



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Gravitation

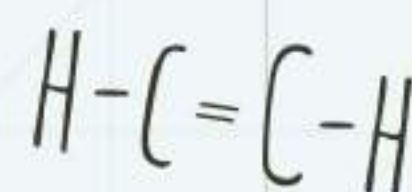
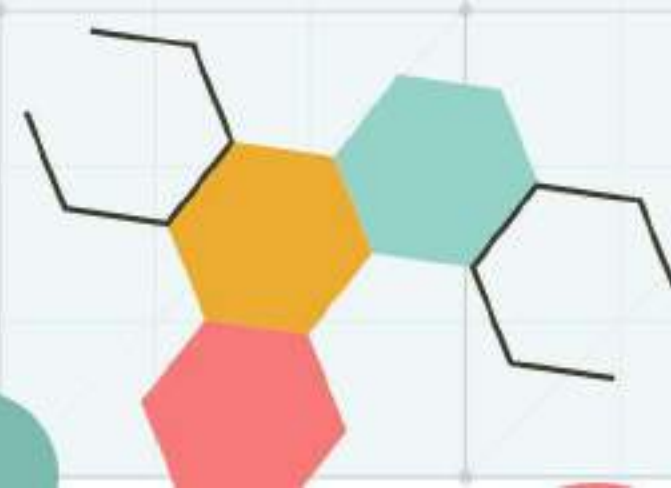
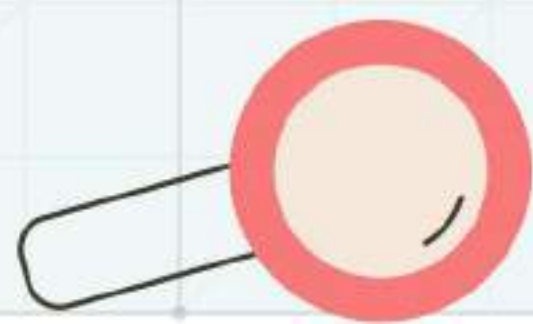
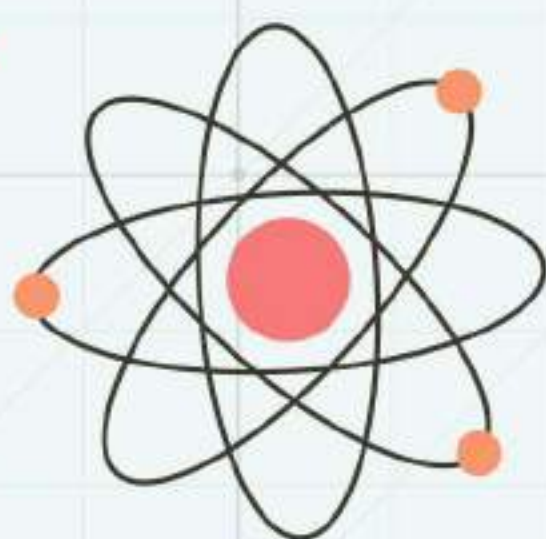
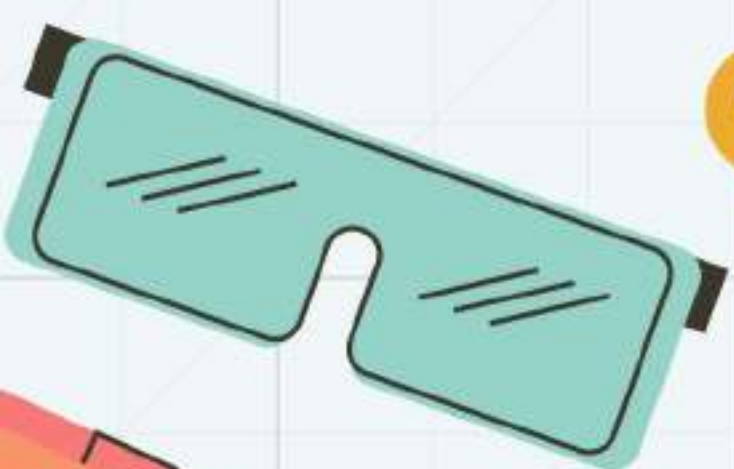
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GRAVITATION

Force of attraction between any two bodies of the universe is known as Gravitation.

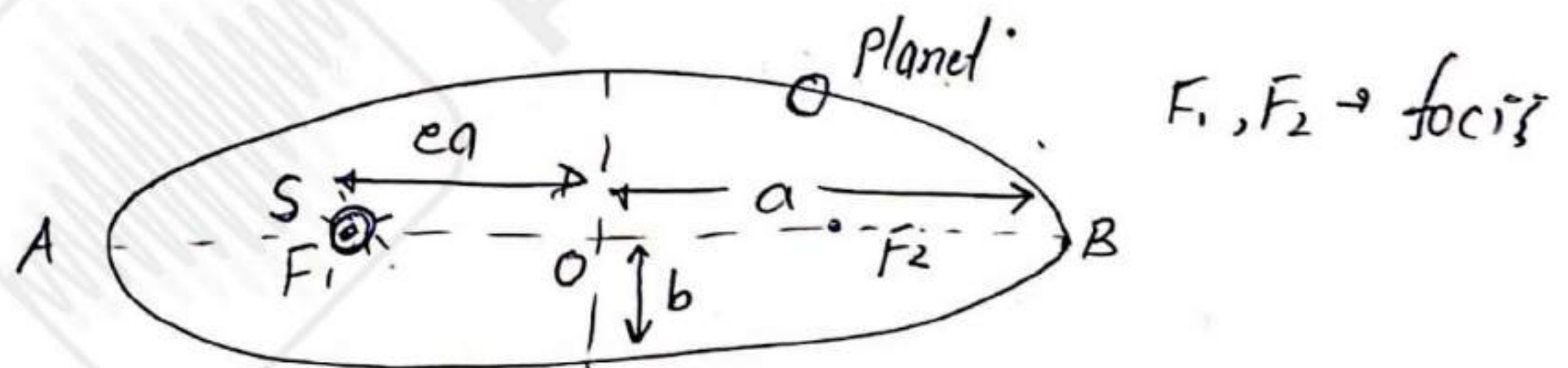


weakest force among the four fundamental forces of nature.

KEPLER'S LAWS [for heavenly body revolving around planet/star in an orbit, is natural satellite.]

