



SHORT NOTES

C H A P T E R

FLUIDS

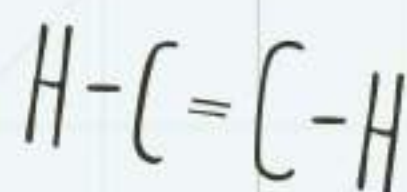
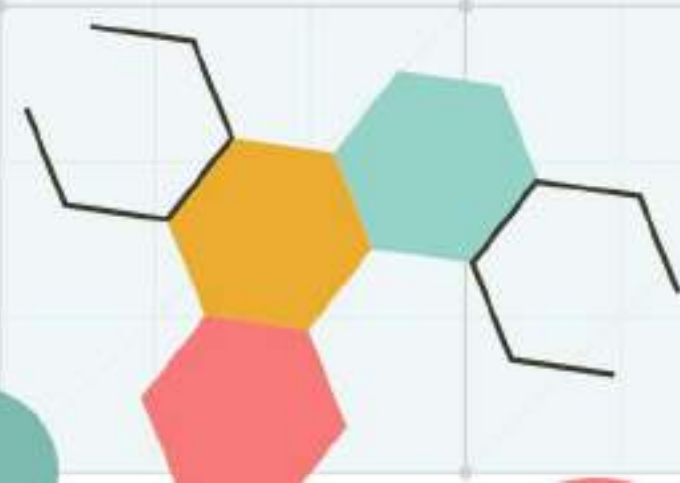
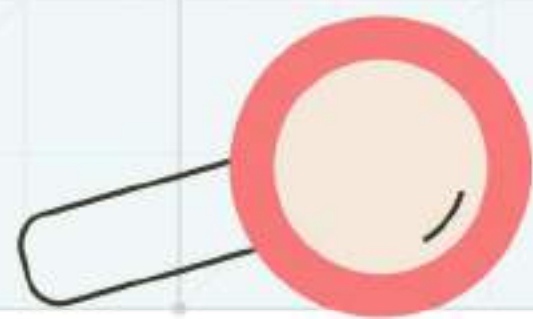
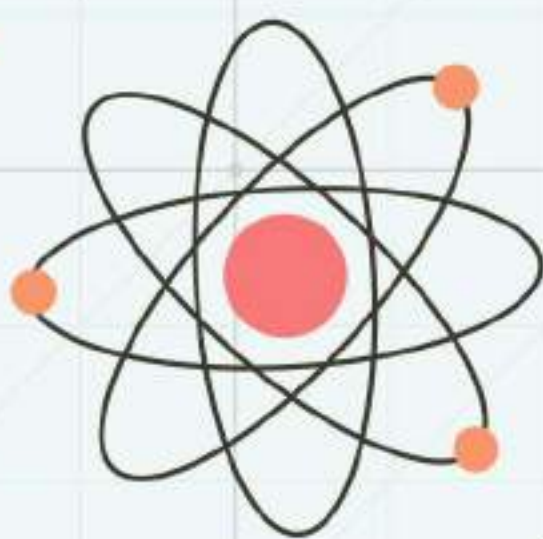
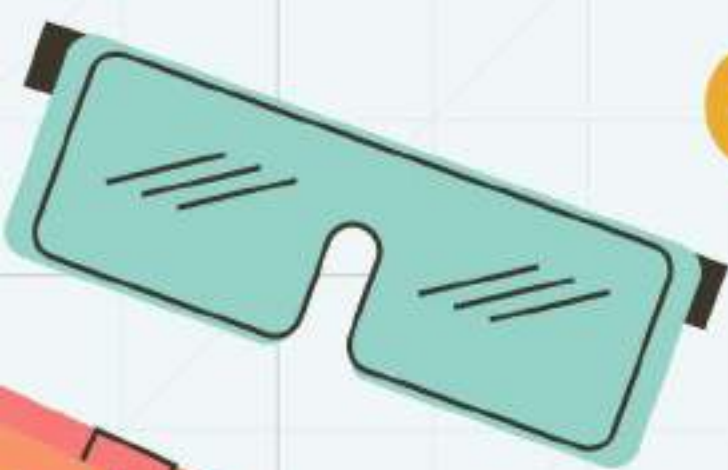
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Fluids



Fluid is a substance that continually deforms (flows) under an applied shear stress.

Pressure:

$$P = \frac{F}{A}$$

Applied force

Area of contact

- (i) for solid, Area of contact $\neq$ Apparent Area.
And hence pressure is not useful concept for solid analysis.
- (ii) for liquid, gas, Area of contact = Apparent Area.

$$\Rightarrow 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

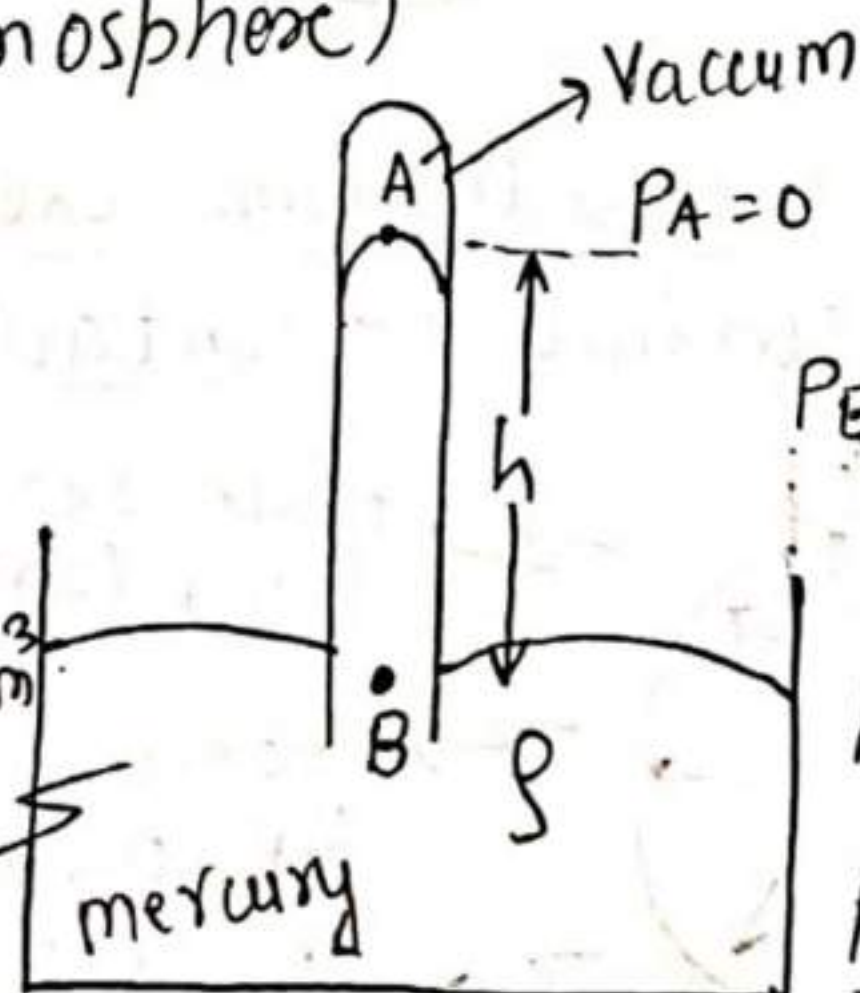
$$1 \text{ bar} = 10^5 \text{ Pa}$$

Measuring pressure: (Atmosphere)

⇒ Mercury barometer

Invented by Torricelli

$$\rho = 13.6 \times 10^3 \text{ kg/m}^3$$



$$P_B = P_A + \rho g h$$

$$h = 76 \text{ cm}$$

$$P_B = \rho \times 9.8 \times 0.76$$

$$P_B = 1.013 \times 10^5 \text{ Pa}$$



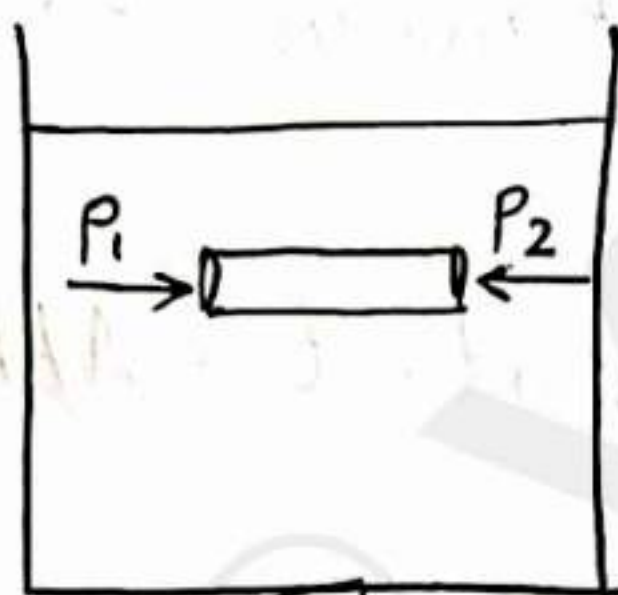
Properties of Pressure:

1. Pressure is isotropic:



pressure exerted by fluid at a point is same from all direction.

2. Pressure is uniform on a horizontal plane (hydrostatic)

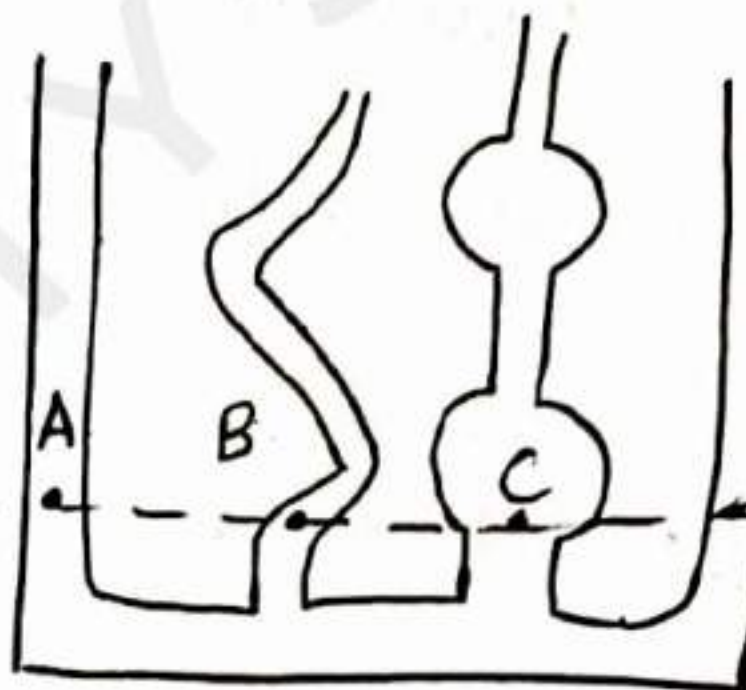


$$P_1 = P_2$$

when fluid is at rest

Hydrostatics

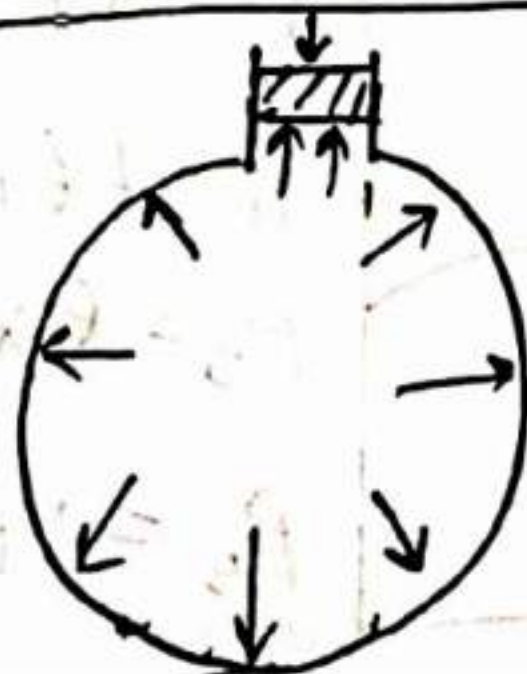
Pressure is same on horizontal line.



$$P_A = P_B = P_C = P_D$$

When fluid is same

3. Fluids in equilibrium exert pressure at right angles to the surface of contact.

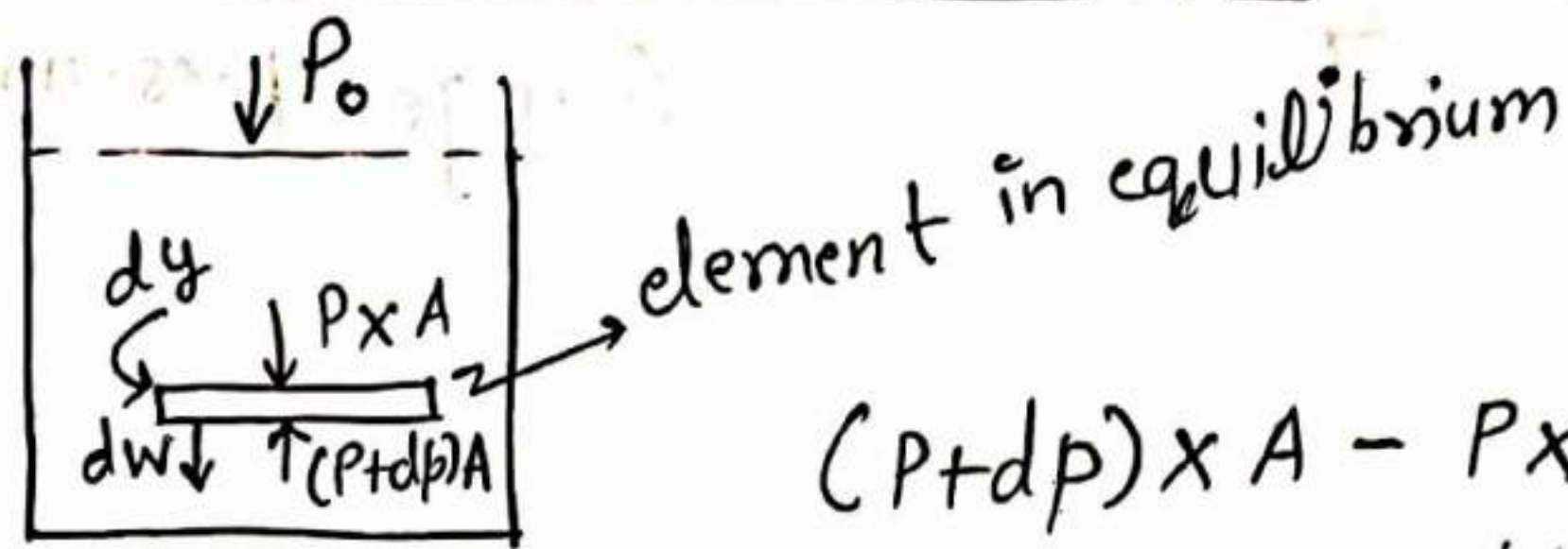


Force exerted perpendicular to surface in contact

force exerted by a liquid on any surface in contact called thrust

Thrust/Area = pressure

4. Pressure varies with depth and height:

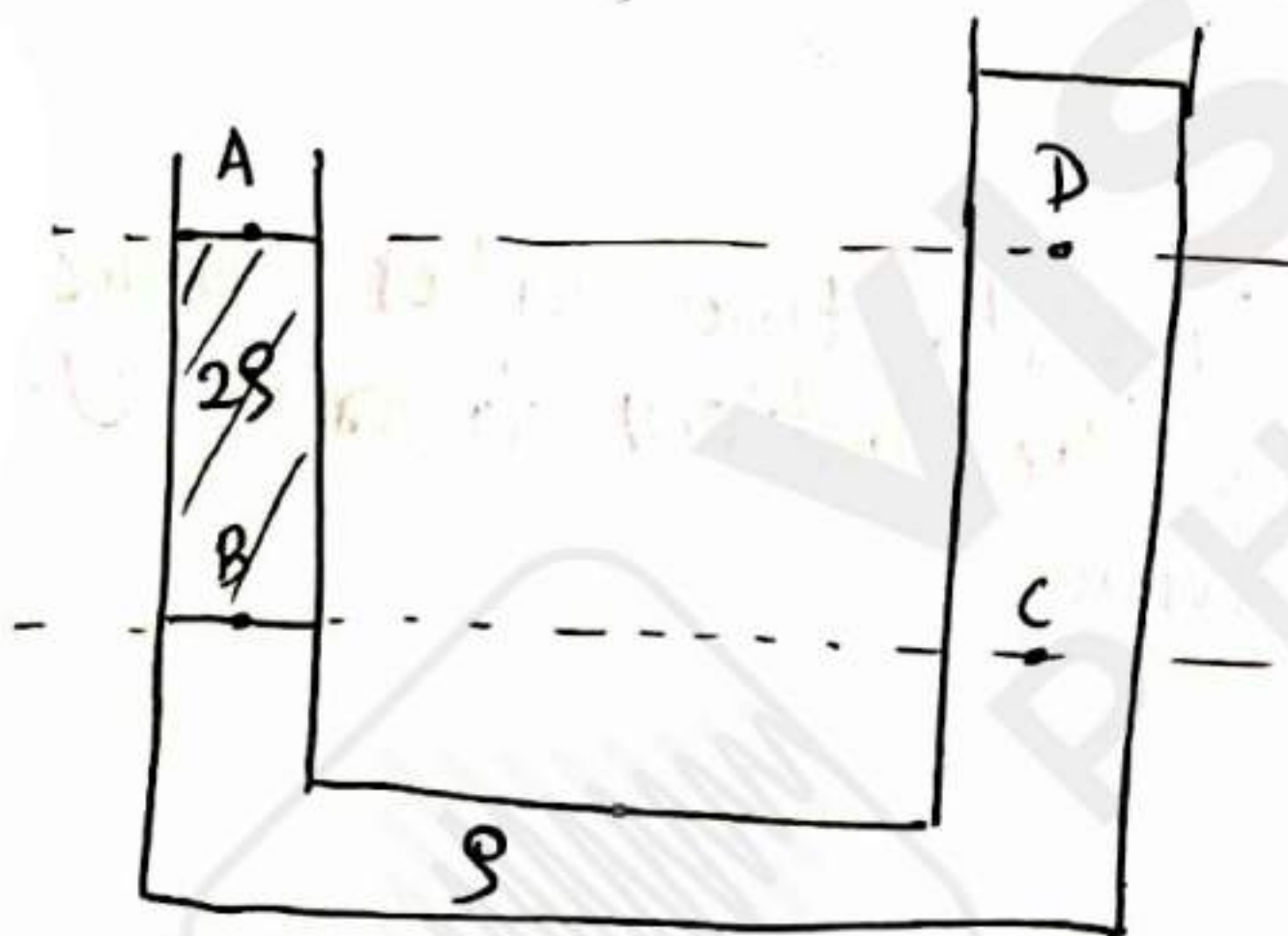


$$(P + dp) \times A - P \times A = dw$$

$$dw \Rightarrow \text{weight} = \rho \times A \times dy \times g$$

$$\int_{P_0}^P dp = \int_0^h \rho g dy$$

$$P = P_0 + \rho g h$$



$$P_B = P_C \quad \text{not}$$

$$P_A \neq P_D$$

Absolute pressure and Gauge pressure:

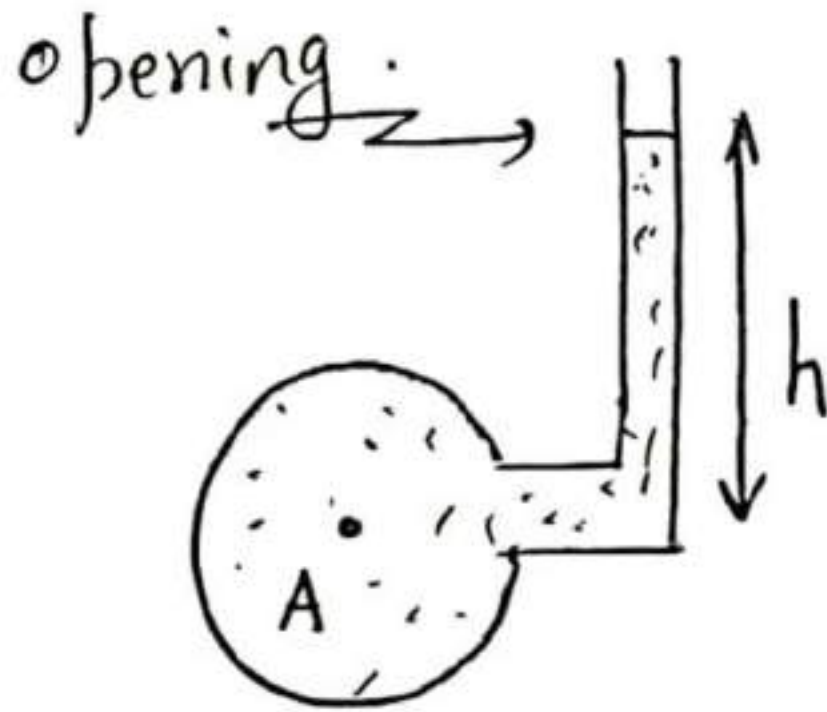
$$P_{\text{absolute}} = P_{\text{gauge}} + P_{\text{atm}}$$

total
pressure

reference
pressure from
atmosphere

Manometers → device used to measure Gauge pressure.

