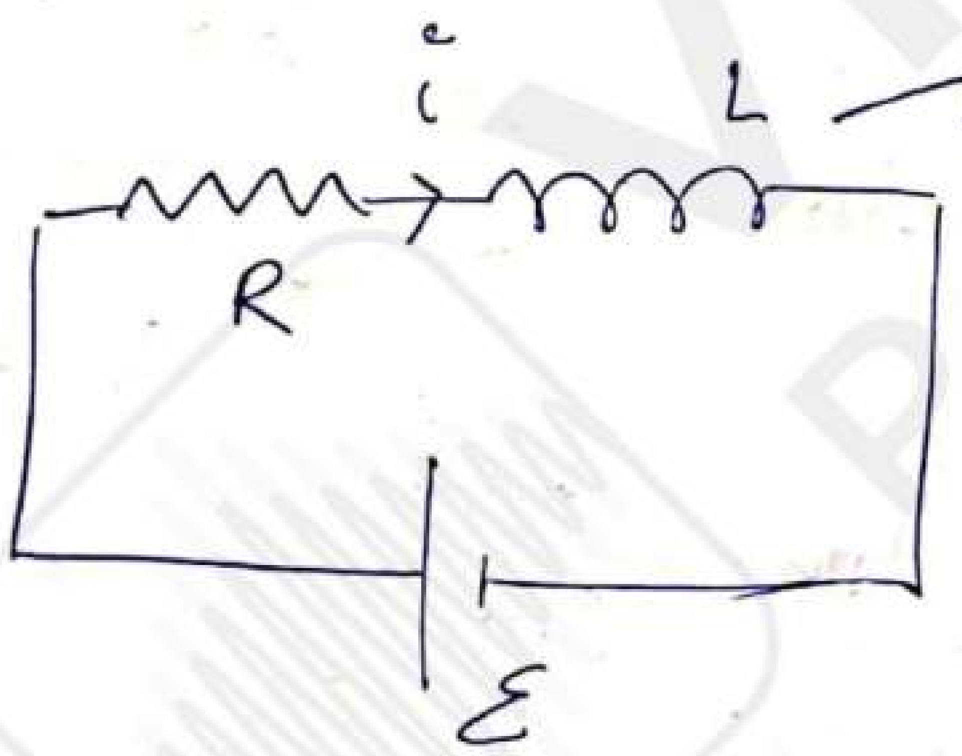
means, $\mathcal{E}_L = L\frac{di}{dt} \Rightarrow \infty$

hence infinite back emf.

sparking

damage to Inductor etc.

Energy in RL Circuit:



it has magnetic field

so store energy in form of magnetic field

$\left(\mathcal{E} - iR - L\frac{di}{dt} = 0\right) \times i$

$\Rightarrow i\mathcal{E} - i^2R - Li\frac{di}{dt} = 0$

Power delivered by Battery

Power dissipated in Resistor

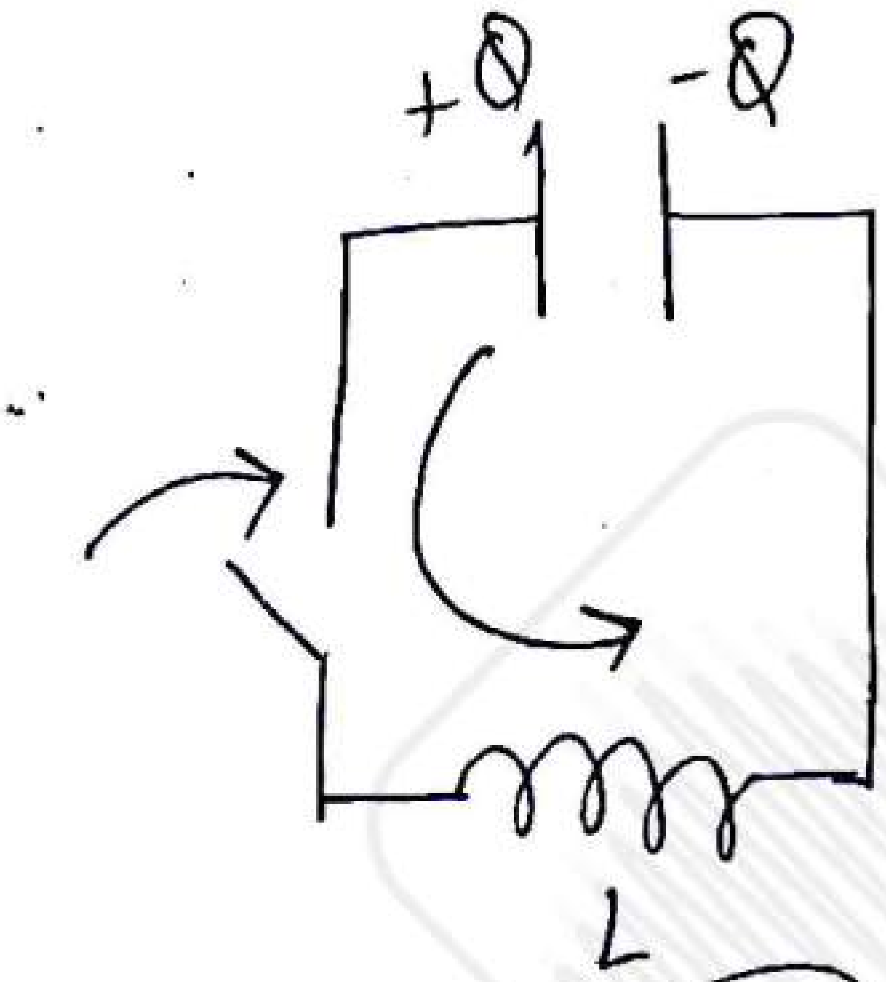
$\frac{dU_L}{dt} = Li\frac{di}{dt}$

$\boxed{U_L = \frac{Li^2}{2}}$

similar for the capacitor, in form of Electric field

$$\boxed{U_c = \frac{1}{2}\frac{Q^2}{C}}$$

LC Circuit-1



so sudden release of charge → current

Inductor opposse sudden change.

So capacitor is not instantly discharged but takes a finite time.

So after some time polarity of capacitor reveresed

it keeps on repeating

so there would be electric oscillation
↳ LC oscillation.

as energy is not lost ⇒ as $R=0$

↳ Energy oscillates in Electrical energy in capacitor

to magnetic energy in inductor

at some time t,

charge → q

current → i

$q \downarrow \Rightarrow i \uparrow$

$$\boxed{i = -\frac{dq}{dt}}$$

so, $\boxed{-L\frac{di}{dt} + \frac{q}{C} = 0}$

so $-L\frac{d^2 i}{dt^2} + \frac{1}{C}\frac{dq}{dt} = 0$

$$\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0$$

SHM equation

so, $\boxed{\omega = \frac{1}{\sqrt{LC}}}$

max current

$\Rightarrow i = i_0 \sin(\omega t + \phi)$

$t=0,\ i=0 \Rightarrow \boxed{\phi = 0}$

$\boxed{i = i_0 \sin \omega t}$

as, $-L\dfrac{di}{dt} + \dfrac{q}{c} = 0$

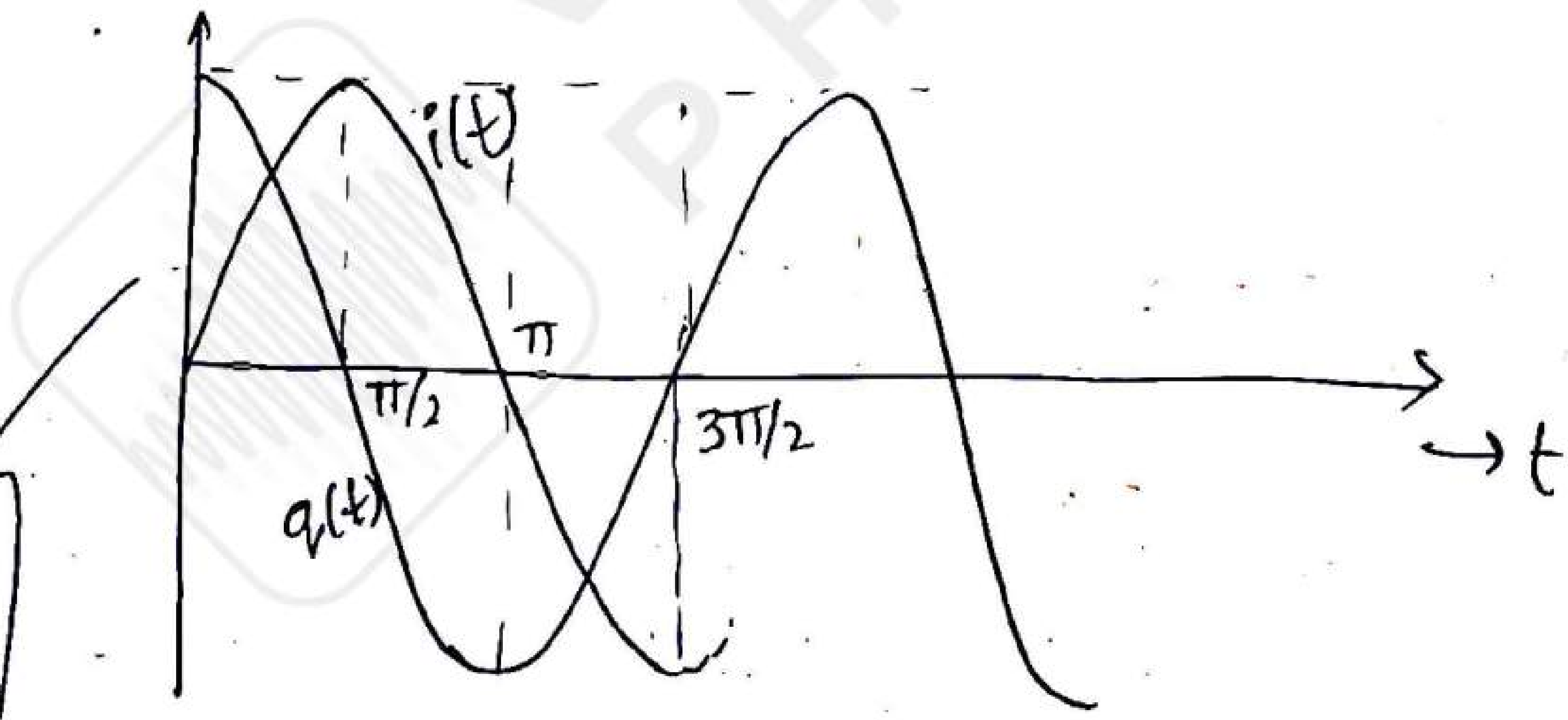
$$-L\left[-\frac{d^2q}{dt^2}\right] + \frac{q}{c} = 0$$