I. LAW of orbit:

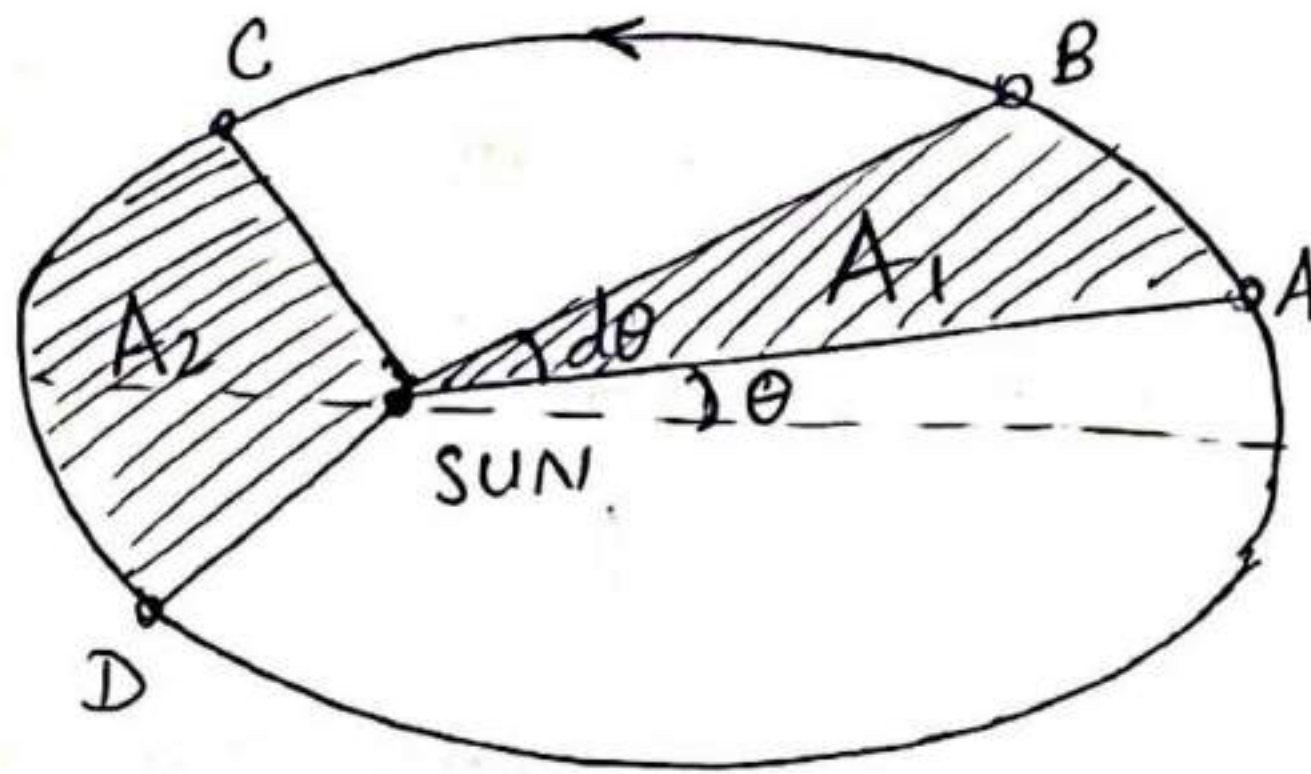
Each planet moves around the sun in an elliptical orbit, with sun at one focus



$$e = \frac{SO}{a}, \quad SO = ea$$

→ Greatest distance (BS) of the planet from sun is called ~~the~~ "Apogee"

II. LAW OF AREAS :



for a time 't' ; $A_1 = A_2$

In a given by equal Areas are covered that means, planet is moving faster at 'C' as compared with 'A'.

$$\boxed{\text{Areal velocity} = \frac{\text{Area Swept}}{\text{Time}} = \text{Constant.}}$$

$$\frac{\frac{1}{2} r(r d\theta)}{dt} = \text{Constant}$$

$$\boxed{\frac{1}{2} r^2 (\omega) = \text{Constant}}$$

if we multiply, m both side
 $\rightarrow$ mass of planet

$$\Rightarrow m r^2 \omega = \text{Constant}$$

$$\boxed{I \omega = \text{angular momentum} = \text{Constant}}$$

III Law of periods:

$$T^2 \propto a^3$$

$T \rightarrow$ Time period of revolution of planet

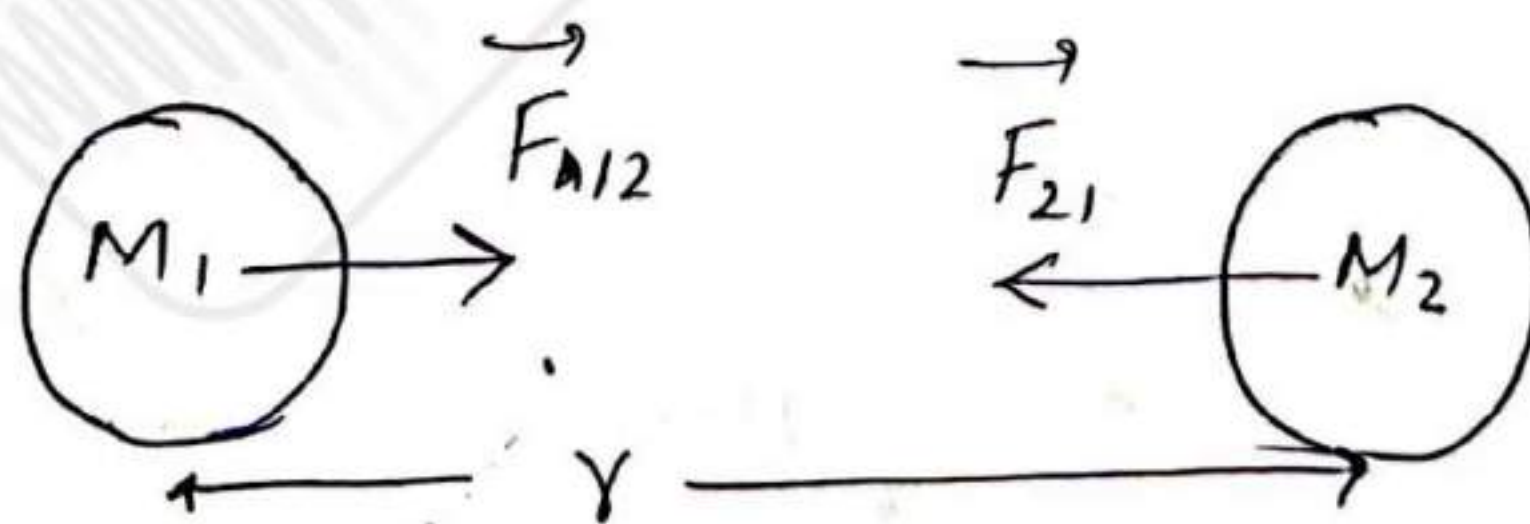
$a \rightarrow$ Semi-major axis of ellipse.

$$\Rightarrow T^2 = k r^3$$

$\rightarrow$ radius (considering orbit to be circular)

* Kepler's laws are applicable not only for solar system but also equally well for satellites orbiting around a planet.

