VISUAL PHYSICS

SHORT NOTES

CHAPTER

Electric Power

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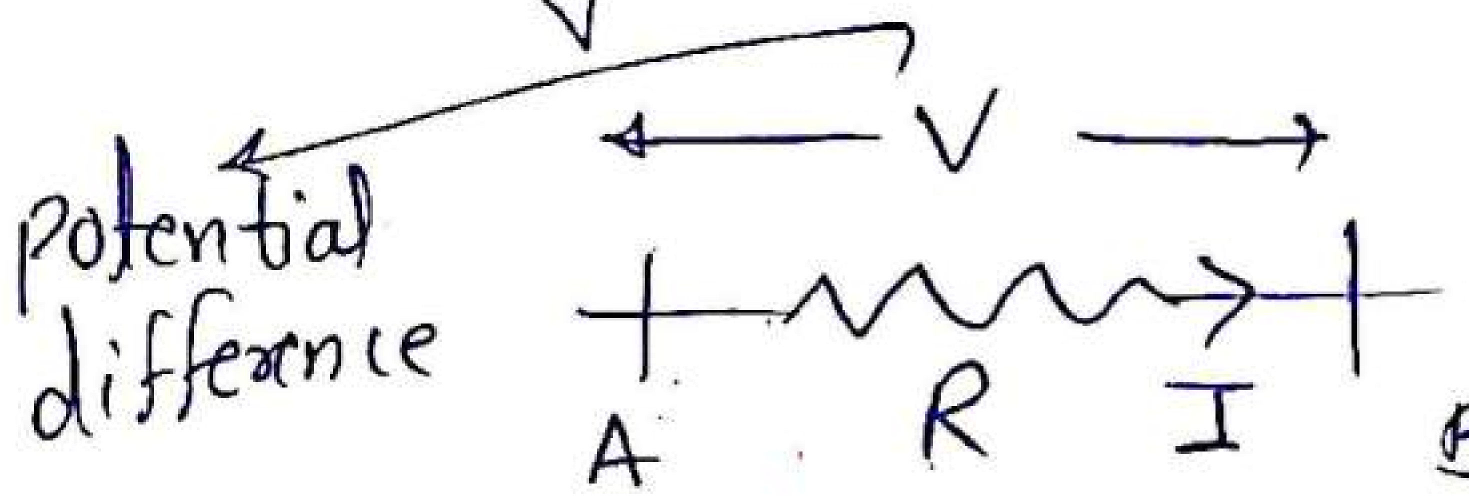
ELECTRIC POWER

heating effect → when current pass through a conductor it becomes hot.

due to collisions of drifting electrons inside the conductor, loss of electrons K.E results in heating of conductor.



amount of charged pass in 't' seconds $q = it$

loss in potential energy $= qV$
$= iVt$

as $V = iR$

$$\Rightarrow \boxed{\text{Energy loss} = i^2Rt}$$

or Energy loss $= \dfrac{V^2}{R}t$

So in terms of energy loss per seconds

i.e. $\boxed{\text{power loss} = i^2R = \dfrac{V^2}{R}}$

Joules law of heating,

Heat produced $\propto i^2$
$\propto t$
$\propto R$

$\Rightarrow$ heat lost $= i^2Rt$

$1\,cal = 4.18\,J = 4.2\,J$

$1\,kWh = 1000\,W \times hour = 1000\,W \times 3600\,s$
$= 3.6 \times 10^6\,J$

1kWh → power consumed 1kW for one hour

$1\,hp = 746\,W$

→ In series → as current is same in series

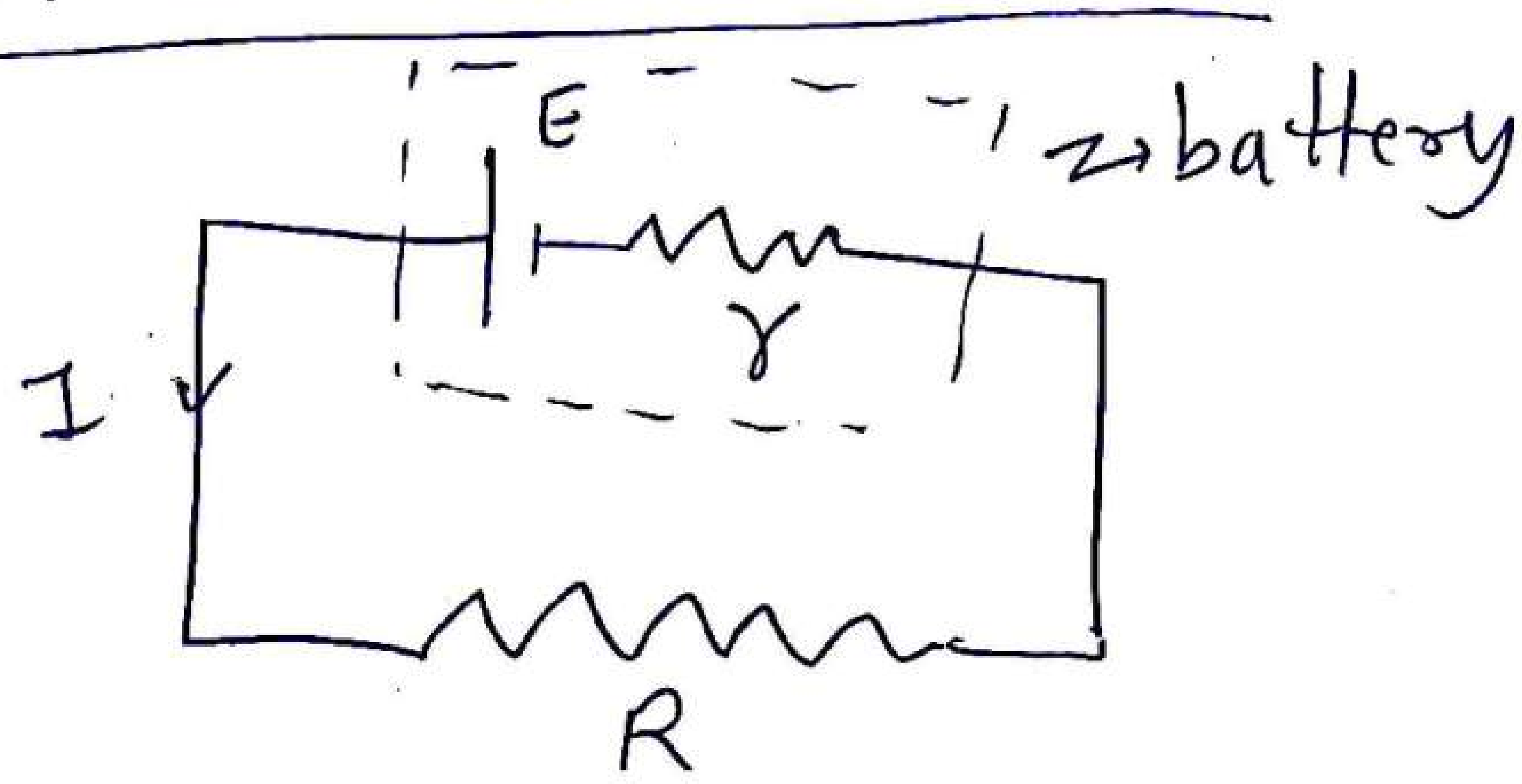
$P = I^2R$, more R more Power loss

→ In parallel:

$P = \frac{V^2}{R}$, less R, more power loss.

as resistor have same potential difference in parallel.

Maximum power Transfer



As power loss in external resistor $= I^2 R$

$$I = \frac{E}{R+r} \quad (E \rightarrow \text{emf})$$

$$P = \frac{E^2 R}{(R+r)^2}$$

$$\frac{dP}{dR} = 0, \quad E^2\left[\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4}\right] = 0$$

$$\Rightarrow (r+R) = 2R \text{ or } \boxed{r = R}$$

So if external resistance = internal resistance than maximum power output to external system R.

