VISUAL PHYSICS

# SHORT NOTES

CHAPTER

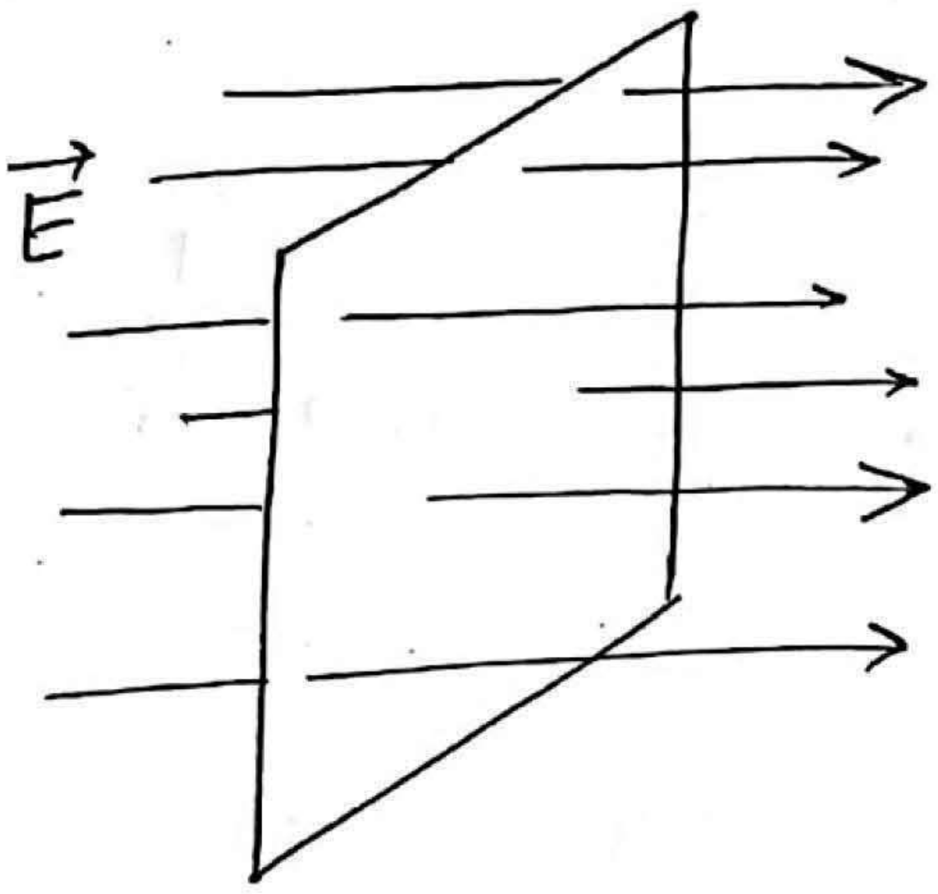
# Gauss's Law

Available at:

9999021287

www.visualphysics.in

# ELECTRIC FLUX
# &
# GAUSS LAW

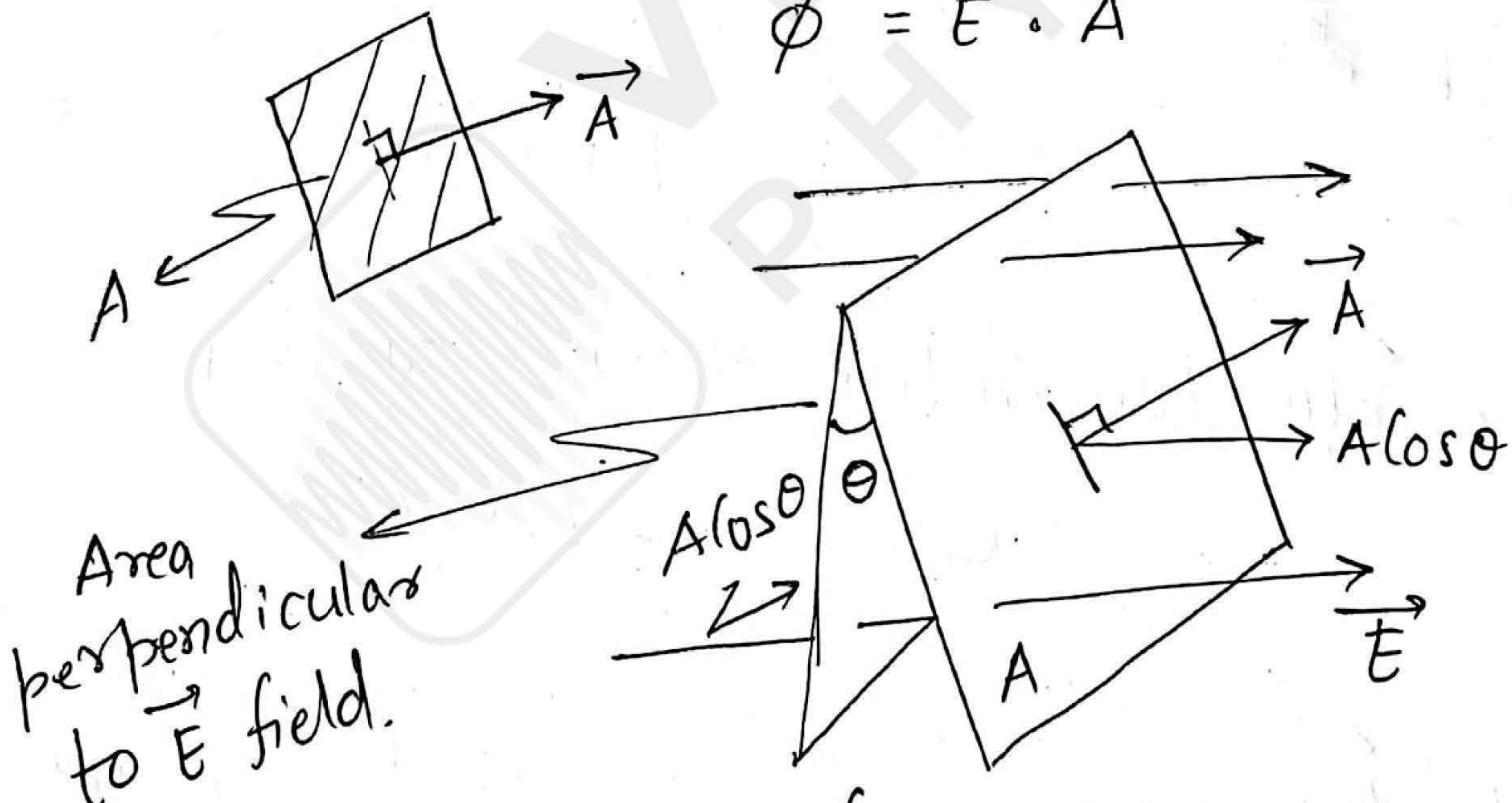



$\phi = \vec{E} \cdot \vec{A}$

$\phi$ → flux → no. of Electric field line passing through a given Area $A$.

Area ($\vec{A}$) is perpendicular to the surface.