NEWTON'S LAW OF GRAVITATION:



$M_1, M_2 \rightarrow$ two point masses

$F_{12} \rightarrow$ force on 1 due to 2

$F_{21} \rightarrow$ force on 2 due to 1

$$|\vec{F}_{12}| = |\vec{F}_{21}| \rightarrow \text{Newton's third law}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

↪ opposite in direction.

$$F \propto \frac{m_1 m_2}{r^2}$$

↪ Newton's Gravitation law

$$F = G \frac{M_1 M_2}{r^2}$$

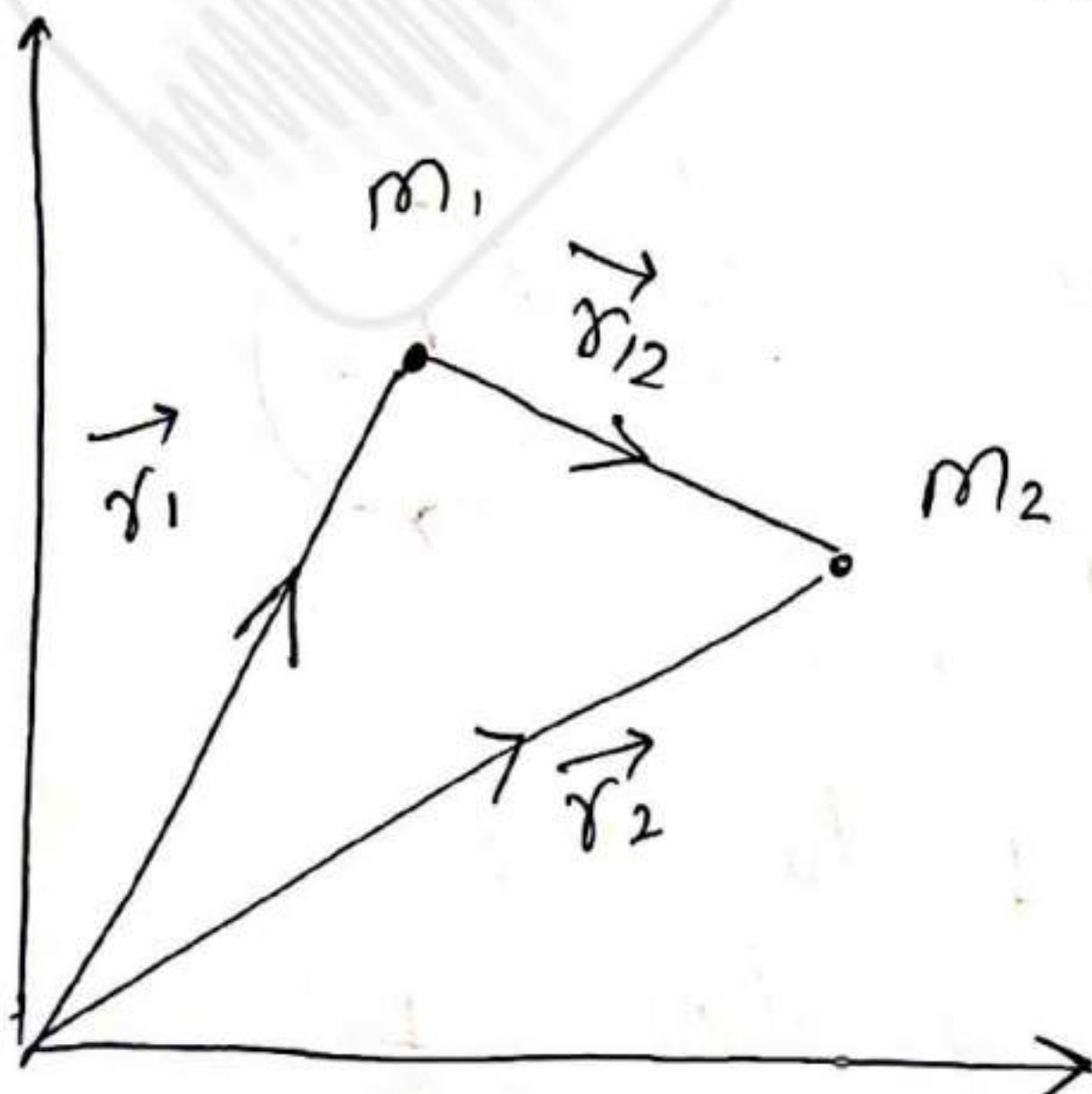
$G \rightarrow$ Universal gravitational constant.

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

↪ directly proportional to masses product

↪ Inversely proportional to square of distance between the point masses.

Vector form



$$\begin{aligned} \vec{F}_{12} &= \frac{G m_1 m_2}{(\vec{r}_{12})^2} \hat{r}_{12} \\ &= \frac{G m_1 m_2}{(\vec{r}_{12})^2} \frac{\vec{r}_{12}}{(\vec{r}_{12})} \\ &= \frac{G m_1 m_2}{(\vec{r}_{12})^3} \vec{r}_{12} \end{aligned}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\& \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

$$\text{So, } \vec{F}_{12} = \frac{G m_1 m_2}{(r_{12})^3} (\vec{r}_2 - \vec{r}_1)$$

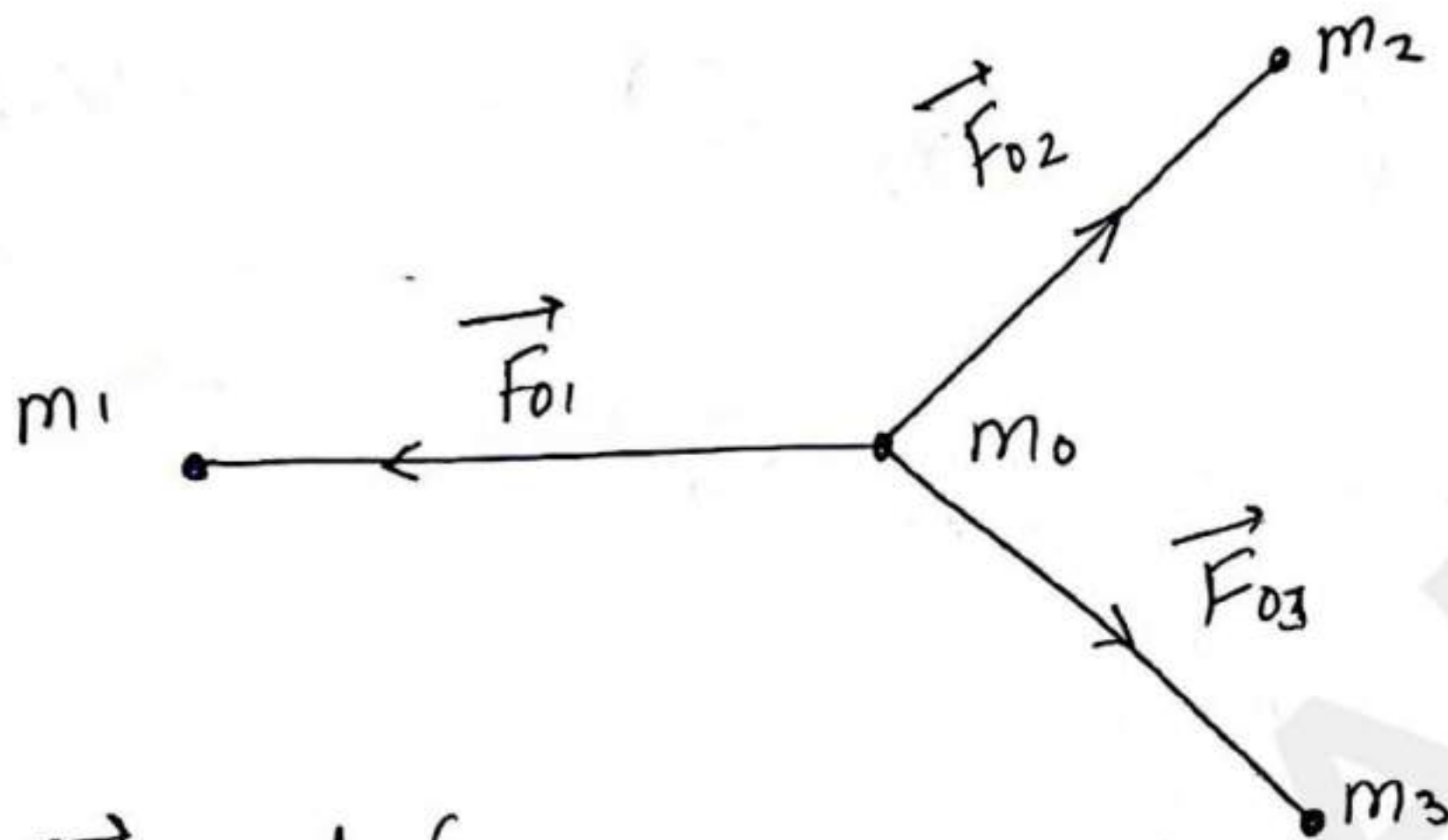
$$\boxed{\begin{aligned} |\vec{r}_{12}| &= r_{12} \\ &= |\vec{r}_{21}| \\ &= r_{21} \end{aligned}}$$