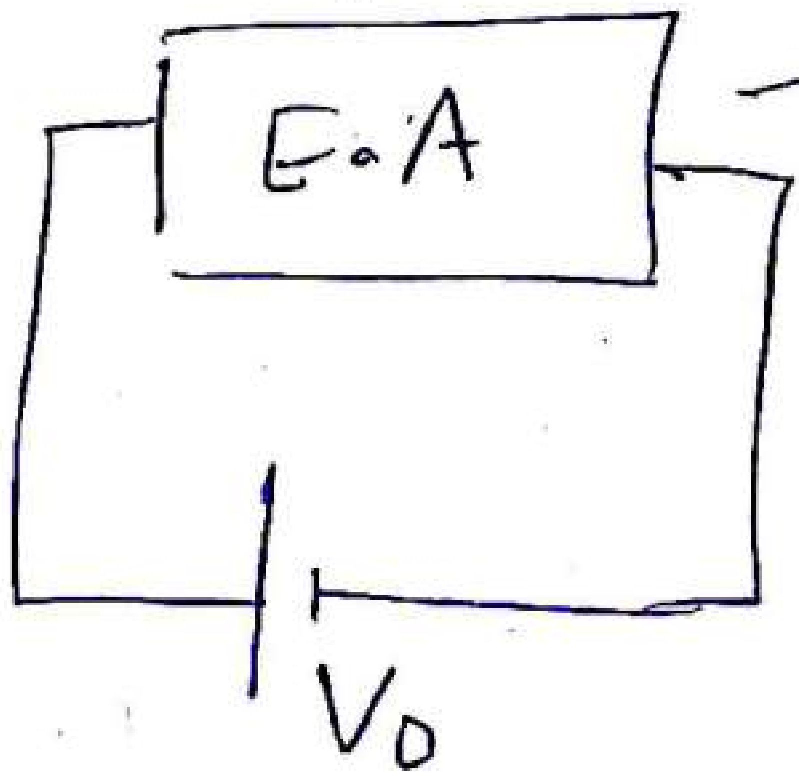
Electric Appliances :

Each Appliance requires power

↳ Rating $[P_0, V_0]$ — P_0 → Rated power; V_0 ↳ Rated voltage

that means:

→ Electric Appliance



→ P_0

$$\Rightarrow P_0 = \frac{V_0^2}{R}$$

or

$$\boxed{R = \frac{V_0^2}{P_0}}$$

if power supplied by V

$$\Rightarrow P = \frac{V^2}{R} = \frac{V^2}{V_0^2} P_0$$

$$\Rightarrow \boxed{P = \left(\frac{V}{V_0}\right)^2 P_0}$$

↗ as appliance work on specific maximum current.

if $\boxed{V > V_0 \Rightarrow P > P_0}$ → damage the appliance.

$\boxed{V < V_0 \Rightarrow P < P_0}$

Long distance power transmission:

Power loss $= I^2R = P$

also $P_0' = VI$

$\hookrightarrow$ given power

$\Rightarrow I = \frac{P_0'}{V}$

$\Rightarrow$ Power loss $= \frac{P_0'^2}{V^2} R$

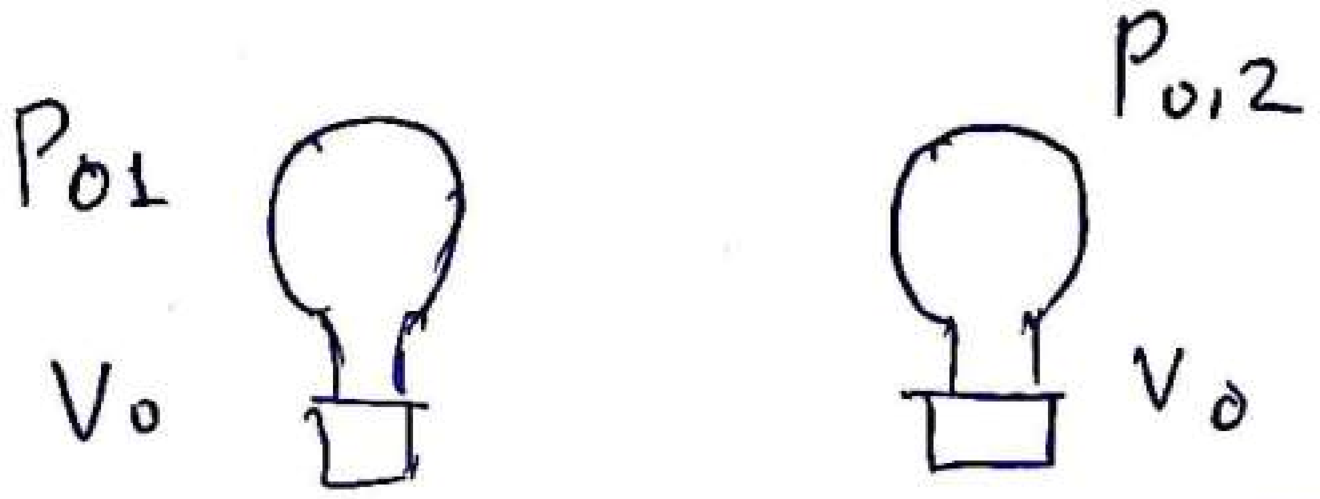
$\Rightarrow$ power loss $\propto \frac{1}{V^2}$

more the potential drop, less the power loss.

OR say less the current, less the power loss. So for given power P_0', V should be high, I should be low

$\Rightarrow$ so transmission lines are high voltage power supplie

Appliance Combination

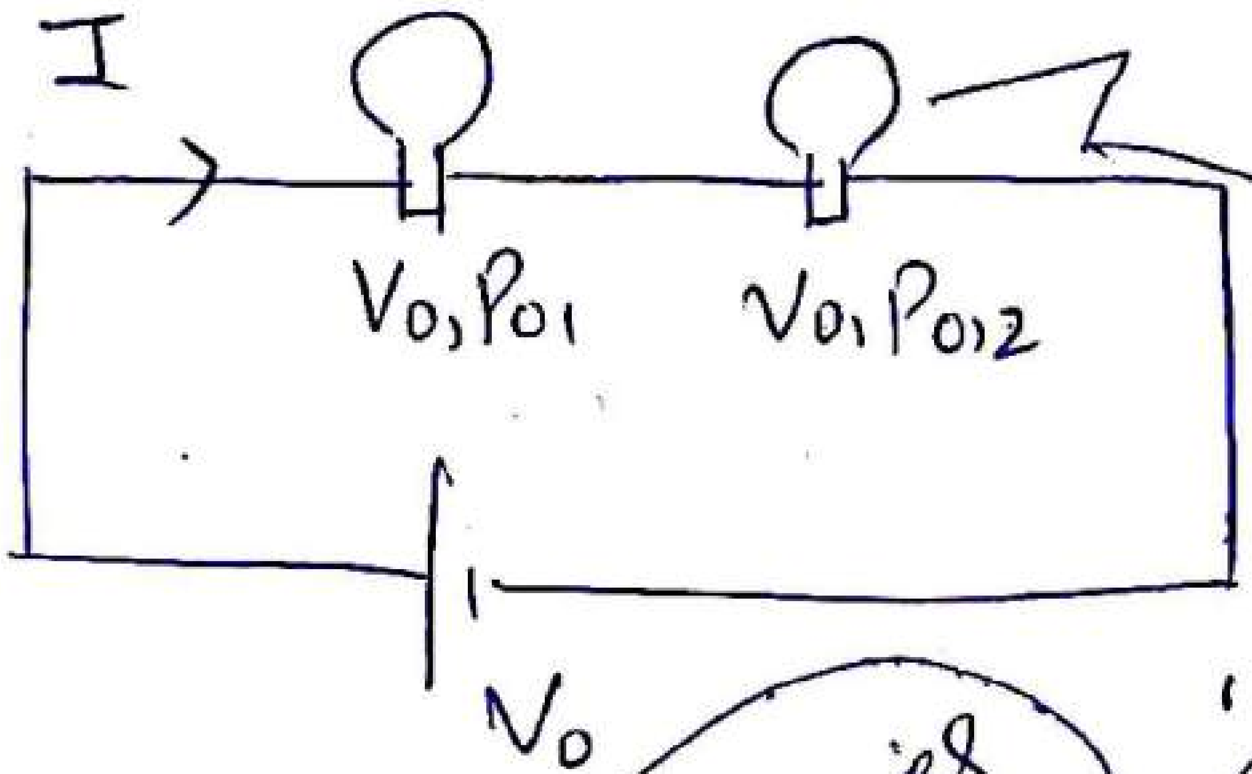


$P_{01} > P_{0,2}$ | $R_1 < R_2$

$\rightarrow \frac{V_0^2}{R_1} > \frac{V_0^2}{R_2}$

→ higher power rating for same V_0, means less Resistance.