$\phi = \vec{E} \cdot \vec{A}$




$\phi = EA\cos\theta$

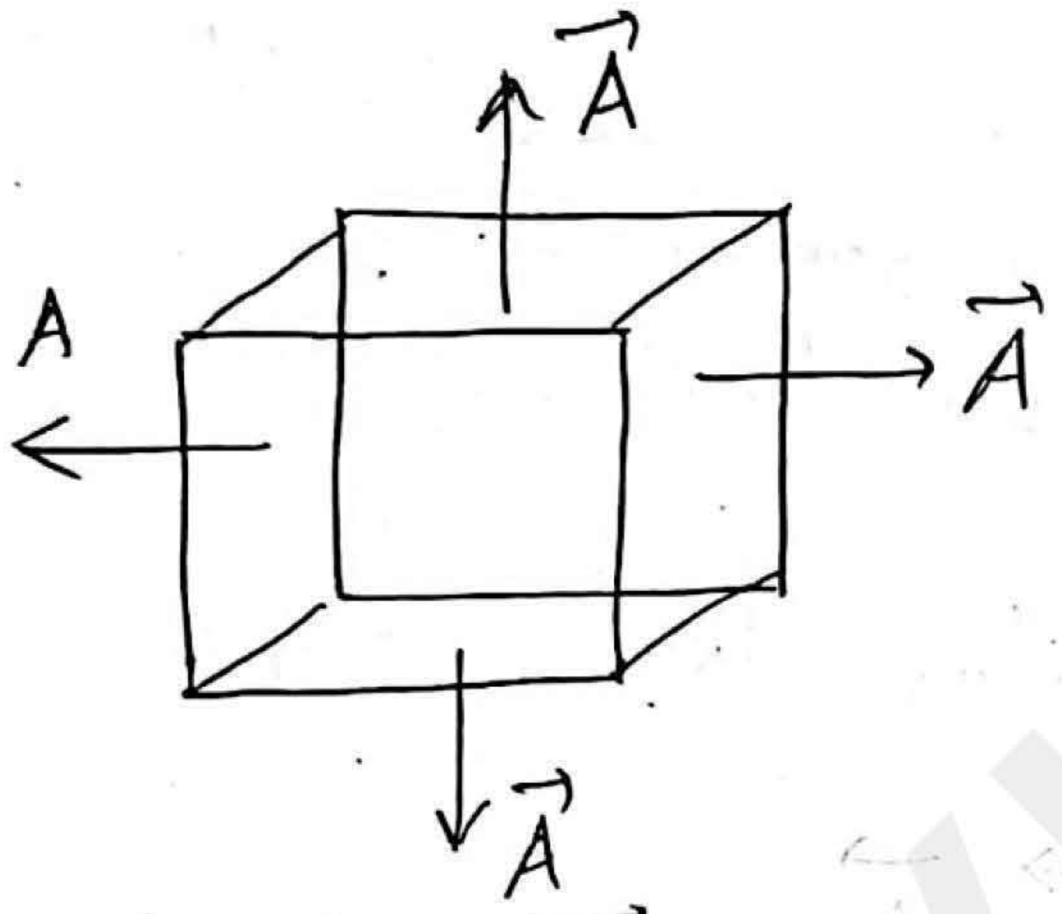
$$\boxed{\phi = \vec{E} \cdot \vec{A}}$$

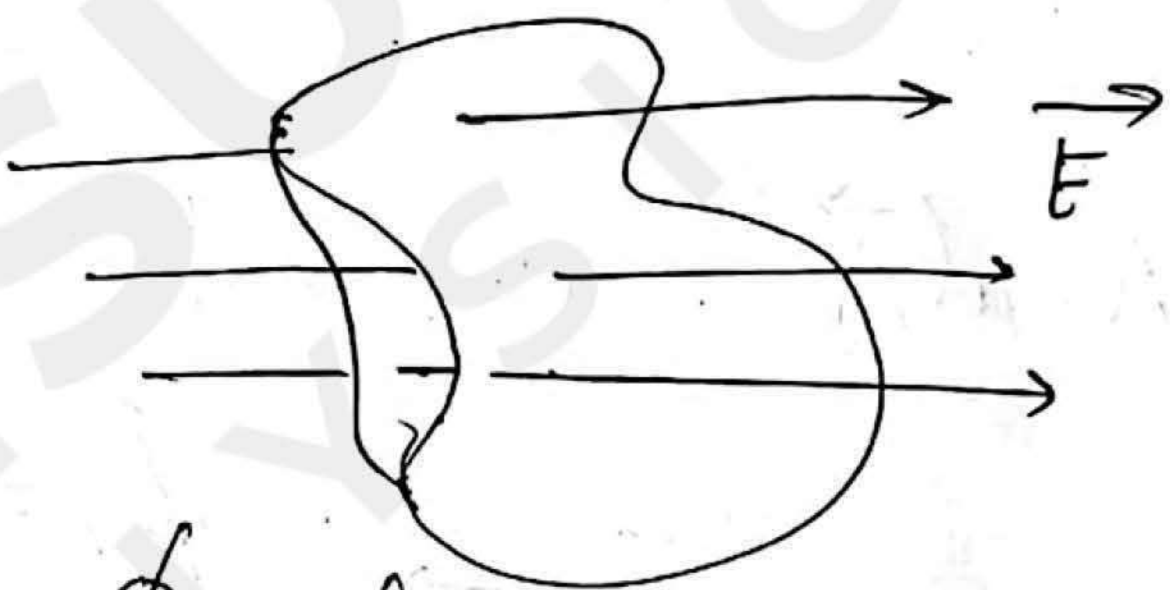
Closed surface → enclosed a volume

open surface → Just a surface and not encloses any volume

direction of $\vec{A}$ not important

for closed surface → direction of $\vec{A}$ always outwards




$\phi = \int \vec{E} \cdot \vec{dA}$

$\phi = \int \vec{E} \cdot \vec{dA}$

sum of flux from each infitestimal surfaces.

GAUSS'S LAW

Net flux through an enclosed surface (closed surface) is equal to $\frac{1}{\epsilon}$ times the net charge inside the closed surface.

$$\phi = \oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\epsilon}$$

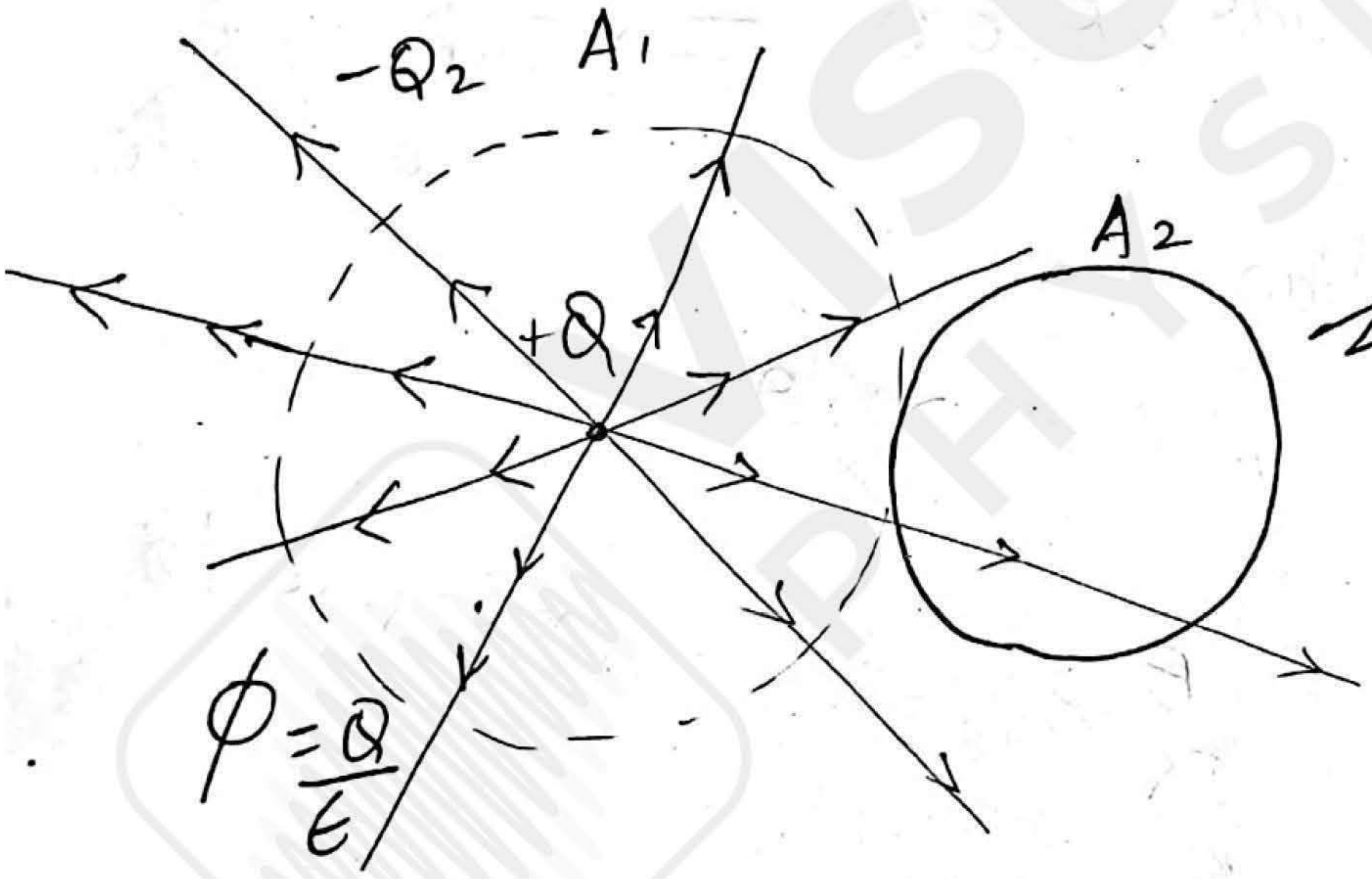
$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

closed integeral.

charge that is only inside the closed surface

$\epsilon = \epsilon_0 \epsilon_r$

→ positive flux → charge enclosed is positive
negative flux → charge enclosed is negative
Zero flux → No charge inside the surface.