$$\vec{F}_{21} = \frac{G m_1 m_2}{(r_{21})^3} (\vec{r}_1 - \vec{r}_2)$$

$$\Rightarrow \boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

- Gravitational force is independent of nature of intervening medium.
- Gravitational force is independent of nature and size of bodies till their masses remain same and distance between their centres is fixed.
- Gravitational force acts along the line joining the centres of two bodies
- Gravitational force is conservative in nature

Principle of Superposition of Gravitation:



$\vec{F}$ = net force

$$\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n}$$

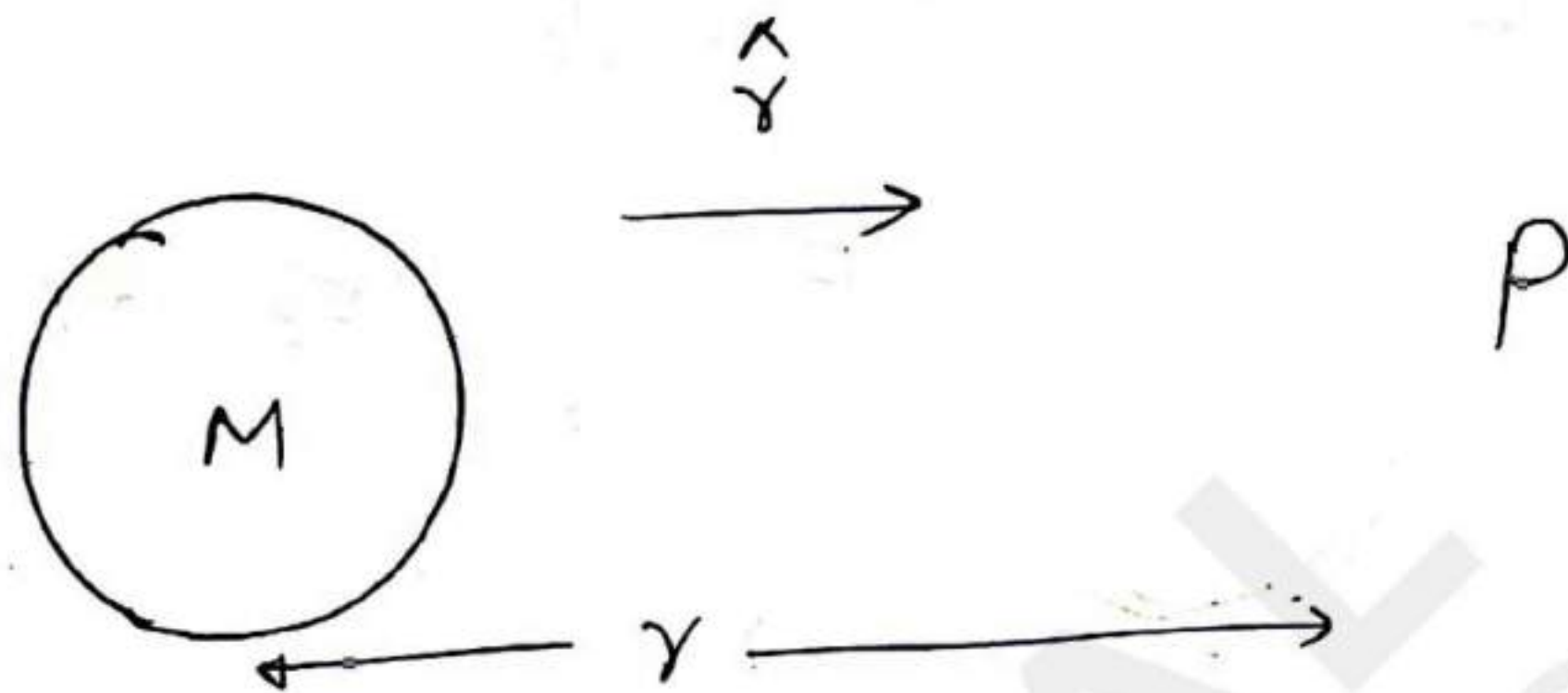
$$\vec{F} = G m_0 \left[\frac{m_1 \hat{r}_{01}}{|\vec{r}_{01}|^2} + \frac{m_2 \hat{r}_{02}}{|\vec{r}_{02}|^2} + \dots \right]$$

GRAVITATIONAL FIELD:

Region around a mass in which another mass can be influenced is field around the mass.