$R_1 < R_2$



I is same

so, $P_1 : P_2 = R_1 : R_2$

$P_1 < P_2$

hence, that means bulb 2 glow brighter

In series low power rating glow brighter

$I = \frac{V_0}{R_1 + R_2}$

$P_1 = I^2 R_1$

$P_2 = I^2 R_2$

$P_{net} = I^2 (R_1 + R_2) = P$ → total power consumed

$$P = \frac{V_0^2}{(R_1+R_2)^2}(R_1+R_2)$$

$$P = \frac{V_0^2}{(R_1+R_2)}$$

or $$\frac{1}{P} = \frac{R_1+R_2}{V_0^2} = \frac{R_1}{V_0^2} + \frac{R_2}{V_0^2}$$

$$\frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$$

or $$\boxed{P = \frac{P_{01} \times P_{02}}{P_{01} + P_{02}}}$$

only when applied voltage $V = V_0$

if $V \neq V_0$

$$P' = \left(\frac{V}{V_0}\right)^2 P$$

$$\boxed{P' = \left(\frac{V}{V_0}\right)^2 \left[\frac{P_{01} \times P_{02}}{P_{01} + P_{02}}\right]}$$

or In general

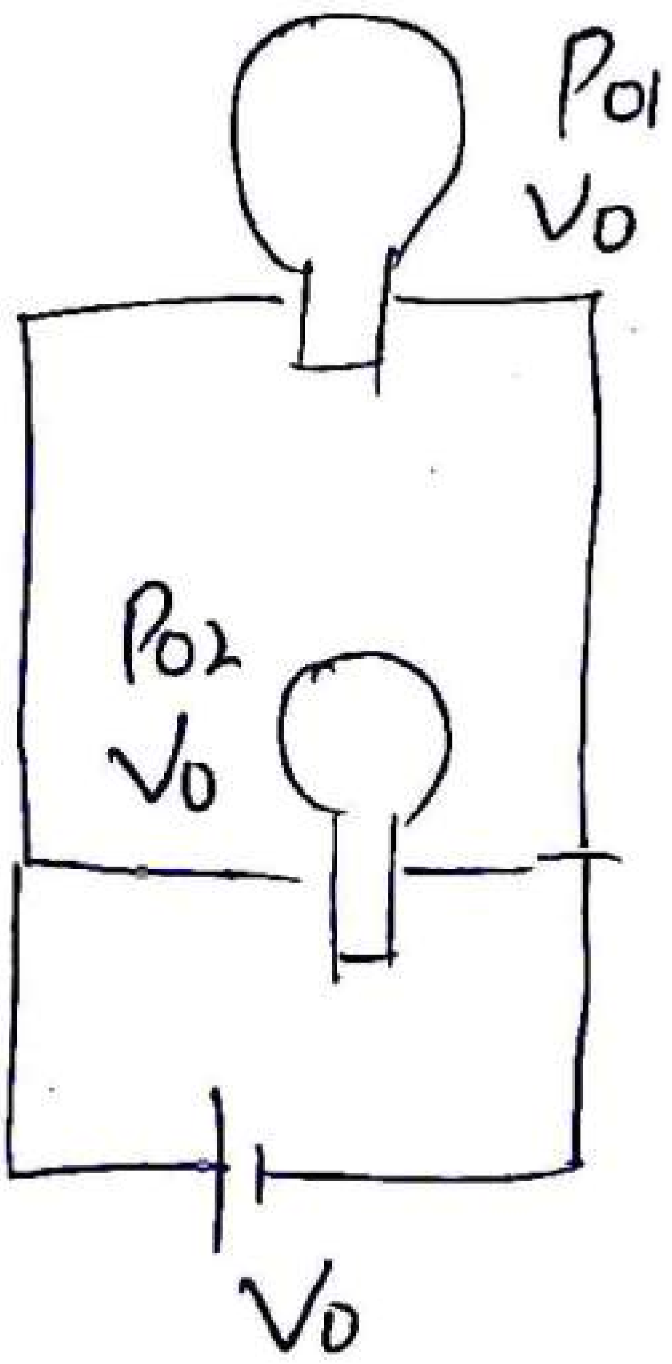
$$\boxed{\frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}} + \dots \frac{1}{P_{0n}}}$$

P → Power consumed when $V = V_0$

V_0 → Rated voltage

V → applied voltage

$P_{01} > P_{02}$



$R_1 < R_2$ ⇝ for parallel

$$P_1 : P_2 = \frac{V^2}{R} : \frac{V^2}{R_2}$$

$$P_1 : P_2 = \frac{1}{R_1} : \frac{1}{R_2}$$

$$\Rightarrow \boxed{P_1 > P_2}$$

so 1 glow brighter in parallel

$$P_1 = \frac{V_0^2}{R_1} = P_{01}$$

$$P_2 = \frac{V_0^2}{R_2} = P_{02}$$

$$\boxed{P = P_1 + P_2 = P_{01} + P_{02}}$$

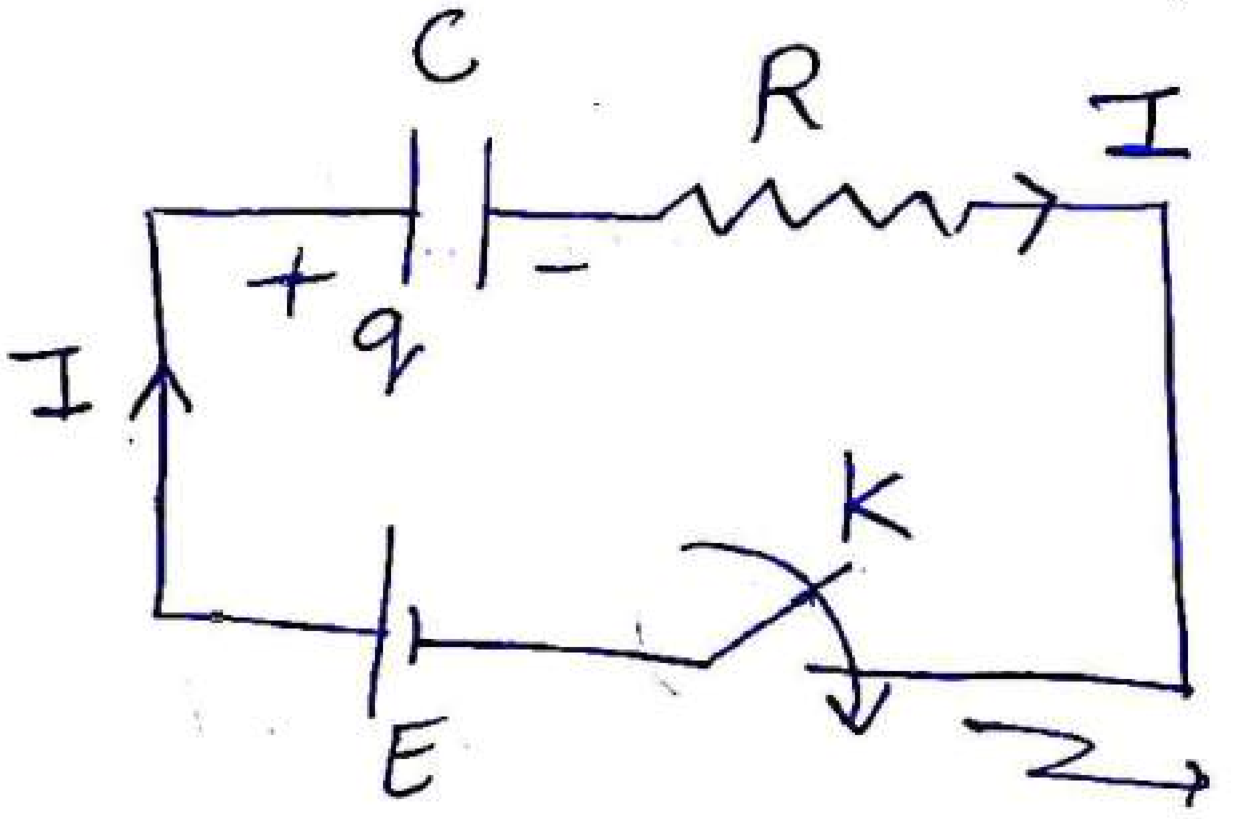
→ if $V = V_0$

if $V \neq V_0$

$$P' = \left(\frac{V}{V_0}\right)^2 P = \left(\frac{V}{V_0}\right)^2 [P_{01} + P_{02}]$$

in general ⇒ $\boxed{P = P_1 + P_2 + \ldots P_n}$

Charging of Capacitor :



initial Conditions:

$t=0, q=0$

$I = \frac{dq}{dt}$

→ key is closed

Applying kirchhoff's law

$$E = \frac{q}{C} + IR$$

$$E = \frac{q}{C} + \frac{dq}{dt}R$$

$$\Rightarrow \int R\frac{dq}{dt} = \int -\frac{q}{C} + E$$

$$\Rightarrow \int_0^{q_0} \frac{dq}{(EC-q)} = \int_0^t \frac{dt}{RC}$$

$$\ln(EC-q) = \ln EC - \frac{t}{RC}$$

$$\ln\left[\frac{EC-q}{EC}\right] = -\frac{t}{RC}$$

$$\frac{EC-q}{EC} = e^{-t/RC}$$

$$\boxed{q = EC\left[1-e^{-t/RC}\right]}$$

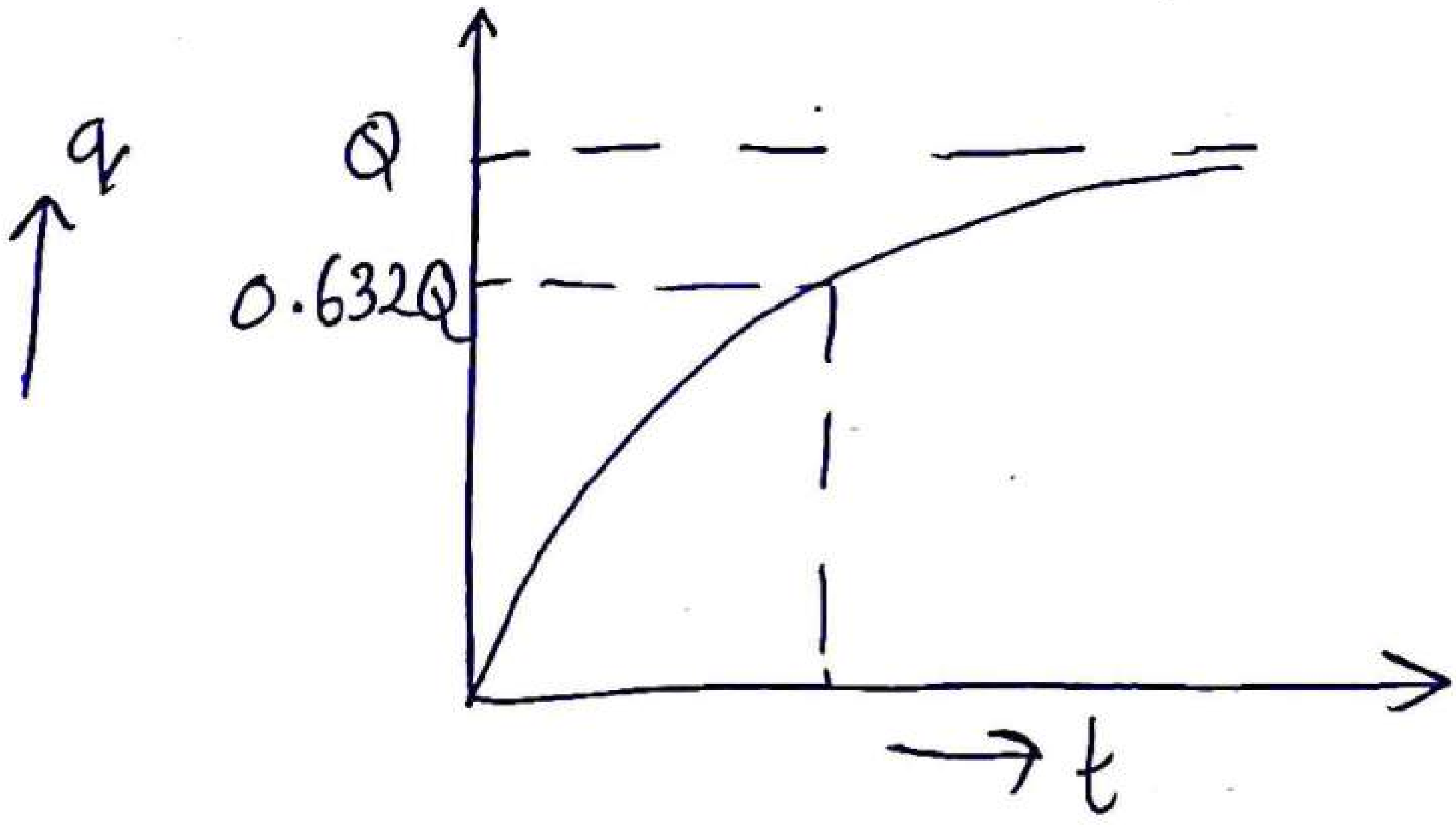
$\tau = RC \rightarrow$ time constant

Whe $t = \tau$

$$q = Q\left(1-e^{-\tau/\tau}\right) = Q\left(1-e^{-1}\right)$$

$$\boxed{q = 0.632Q}$$

So, $t \rightarrow \tau$, charging to 63.2%

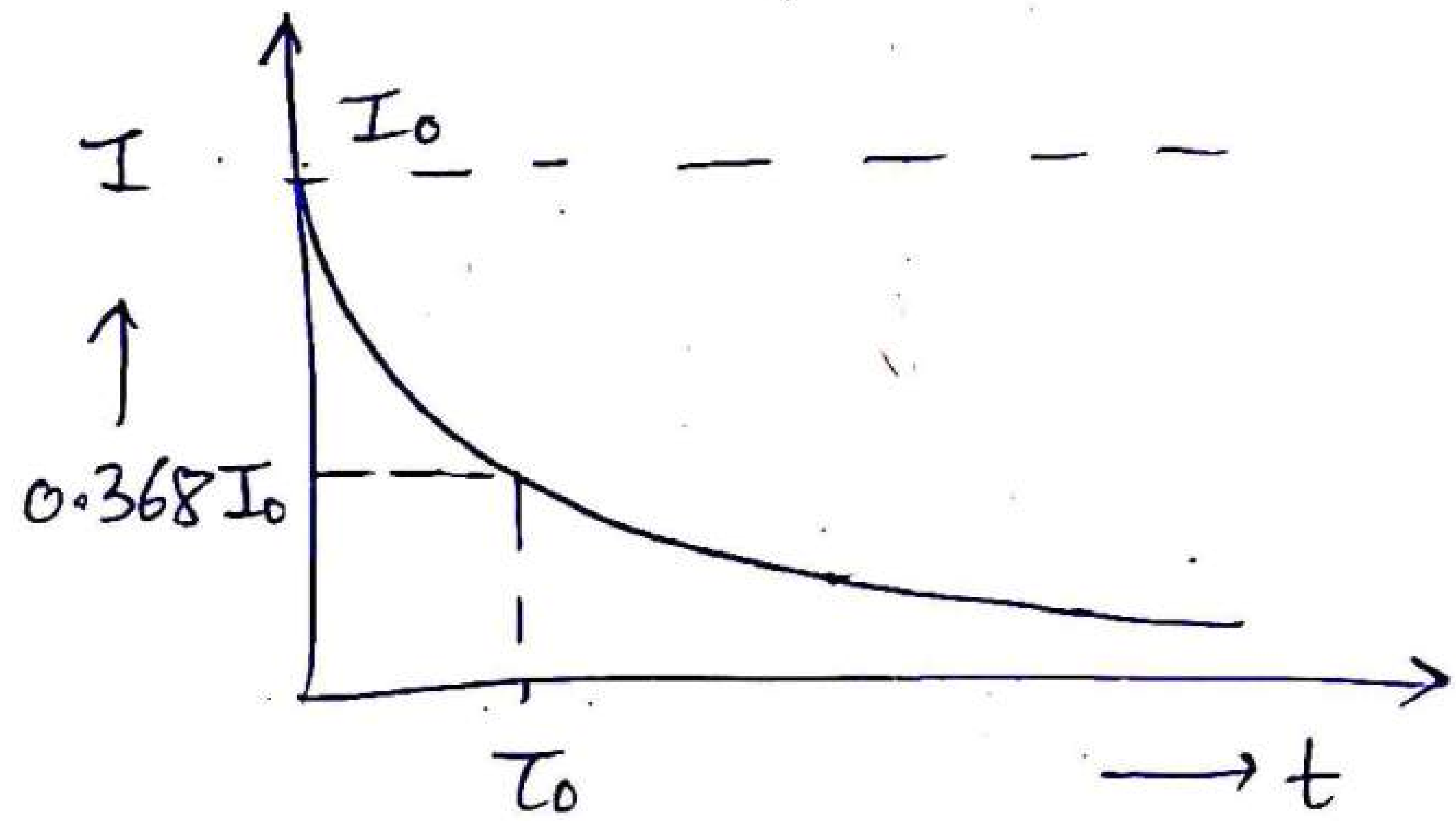


$$I = \frac{dq}{dt} = Q\left[0 - e^{-t/\tau}\left(-\frac{1}{\tau}\right)\right]$$