$\phi = 0$ → No enclosed charged.

## Solid angle :

$$\boxed{\Omega = \frac{A}{r^2}}$$

$\Omega \to$ solid angle

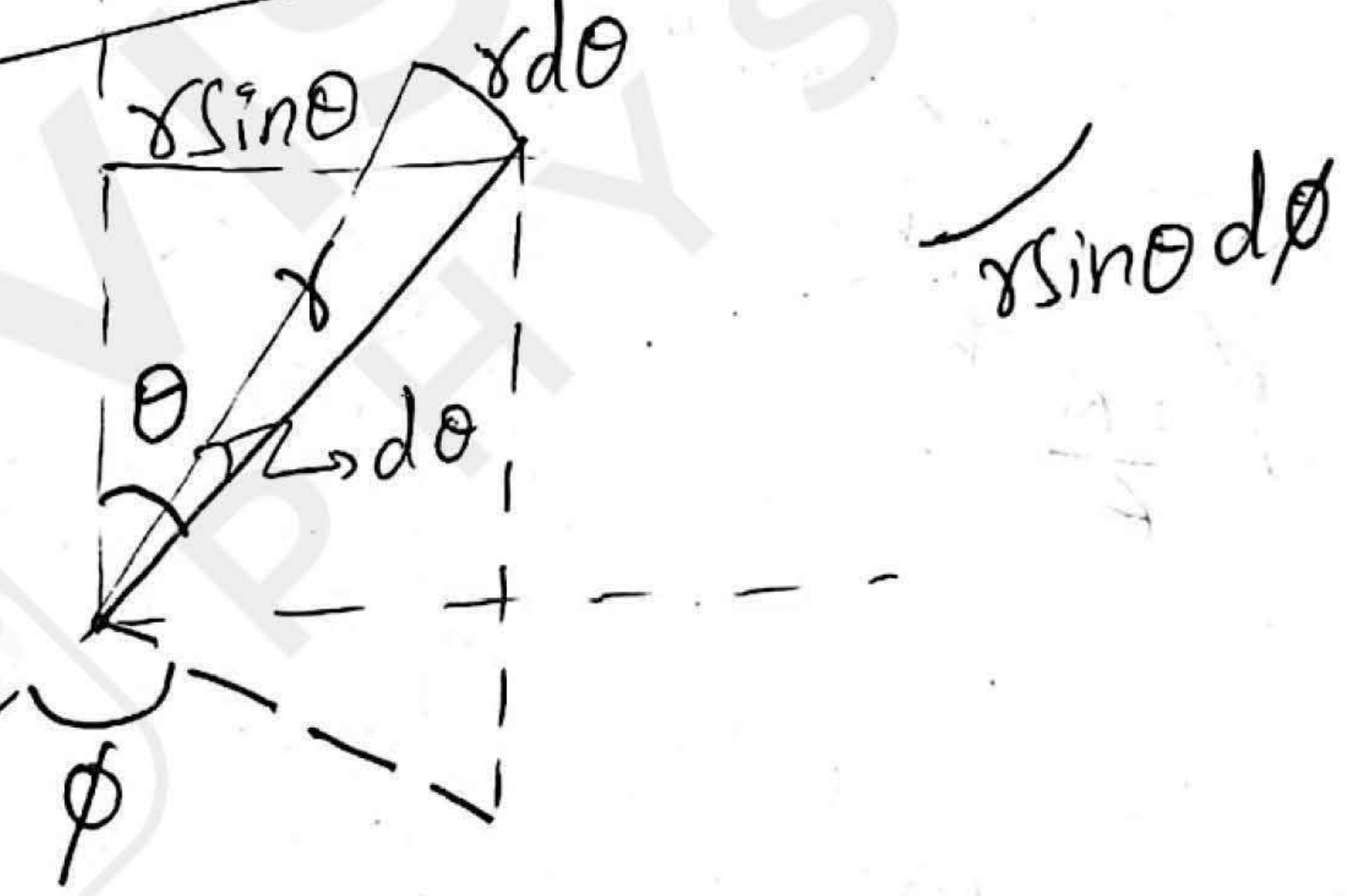
$A \to$ Projected Area

$r \to$ radius


if we take small area element.

$$\Omega = \iint_S \sin\theta \, d\theta \, d\phi \to \text{Spherical cordinate}$$