$$q = Q_0 \sin(\omega t + \phi)$$

$$t = 0,\ q = Q_0 \Rightarrow \phi = \pi/2$$

$$\Rightarrow \boxed{q = Q_0 \cos\omega t}$$

So, charge and current in LC oscillates by $\pi/2$ phase difference.



when $i \to i_{max}$ $\hookrightarrow q = 0$ $\hookrightarrow$ and vice-versa

Now
at any instant

$$U_c + U_L = \text{Constant} = E$$

so, when $U_c = 0,\ q = 0$

$\hookrightarrow U_L = E$; max energy in Inductor

Now, $(U_c)_{max} = \frac{Q_0^2}{2C}$

and $(U_L)_{max} = \frac{1}{2} L I_{max}^2$

$$\Rightarrow (U_c)_{max} = (U_L)_{max}$$

$$\Rightarrow I_{max}^2 = \frac{Q_0^2}{LC}$$

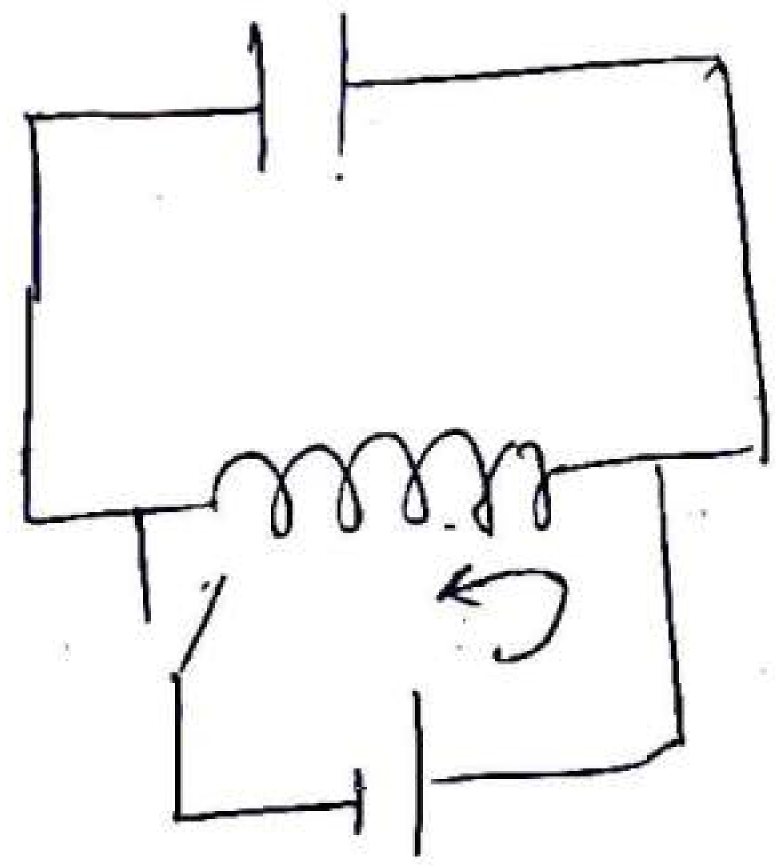
as $\omega = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \boxed{I_{max} = I_0 = \omega Q_0}$$