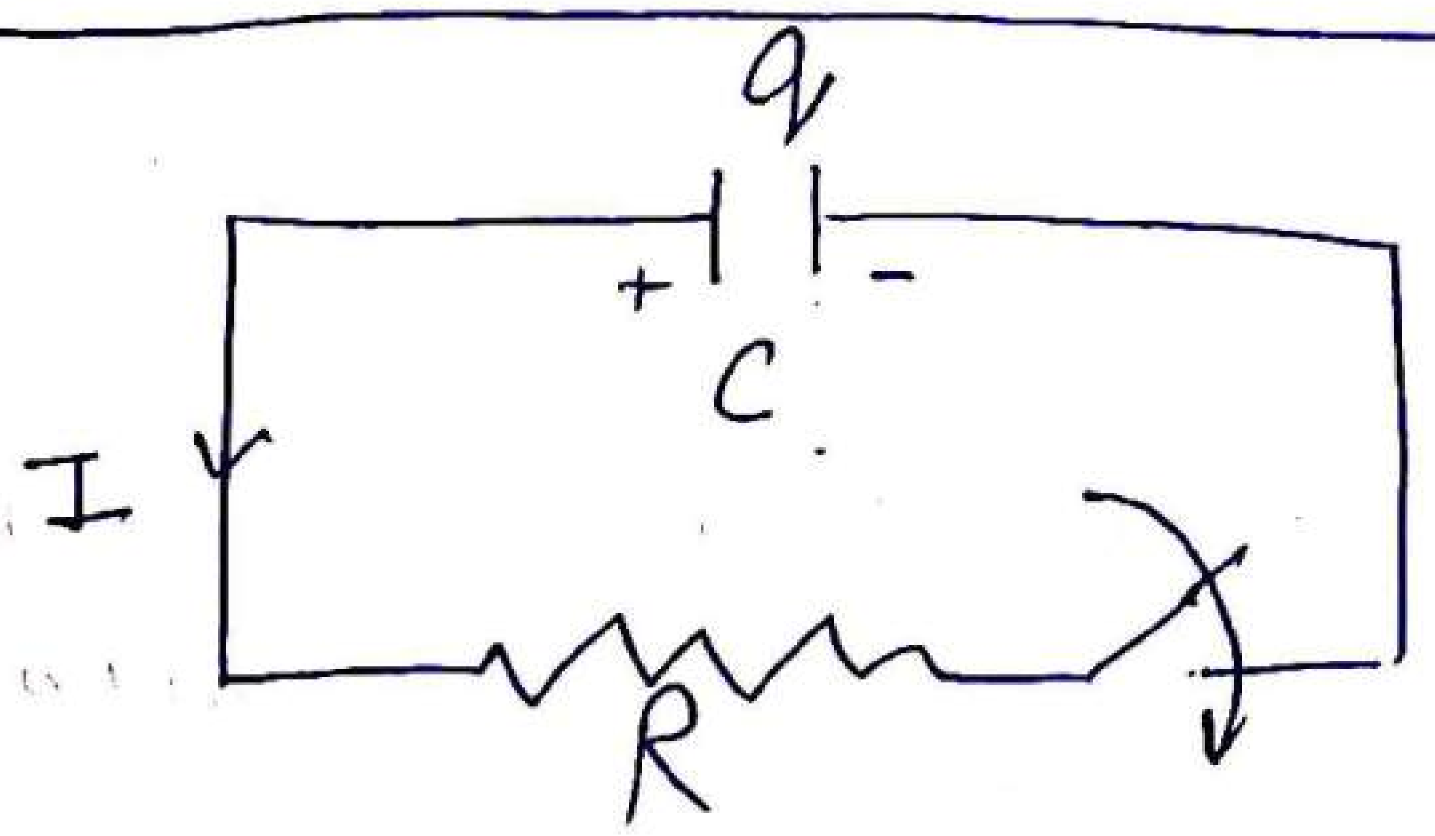
$$I = \frac{Q}{\tau} e^{-t/\tau} \Rightarrow \boxed{I = \frac{EC}{RC} e^{-t/\tau}}$$

$$\boxed{I = I_0 e^{-t/\tau}}$$

When $t \to \tau \Rightarrow \boxed{I = \frac{I_0}{e} = 0.368 I_0}$



DISCHARGING OF CAPACITOR:



At $t=0$,
$q = Q \Rightarrow$ initial charge

$$I = -\frac{dQ}{dt}$$

Applying kirchhoff's law:

$$\frac{q}{C} = IR \Rightarrow \frac{q}{C} = -\frac{dq}{dt}R$$

$$\int_Q^q \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\Rightarrow \ln\left(\frac{q}{Q}\right) = -t/RC$$