$\boxed{\text{unit} \to \text{steradian, } \underline{\text{str}}}$



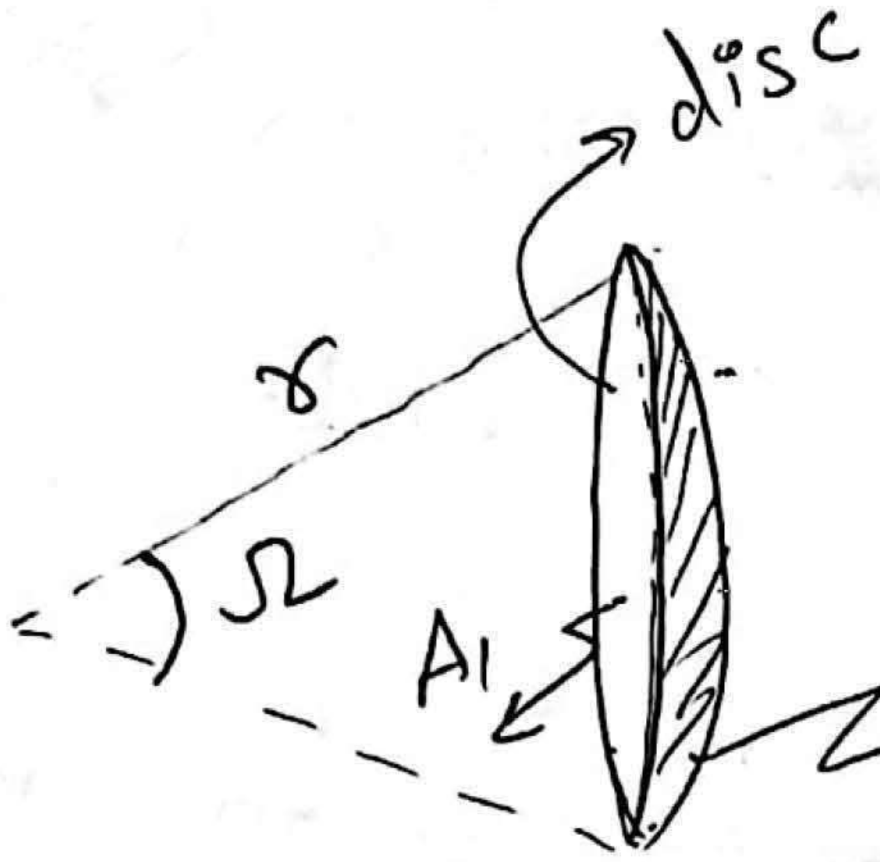

so, $dA = r\sin\theta \times r\,d\theta\,d\phi$

$\boxed{\text{total solid angle} = 4\pi}$

so, $\Omega = \iint_S \frac{dA}{r^2}$

$$\boxed{\Omega = \iint_S \sin\theta \, d\theta \, d\phi}$$

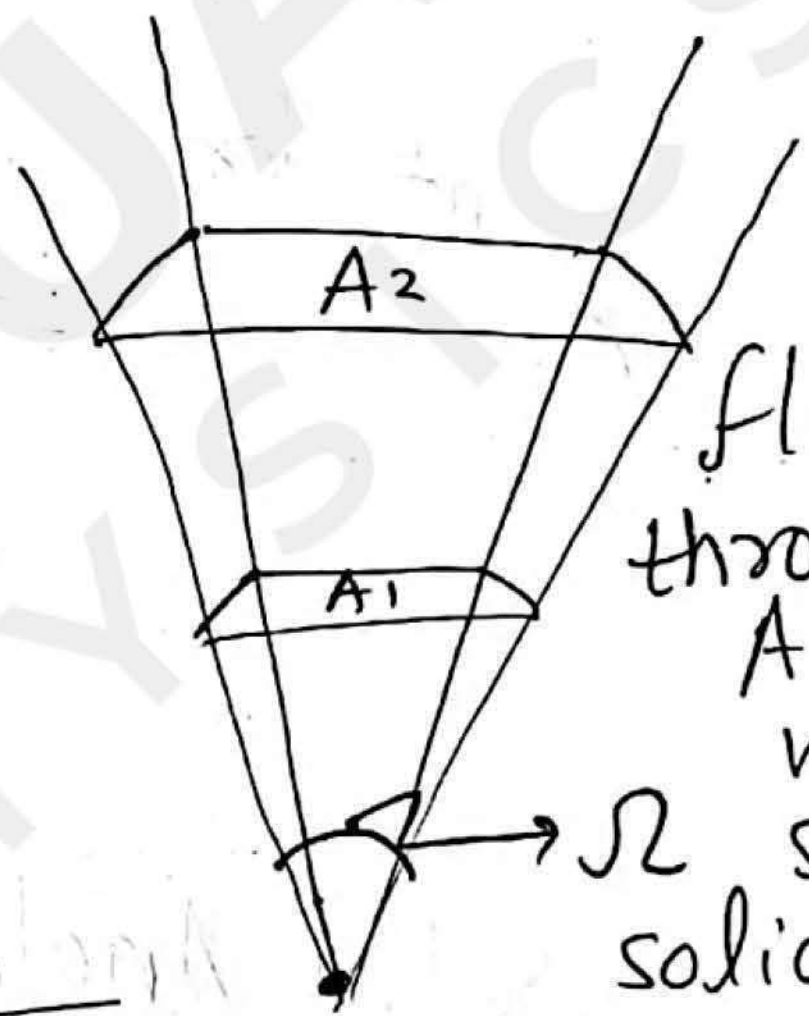
total solid angle outside the closed surface $= 0$



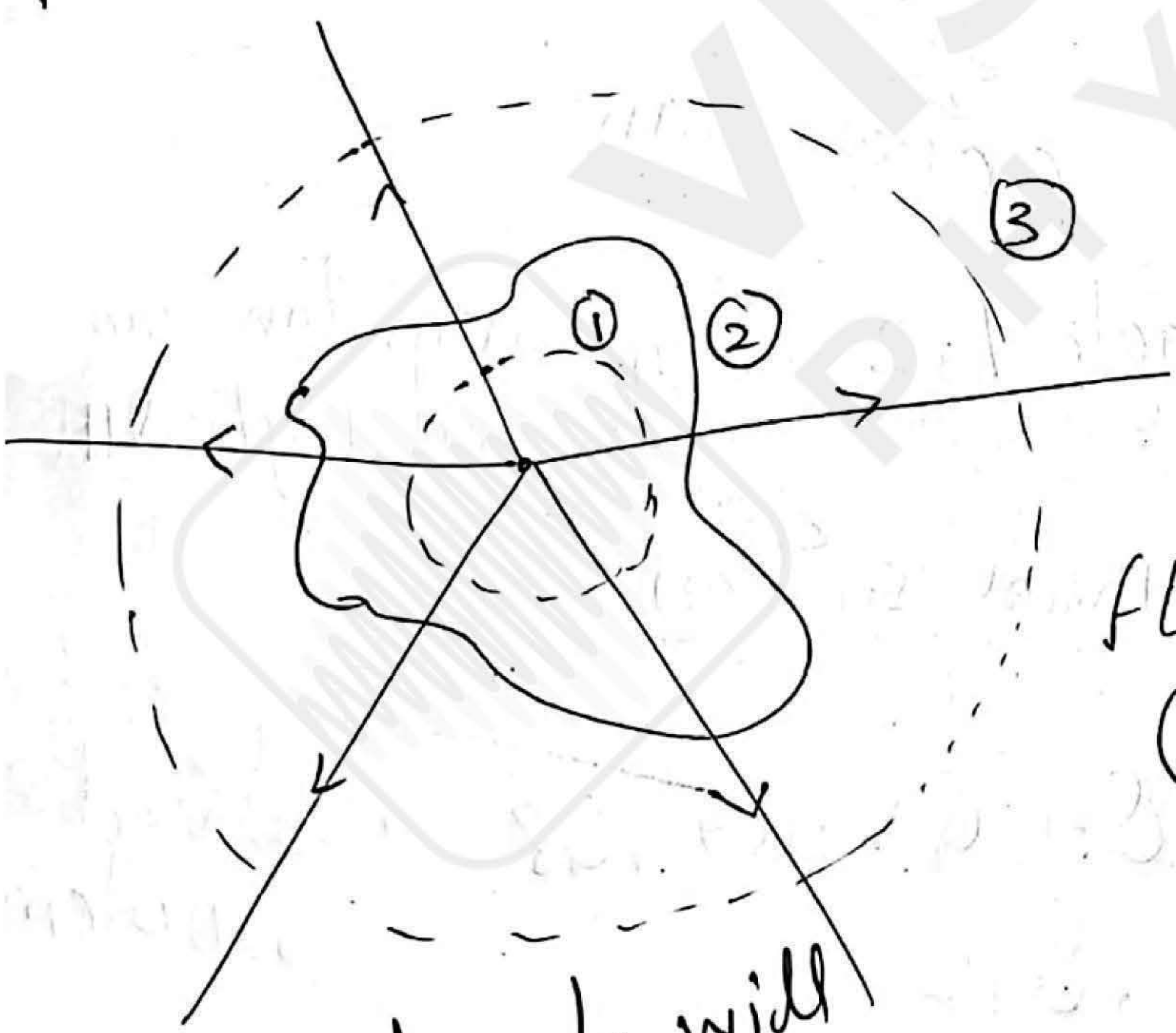

A (Area of curved part) → part of sphere with radius r

$$\Omega = \frac{A}{r^2} \checkmark \qquad \Omega = \frac{A_1}{r^2} \times$$

$$\boxed{\underset{\text{Any closed surface}}{\text{total } \Omega} = 4\pi \text{ str}}$$