Simple monometer



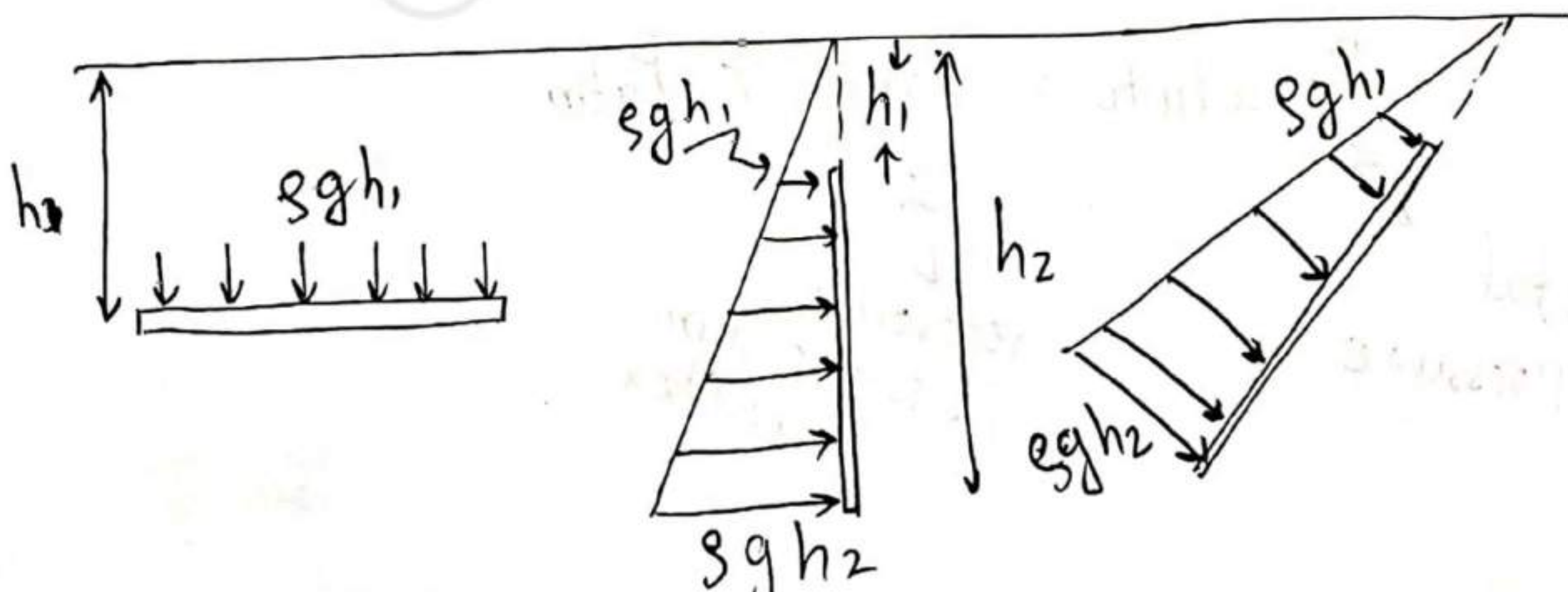
$$P_A = \rho g h$$

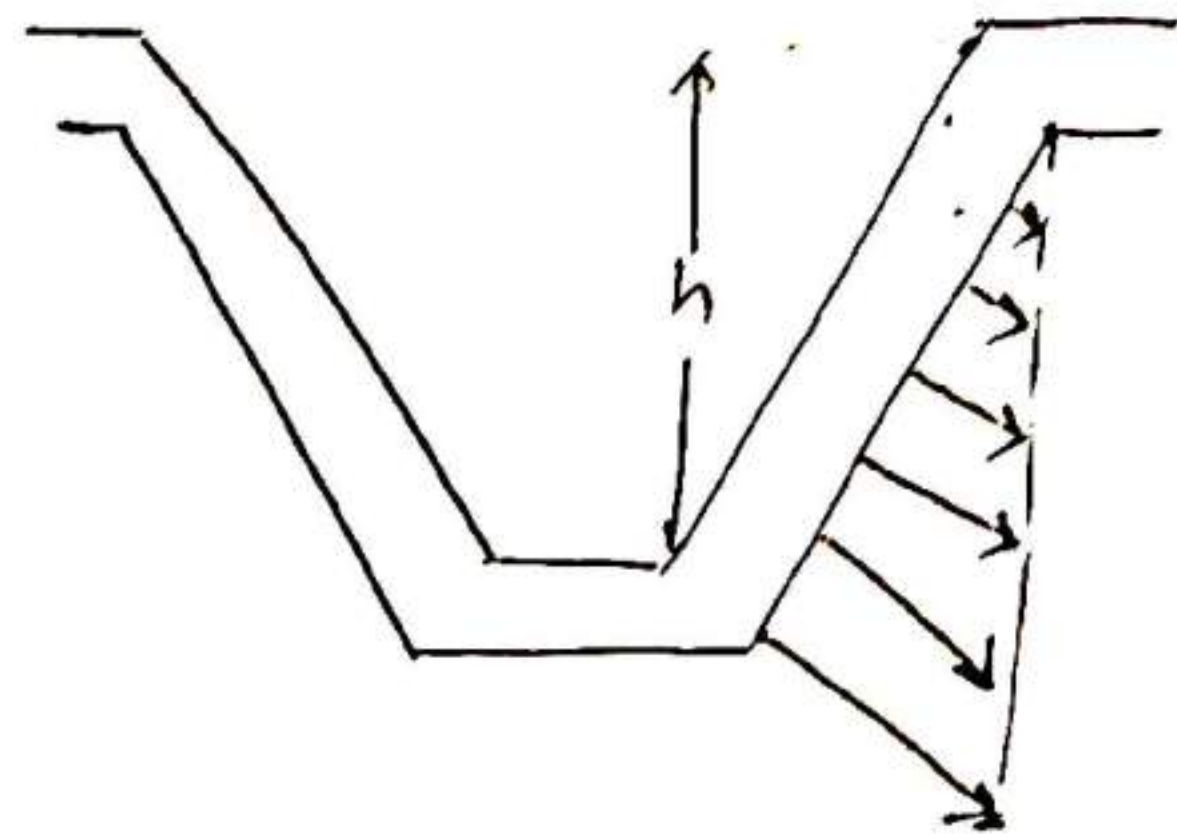
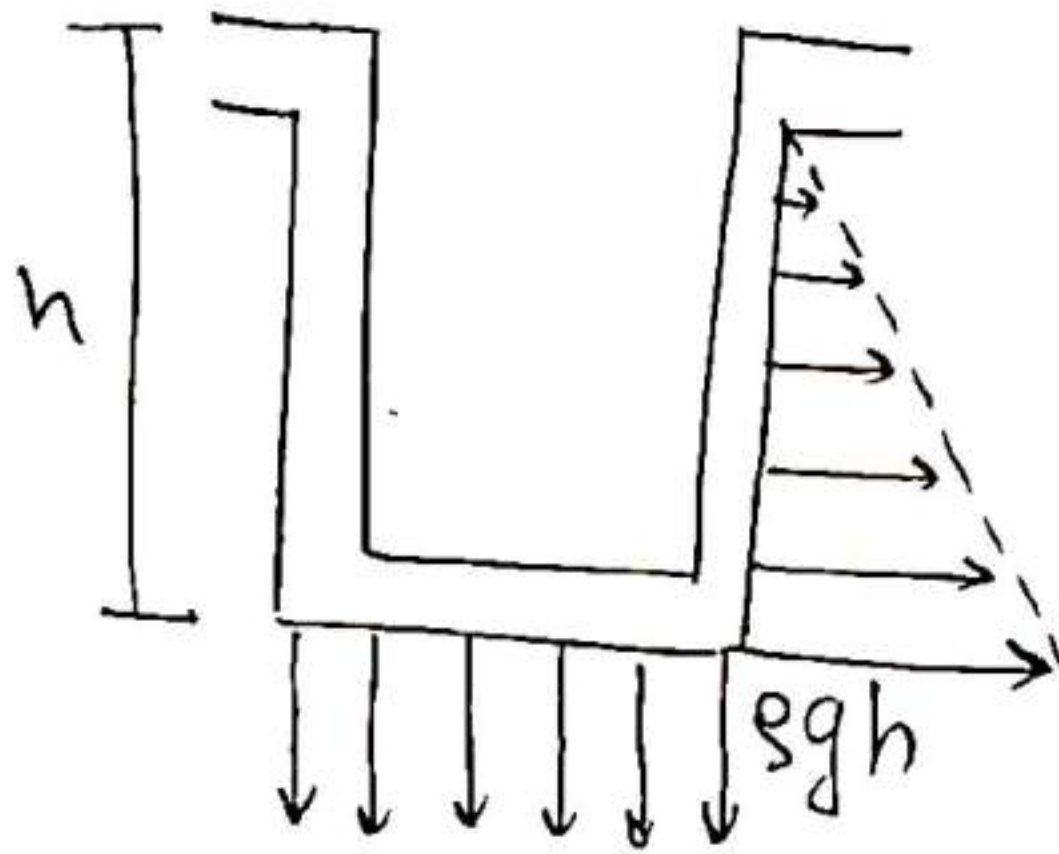
- Cannot measure pressure in a gas
- Can measure only positive gauge pressure.
- Cannot measure large pressure as tube will be long.

U-tube manometer → Consists two tubes connected at the bottom to form U-shape tube

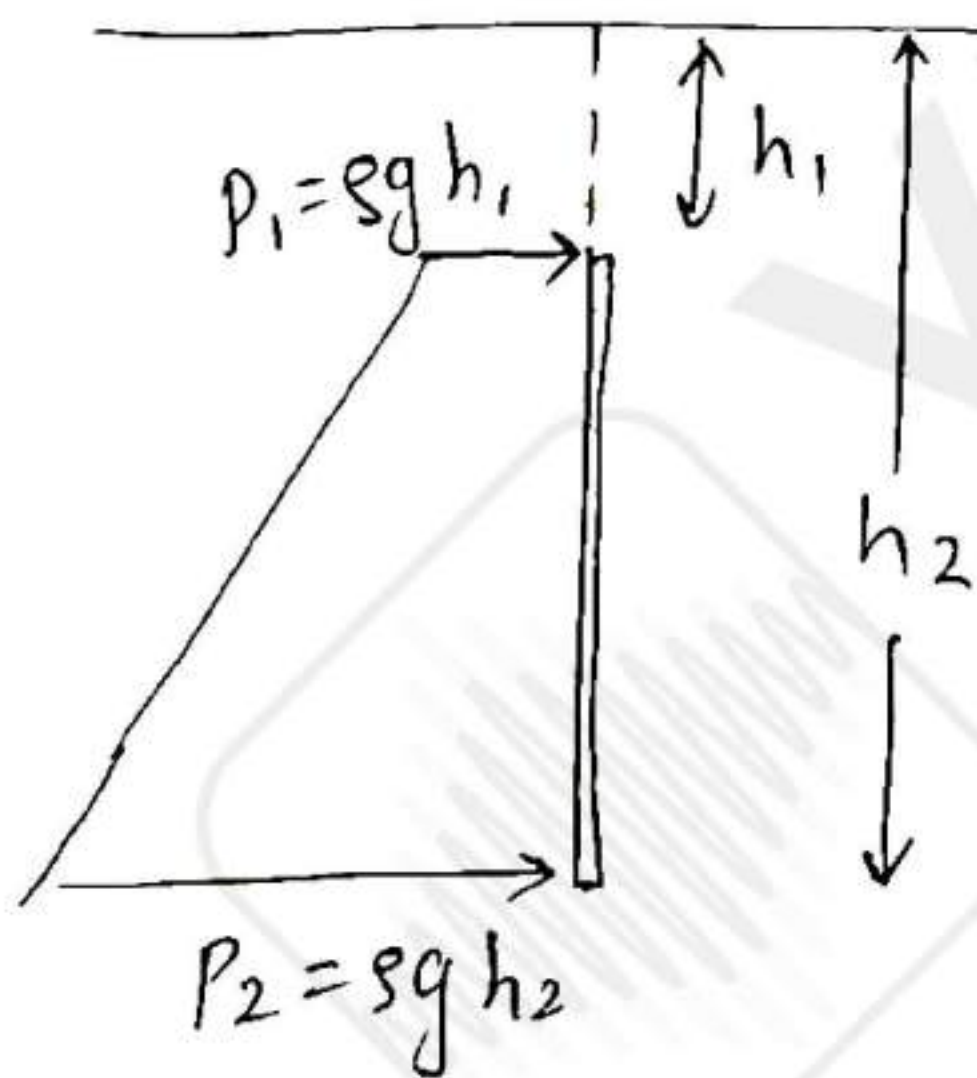
↓
Can measure high pressure difference.

Pressure Diagrams and Force on Boundaries?





Force on Surface Immersed in a liquid and Centre of force
for any pressure diagram.



$$P_{\text{average}} = \frac{P_1 + P_2}{2} = P_{\text{av}}$$

force on the surface
 $= P_{\text{av}} \times \text{Area}$

$$F = \rho g \left(\frac{h_1 + h_2}{2} \right) \times A$$

Thrust

$$\text{Thrust} = \int \rho g h b dh = \rho g \int b h dh$$

as $\bar{h}$ = centroid depth of the area $= \frac{\int (b dh) h}{A} = \bar{h}$

$$\Rightarrow \boxed{\text{Thrust} = (\rho g \bar{h}) A}$$

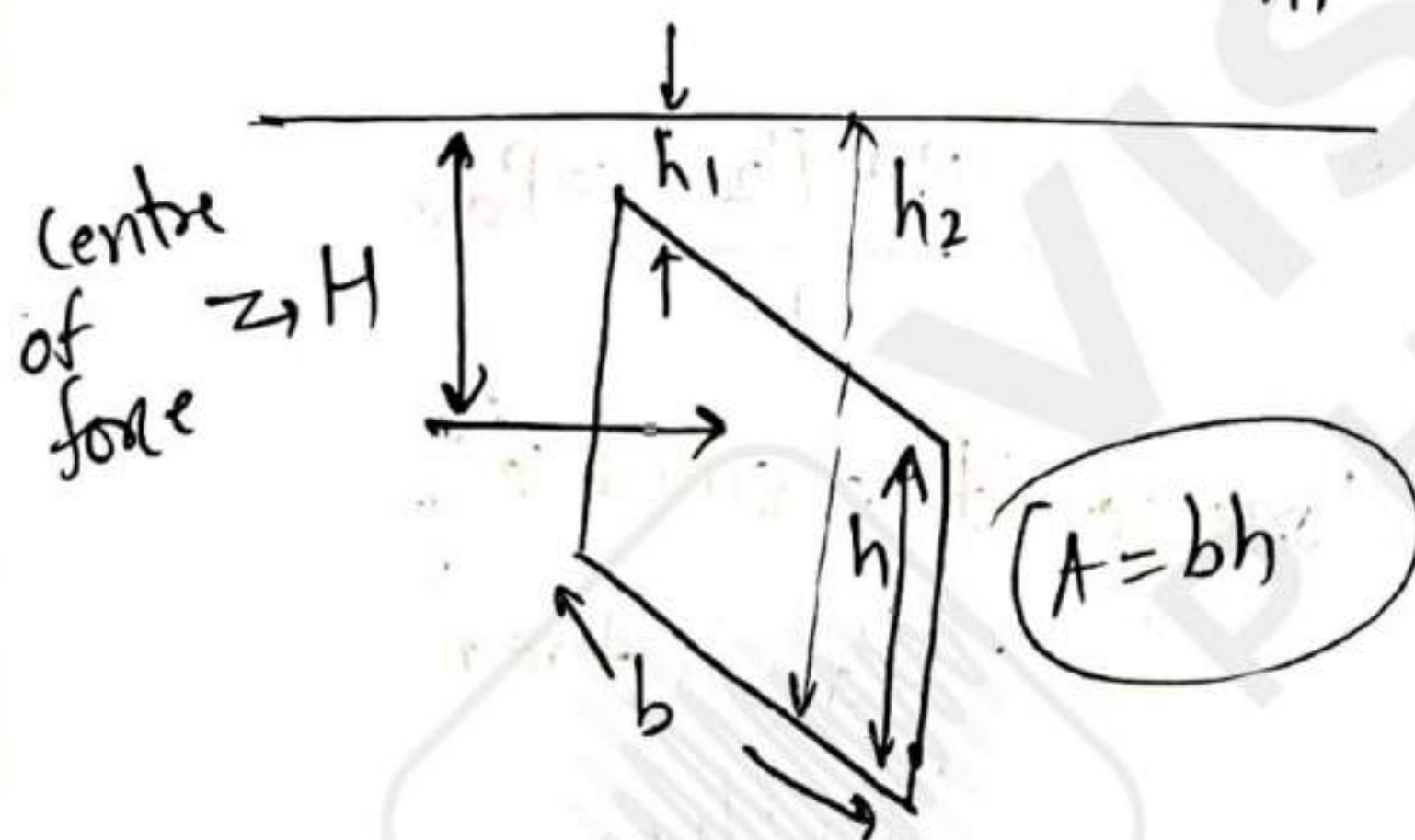
Centre of Force

→ The point where the entire force can be thought of acting.