The magnitude at a point 'P' around Mass 'M' is given by:

$$\vec{E} = \frac{GM}{r^2} \hat{r}$$



So for mass \$m_1\$,

$$\vec{F} = \frac{GM m_1}{r^2} \hat{r}$$

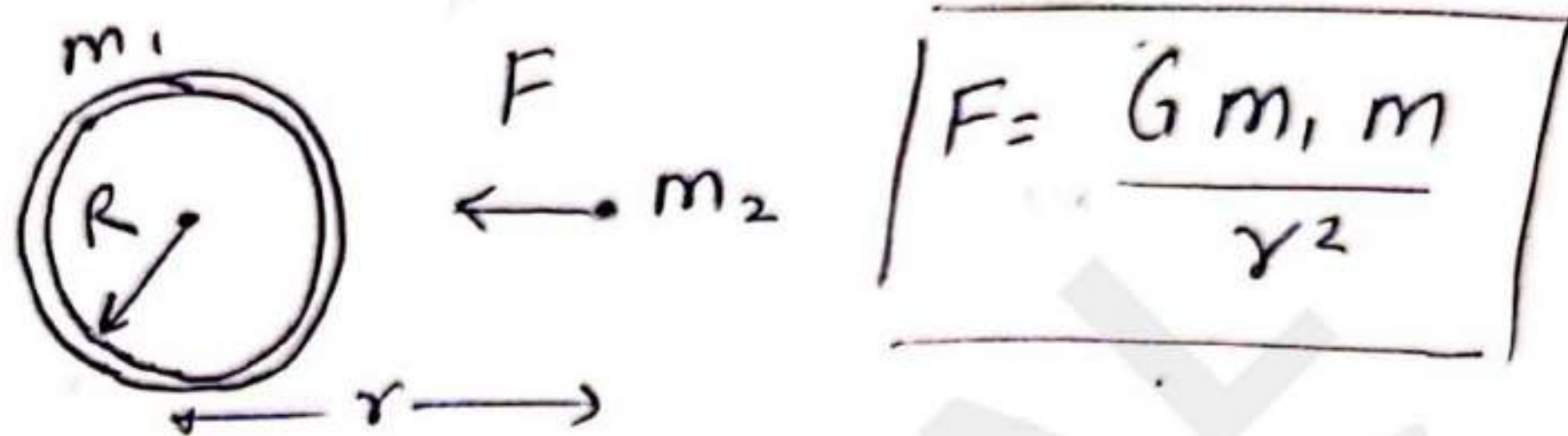
So, $\boxed{\vec{E} = \frac{\vec{F}}{m_1}} \Rightarrow \boxed{\vec{E} = \frac{GM}{r^2} \hat{r}}$

If gravitational field is due to earth at point 'P' lies on surface of earth then, \$r = R\$

$$\Rightarrow |E| = \frac{GM}{R^2} = g \quad (\text{acceleration due to gravity at surface of earth})$$

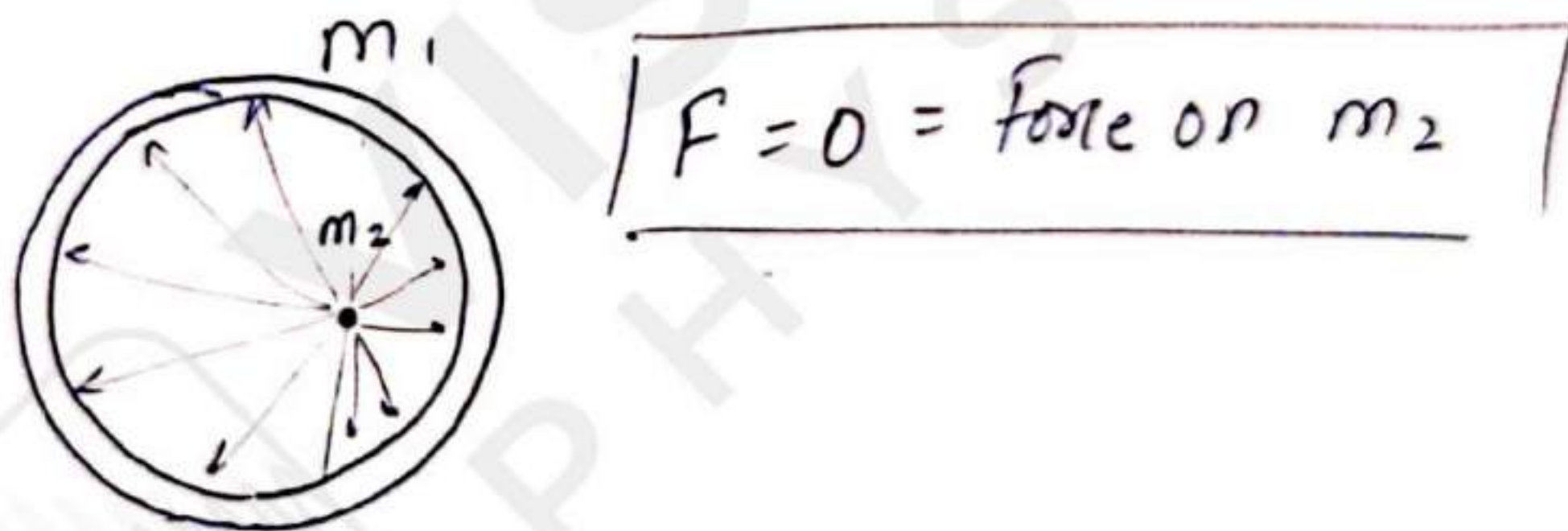
Force of attraction between hollow spherical shell of uniform density and a point mass

Case I: point mass is outside the shell.



$r \rightarrow$ from center of mass

Case II: point mass inside the shell.



$\rightarrow$ Gravitational force on a point mass situated inside spherical shell of uniform density will be zero.

* GRAVITY $\rightarrow$ force exerted by earth towards its centre on a body lying on or near to surface of earth.

* Gravitation is force of attraction acting between any two bodies of universe. Gravity is earth's gravitational pull on body lying on near the surface of earth.

Acceleration due to Gravity:

Acceleration due to gravity is defined as force of gravity acting on a unit mass of the body placed near the surface of the earth.

$$g = \frac{F}{m} = \frac{GMm}{R^2 m}$$

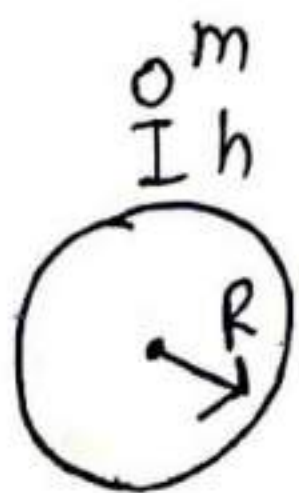
$$g = \frac{GM}{R^2}$$

$R \rightarrow$ Radius of earth
 $M \rightarrow$ Mass of earth

Variation of Acceleration due to Gravity:

I. Effect of Altitude:

$$g_1 = \frac{F}{m} = \frac{GMm}{(R+h)^2 m}$$



$$g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{GM R^2}{(R+h)^2 GM} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{1}{\left(1 + \frac{h}{R}\right)^2} = \left(1 + \frac{h}{R}\right)^{-2}$$