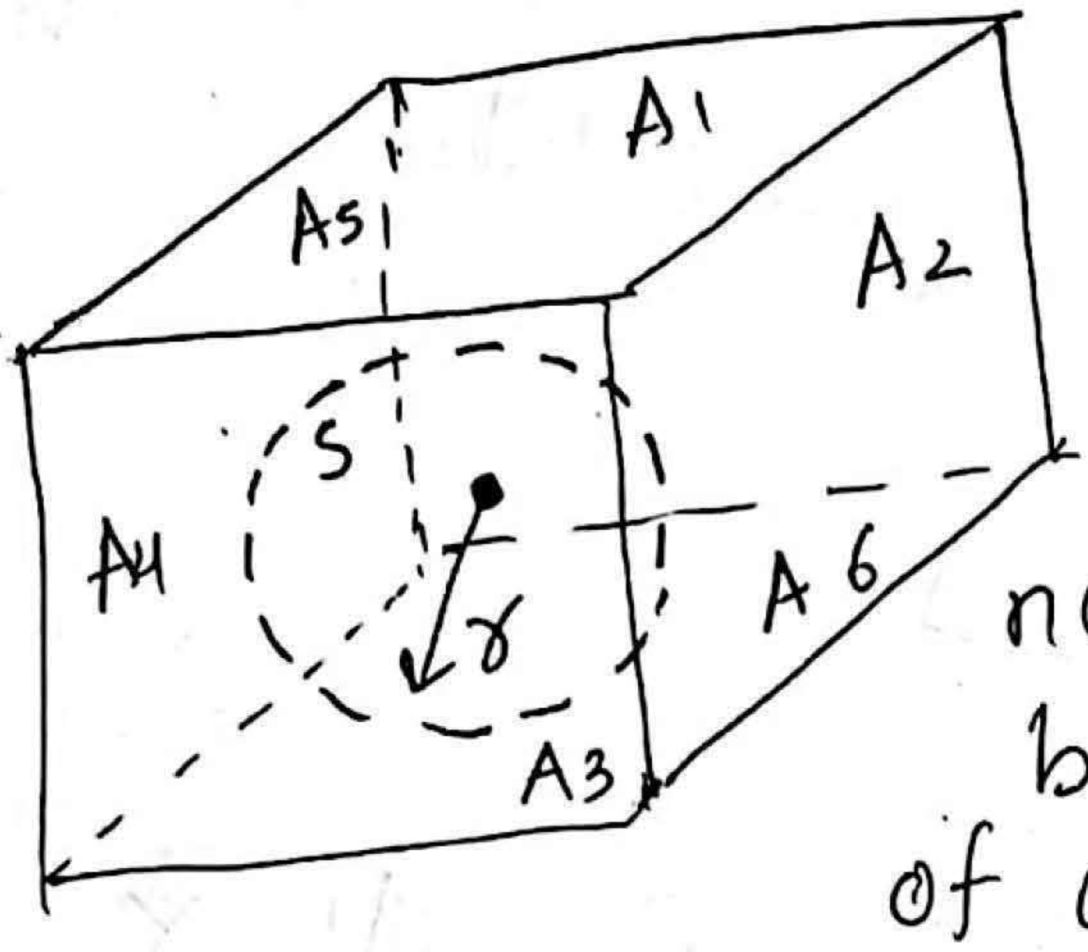



flux through Area with same solid angle is same




flux through surface (2) = flux through its projection on surface (1)

As solid angle will be same for all surfaces → which is equal to surface (3) flux




$\phi$ = flux through $A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

now same flux will be through projection of area of cube on sphere S

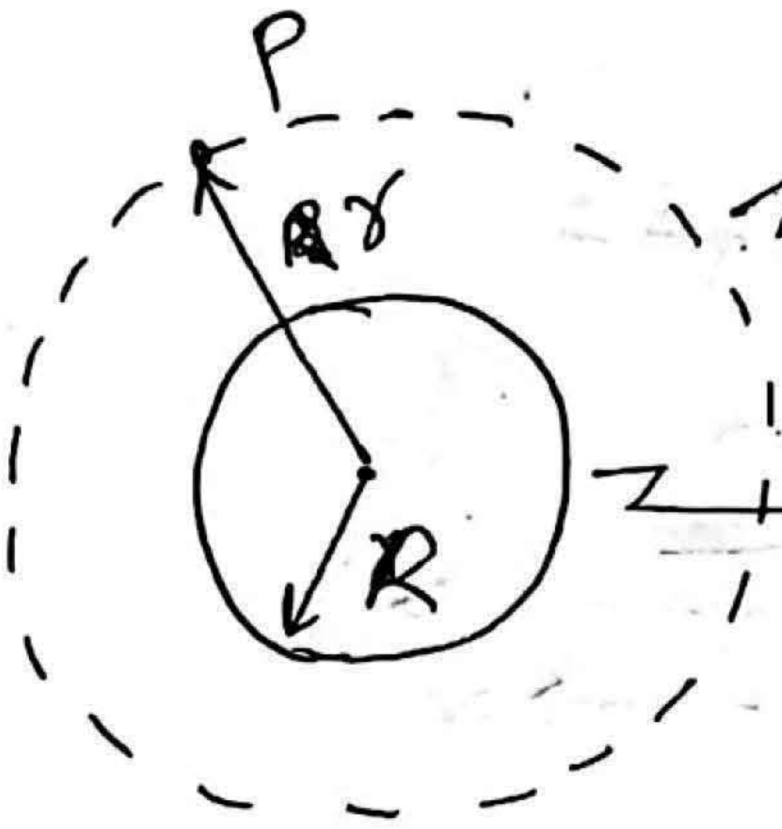
so,
$$\phi = \int_S \vec{E} \cdot \vec{ds} = E \int ds$$
$$= E \times 4\pi r^2$$
$$= \frac{Q}{4\pi r^2 \epsilon} \times 4\pi r^2$$

E is constant for given r

$$\Rightarrow \boxed{\phi = \frac{Q}{\epsilon}}$$

gauss's law

## Electric Field outside the sphere (shell) & (solid)




Gauss' surface

total charge on sphere = Q

Electric field at point 'p' is:

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q}{\epsilon} \quad \text{[gauss' law]}$$

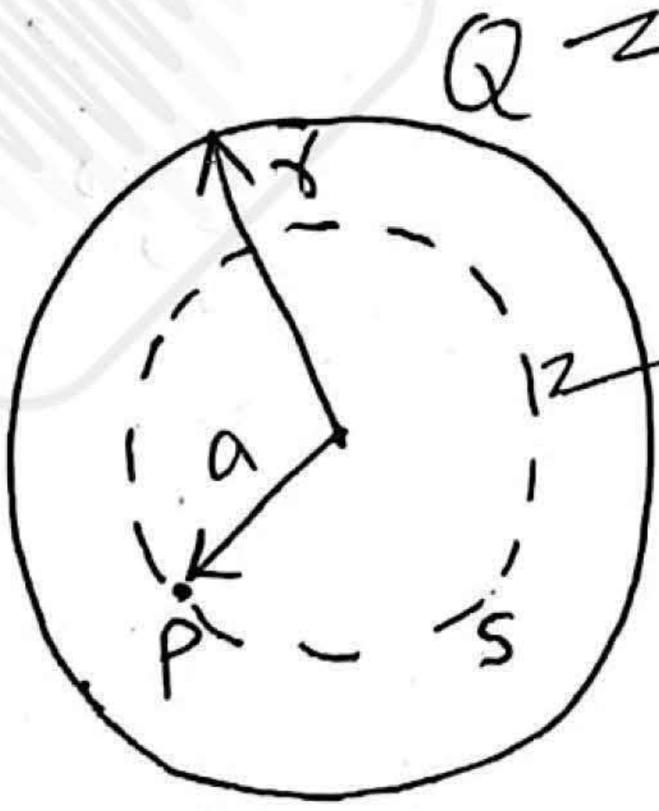
$$E \times 4\pi r^2 = \frac{Q}{\epsilon} \quad (A = 4\pi r^2)$$

$$\Rightarrow \boxed{E = \frac{Q}{4\pi r^2 \epsilon}} \quad \boxed{E = \frac{Q}{4\pi r^2 \epsilon}}$$