→ producing same rotational effect as the individual thrust on each elemental strip.

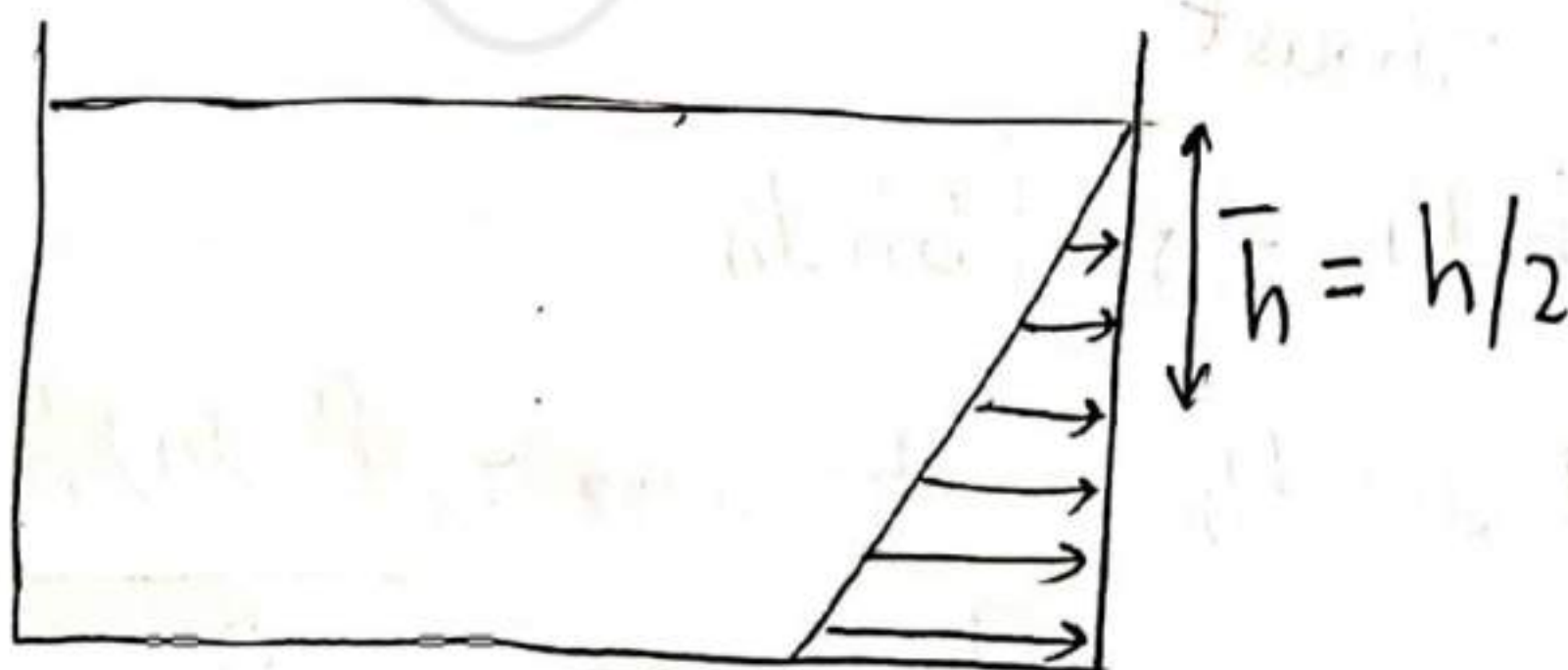
$$M = \int_{h_1}^{h_2} (b dh) (h sg) h = sg \bar{h} A H \quad \rightarrow \text{Centre of force}$$

$$\Rightarrow H = \frac{1}{A \bar{h}} \int_{h_1}^{h_2} (b h^2) dh = \frac{1}{b \bar{h}} \left(\frac{2}{h} \right) \times b \frac{h^3}{3}$$



$$H = \frac{2}{3} h$$

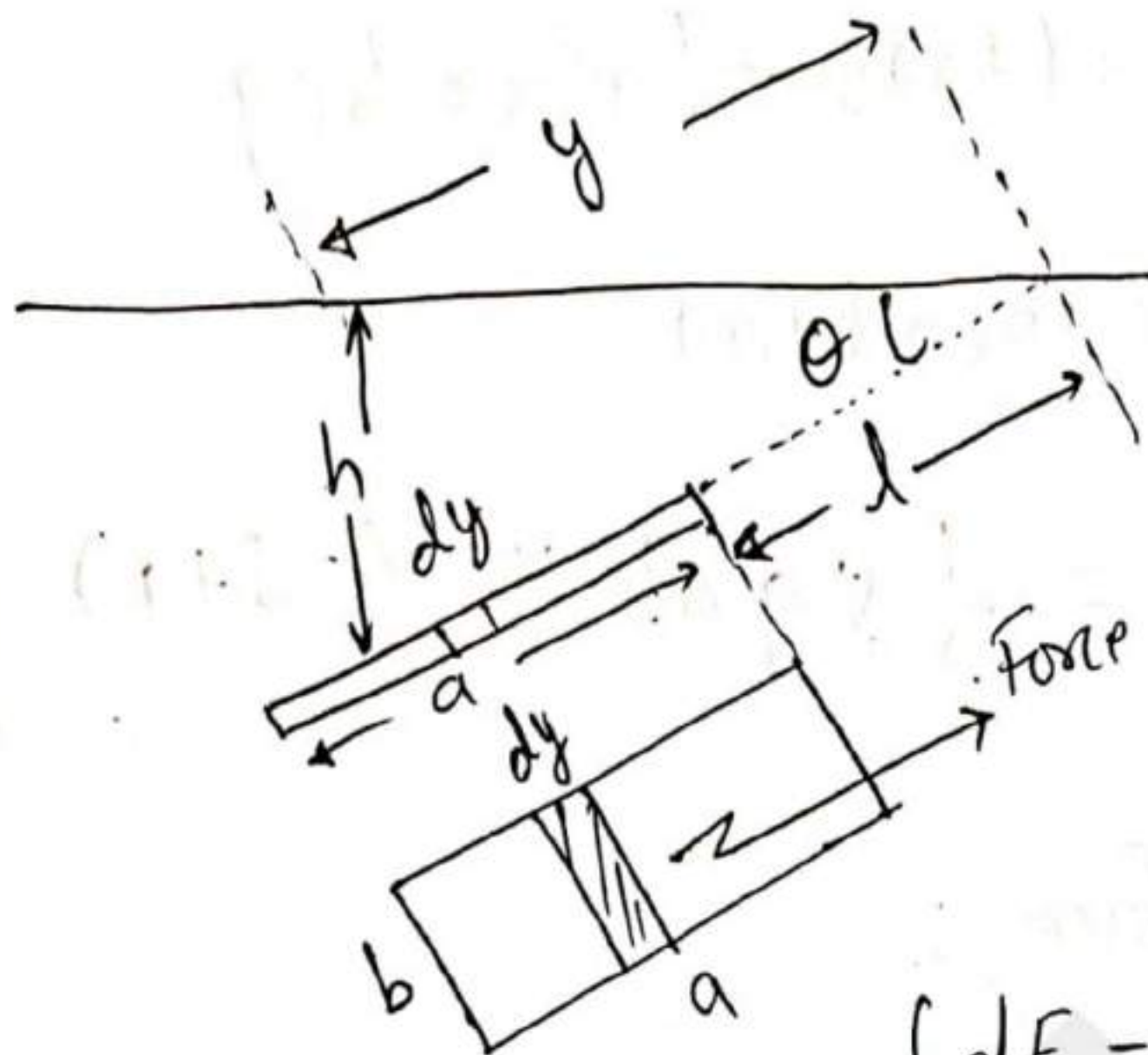
Force on vertical wall:



$$\text{Force} = \text{Thrust} = (sg A) \bar{h}$$

$$\text{Force} = \frac{sg A h}{2}$$

Hydrostatic Force on Inclined Surfaces Immersed in Liquid



$$h = y \sin \theta$$

Force on element

$$dF = P(b dy)$$

$$dF = \rho g h b dy$$

$$\int dF = F = \rho g b \sin \theta \int_l^{a+l} y dy$$

$$F = \rho g \frac{b \sin \theta}{2} \left[y^2 \right]_l^{a+l}$$

$$F = \rho g b \frac{\sin \theta}{2} [(l+a)^2 - l^2]$$

$$F = \frac{\rho g a b}{2} [a + 2l]$$

Other method:

Projected area = $a b \sin \theta$

F_H = Force on area projected on vertical plane

$$F_H = \frac{1}{2} [\rho g l \sin \theta + \rho g (l+a) \sin \theta] \times (a b \sin \theta)$$

Projected area

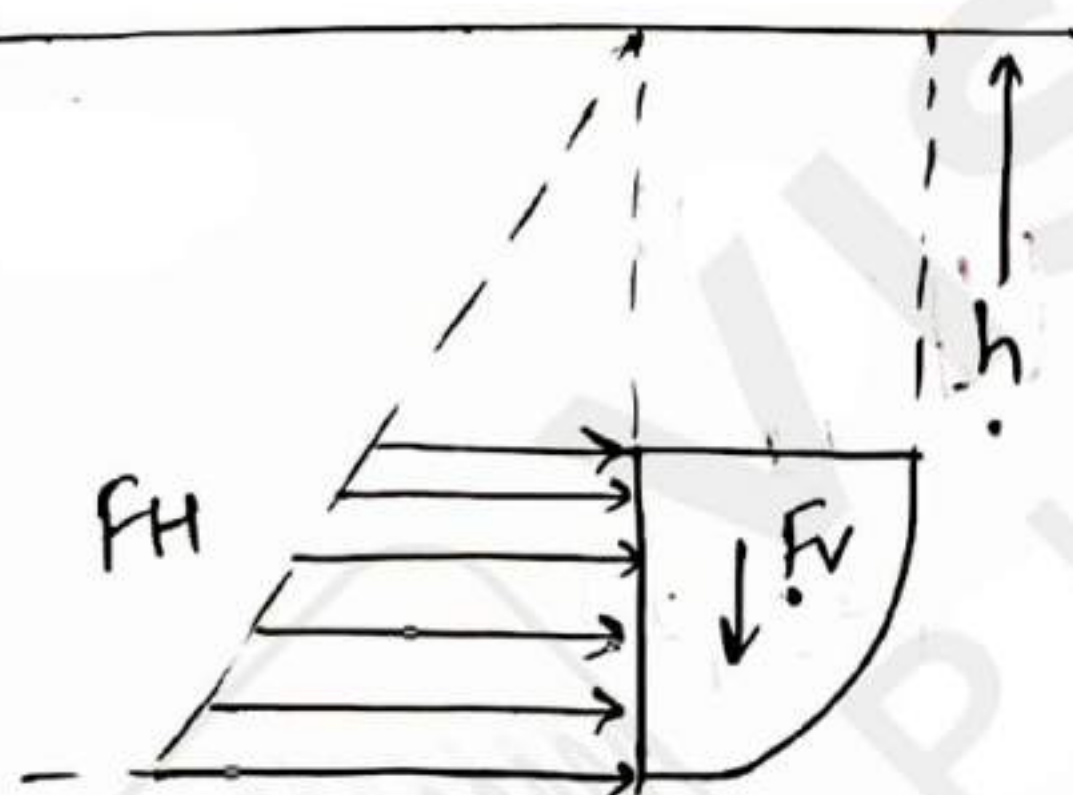
$$F_v = \text{Weight of water prism above inclined surface}$$

$$= \frac{1}{2} [l \sin \theta + (l+a) \sin \theta] a \cos \theta b \rho g$$

$$F_v = \frac{1}{2} \rho g a b \sin \theta \cos \theta (2l+a)$$

$$F = \sqrt{F_H^2 + F_v^2} = \frac{1}{2} \rho g a b \sin \theta (2l+a)$$

Force on Curved Surface :



$$dF_H = dF \sin \theta = \rho g h dA \sin \theta$$

or force on projected area:

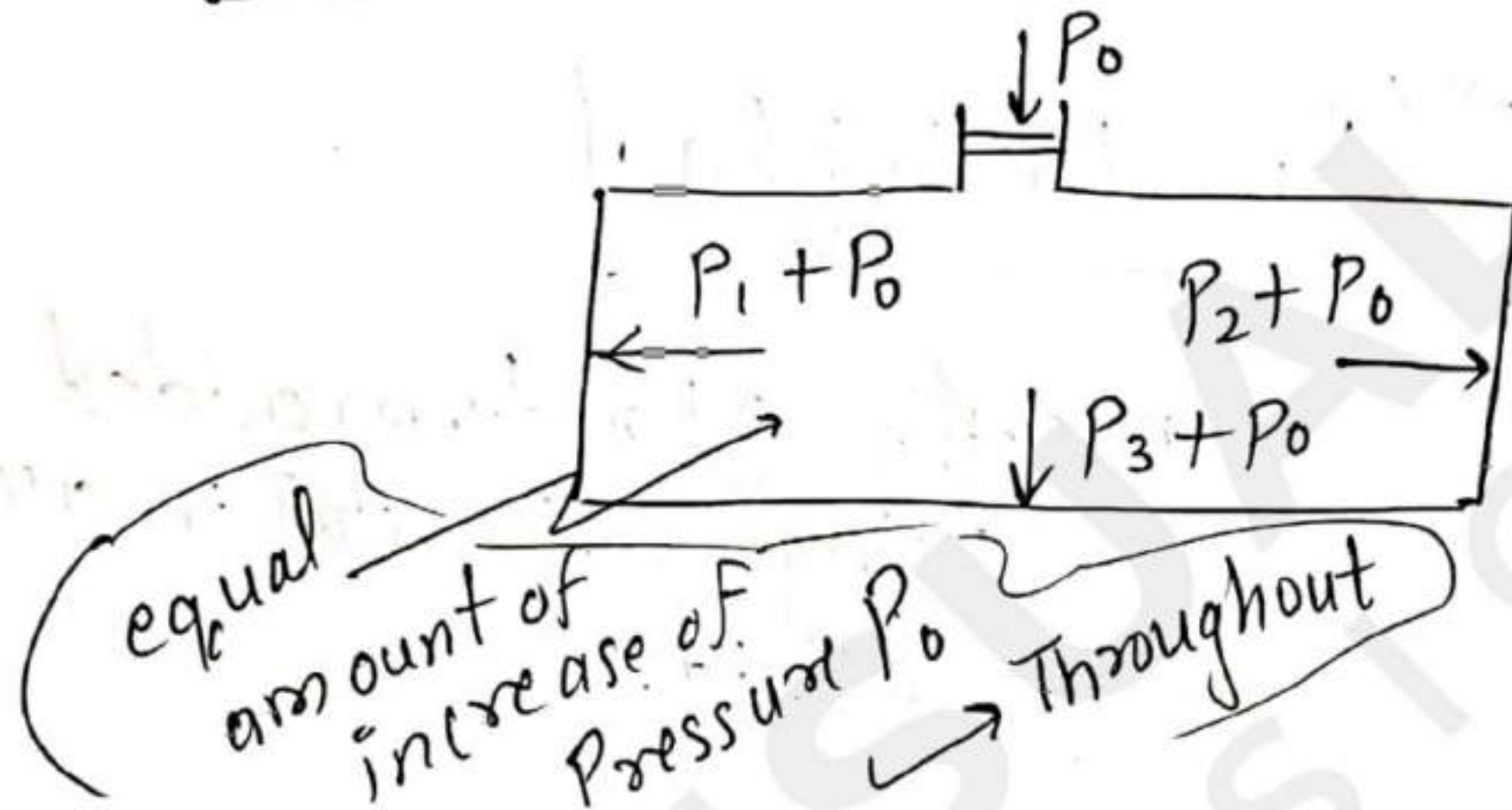
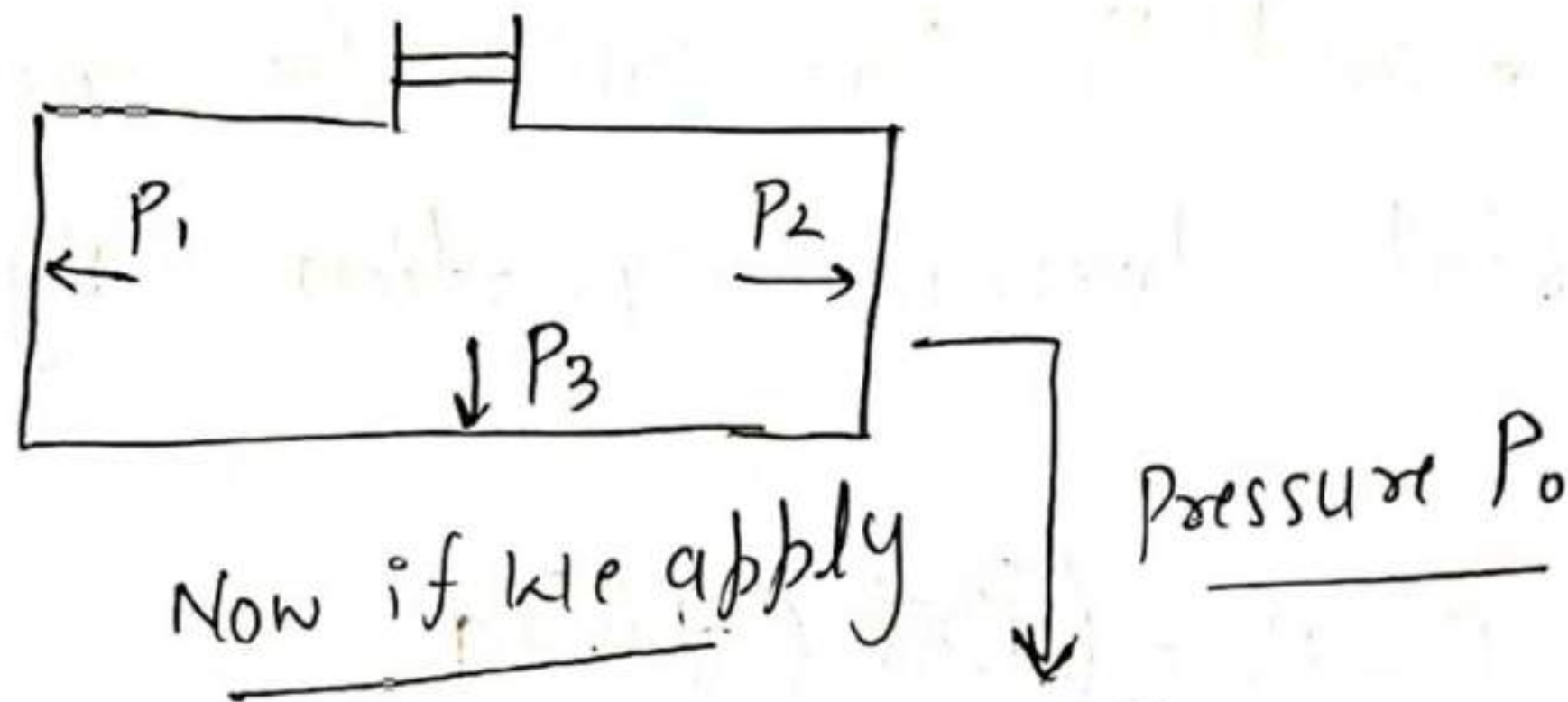
$$F_H = \rho g \bar{h} A \sin \theta$$