☆

when $q \downarrow \rightarrow i \uparrow$
and $q \uparrow \rightarrow i \downarrow$

if $t=0$, $q=0$ and current was there in inductor



&

$$i(t) = I_0 \cos\omega t$$

$$q(t) = \left(\frac{I_0}{\omega}\right) \sin\omega t$$

just add by phase of $\pi/2$

$t=0, i=0, q=Q_0$	$t=0, i=I_0, q=0$
$i(t) = \omega Q_0 \sin\omega t$ $q(t) = Q_0 \cos\omega t$	$i(t) = I_0 \cos\omega t$ $q(t) = \left(\frac{I_0}{\omega}\right) \sin\omega t$

$$V_c(t) = \frac{q(t)}{C} = \frac{Q_0}{C}\cos\omega t$$ Varies sinusodialy

$$V_L(t) = L\frac{di}{dt}$$

$$= L\frac{Q_0}{LC}\cos\omega t$$

$$\Rightarrow \boxed{V_L = L\omega^2 Q_0 \cos\omega t}$$

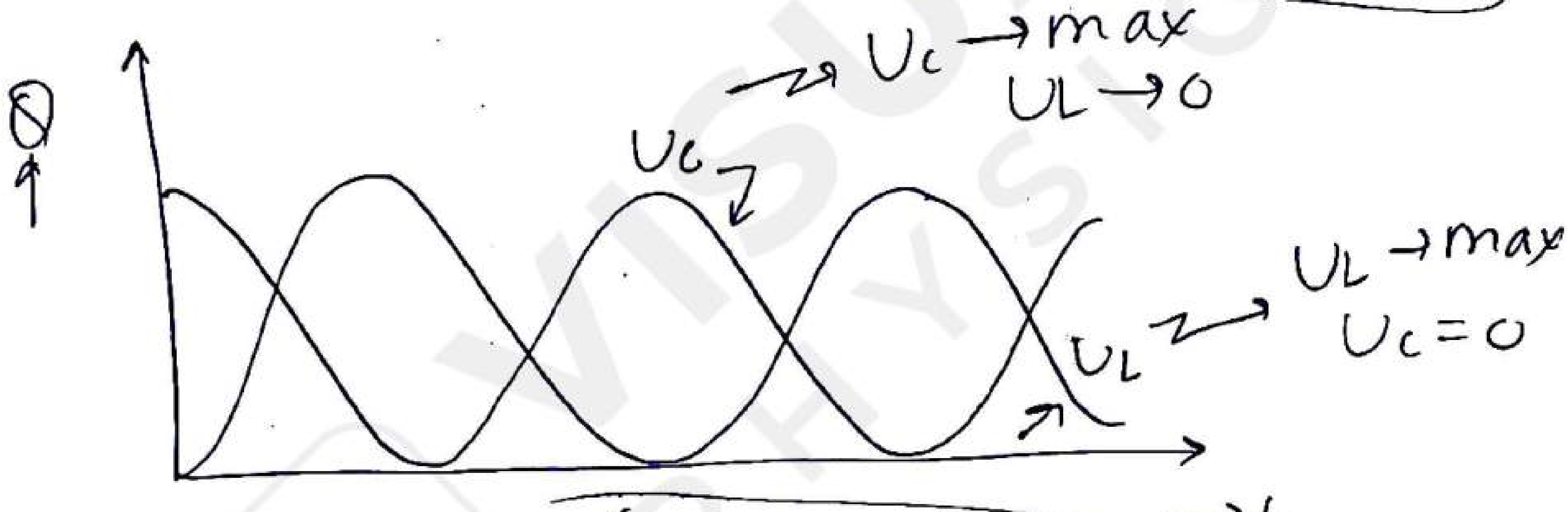
Varies sinusodialy

Now, $U_c = \frac{q^2}{2C}$, $U_L = \frac{1}{2} L i^2$

$\Rightarrow$ $\boxed{U_c = \frac{Q_0^2}{2C} \cos^2 \omega t}$ $\boxed{U_L = \frac{1}{2} L I_0^2 \sin^2 \omega t}$

OR

$\boxed{U_L = \frac{Q_0^2}{2C} \sin^2 \omega t}$



always at any 't'

Now,
$$U = U_c + U_L = \text{Constant}$$

$$U = \frac{Q_0^2}{2C} \left[\cos^2 \omega t + \sin^2 \omega t \right]$$

1

$\boxed{U = \frac{Q_0^2}{2C}}$ $\rightarrow$ similar to spring-block

$\rightarrow$ Energy $\rightarrow$ between potential and kinetic

★