Same for both Shell & solid sphere

## Electric field inside the shell:

Q → total charge on sphere = Q

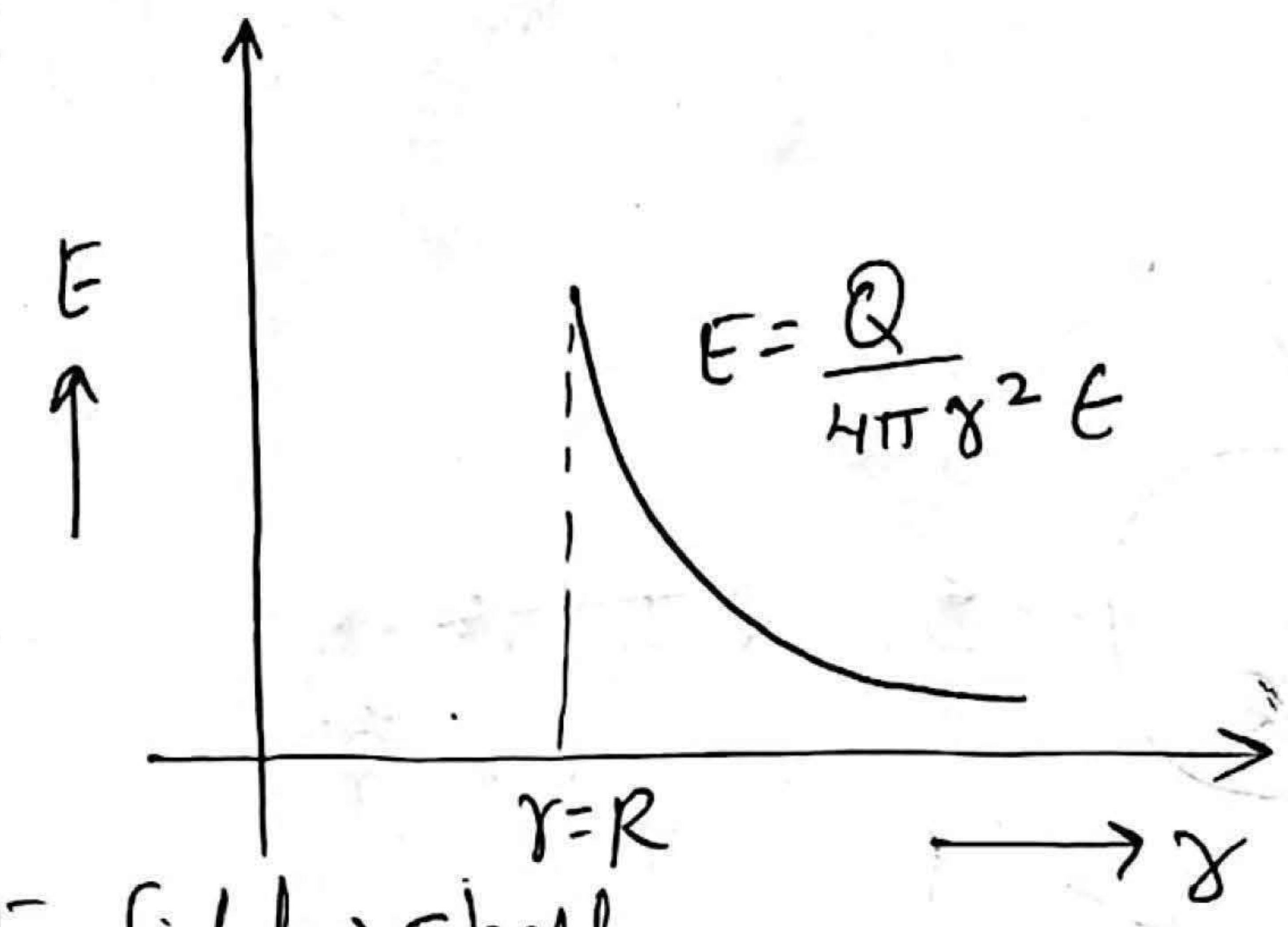



Gauss surface

$$\oint \vec{E} \cdot dA = \frac{q_{in}}{\epsilon}$$

$$E(4\pi a^2) = \frac{0}{\epsilon}$$

$$\Rightarrow \boxed{E = 0}$$



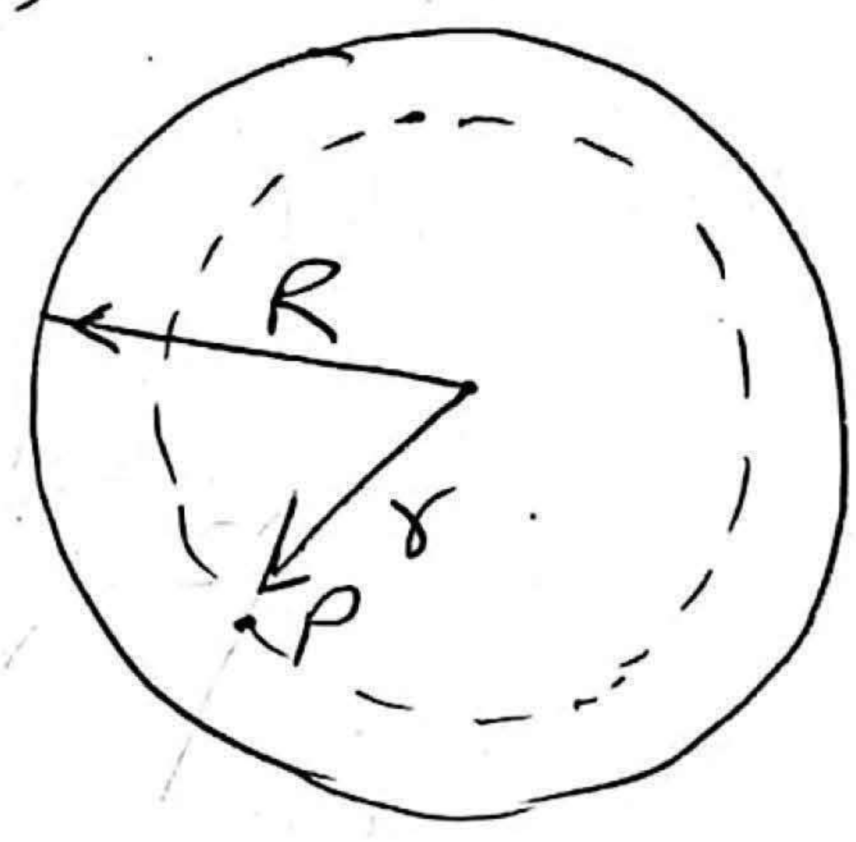

E-field → shell

## Electric field inside a solid uniformly charged sphere :

$\rho \rightarrow$ charge density $= \dfrac{Q}{\frac{4}{3}\pi R^3}$

$q_{in} \rightarrow \dfrac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \dfrac{Q r^3}{R^3}$

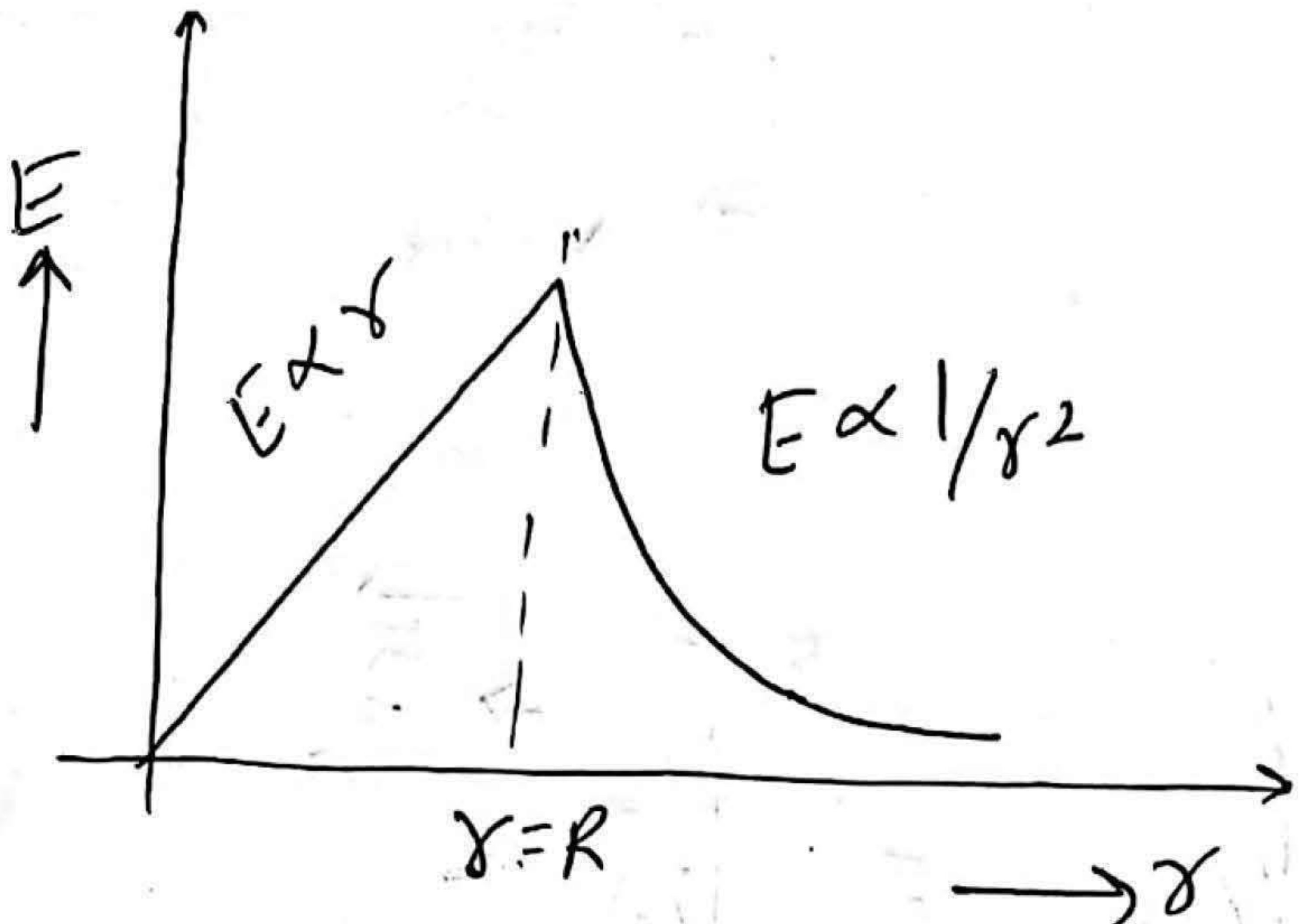
Electric field at P :