$F_v = \text{Weight of water prism over the surface}$

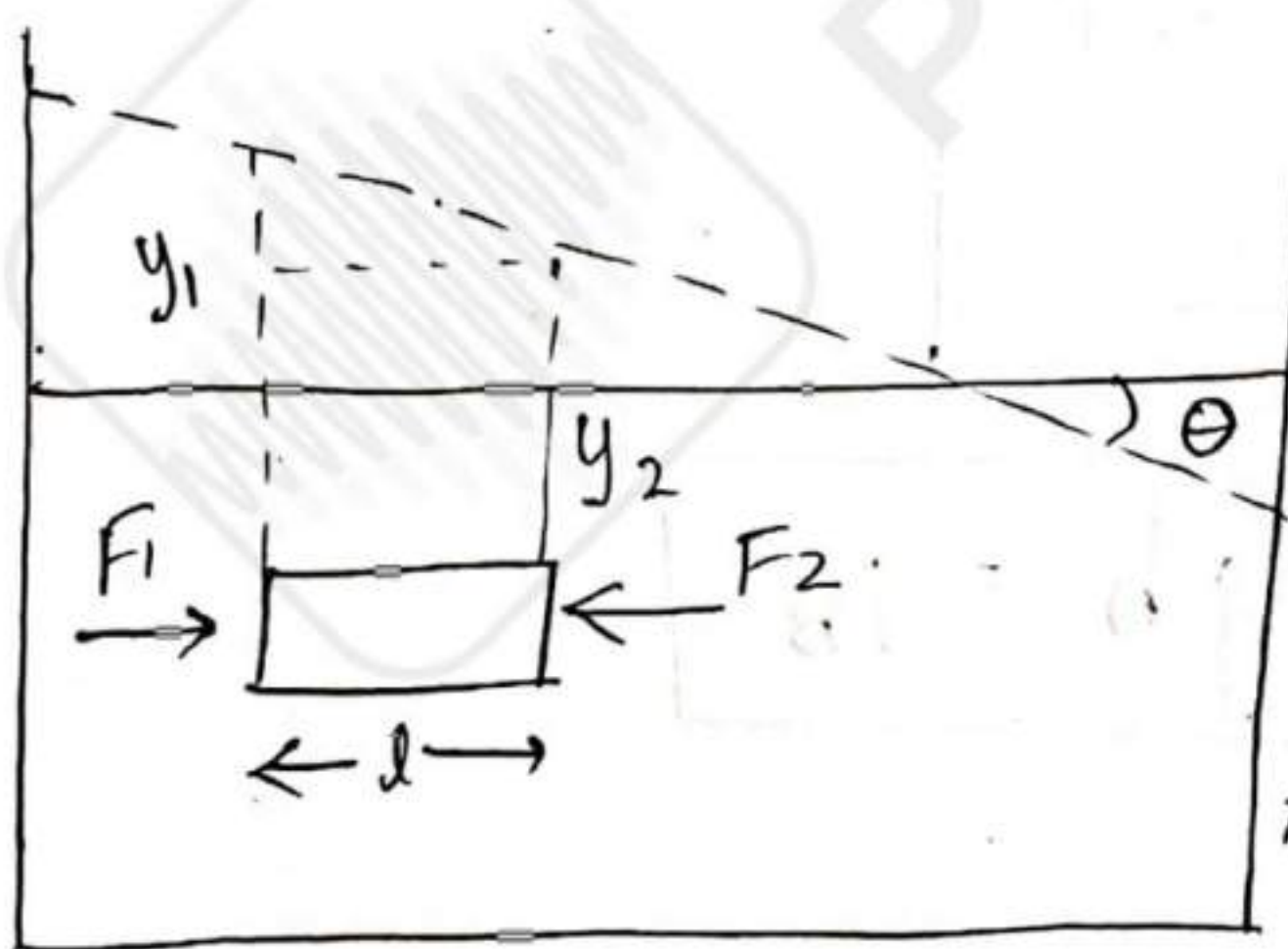
$$F_v = \rho g \int h (dA) \cos \theta$$

Pascal's law

"Any change in the pressure applied to an enclosed incompressible fluid is transmitted to every point of the fluid."



Fluids in Uniformly Accelerating Motion



$$F_1 - F_2 = ma$$

$$P_1 A - P_2 A = ma$$

$$A(\rho g y_1 - \rho g y_2) = ma$$

as $(m = \rho A l)$

$$\text{as } \frac{y_1 - y_2}{l} = \frac{ma}{\rho g l} = \frac{\rho A l a}{\rho g l}$$

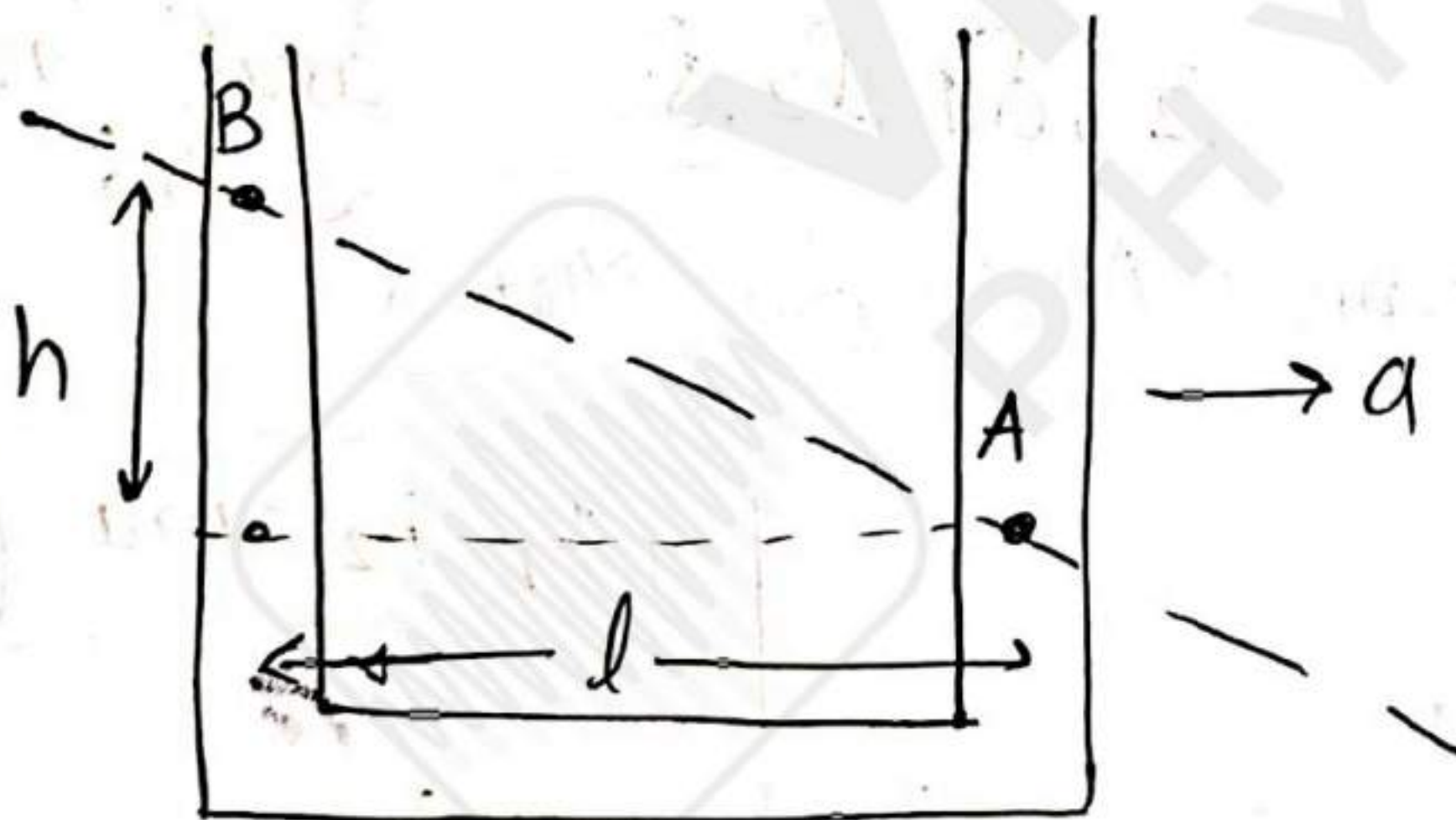
$$\Rightarrow \left| \frac{y_1 - y_2}{l} \right| = \tan \theta = a/g$$

* now P_2 and P_1 can easily be found using static pressure equation $= \rho g h$

$$\text{as } P_1 - P_2 = (\rho l) a$$

$$\Rightarrow \boxed{P_1 = P_2 + \rho l a}$$

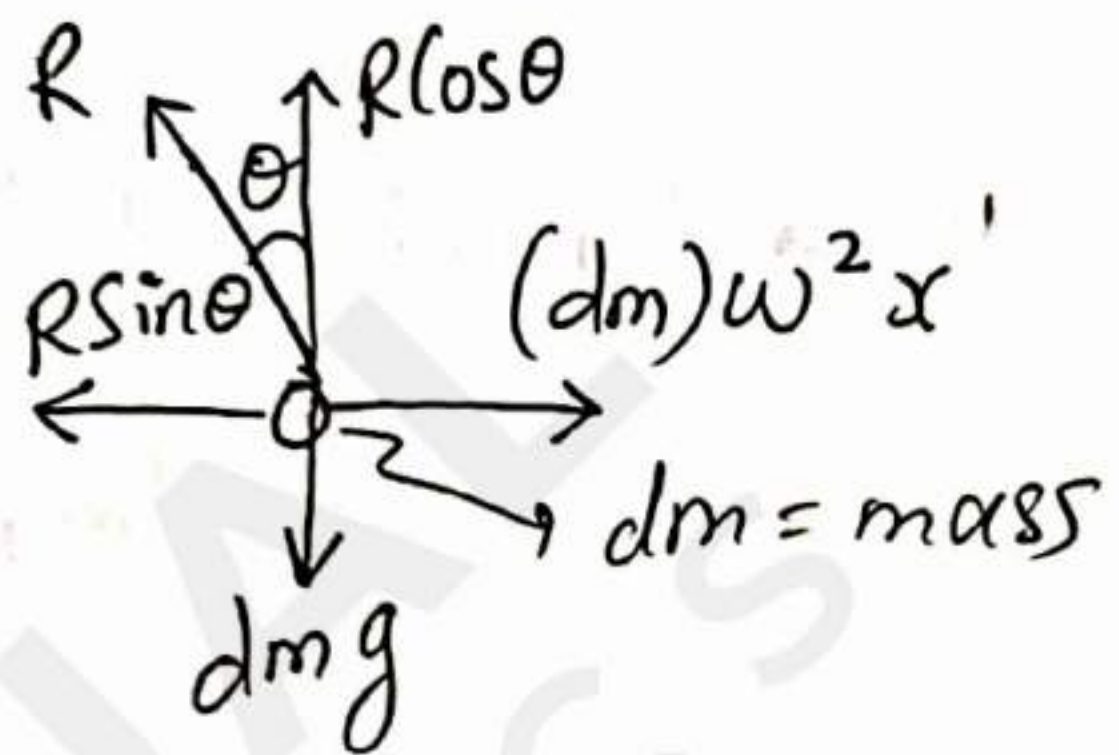
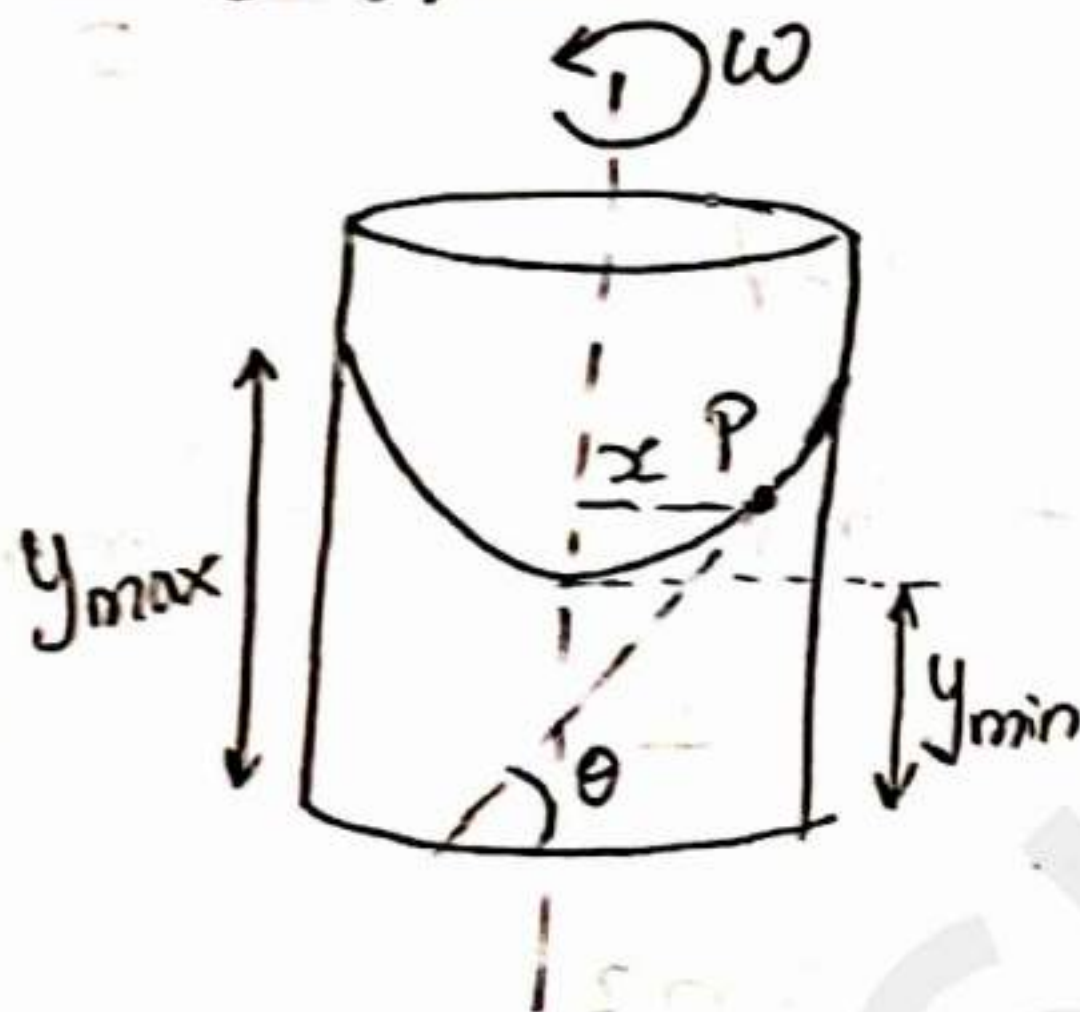
an extra $\rho l a$ term added while going horizontal in accelerating fluid.



$$\boxed{P_A + \rho a l + \rho g h = P_B}$$

Liquid in a Vessel Rotating with constant angular velocity:

from vessel frame:



$R \rightarrow$ force exerted on the element due to neighbouring elements of fluid.

$$\text{So, } R \cos \theta = dm g$$

$$R \sin \theta = (dm) \omega^2 x$$

$$\boxed{\tan \theta = \frac{\omega^2 x}{g}} = \frac{dy}{dx}$$

$$\Rightarrow \int dy = \int \frac{\omega^2 x}{g} dx$$

$$y = \frac{\omega^2 x^2}{2g} + C$$

$$\text{at } x=R, \\ y = y_{\max}$$

$$\text{at } x=0, y = y_{\min} = C$$

$$\Rightarrow \boxed{y_{\max} = \frac{R^2 \omega^2}{2g} + y_{\min}}$$

Volume of paraboloid of revolution is half that of circumscribing cylinder

$$\text{Volume of paraboloid} = \frac{(y_{\max} - y_{\min}) \times A}{2}$$

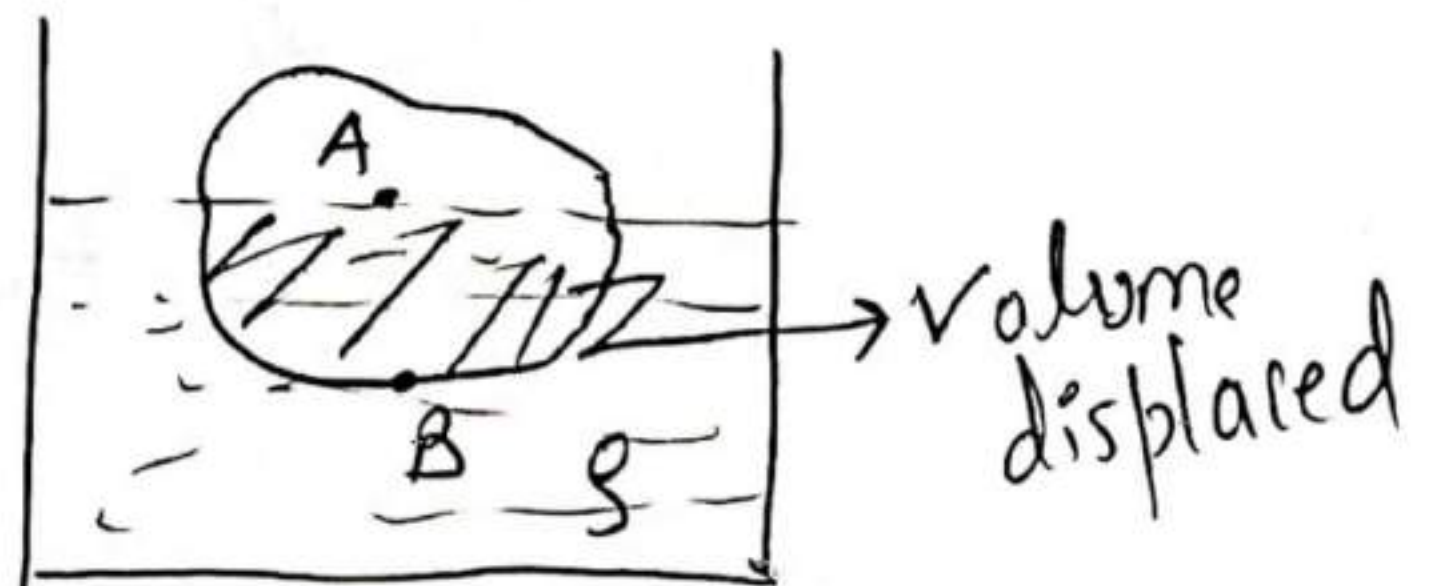
$y_0 \rightarrow$ liquid initially filled

$$y_0 = y_{\min} + \frac{y_{\max} - y_{\min}}{2} = \frac{y_{\max} + y_{\min}}{2}$$