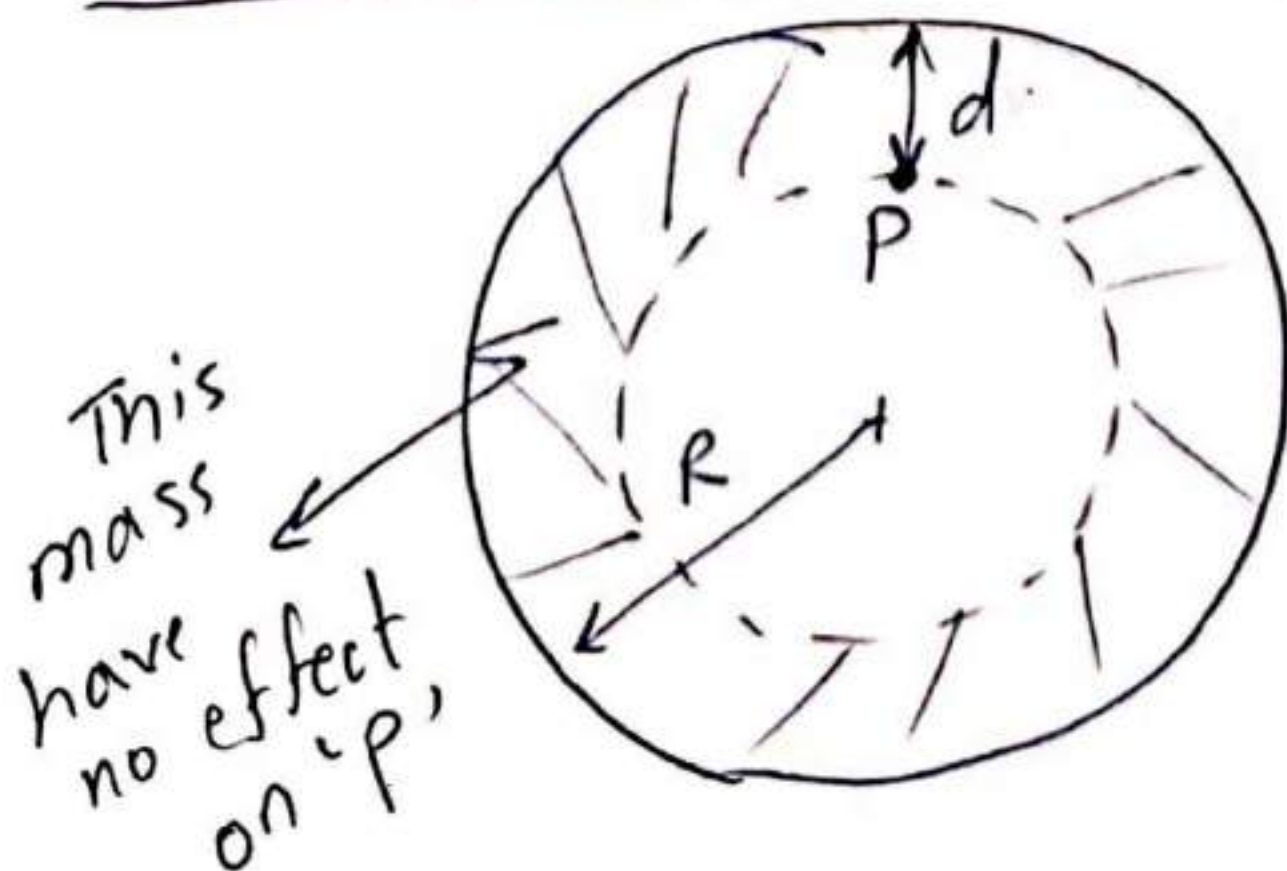
if and only if $h \ll R$

$$\left(1 + \frac{h}{R}\right)^{-2} \rightarrow 1 - \frac{2h}{R}$$

so, $\frac{g'}{g} = \left(1 - \frac{2h}{R}\right)$ [when $h \ll R$]

$\Rightarrow g' = g \left(1 - \frac{2h}{R}\right)$ $\Rightarrow$ only valid when $h \ll R$

Effect of Depth



$M \rightarrow$ net mass

$$g = \frac{GM}{R^2}$$

$$M = \frac{4}{3} \pi R^3 \rho \rightarrow \text{density of earth (assumed uniform)}$$

M' = Mass of unshaded part

$$M' = \frac{4}{3} \pi (R-d)^3 \rho$$

$$M' = \frac{4}{3} \pi (R-d)^3 \frac{M}{\frac{4}{3} \pi R^3}$$

$$\boxed{M' = \frac{M (R-d)^3}{R^3}}$$

$$g'' = \frac{G M'}{(R-d)^2} = \frac{G M (R-d)^3}{(R-d)^2 R^3}$$

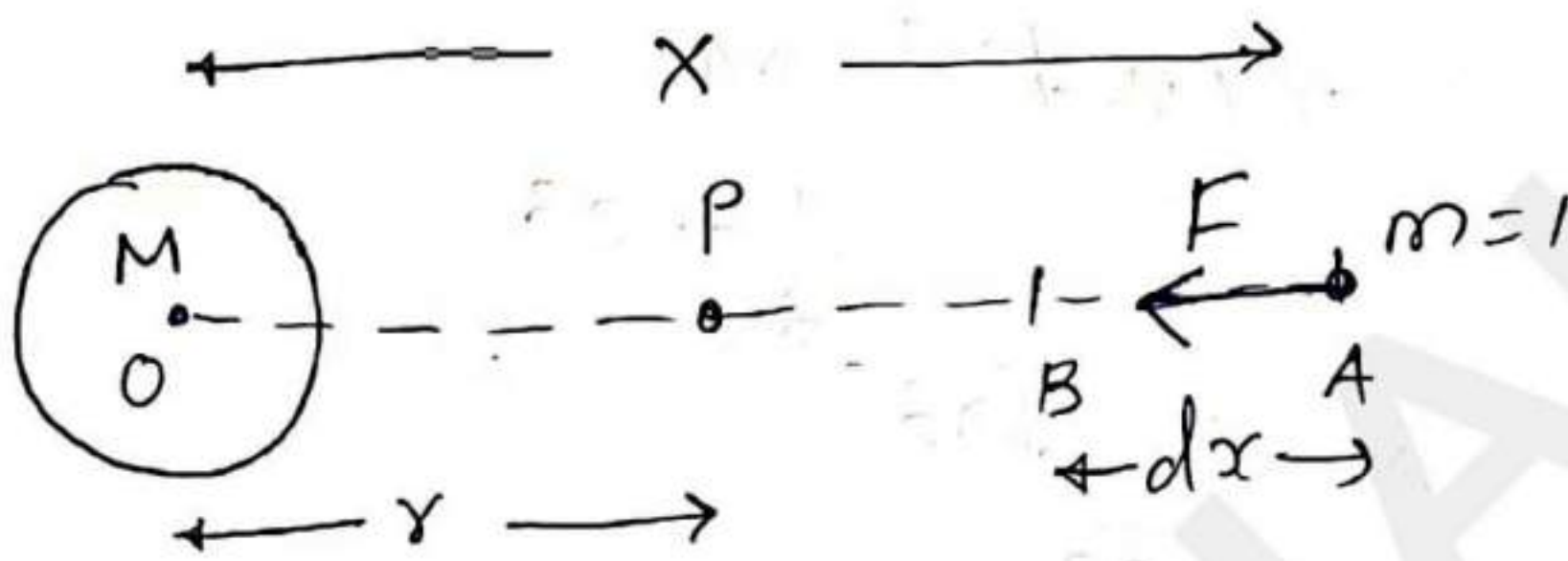
$$= \left(\frac{G M}{R^2} \right) \frac{(R-d)}{R}$$

$$\boxed{g'' = g \left(1 - \frac{d}{R} \right)}$$

Valid for any d.

GRAVITATIONAL POTENTIAL:

Work done in bringing a unit mass from infinite to gravitational field of body.



$$F = \frac{GM(1)}{x^2} = \frac{GM}{x^2}$$

$$\int_{\infty}^r dw = F dx = \int_{\infty}^r \frac{GM}{x^2} dx$$

$$W = \int_{\infty}^r \frac{GM}{x^2} dx = -GM \left[\frac{1}{x} - \frac{1}{\infty} \right]$$

$$W = -\frac{GM}{r}$$

$$V_p = \text{potential at point 'P'} = -\frac{GM}{r}$$

Gravitational potential at a point due to earth:

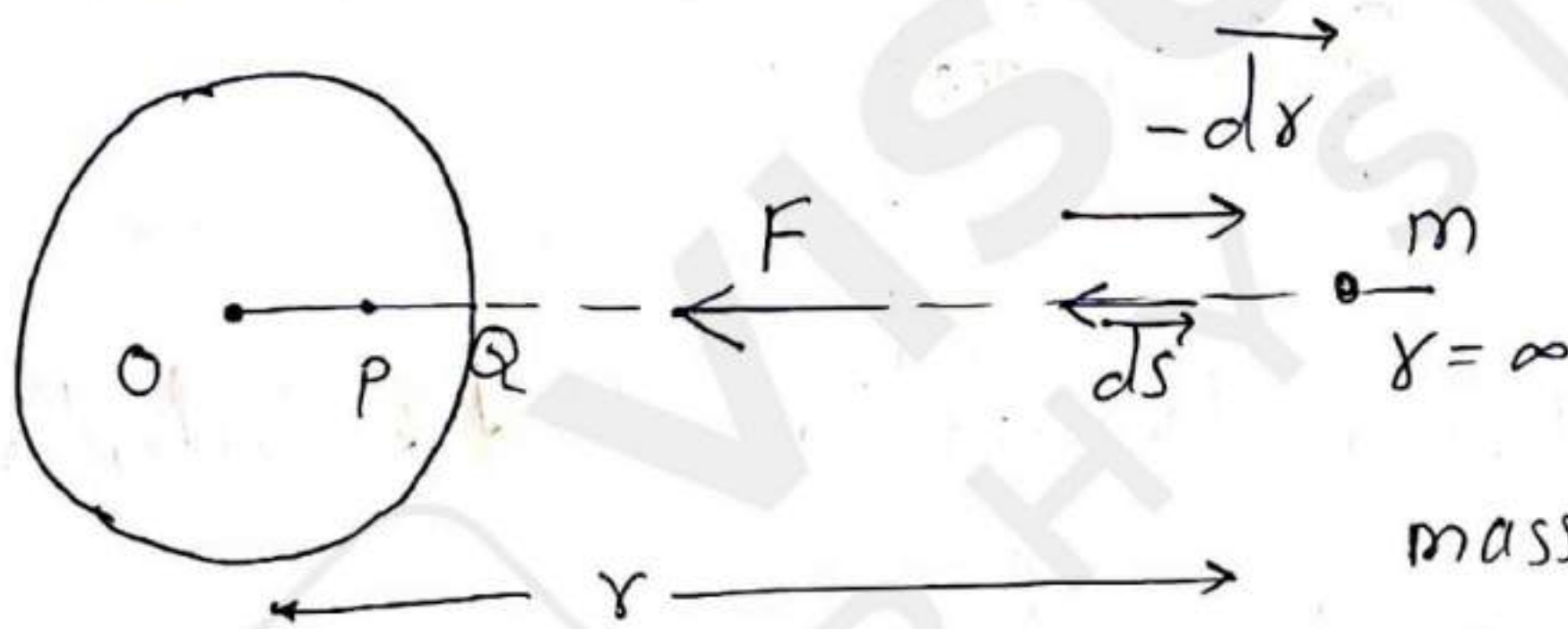
Outside the earth:

$$r > R$$

$$\boxed{V = -\frac{GM}{r}}$$

→ as earth have mass M ,
similar in derivation as we
just did

Inside the earth:



$$V_p = \frac{[W_{\infty \rightarrow P}]}{m}$$

$$V_p = + \left[\frac{W_{\infty \rightarrow Q} + W_{Q \rightarrow P}}{m} \right]$$

$$= + \frac{1}{m} \left[\int_{\infty}^R \frac{GMm}{r^2} dr + \int_R^r \frac{GMm}{R^3} r(dr) \right]$$

$$\boxed{V_p = -GM \left[\frac{3R^2 - r^2}{2R^3} \right]}$$

mass of ' r ' radius

$$\rho \times \frac{4}{3} \pi r^3$$
$$\boxed{M' = \frac{Mr^3}{R^3}}$$

$$\rightarrow V = -\frac{GM}{r}, r > R$$

$$\rightarrow V = -\frac{GM}{R}, r = R$$

$$\rightarrow V = -GM \left[\frac{3R^2 - r^2}{2R^3} \right] \quad r < R$$

$$r = 0$$

$$\rightarrow V = -\frac{3GM}{2R}$$

$$\boxed{V_{\text{centre}} = \frac{3}{2} V_{\text{surface}}}$$

Relation between Gravitational Field & potential:

$$\boxed{E = -\frac{dV}{dr}}$$

Gravitational potential energy = work done in bringing 'm' from $\infty \rightarrow P$.

$$\Rightarrow \boxed{U_P = mV_P}$$

$$\text{or } \boxed{U_P = -\frac{GMm}{r}}$$

→ negative sign shows potential energy is due to attractive gravitational force. system is bounded.

→ for earth, $r_1 = R$, $r_2 = R+h$

$$\Delta U = \underbrace{-\frac{GMm}{(R+h)}}_{\text{final}} - \underbrace{\left(-\frac{GMm}{R}\right)}_{\text{Initial}}$$

change in potential energy

$$\Delta U = \frac{GMm}{R} \left[1 - \frac{R}{(R+h)} \right]$$
$$= \frac{GMm}{R} \left[1 - \frac{1}{(1+h/R)} \right]$$

if $h \ll R$

$$\Rightarrow \Delta U = \frac{GMm}{R} \left[1 - \left(1 - \frac{h}{R} \right) \right]$$

$$\Delta U = \left(\frac{GMm}{R^2} \right) h$$

$$\boxed{\Delta U = m \left(\frac{GM}{R^2} \right) h = mgh}$$

if there are $m_1, m_2, \dots, m_n$ masses,
so U of m_0 in this system will be

$$U = - \left[\frac{G m_1 m_0}{r_{01}} + \frac{G m_2 m_0}{r_{02}} + \dots \right]$$

distance b/w
 m_1 & m_0

distance b/w
 m_2 & m_0

→ if there are three particles, m_1, m_2, m_3
 U of system will be:

$$U = - \left[\frac{G m_1 m_2}{r_{12}} + \frac{G m_2 m_3}{r_{23}} + \frac{G m_1 m_3}{r_{13}} \right]$$

* for isolated system, gravitational potential energy for all possible pairs of its constituent particle.