$E \times 4\pi r^2 = \dfrac{q_{in}}{\epsilon}$

$\Rightarrow E = \dfrac{Q}{4\pi R^3 \epsilon} r$

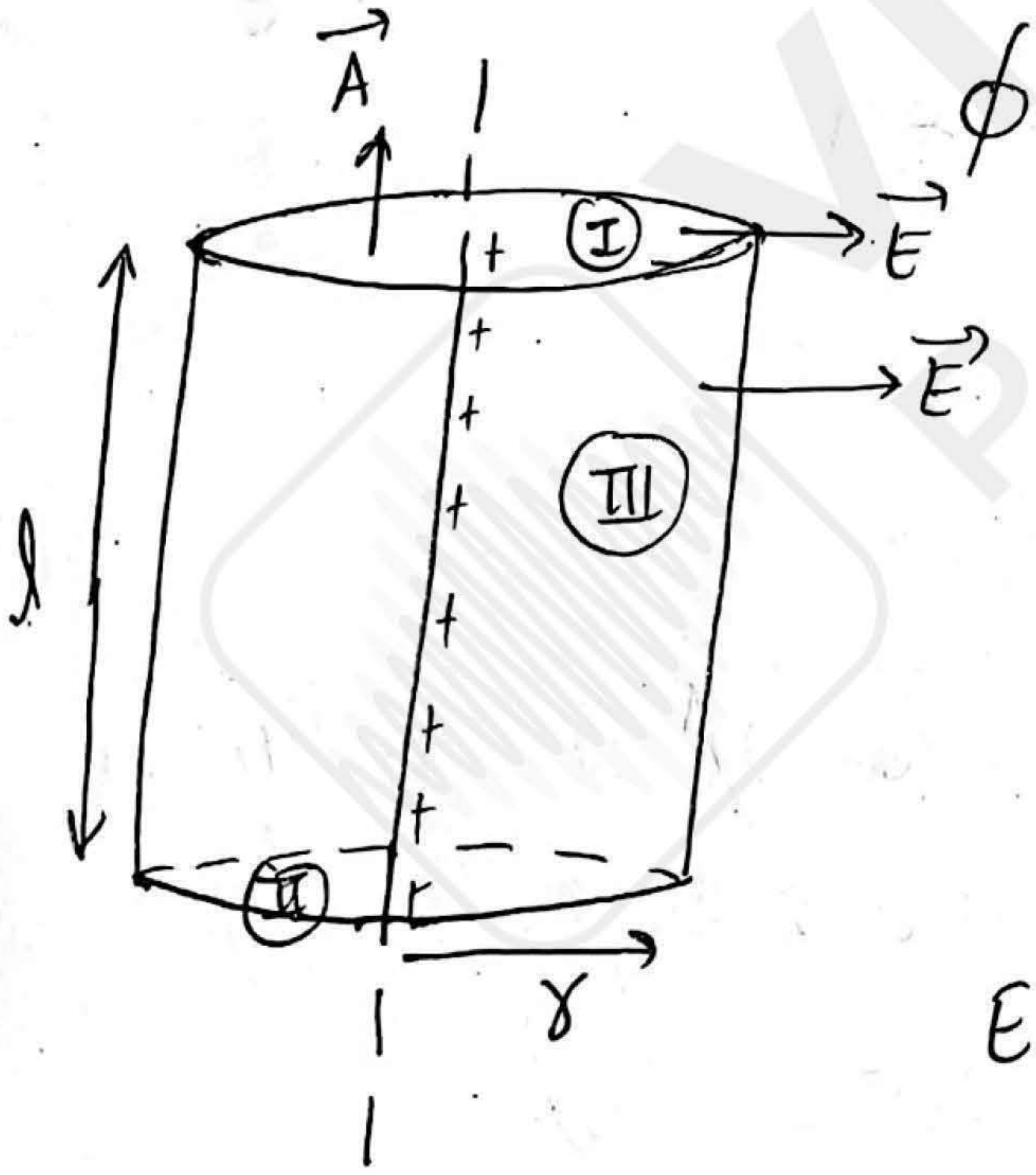
$\Rightarrow E \propto r$




E-field for solid sphere

## FIELD OF A LINE CHARGE:

$\lambda \rightarrow$ line charge density $\Rightarrow$ (assumed positive)




$$\phi = \vec{E}\cdot\vec{A}_{I} + \vec{E}\cdot\vec{A}_{II} + \vec{E}\cdot\vec{A}_{III}$$

$$= EA_{I}\cos 90 + EA_{II}\cos 90^\circ + EA\cos 0$$

$$\phi = \frac{q_{in}}{\epsilon} = \frac{\lambda l}{\epsilon}$$

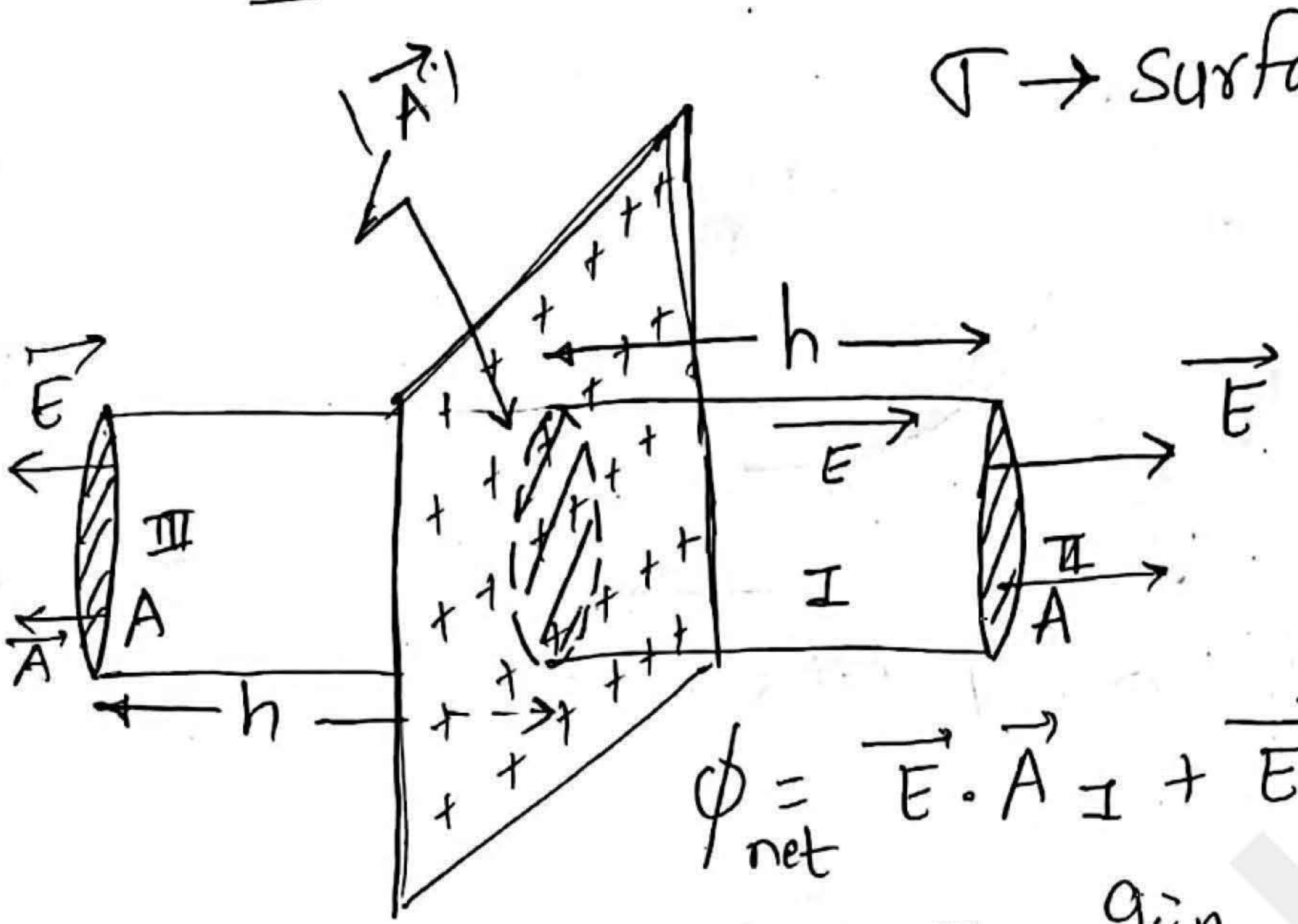
$$EA\cos 0 = \frac{\lambda l}{\epsilon}$$