$$\Rightarrow \boxed{y_{\max} = y_0 + \frac{\omega^2 R^2}{2g}}$$

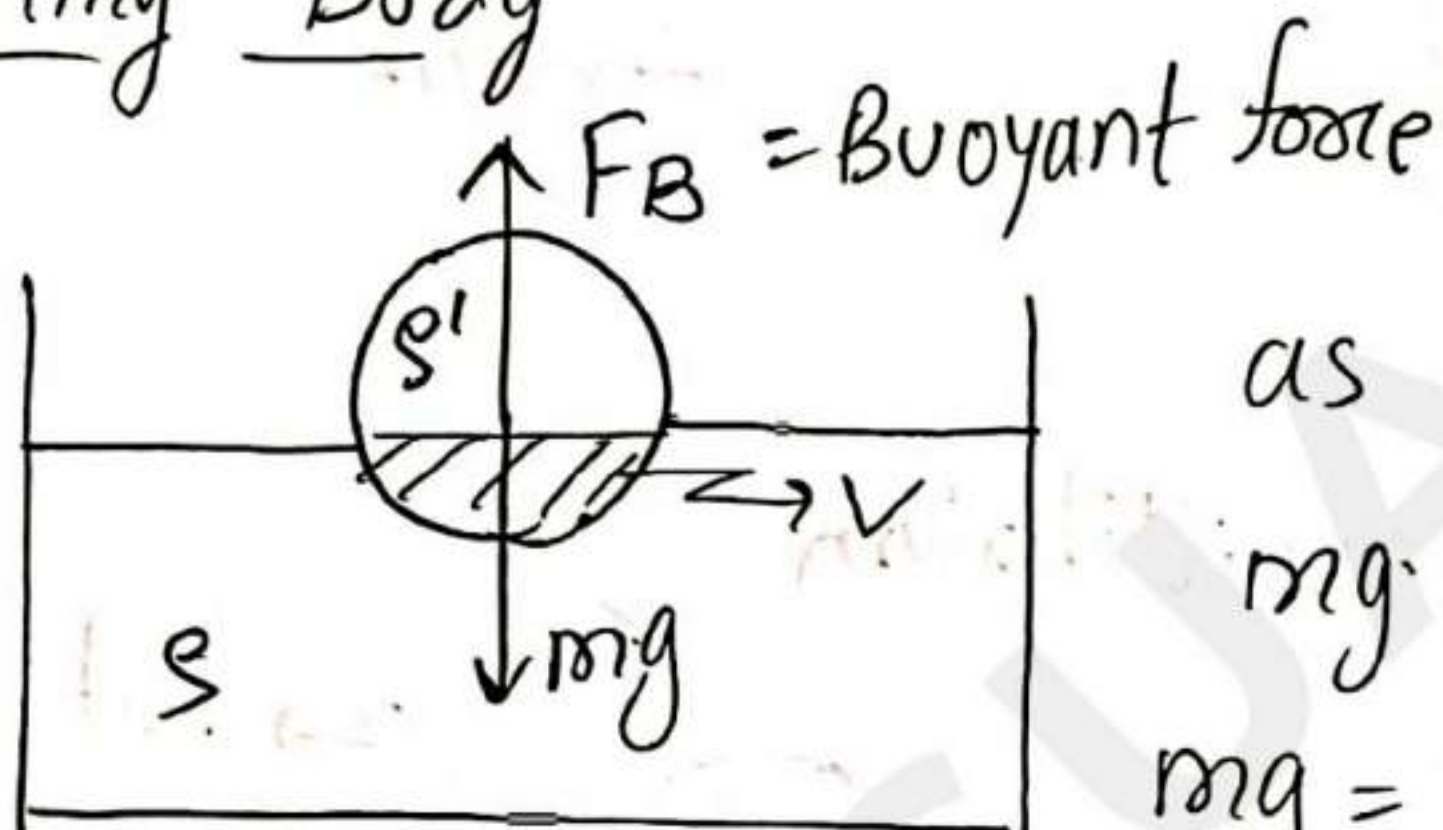
Archimede's Principle

magnitude of upward force
 = weight of liquid displaced
 = Buoyant Force
 = $\rho V g$ = weight displaced



Buoyant force $\rightarrow$ upward force
 = Weight of liquid displaced
 or gas

Floating Body



as body is floating

$$mg = F_B$$

$$mg = \rho V g$$

$V' =$ net volume of body

$$\Rightarrow \rho' V' g = \rho V g$$

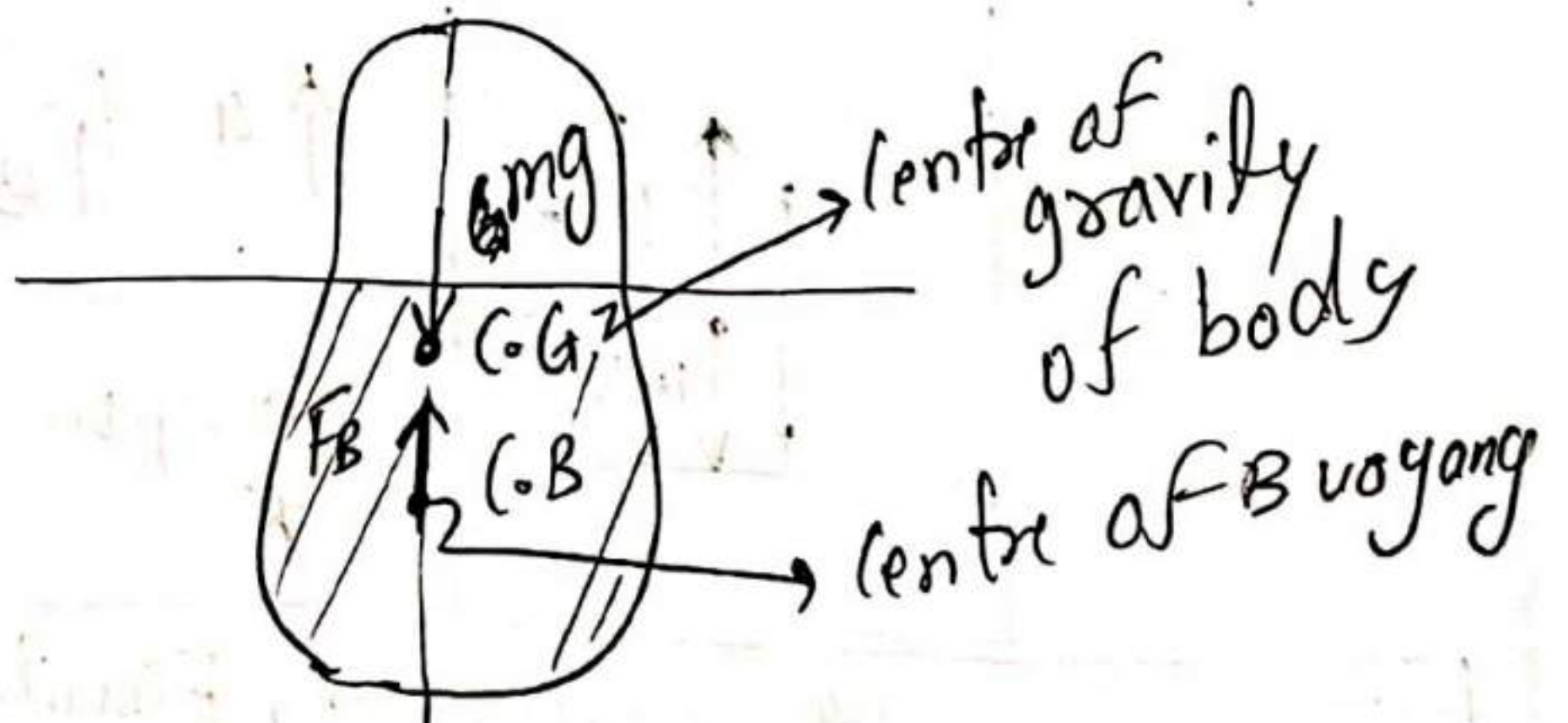
$V =$ volume displaced

$$V = \frac{\rho' V'}{\rho}$$

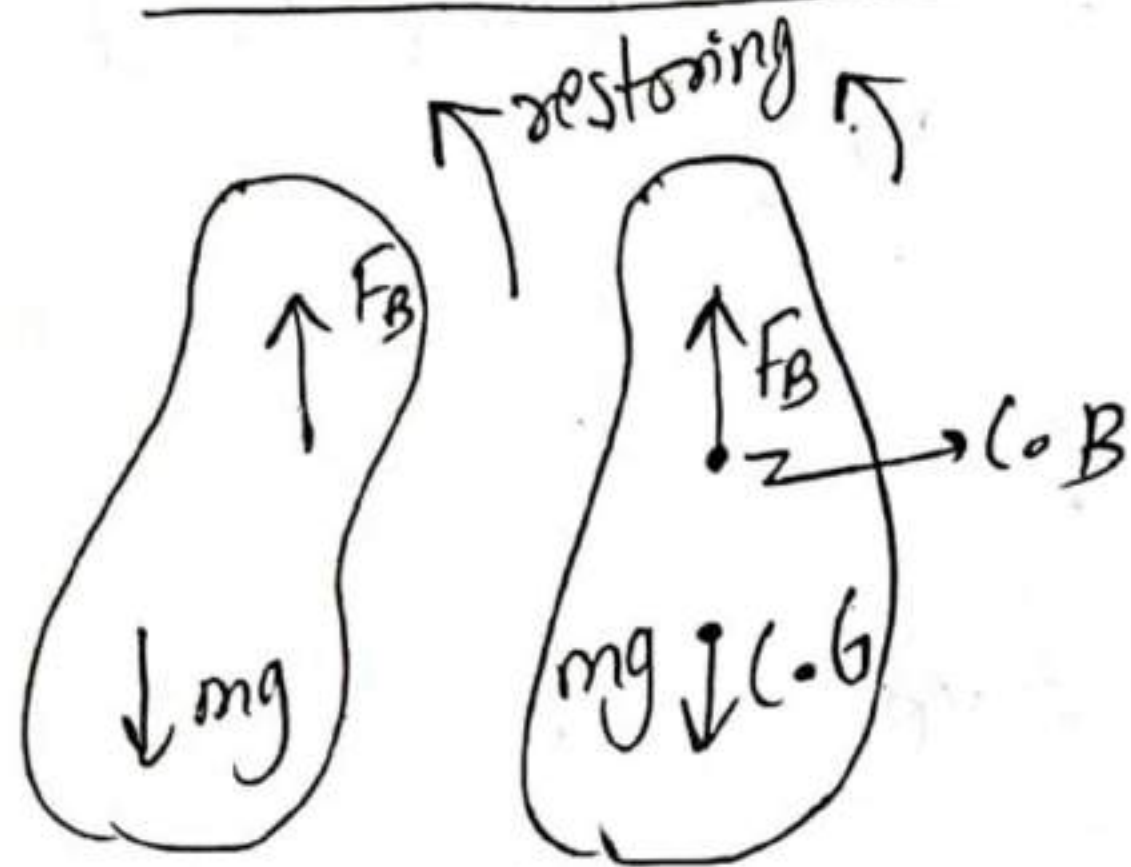
Centre of Buoyancy:

$\rightarrow$ Centre of gravity of displaced fluid

Point where all force acts.



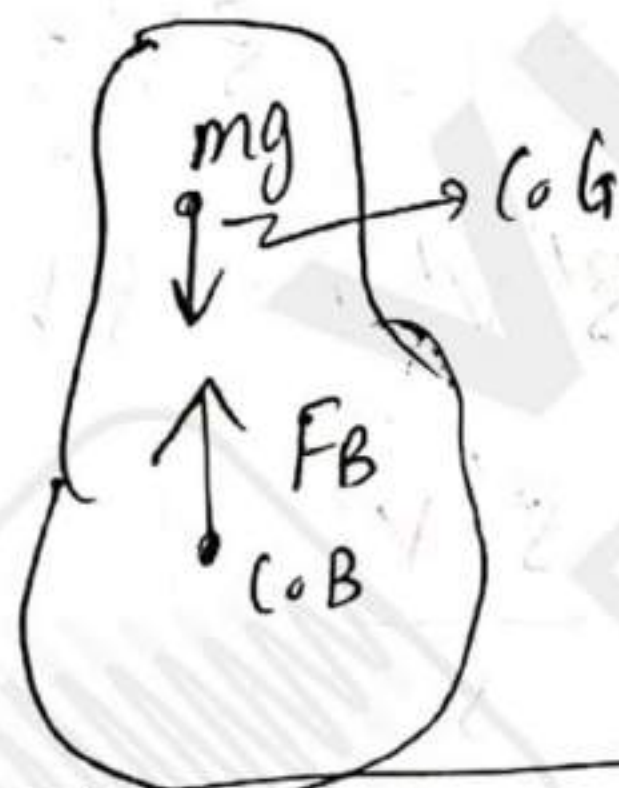
for stable equilibrium:



(C.B should lie above C.G)

hence, always restoring torque

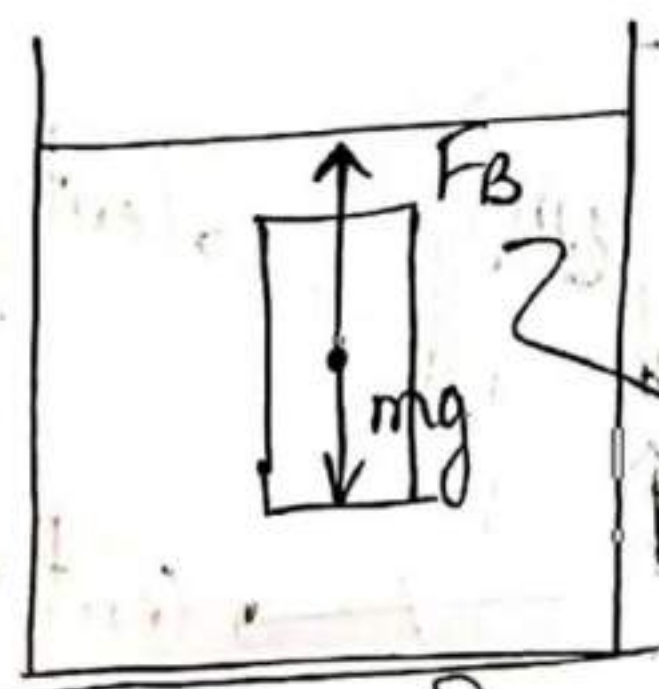
for unstable equilibrium



→ not restoring

★ C.B lies below C.G

Buoyant force in accelerating fluid



↑ a $F_B = mg + ma$

★ upward force = buoyancy force

Hydro dynamics :

↳ Fluids in motion (bulk motion)

Type of flow:

(i) steady and unsteady flow:

(Steady) → not change in velocity, pressure, density with time.

$$\frac{dv}{dt} = 0, \frac{dp}{dt} = 0 \text{ \& } \frac{d\rho}{dt} = 0$$

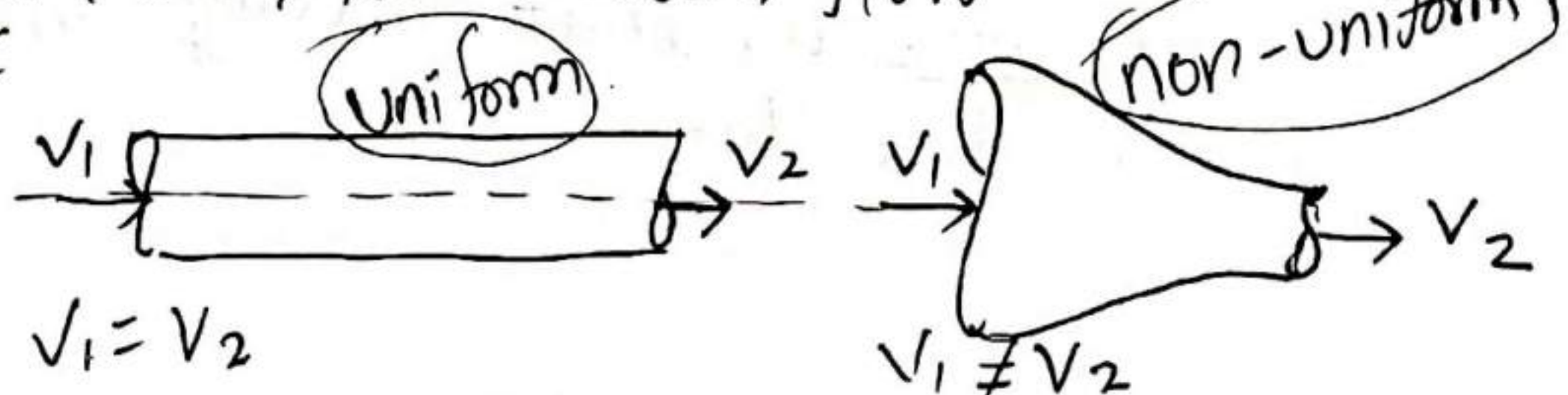
(Unsteady) → velocity at a point in the flow varies with time.
i.e. $\frac{dv}{dt} \neq 0$

(ii) Uniform & non-uniform flow:

★ Flow is uniform at any instant of time if the velocity does not vary along the direction of flow

$$\frac{dv}{dx} = 0$$

★ if $\frac{dv}{dx} \neq 0 \Rightarrow$ non-uniform flow



Stream line flow:

if every point of a steadily flowing liquid follows exactly the same path followed by particles preceding it.

→ Flow is streamlined.



Turbulent Flow

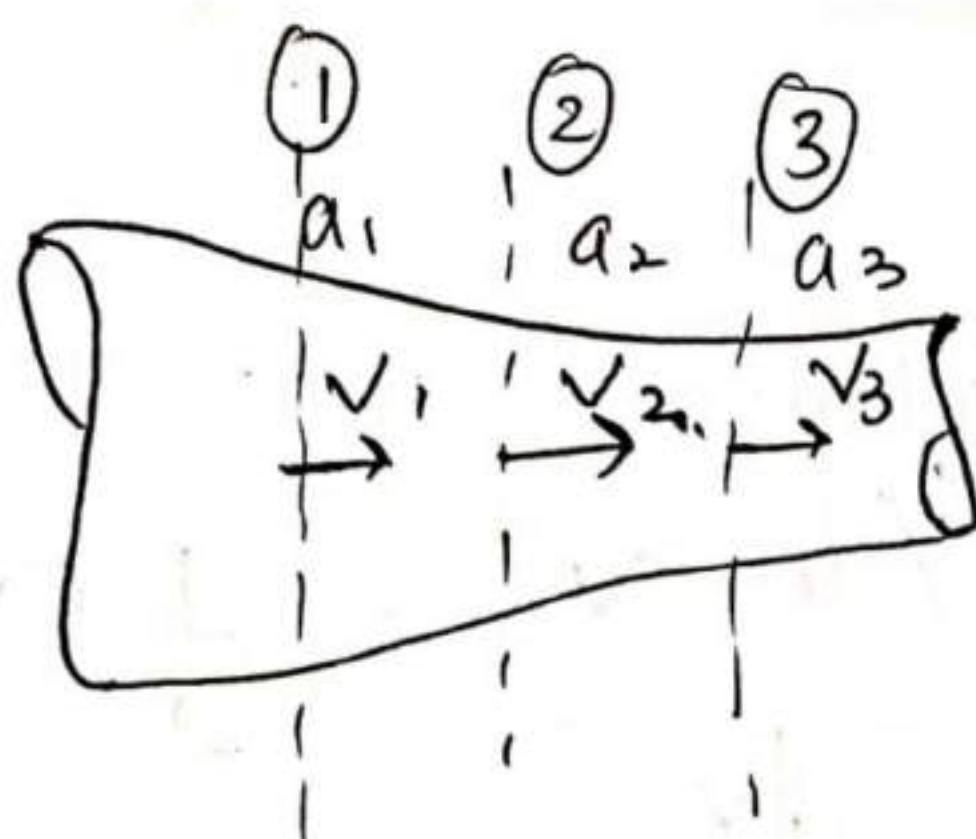
If velocity exceeds a certain value, flow becomes complicated. Random, irregular, local currents develop throughout fluid.