→ This is superposition principle

ESCAPE VELOCITY:

↳ minimum speed required to project, so that it never return to the surface is called Escape velocity.

means, $r_f \rightarrow \infty$ [no potential or say Gravitation effect]

$r_i \rightarrow R$
↳ Radius of earth

⇒ All from mechanical energy conservation

$$E_i = E_f$$

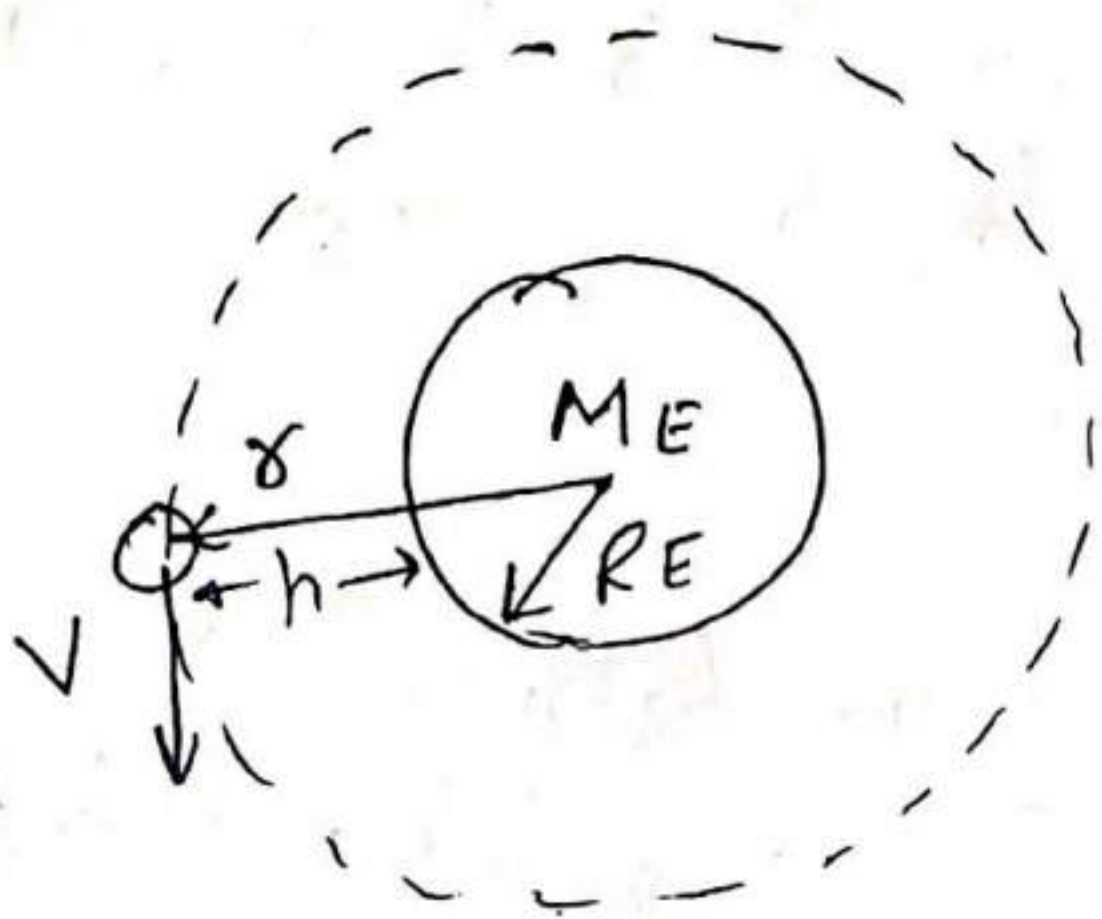
$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{\infty} + 0$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2GM}{R}}}$$

on putting values

$$\boxed{v = 11.2 \text{ km/s}}$$

ORBITAL SPEED of satellite :



as Centripetal force is provided by Gravitational force

$$\Rightarrow \frac{mv^2}{r} = \frac{GM_E m}{r^2}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

$$g = \frac{GM_E}{R_E^2}$$

$$\Rightarrow v = \sqrt{\frac{GM_E}{R_E^2} \frac{R_E^2}{r}} = R_E \sqrt{\frac{g}{r}}$$

$$v = R_E \sqrt{\frac{g}{r + R_E}}$$

→ Independent of mass of satellite
→ Depend on mass of planet & Radius

Time period of satellite :

$$T = \frac{2\pi r}{v} = \frac{\text{distance}}{\text{orbital velocity}}$$

$$\boxed{T = \frac{2\pi r}{R} \sqrt{\frac{r}{g}} = \frac{2\pi}{R} \sqrt{\frac{r^3}{g}} \Rightarrow T^2 \propto r^3}$$

$$M = \frac{4}{3}\pi R^3 \rho, \quad g = \frac{GM}{R^2} = \frac{G}{R^2} \left(\frac{4}{3}\pi R^3 \rho \right)$$

$$g = \frac{4\pi \rho R G}{3}$$

$$T = \frac{2\pi}{R} \sqrt{\frac{3(R+h)^3}{4\pi \rho R G}}$$

$$\boxed{T = \sqrt{\frac{3\pi (R+h)^3}{G \rho R^3}}}$$

for $h \ll R$

$$h+R \rightarrow R$$

$$\boxed{T = \sqrt{\frac{3\pi}{G \rho}} \quad \text{or} \quad 2\pi \sqrt{\frac{R}{g}}}$$

Altitude or Height of satellite

$$T^2 = \frac{4\pi^2 (R+h)^3}{R^2 g} = \frac{4\pi^2 r^3}{R^2 g}$$

$$\Rightarrow (R+h)^3 = \frac{T^2 R^2 g}{4\pi^2}$$

$$R+h = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{1/3}$$

$$h = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{1/3} - R$$

Angular momentum, L

$$L = mvr = mr \sqrt{\frac{GM}{r}}$$
$$L = \left(m^2 G M r \right)^{1/2}$$

↙
dependant on M, m, r

Energy of a satellite:

↳ Kinetic Energy + potential Energy

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{GM}{r} \right)$$

$$\& \quad U = - \frac{GMm}{r}$$

SO, $E = K + U$

$$E = - \frac{GMm}{2r}$$

★ → That means system is bounded as total energy is negative

GEOSTATIONARY OR GEOSYNCHRONOUS:

↳ satellite that appears to be fixed at a definite heights.

→ This satellite revolves around the earth with same angular speed in the same direction as done by earth.

$$h = \left(\frac{T^2 R^2 g}{4\pi^2} \right)^{1/3} - R$$

$$R = 6.4 \times 10^6 \text{ m}, \quad g = 9.8 \text{ m/s}^2, \quad T = 24 \text{ h}$$

$$T = 24 \times 60 \times 60 \text{ s}$$

on solving we get

$$h = 3.6 \times 10^7 \text{ m}$$

$$\rightarrow \text{for this } v = 3.1 \text{ km/s}$$

→ Sense of rotation is same as that of earth.

→ It should revolve in an orbit concentric and coplanar with equatorial plane.