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon}$$

$$\boxed{E = \frac{\lambda}{2\pi r \epsilon}}$$

## Electric field Thin sheet [uniformly charged]

$\sigma \rightarrow$ surface charge density




$$\phi_{net} = \vec{E}\cdot\vec{A}_{I} + \vec{E}\cdot\vec{A}_{II} + \vec{E}\cdot\vec{A}_{III}$$

$$= \frac{q_{in}}{\epsilon}$$

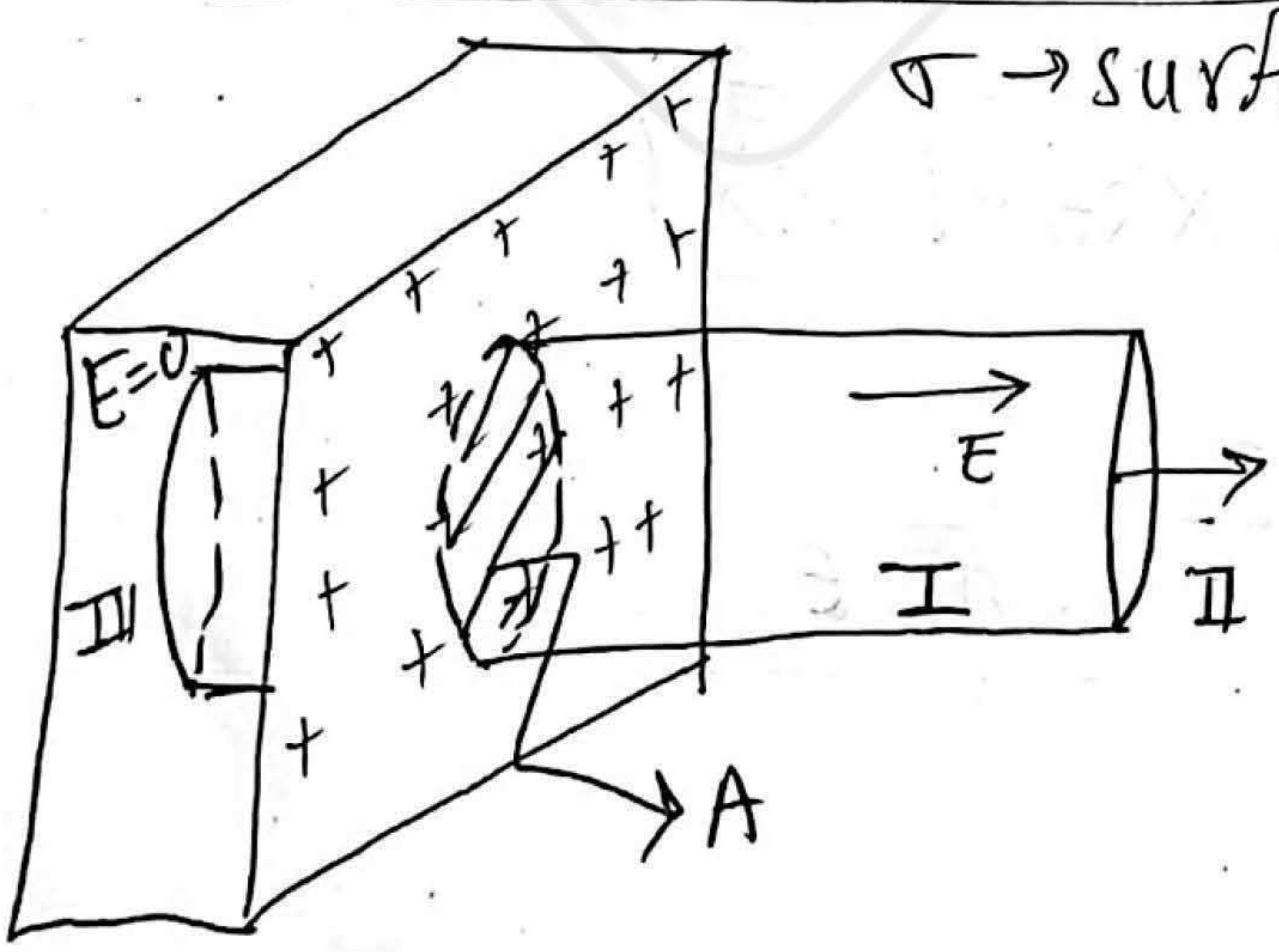
$$\Rightarrow \quad EA\cos 90 + EA\cos 0 + EA\cos 0 = \frac{\sigma A}{\epsilon}$$

$$2EA = \frac{\sigma A}{\epsilon}$$

$$\boxed{E = \frac{\sigma}{2\epsilon}}$$

## FIELD AT The surface of a Conductor:

$\sigma \rightarrow$ surface charge density




$$\phi = \frac{q_{in}}{\epsilon}$$

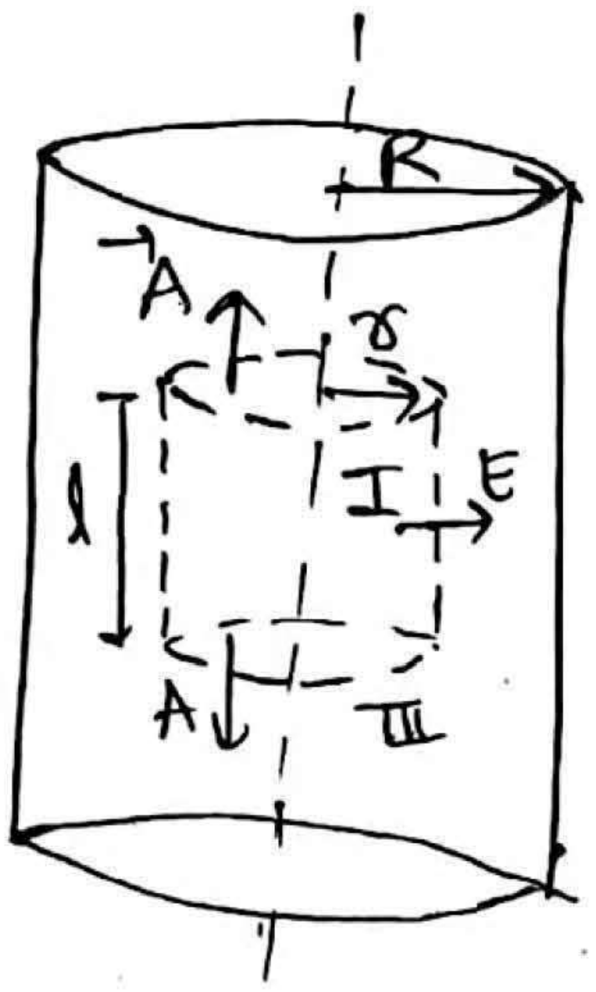
$$EA = \frac{\sigma A}{\epsilon}$$

$$\boxed{E = \frac{\sigma A}{A\epsilon} = \frac{\sigma}{\epsilon}}$$

# Electric field due to a long uniformly charged cylinder:

$\rho \rightarrow$ volume charge density

$R \rightarrow$ radius




$$q_{in} = \rho \times \pi r^2 l$$

$$\phi = q_{in}/\epsilon$$

$$\underset{\mathrm{I}}{E(2\pi r l)} + \underset{\mathrm{II,III}}{EA\cos 90} = \frac{\rho \pi r^2 l}{\epsilon}$$

$$\Rightarrow \boxed{E = \frac{\rho r}{2\epsilon}} \quad \boxed{E \propto r}$$

and outside the cylinder:

$$q_{in} = \rho \times \pi R^2 l$$

$$\Rightarrow E(2\pi r l) = \frac{\rho \pi R^2 l}{\epsilon}$$

$$E = \frac{\rho R^2}{2\epsilon r} \Rightarrow \boxed{E \propto \frac{1}{r}}$$