Principle of Continuity: The continuity equation:

Mass Conservation

In steady state flow:

→ amount of liquid entering a section is the same as that leaving another section in a given time interval.



a_1, a_2, a_3
 $\rightarrow$ Area of cross-section

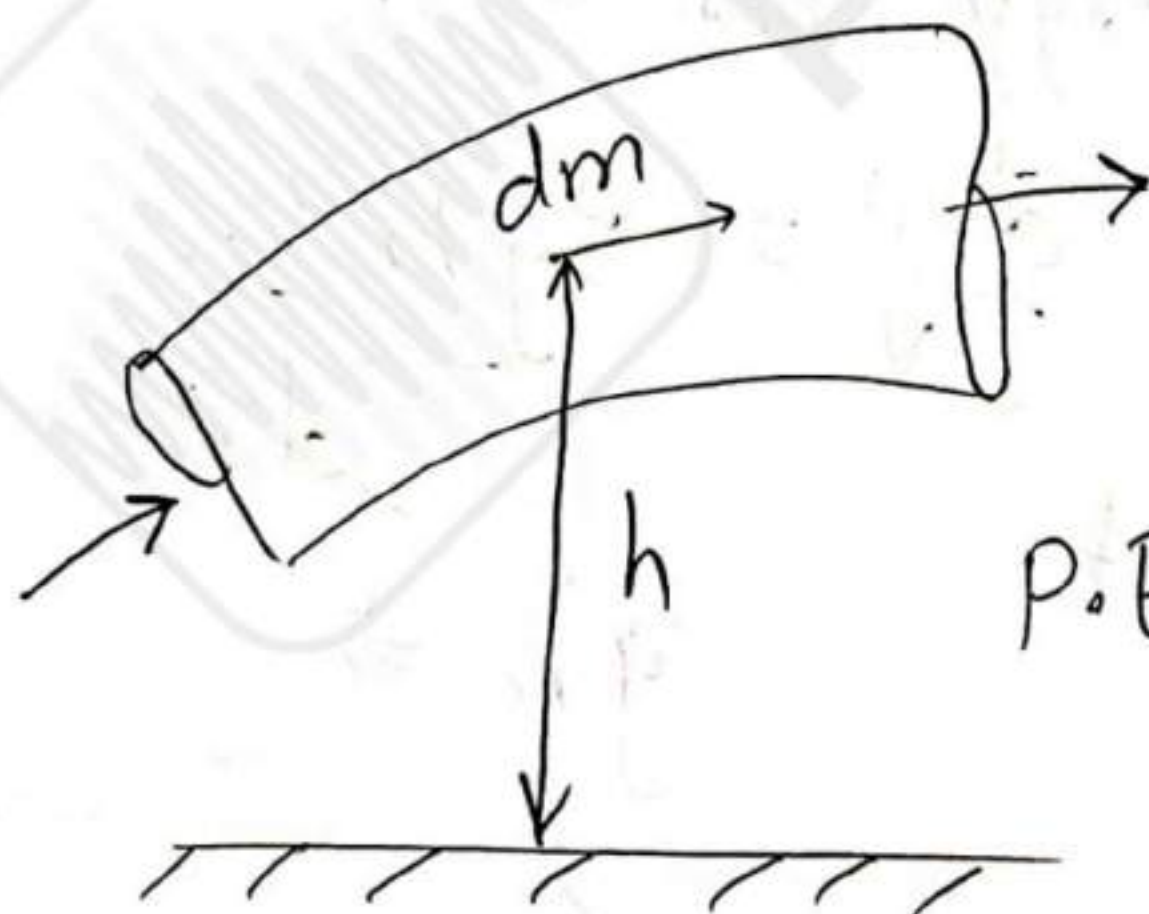
$$m_1 = m_2 = m_3 \Rightarrow \rho_1 v_1 a_1 \Delta t = \rho_2 v_2 a_2 \Delta t = \rho_3 v_3 a_3 \Delta t$$

$$\Rightarrow \boxed{v_1 a_1 = v_2 a_2 = v_3 a_3}$$

Continuity equation

Energy Associated with moving Liquid :

Potential Energy:



$$P.E = dmgh$$

P.E per unit volume

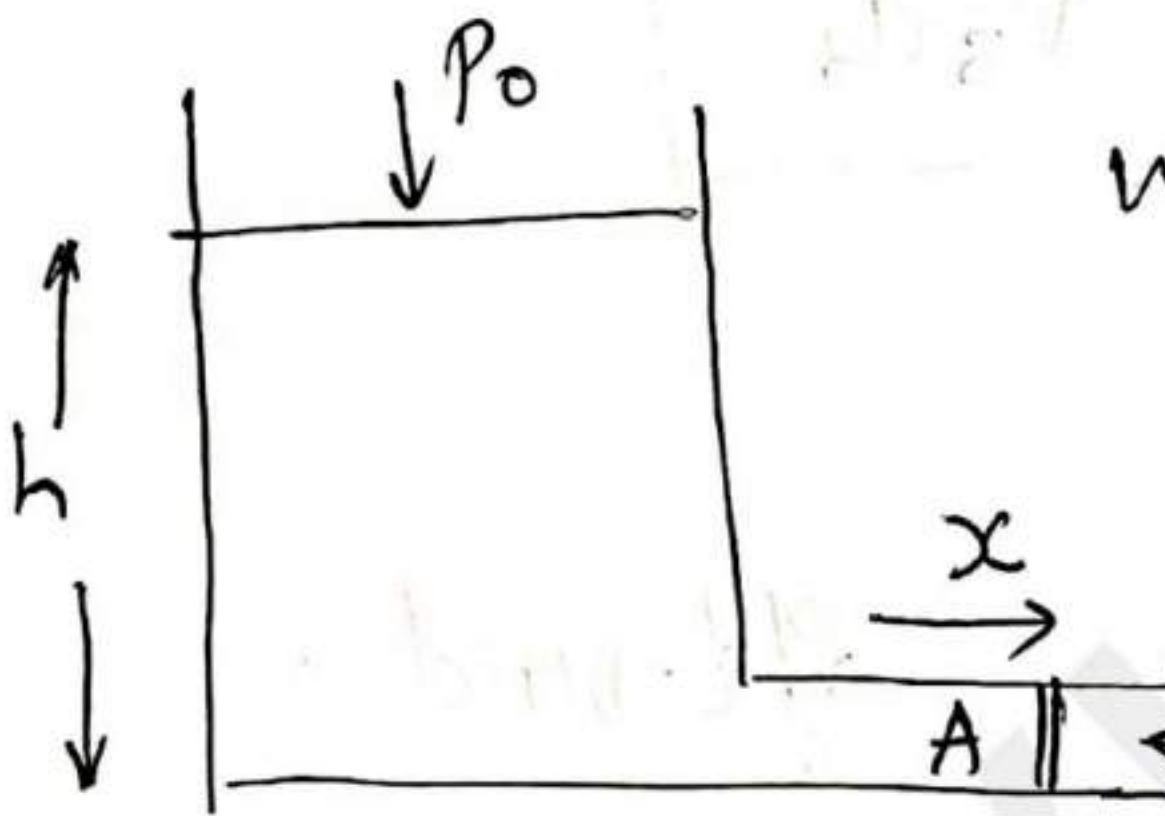
$$\Rightarrow \boxed{\frac{P.E}{dV} = \frac{dm}{dV} gh = \rho gh}$$

Kinetic Energy:

energy per unit volume:

$$= \frac{1}{2} \frac{(dm) v^2}{dv} = \frac{1}{2} \rho v^2$$

Pressure Energy:



work done by displacement - 'x'

$$\Rightarrow dw = (P_0 + \rho g h - P_0) A \cdot dx$$

$$\text{Force} = (P_0 + \rho g h - P_0) A$$

$$\Rightarrow dw = \rho g h A dx$$

So energy per unit volume

$$\Rightarrow \frac{dw}{dv} = \frac{\rho g h (A dx)}{dv}$$

Pressure energy per unit volume

$$= \rho g h \Rightarrow \text{gauge pressure}$$

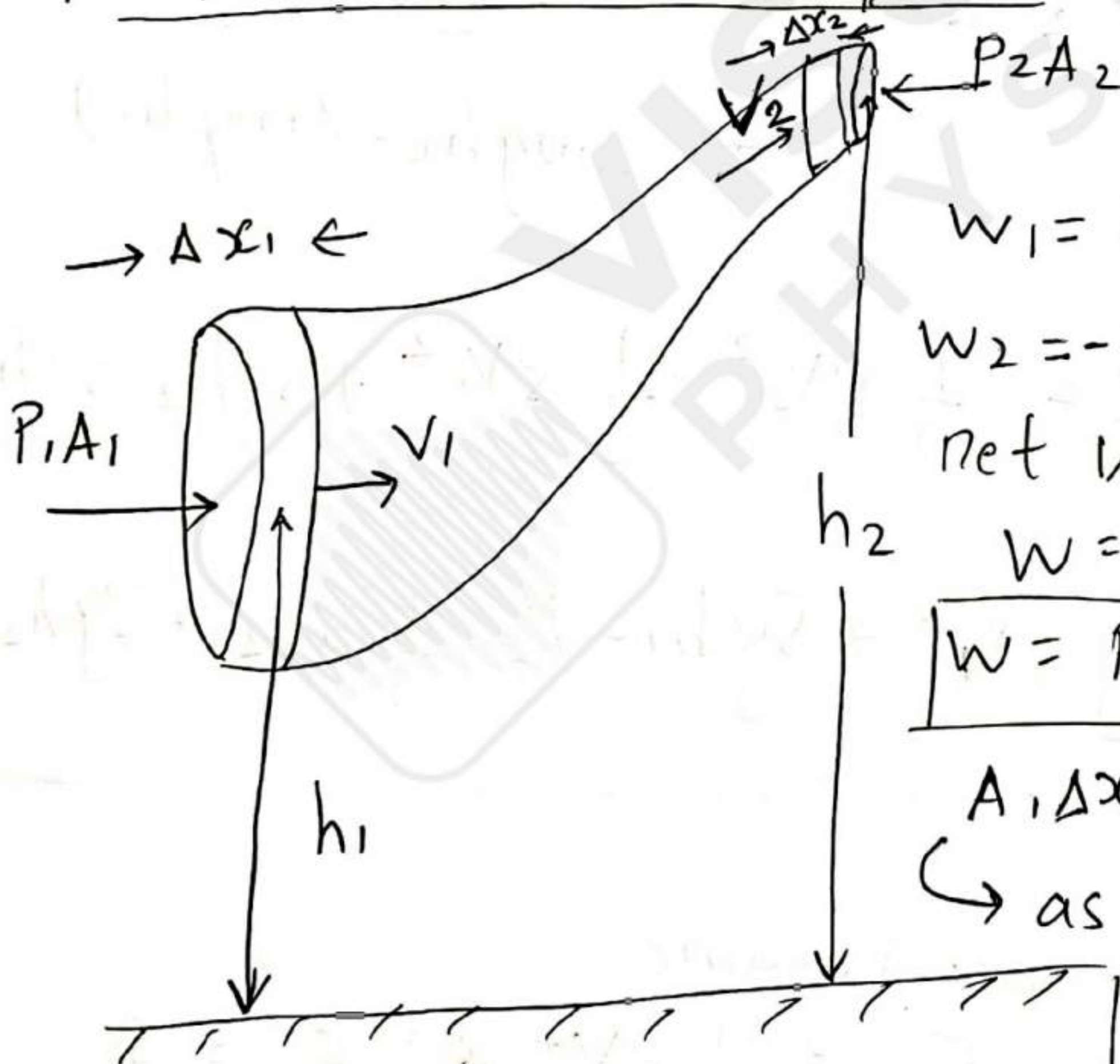
* hence pressure has energy

Bernoulli's Equation :

↳ Sum of total of these three kinds of
★ energy remains constant along a streamline.

$$\Rightarrow \boxed{\frac{1}{2} \rho v^2 + \rho g y + p = \text{Constant}}$$

Proof of Bernoulli's Equation :



$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

$$W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2$$

net work

$$W = W_1 + W_2$$

$$\boxed{W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2}$$

$$A_1 \Delta x_1 = A_2 \Delta x_2 = \Delta V$$

↳ as $A_1 v_1 = A_2 v_2$

$$\boxed{A_1 \frac{\Delta x_1}{\Delta t_1} = A_2 \frac{\Delta x_2}{\Delta t_2}}$$

$$\text{So, } \boxed{W = (P_1 - P_2) \Delta V}$$

change in kinetic-Energy

$$\Rightarrow \Delta K = \frac{1}{2} \Delta m V_2^2 - \frac{1}{2} \Delta m V_1^2$$

$$\& \Delta P.E = \Delta m g h_2 - \Delta m g h_1$$

so Energy conservation:

$$\Rightarrow \Delta W = \Delta K + \Delta U$$

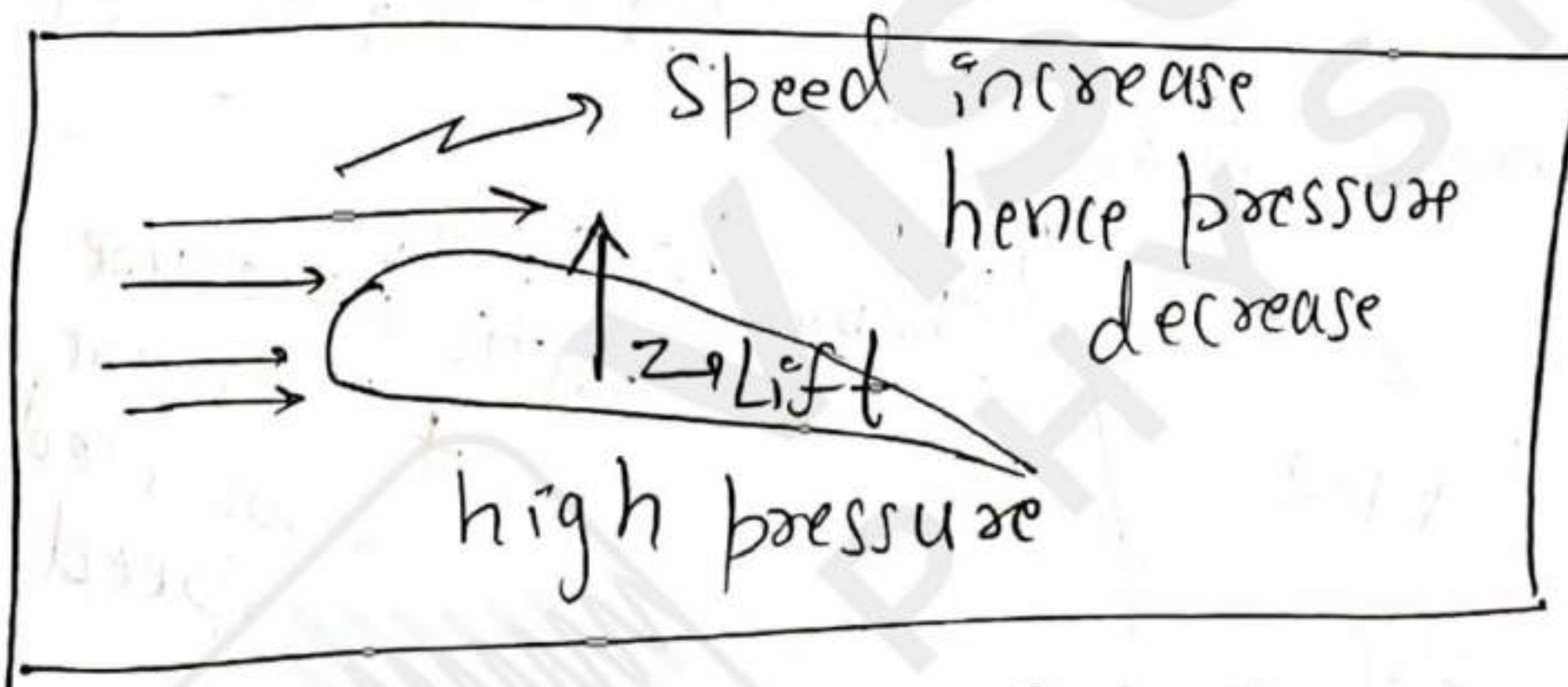
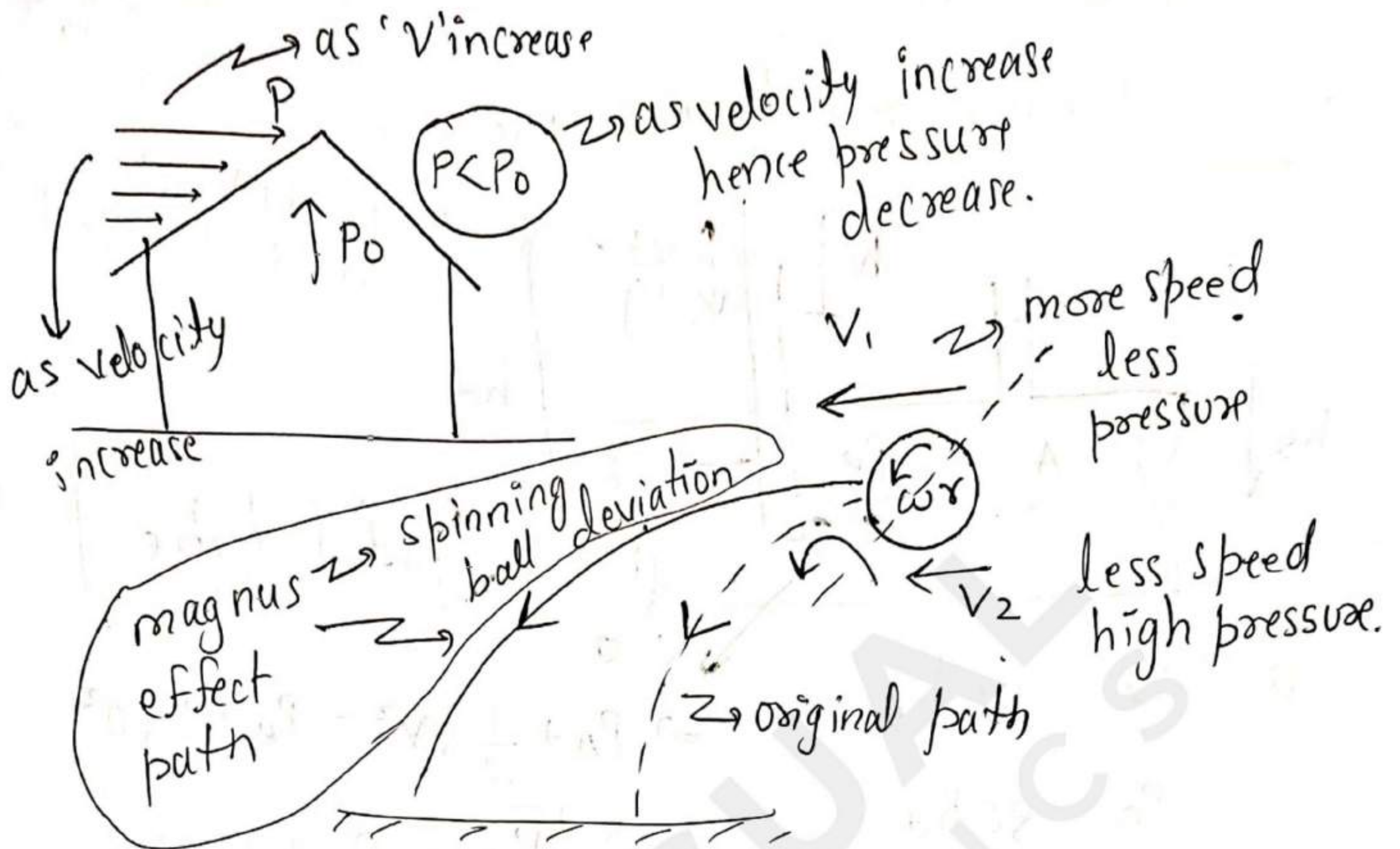
$$\Rightarrow (P_1 - P_2) \Delta V = \left(\frac{1}{2} \Delta m V_2^2 - \frac{1}{2} \Delta m V_1^2 \right) + (\Delta m g h_2 - \Delta m g h_1)$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2 + \rho g h_2 - \rho g h_1$$