$$\Rightarrow \boxed{q = Qe^{-t/RC}}$$

$RC = \tau$ $\Rightarrow \boxed{q = Qe^{-t/\tau}}$

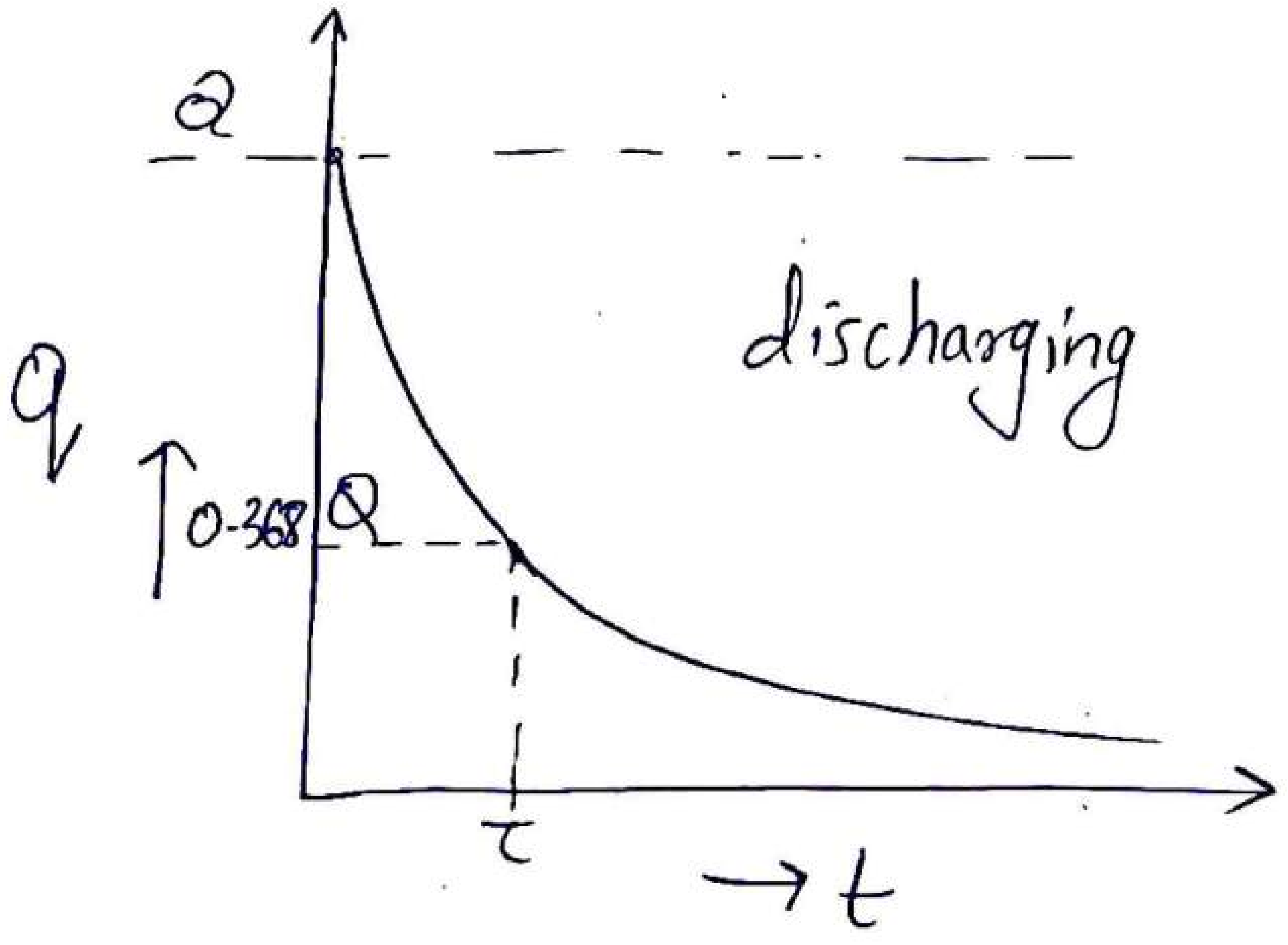
when $\boxed{t = \tau}$

$$\Rightarrow q = Qe^{-\tau/\tau}$$

$$\boxed{q = Qe^{-1} = \frac{Q}{e}}$$

$$\Rightarrow \boxed{q = 0.368Q}$$

so $t \to \tau$, $q \to 36.8\%$ of initial charge



Energy Considerations in charging:

Total work done by battery in fully charging the capacitor

$$W_b = EQ = E^2C \quad [\text{as } Q = EC]$$

↳ emf of battery

$$\boxed{U = \frac{1}{2}CE^2}$$

as $H = \int_0^\infty I^2 R\,dt = \int_0^\infty I_0^2 e^{-2t/\tau} R\,dt$

↙ Heat dissipated in resistor

$$= I_0^2 R\left[\frac{e^{-2t/\tau}}{-2\tau}\right]_0^\infty$$

$$H = \frac{-I_0^2 R\tau}{2}(0-1)$$

so, total $\boxed{W_b = CE^2}$ one half dissipated in heat → $\boxed{H = \frac{1}{2}CE^2}$

Energy Consideration in discharging:

energy loss, $H = \int_0^\infty I^2 R\, dt$

$$\Rightarrow H = \int_0^\infty \left(\frac{Q}{RC}\right)^2 e^{-2t/\tau} R\, dt$$

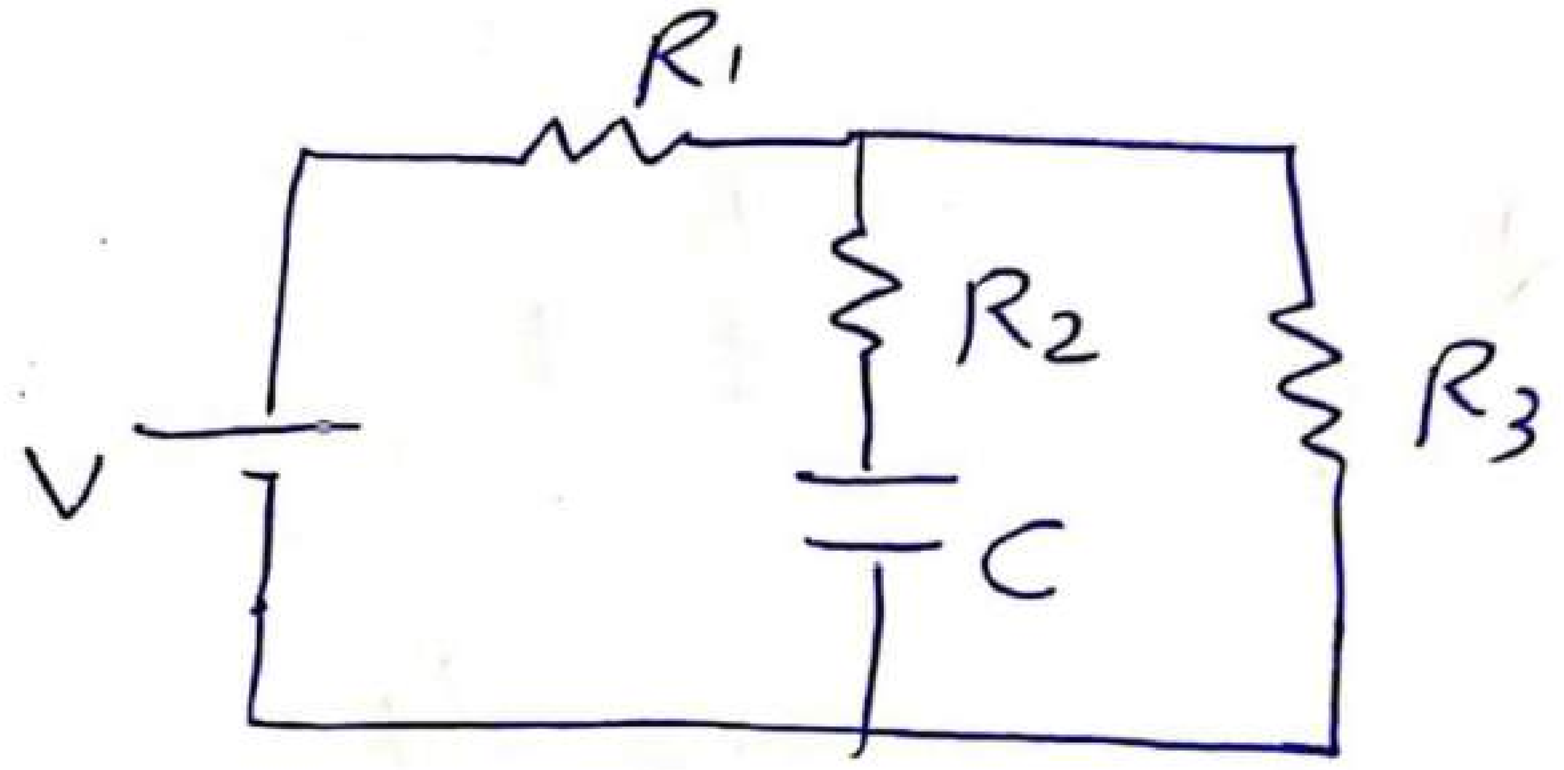
$$H = \frac{-Q^2 RC}{2RC^2}[0-1]$$

$$\boxed{H = \frac{1}{2}\frac{Q^2}{C}}$$

→ initial energy stored in capacitor

So whole energy is dissipated in form of heat.

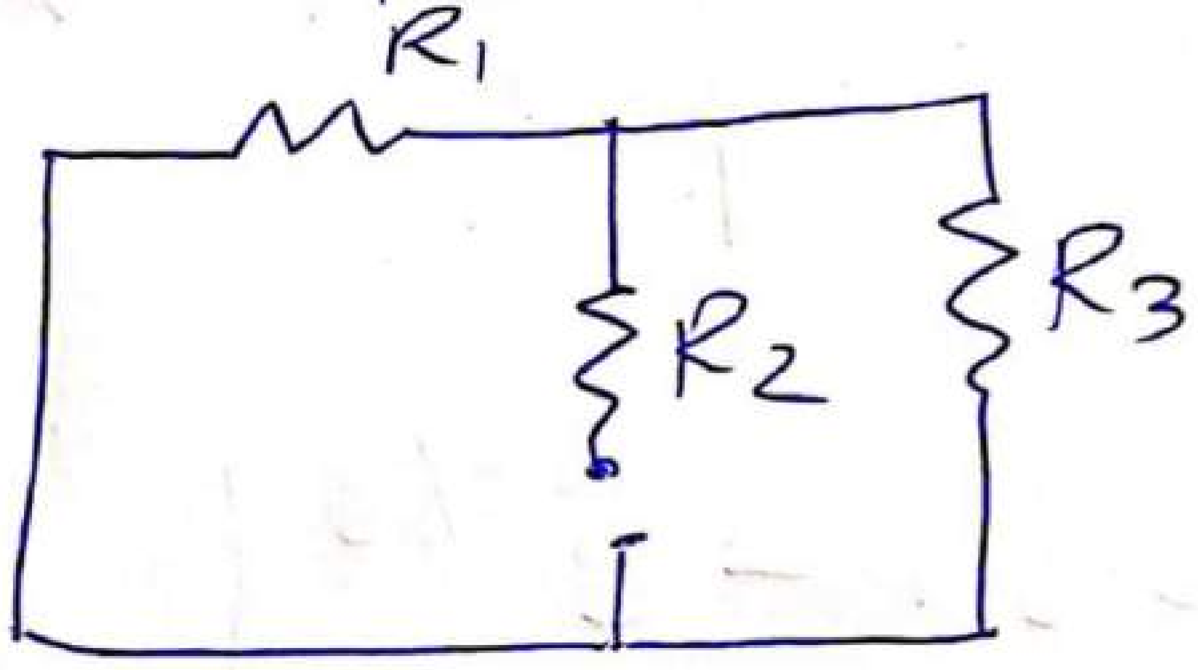
Equivalent Time Constant



$\tau_{eq} = (R_{eq})\, C$

replace battery by short-circuit and capacitor by open circuit and find R_{eq} around C

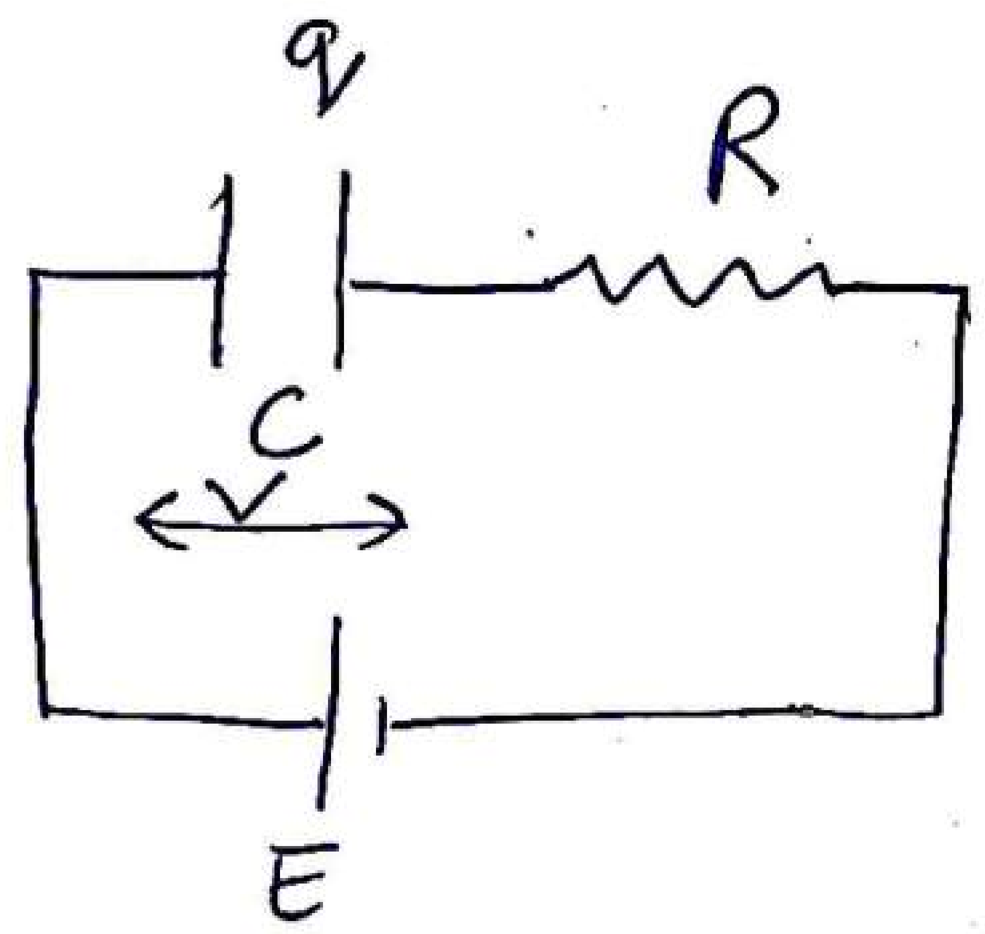
$\Rightarrow$



$\Rightarrow$ $$R_{eq} = \frac{R_1 \cdot R_3}{R_1 + R_3} + R_2$$

So $$\tau_{eq} = R_{eq}\, C$$

for charging RC circuit:



$$V = \frac{q}{C}$$

$$P_C = \frac{dU_C}{dt}$$

Power transferred to capacitor

$$q = Q[1 - e^{-t/\tau}]$$

$$Q = EC$$

$$\Rightarrow i = \frac{E}{R} e^{-t/\tau}$$

$$\Rightarrow P_C = Vi = \frac{q}{C} i$$

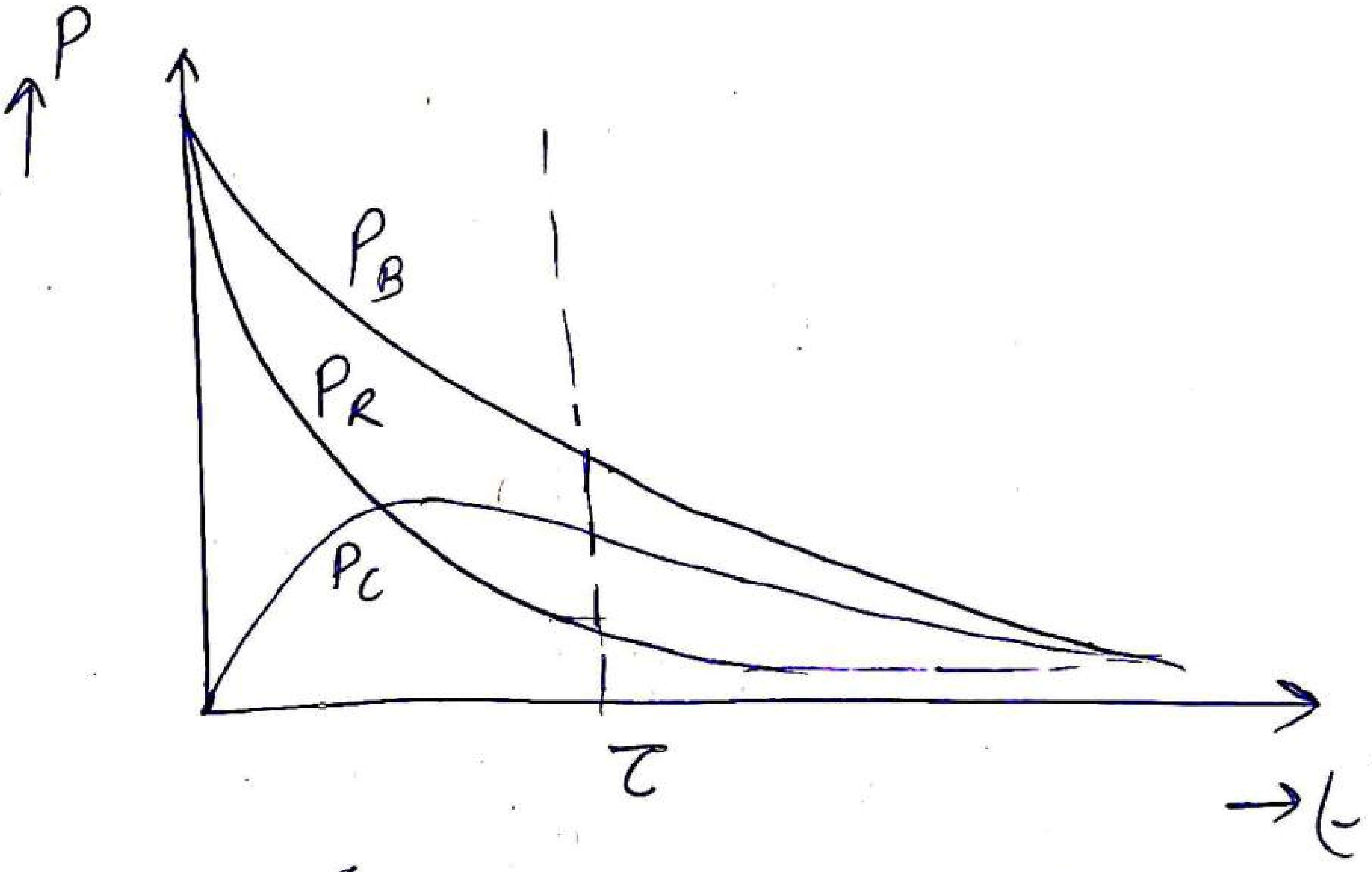
$$P_C = \frac{EC[1 - e^{-t/\tau}]}{C} \frac{E}{R} e^{-t/\tau}$$

$$\boxed{P_C = \frac{E^2}{R}[1 - e^{-t/\tau}] e^{-t/\tau}}$$

$P_B \rightarrow$ power delivered by battery

$= E\,i$

$$P_B = \frac{E^2}{R} e^{-t/\tau}$$



$$P_R = i^2 R = \left[\frac{E}{R} e^{-t/\tau}\right]^2 R$$

$\hookrightarrow$ power dissipated by resistor

$P_C \neq P_R$ at any time t

$\Rightarrow$ But $\boxed{P_R + P_C = P_B}$

$P_c \rightarrow max$

$$\frac{dP_c}{dt} = 0$$

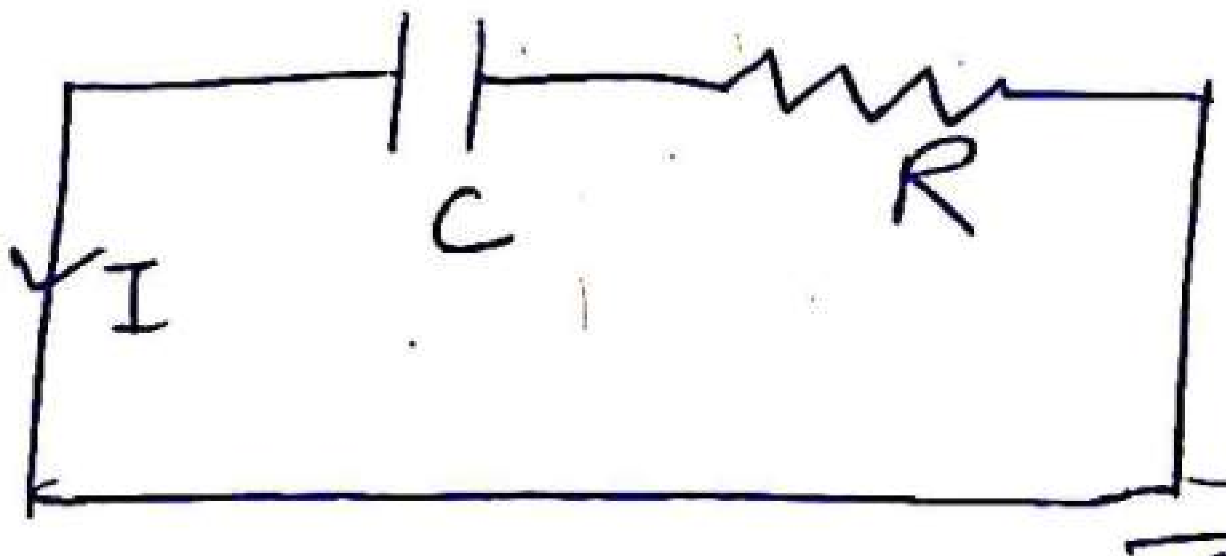
$$\frac{d}{dt}\left[\frac{E^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)\right] = 0$$

$$e^{-t/\tau}\left[\frac{1}{\tau}\right] - e^{-2t/\tau}\left(\frac{2}{\tau}\right) = 0$$

$$\Rightarrow e^{t/\tau} = 2$$

or $\boxed{t = \tau \ln 2}$

For discharging RC circuit :



$$q = Q e^{-t/\tau}$$

$$i = i_0 e^{-t/\tau}$$

$$\boxed{q = EC\, e^{-t/\tau}}$$

$$\boxed{Q = EC} \quad \boxed{i = \frac{E}{R} e^{-t/\tau}}$$

$P_c = \frac{dU}{dt}$ rate at which energy is transffered

$$P_c = \frac{dU}{dt} = \frac{d}{dt}\left[\frac{q^2}{2C}\right]$$

$$= \frac{d}{dt}\left[\frac{E^2C^2}{2C} e^{-2t/\tau}\right]$$

$$= \frac{E^2C}{2}\left[e^{-2t/\tau}\left[\frac{-2}{\tau}\right]\right]$$

$$\boxed{P_C = -\frac{E^2}{R} e^{-2t/\tau}}$$

P_R = energy dissipation by resistor

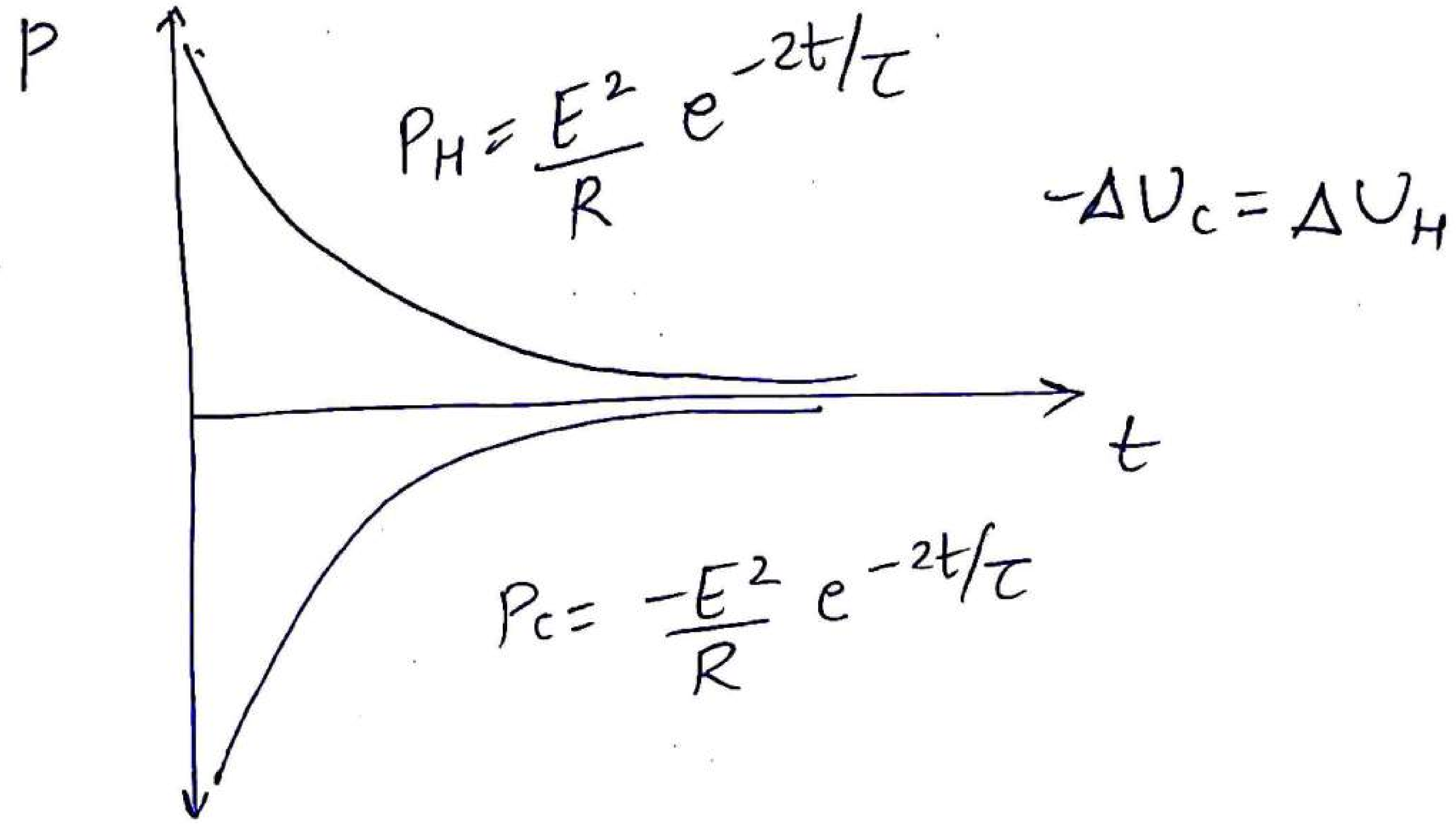
$$P_R = I^2R$$

$$= \frac{E^2}{R^2} e^{-2t/\tau} R$$

$$\boxed{P_R = \frac{E^2}{R} e^{-2t/\tau}}$$

$\Rightarrow$ $P_C = -P_R$ → or rate of energy decrease in capacitor

energy dissipated by capacitor will be lost by resistor.



P
PH = E2/R e-2t/τ
-ΔUc = ΔUH
t
Pc = -E2/R e-2t/τ