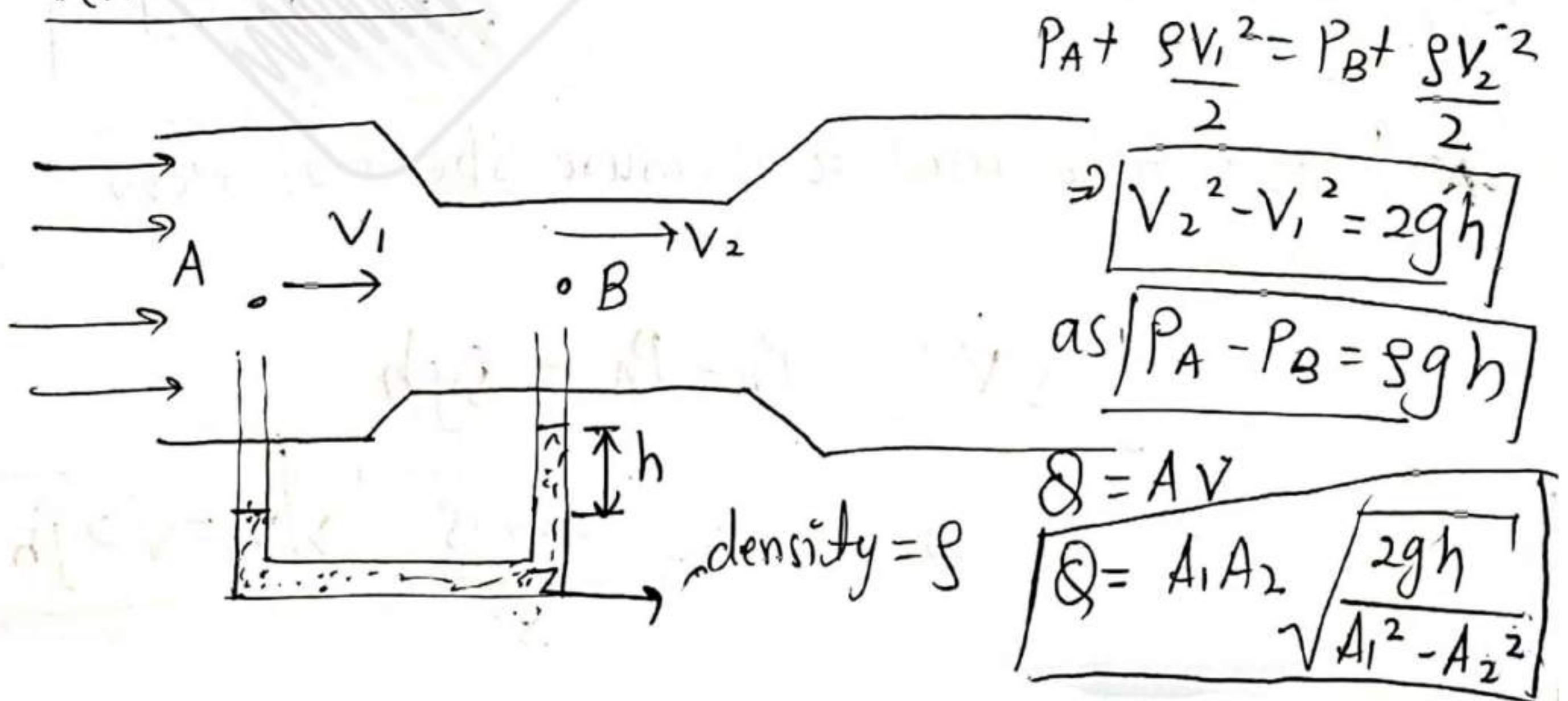
$$\Rightarrow \boxed{P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2}$$

As energy per volume

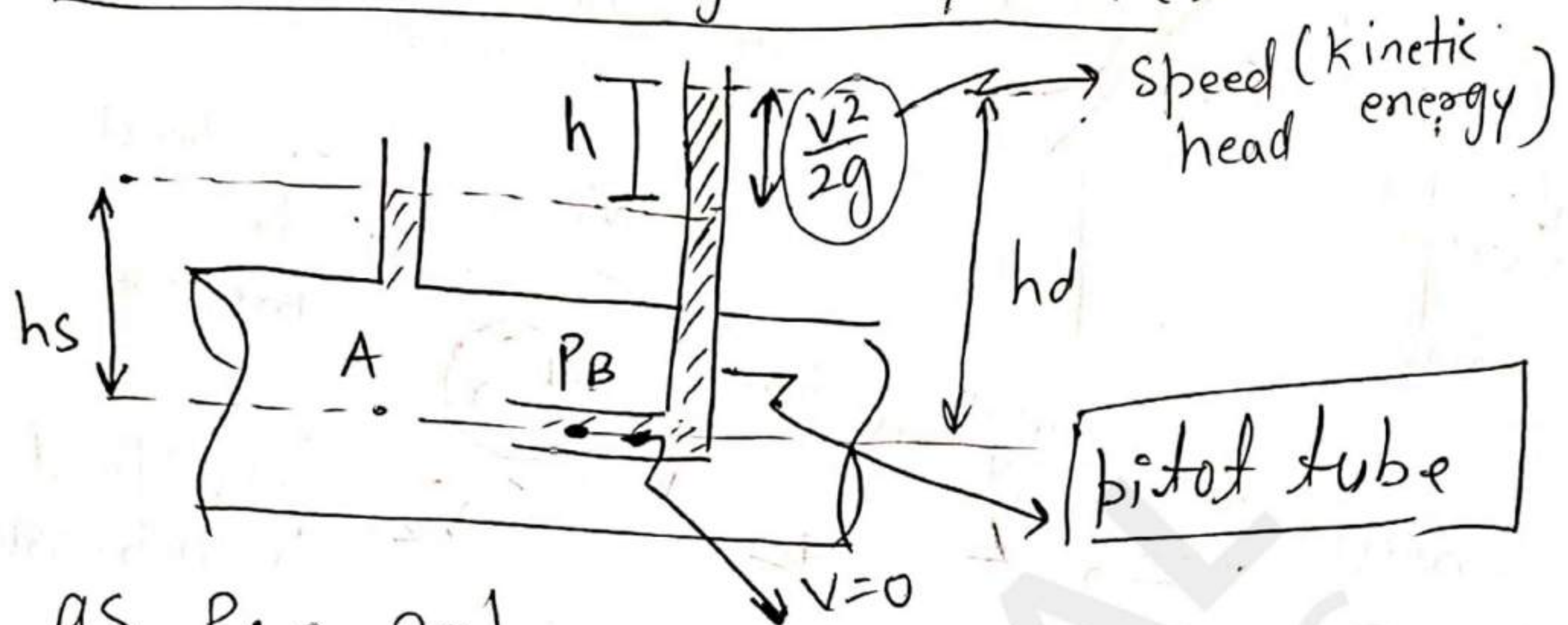
$$\frac{K.E}{\text{Volume}} = \frac{1}{2} \rho V^2 \quad \& \quad \frac{P.E}{\text{Volume}} = \rho g h$$



Venturi-meter: used for measuring rate of flow



Static pressure and dynamic Pressure:



as $P_A = \rho g h_s$

$P_B = \rho g h_d$

$$\Rightarrow P_A + \frac{1}{2} \rho V^2 = P_B + \frac{1}{2} \rho 0^2$$

$$\Rightarrow \boxed{P_B = P_A + \frac{1}{2} \rho V^2}$$

now in terms of head

dynamic pressure = static pressure + pressure associated with speed.

$$\boxed{h_d = h_s + \frac{\frac{1}{2} \rho V^2}{\rho g}}$$

dynamic head = static head + head associated with speed

★ pitot tube used to measure speed of flow

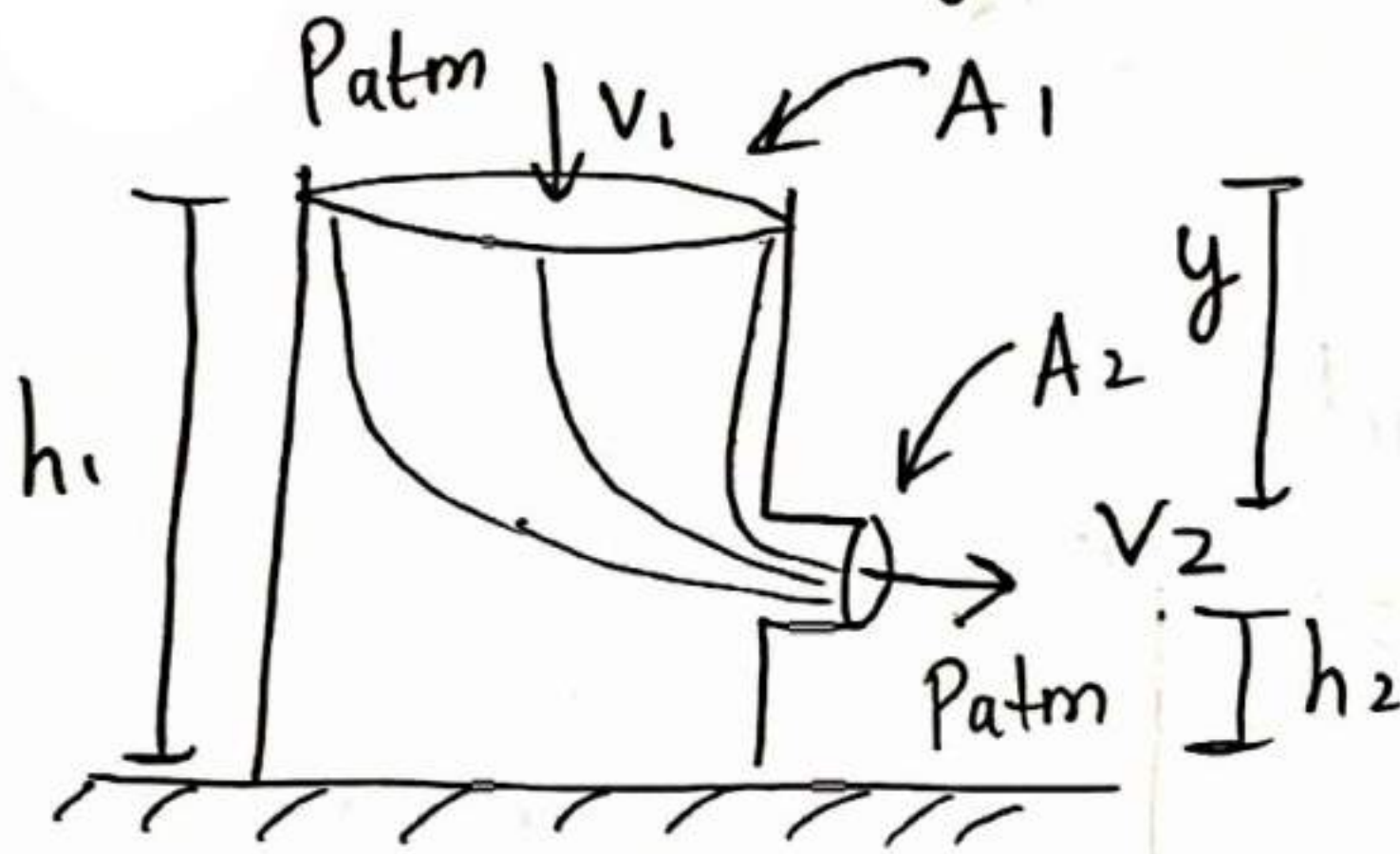
$V_B = 0$

$$\Rightarrow \frac{1}{2} \rho V^2 = P_B - P_A = \rho g h$$

$$\Rightarrow V = \sqrt{\frac{2gh\rho}{\rho}} \Rightarrow \boxed{V = \sqrt{2gh}}$$

Velocity of EFFLUX

liquid comes out of a hole made at a depth y .



$$\text{as } A_1 v_1 = A_2 v_2$$

$$P_{atm} + \frac{1}{2} \rho v_1^2 + \rho g h_1$$

$$= P_{atm} + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\Rightarrow h_1 - h_2 = y$$

$$\Rightarrow v_2^2 = v_1^2 + 2gy$$

$$\Rightarrow v_2 = \sqrt{\frac{2gy}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

using

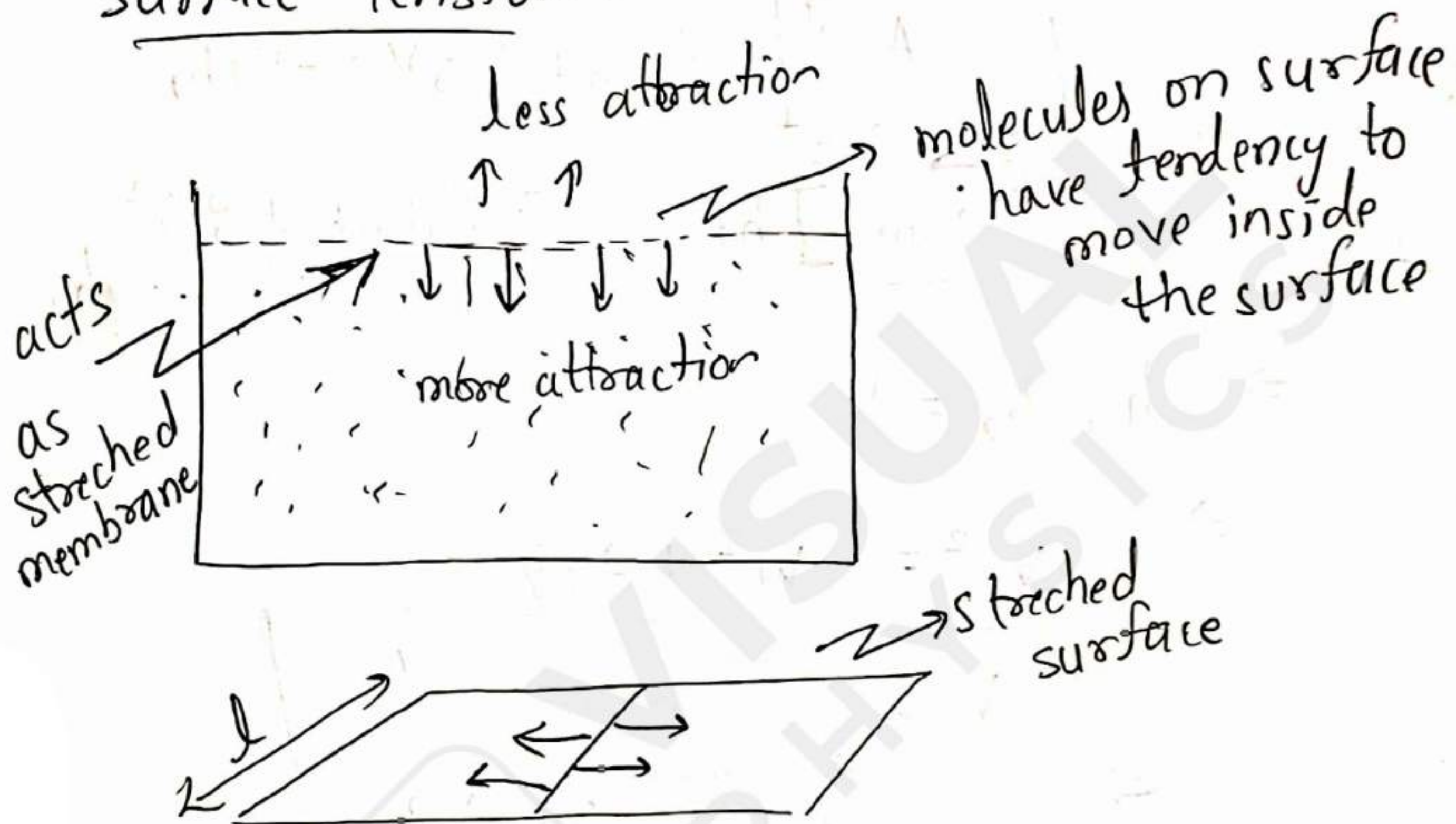
$$v_1 = \left(\frac{A_2}{A_1}\right) v_2$$

so, time to fall on ground = t

$$\Rightarrow t = \sqrt{\frac{2h_2}{g}}$$

$$\text{Range} = v_2 t = \sqrt{\frac{2gy}{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2h_2}{g}}$$

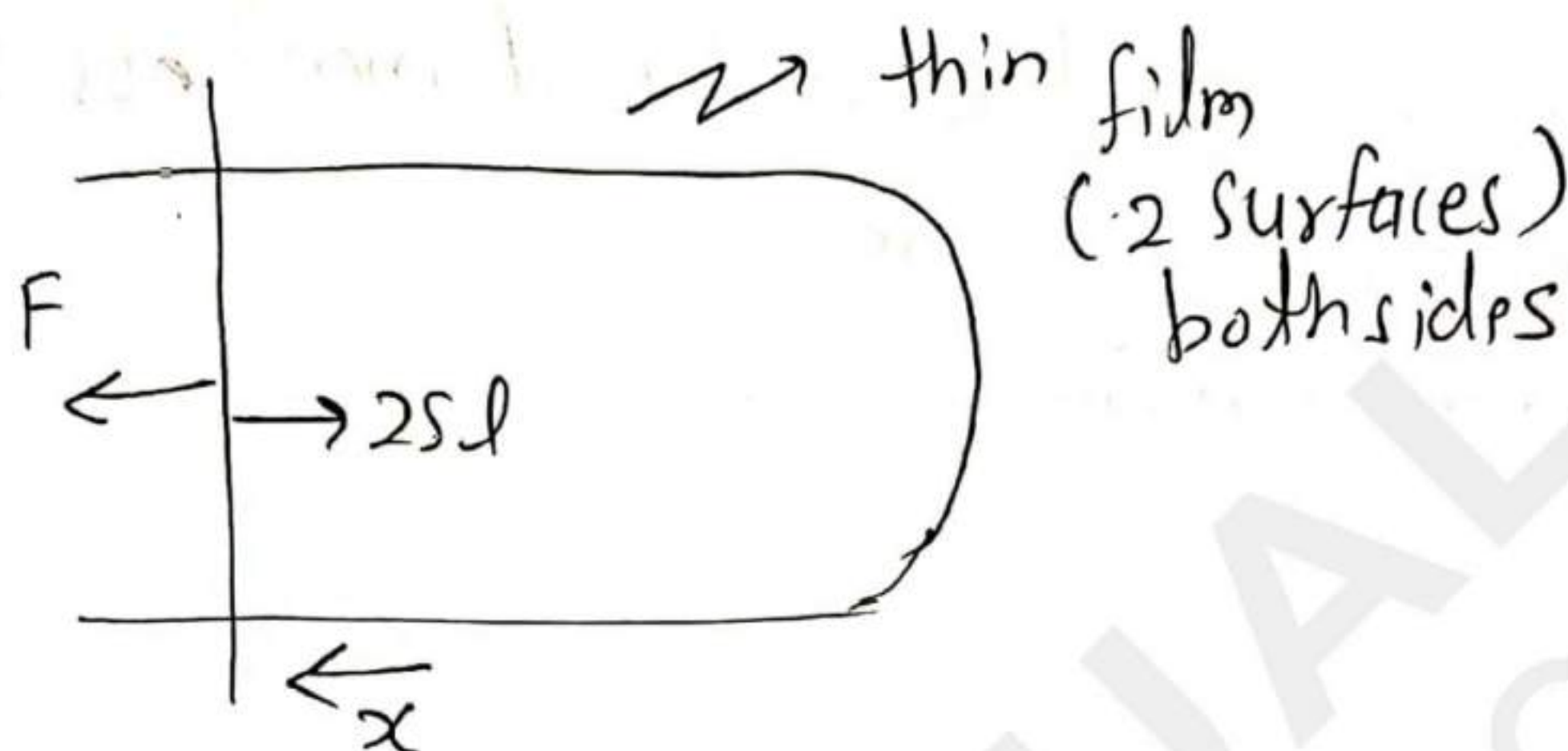
Surface tension:



now $\frac{\text{force}}{\text{length}} = \text{surface tension} = \gamma$

* hence because of this $\rightarrow$ surface tends to have smaller area

Surface Energy: $= \frac{\text{potential energy}}{\text{Area of surface}} = \text{Surface tension}$



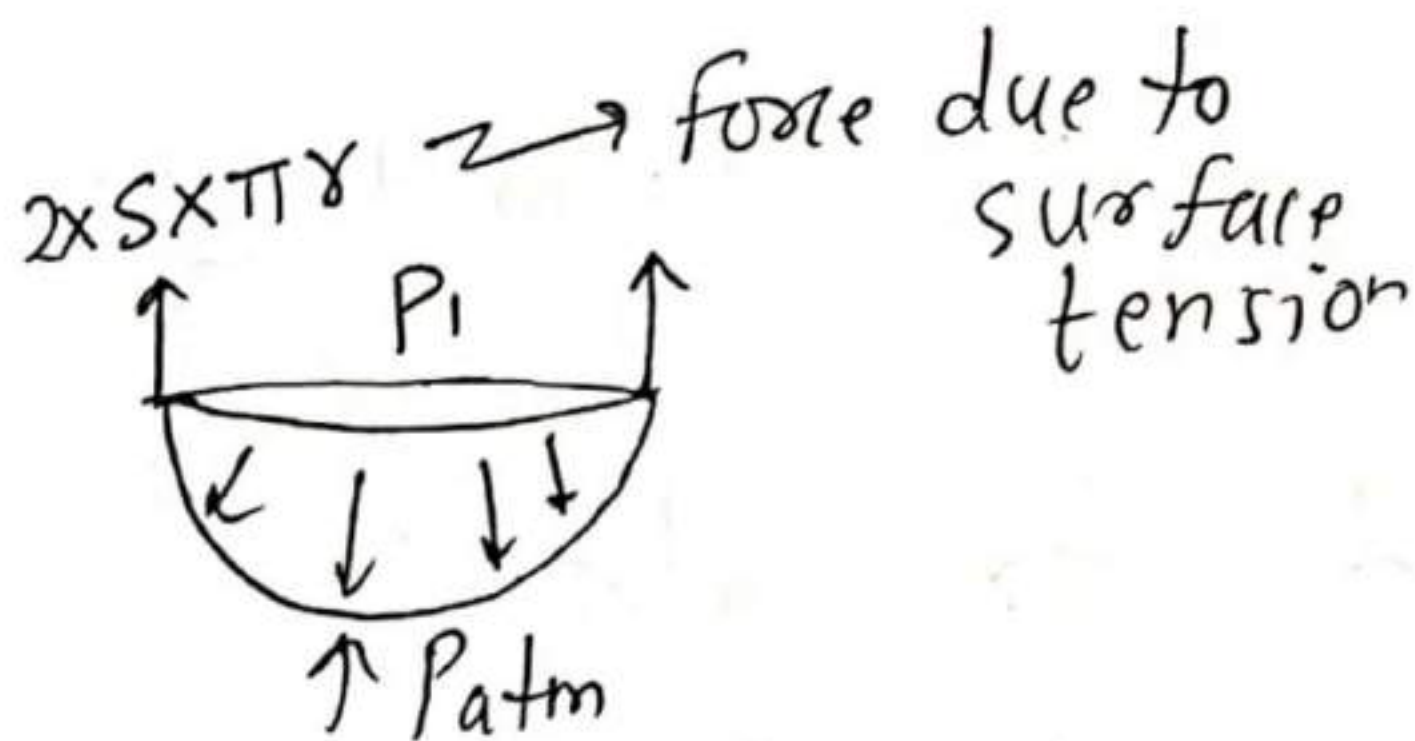
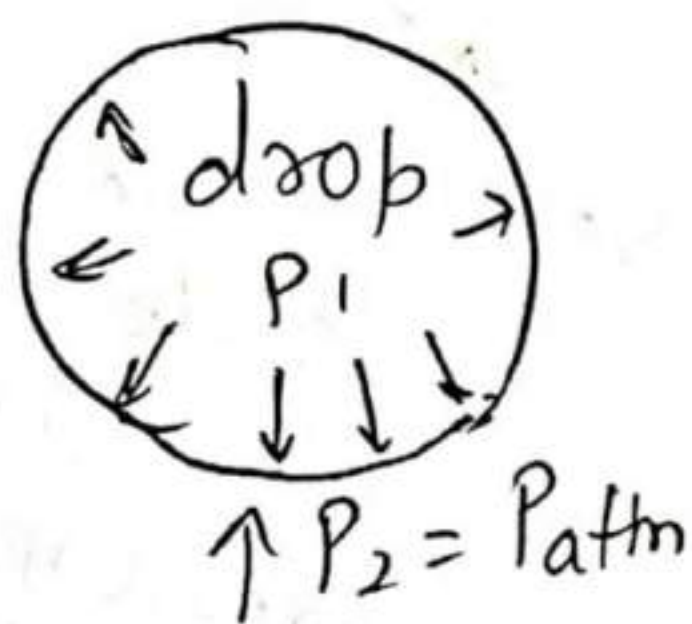
$$\Delta W = (2sl)x = \Delta U$$

$$\Delta A = 2lx$$

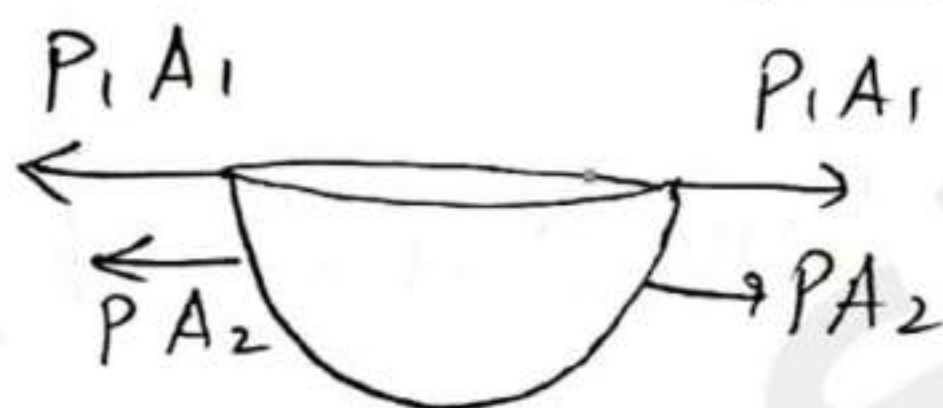
so $\left| \text{surface energy} = \frac{\Delta U}{\Delta A} = S = \text{Surface tension} \right|$

★ for given volume $\rightarrow$ sphere have lowest surface area
 Hence liquid drops are in spherical shape

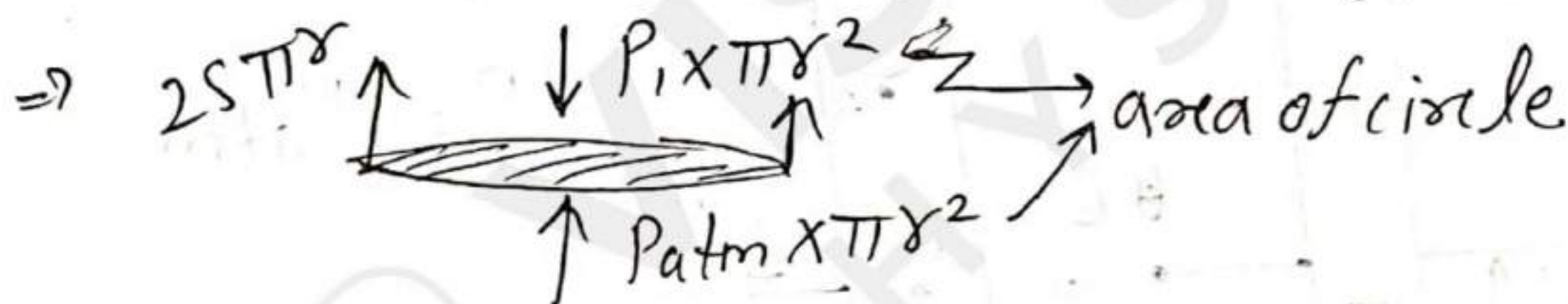
Pressure Inside Bubble and drop



now net force is force on projected area



as force in horizontal direction will cancel out



$$\Rightarrow 2s\pi r + P_{atm} \times \pi r^2 = P_1 \times \pi r^2$$

$$\Rightarrow \boxed{P_1 = P_{atm} + \frac{2s}{r}}$$

Not for bubble there are two surface, inside & outside

$$\text{hence } \boxed{P_1 = P_{atm} + 2\left(\frac{2s}{r}\right) = \left(\frac{4s}{r}\right)}$$

hence for bubble additional pressure

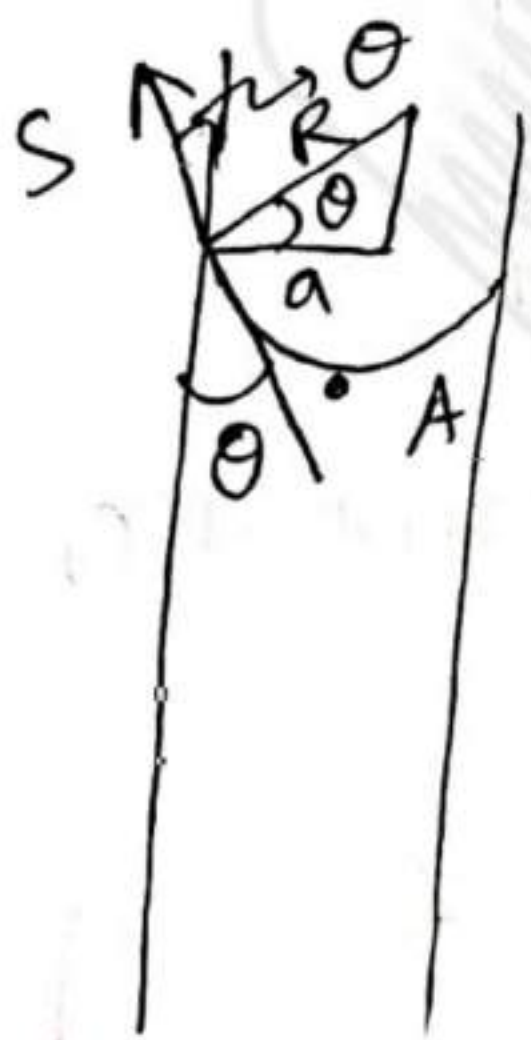
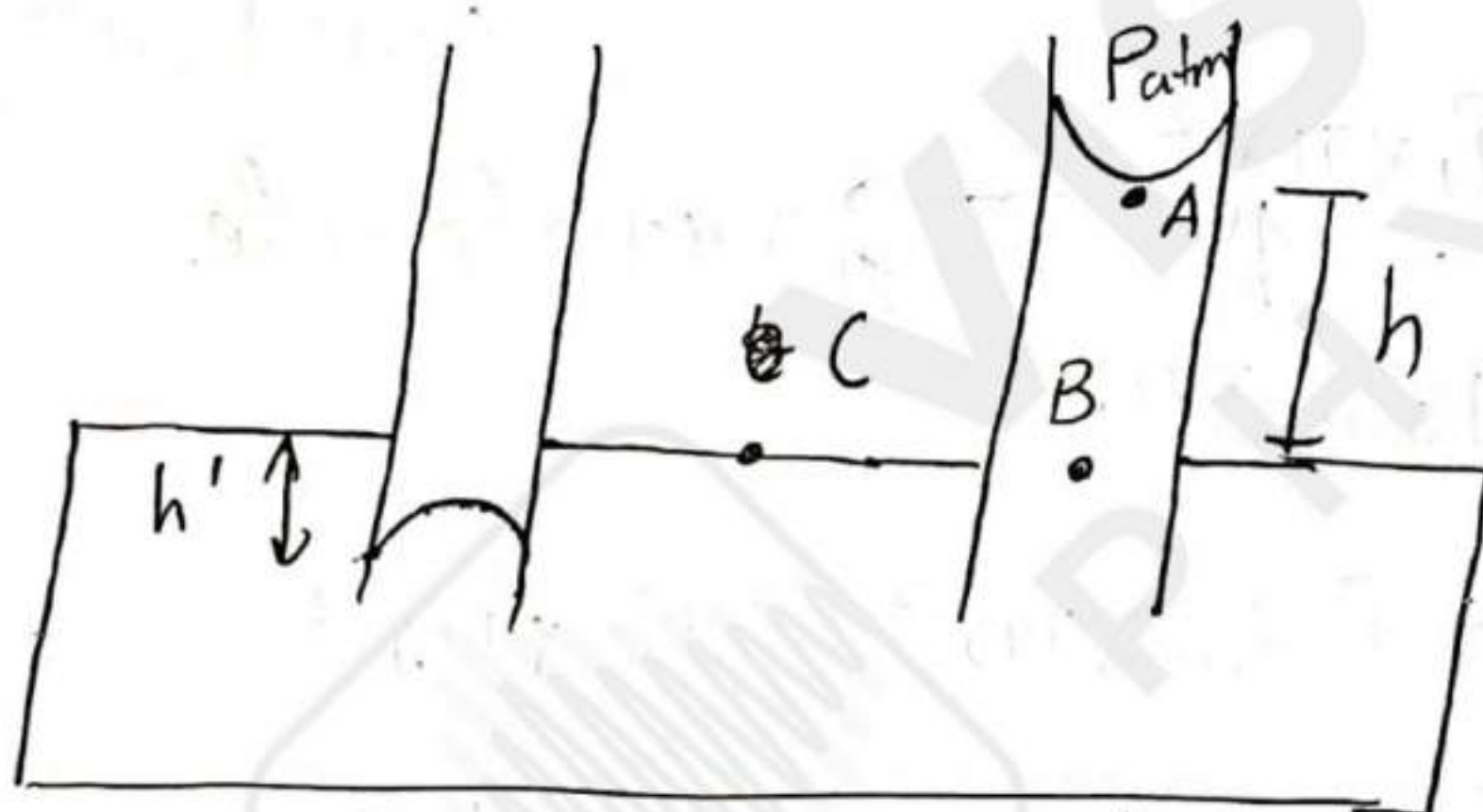
$$P_i = P_{atm} + \frac{4s}{r}$$

So ~~*~~ concave side have additional pressure!

& for drop:

$$P_i = P_{atm} + \frac{2s}{r}$$

Capillary tube (capillary rise/fall)



$a \rightarrow$ radius of tube

$R \rightarrow$ radius of surface

now
$$P_A = P_{atm} - \frac{2s}{R}$$

as concave side pressure have additional pressure $\frac{2s}{R}$

hence convex side pressure

now $P_B = P_C = P_{atm}$

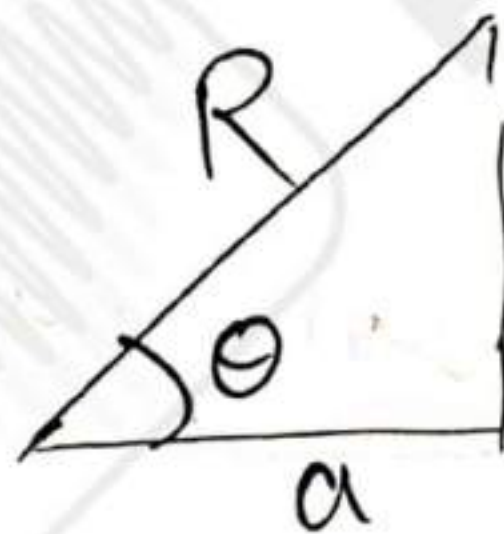
now $P_B = P_A + \rho g h$

$$P_{atm} = P_{atm} - \frac{2s}{R} + \rho g h$$

$$\Rightarrow h = \frac{2s}{\rho g R}$$

★ Same for capillary fall

now



$$a = R \cos \theta$$

$$R = \frac{a}{\cos \theta}$$

$$\Rightarrow h = \frac{2s \cos \theta}{\rho g a}$$

Viscosity:

Ideal fluid $\rightarrow$ no friction

(and the cases derived were for ideal fluid)

Real fluid $\rightarrow$ have internal friction.

It means. layers of fluids applies drag on each other.

★ Now this internal resistance in fluids layers is known as viscosity.

so velocity profile:

