VISUAL PHYSICS

SHORT NOTES

CHAPTER

Electric Potential

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ELECTRIC POTENTIAL

$$\Delta U = -W \Rightarrow U_b - U_a = -W$$

work done from going $a \to b$

potential energy at position 'b'

Potential energy at position 'a'

$$\text{as} \quad W_{a\to b} = \int_{r_a}^{r_b} \vec{F}\cdot\vec{dl} = \int_{r_a}^{r_b}\left(\frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}\hat{r}\cdot\vec{dl}\right)$$

electrostatic force is conservative force

Point function

Work is done

an also

$$\boxed{-\Delta U = W_{a\to b} = \int_{r_a}^{r_b}\vec{F}\cdot\vec{dl} = \int_{r_a}^{r_b} q\vec{E}\cdot\vec{dl}}$$

For any point 'r' from q_0, q have potential $\Rightarrow$

$$\boxed{U = \frac{1}{4\pi\epsilon}\frac{q q_0}{r}}$$

potential energy at a given point in presence of many charges,

$$U = \frac{1}{4\pi\epsilon}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \ldots\ldots\right]$$

As potential is scalar quantity.

For a system, the Net potential of the system is:

$$\boxed{U = \frac{1}{2}\left[\frac{1}{(4\pi\epsilon)}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}}\right]}$$

[$\frac{1}{2}$ is used as each term in summation will appear twice]

Now 'Potential energy' per unit charge at point a is known as potential

$$\Rightarrow \boxed{V_a = \frac{U_a}{q_0} = \frac{1}{4\pi\epsilon}q \times \frac{1}{r_a}}$$

potential difference from point $a \to b$ is = negative of work from $a \to b$

$$\Rightarrow \boxed{\Delta V = V_b - V_a = -\frac{W_{a\to b}}{q_0} = \frac{\Delta U_{a\to b}}{q_0}}$$

$$[V] = \left[\frac{W}{q_0}\right]$$

$$= \left[\frac{ML^2T^{-2}}{AT}\right] = [ML^2T^{-3}A^{-1}]$$

Unit (JC^{-1})

So, net V at a point:

$$\boxed{V = V_1 + V_2 + V_3 = \frac{1}{4\pi\epsilon}\sum\frac{q_i}{r_i}}$$

for $\boxed{\Delta V = 0 \Rightarrow -W = 0}$

Now $\int q_0\vec{E}\cdot d\vec{l} = W$

So for W to be zero

(i) $\vec{dl}$ and $\vec{E}$ must be perpendicular

(ii) for $E = 0$

"Now a surface with same potential is known as equipotential surface"

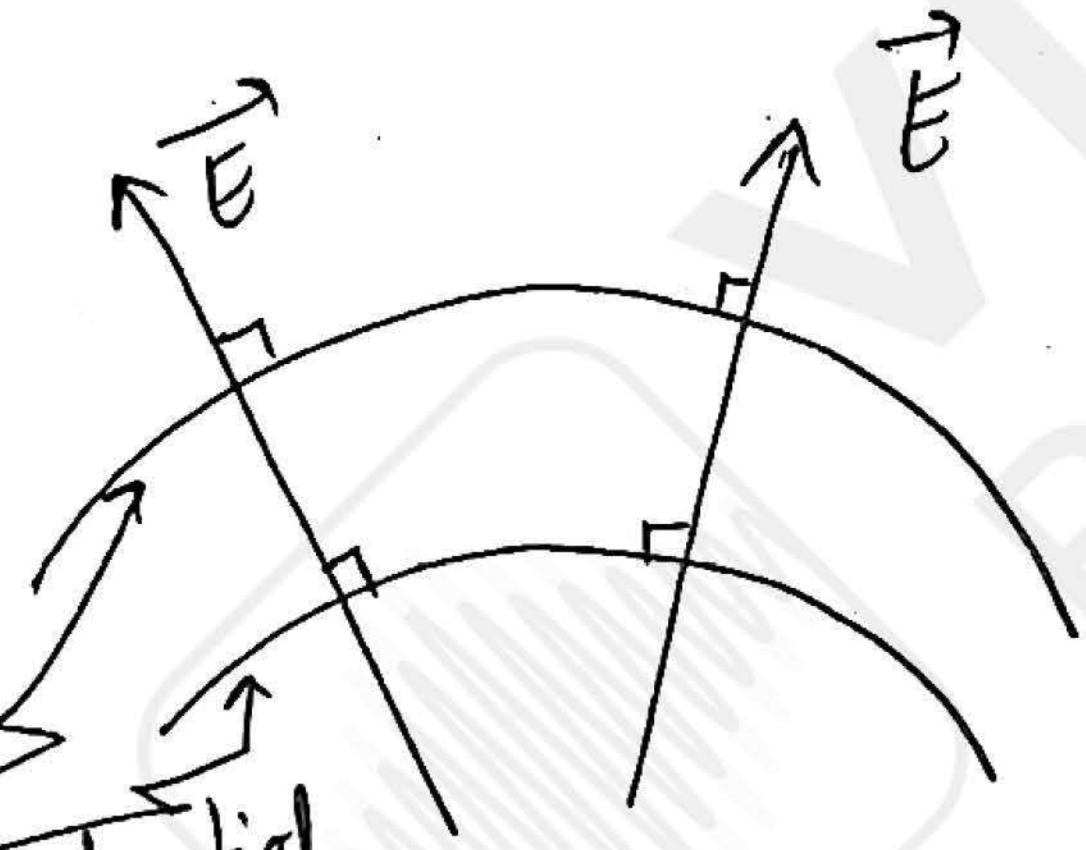
⇒ So for a equipotential surface $\vec{E}$ is perpendicular on to it surface.

As, $V_f - V_i = -\int_i^f \frac{q_0 \vec{E} \cdot \vec{ds}}{q_0}$

$$= -\int_i^f \vec{E} \cdot \vec{ds}$$

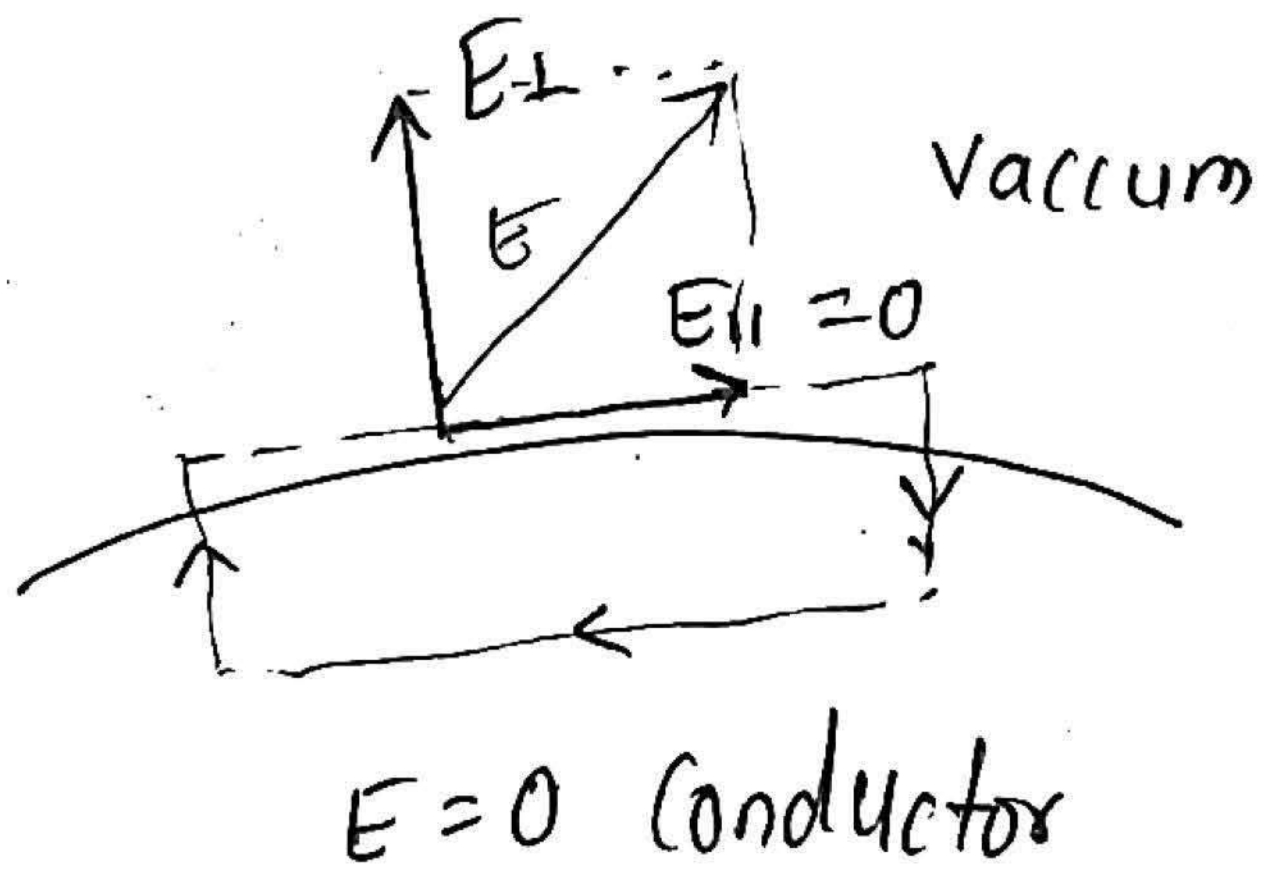
for $i \to \infty$, $V_i \to 0$ [at infinity for our reference, $V = U = 0$ at $r \to \infty$]

$$\Rightarrow \boxed{V_f = V = -\int_\infty^f \vec{E} \cdot \vec{ds}}$$

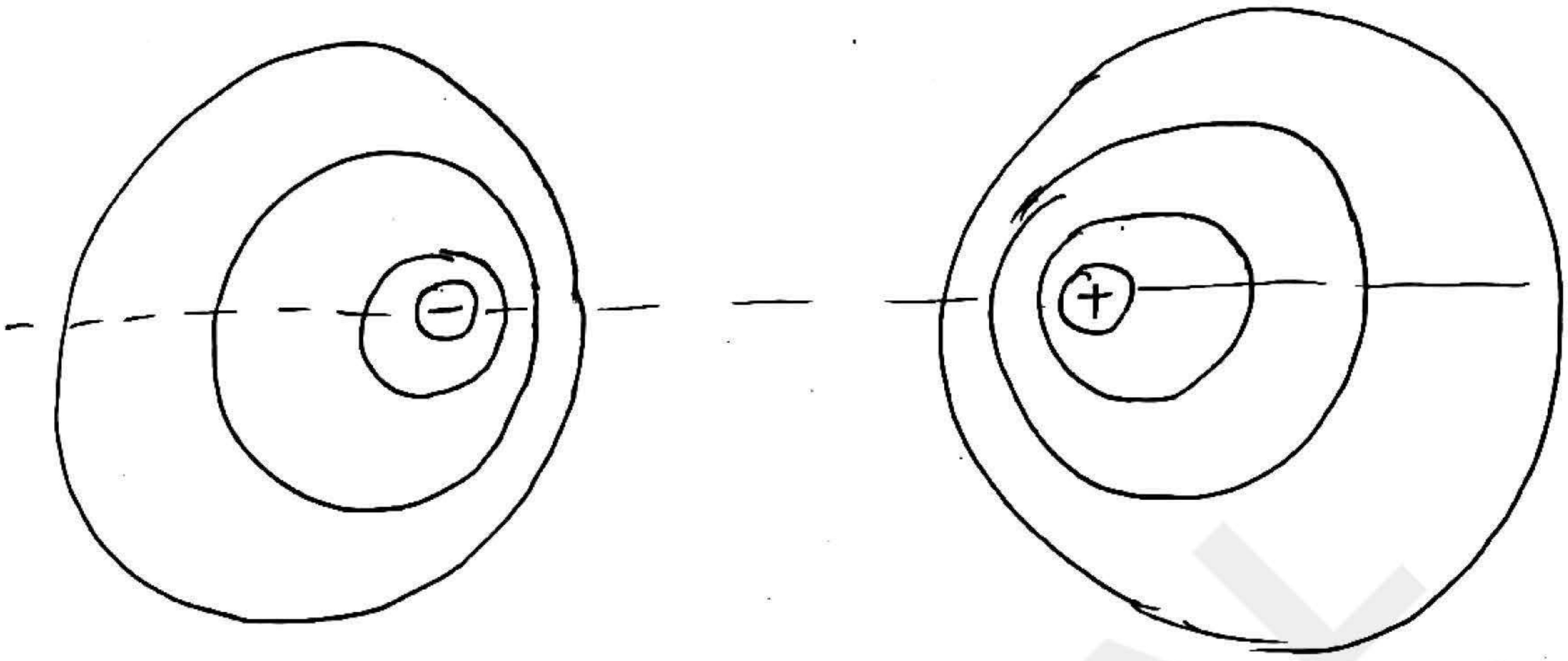


equipotential surface, is perpendicular to $\vec{E}$.

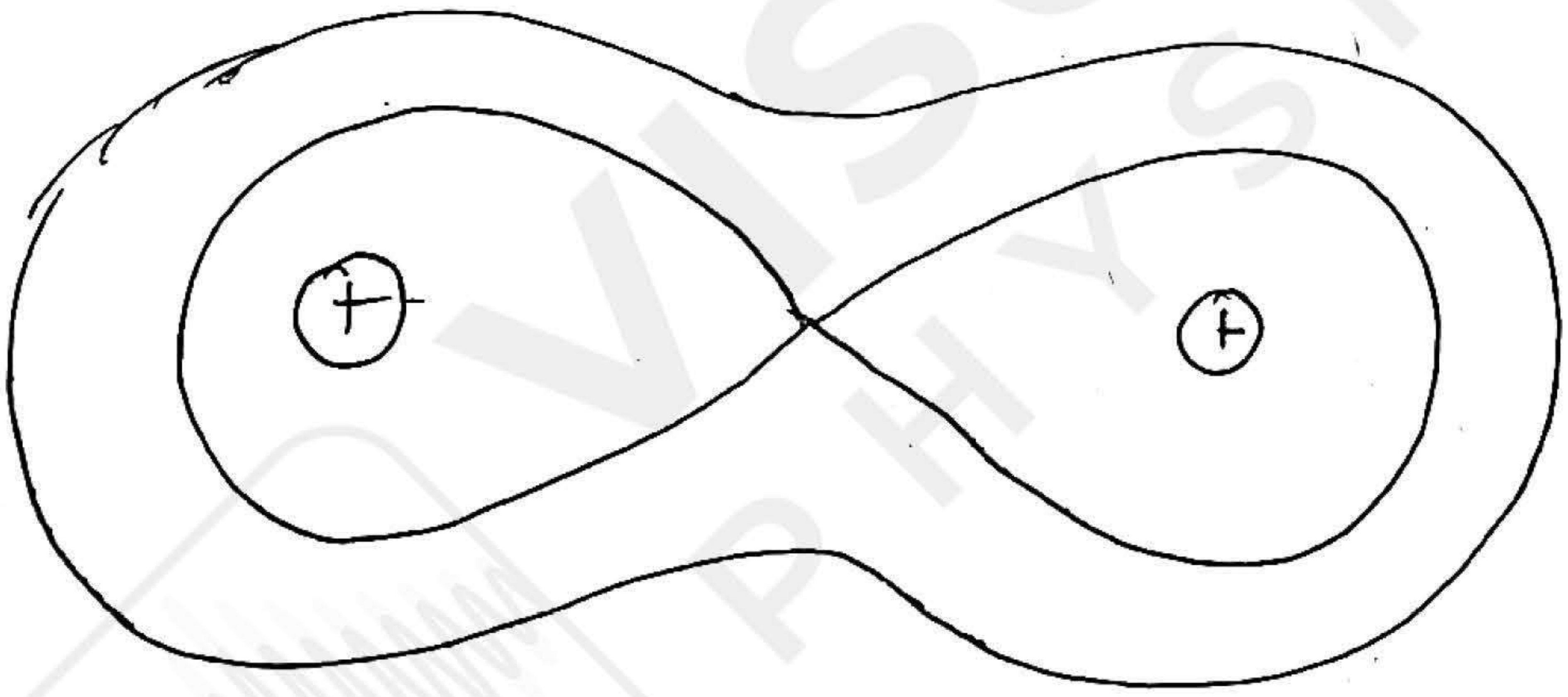
just above the conductor E must be perpendicular to conductor



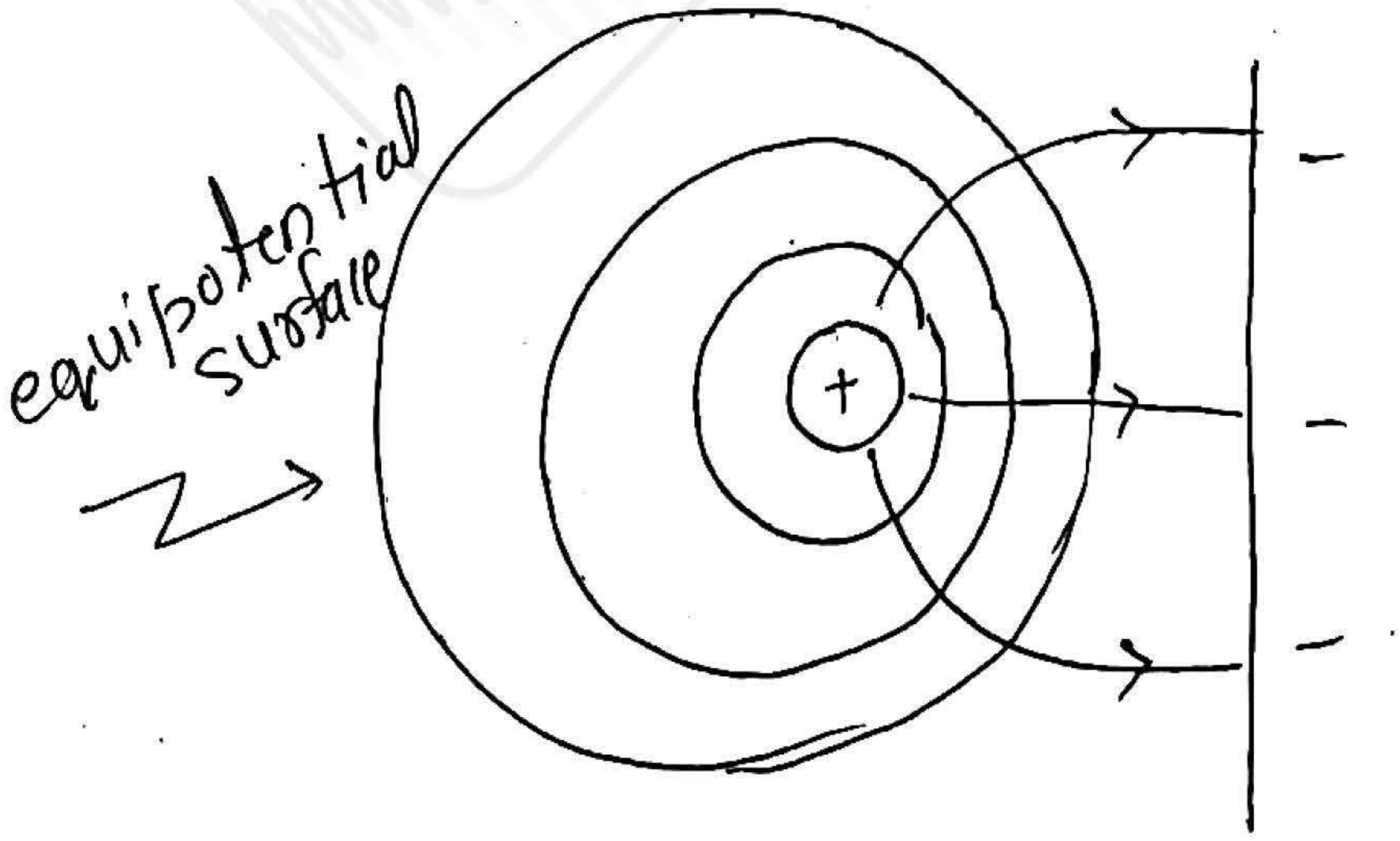
As if $E_\parallel \neq 0$, electron must be moving and hence non zero work by E. but E is conservative so $\underline{E_\parallel = 0}$



Equipotential surface of dipole



Two equal positive charge



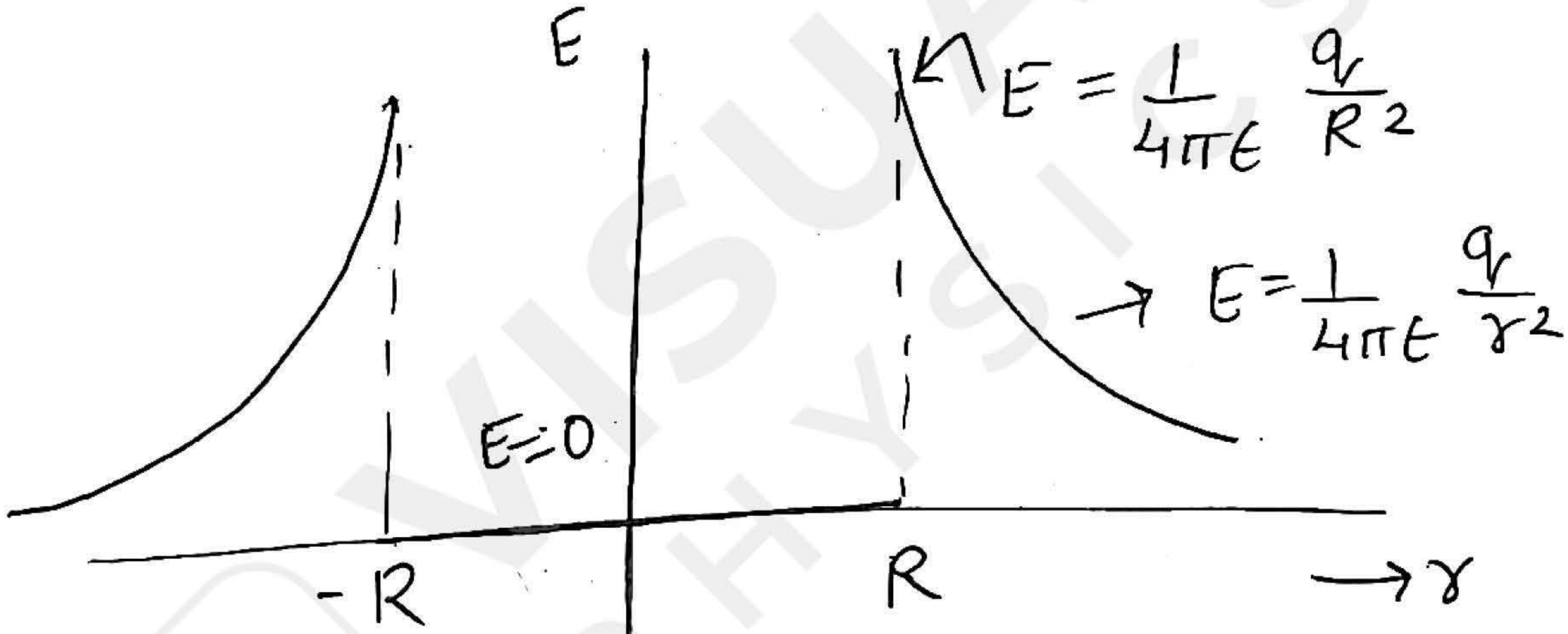
charge at rest near a conductor

Potential due to a conducting shell.

$$\int_{\infty}^{V} dV = \int_{\infty}^{r} -\vec{E}\cdot\vec{dr} \qquad [V_{\infty} = 0]$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon}\frac{q}{r^2}\,dr$$

$$\boxed{V = \frac{1}{4\pi\epsilon}\frac{q}{r}}$$



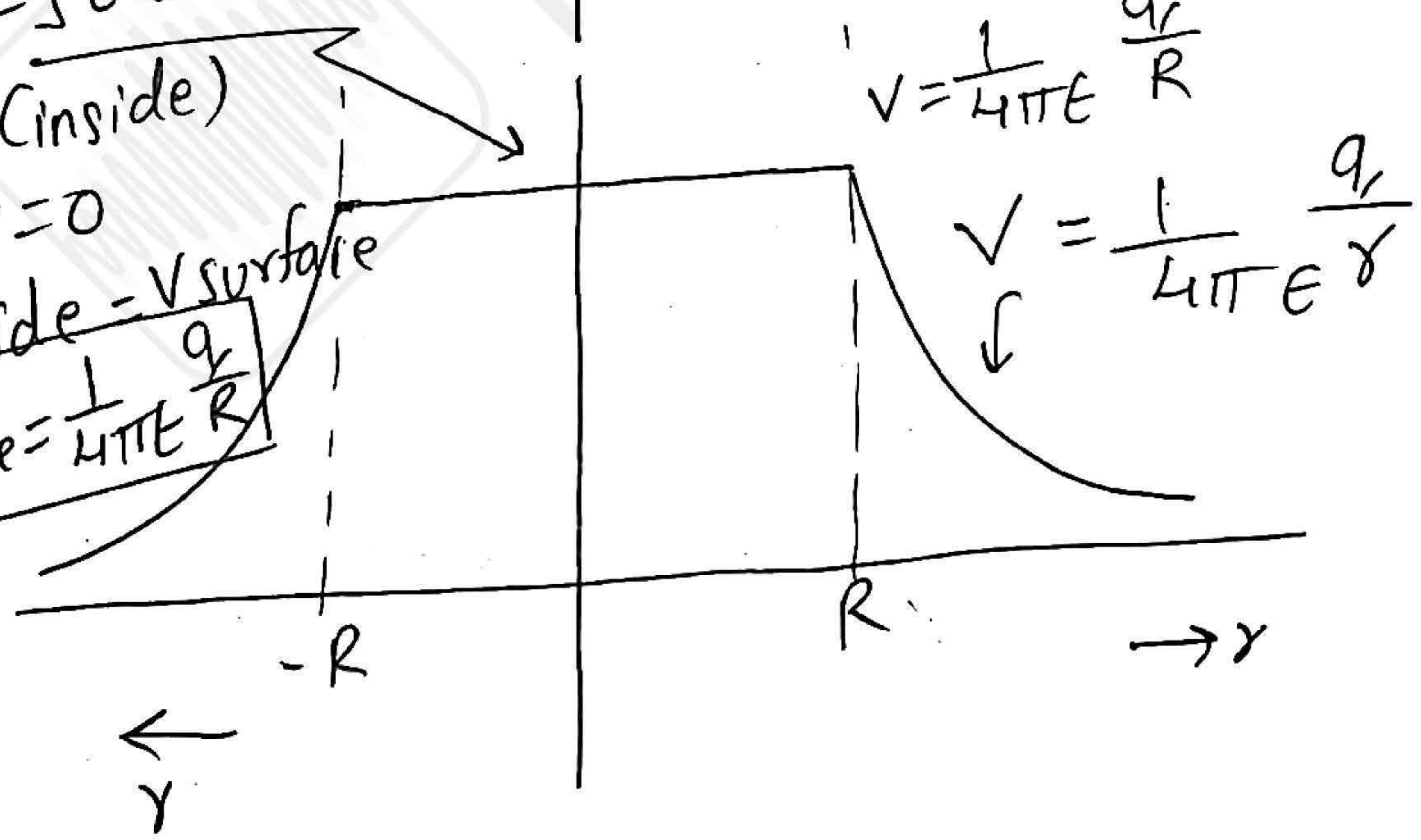
as $dV = -\int \vec{E}\cdot dr$

$E = 0$ (inside)

So, $dV = 0$

So $V_{inside} = V_{surface}$

$\Rightarrow \boxed{V_{inside} = \frac{1}{4\pi\epsilon}\frac{q}{R}}$



A Non-Conducting Solid sphere

outside

$$E_{outside} = \frac{1}{4\pi\epsilon}\frac{q}{r^2}$$

$q \rightarrow$ total charge

$$\int_{\infty}^{V} dV = -\int \vec{E} \cdot \vec{dr}$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon}\frac{q}{r^2}\,dr$$

$$\boxed{V = \frac{1}{4\pi\epsilon}\frac{q}{r}}$$

At surface, $V = \frac{1}{4\pi\epsilon}\frac{q}{R}$ $R \hookrightarrow$ Radius

Inside

$$E_{inside} = \frac{1}{4\pi\epsilon}\frac{q}{R^3}r$$

$$\int_{V_s}^{V} dV = -\int_{R}^{r} \frac{1}{4\pi\epsilon}\frac{q}{R^3}r\,dr$$

$$V - V_s = -\frac{1}{4\pi\epsilon}\frac{q}{R^3}\left[\frac{r^2}{2}\right]_R^r$$

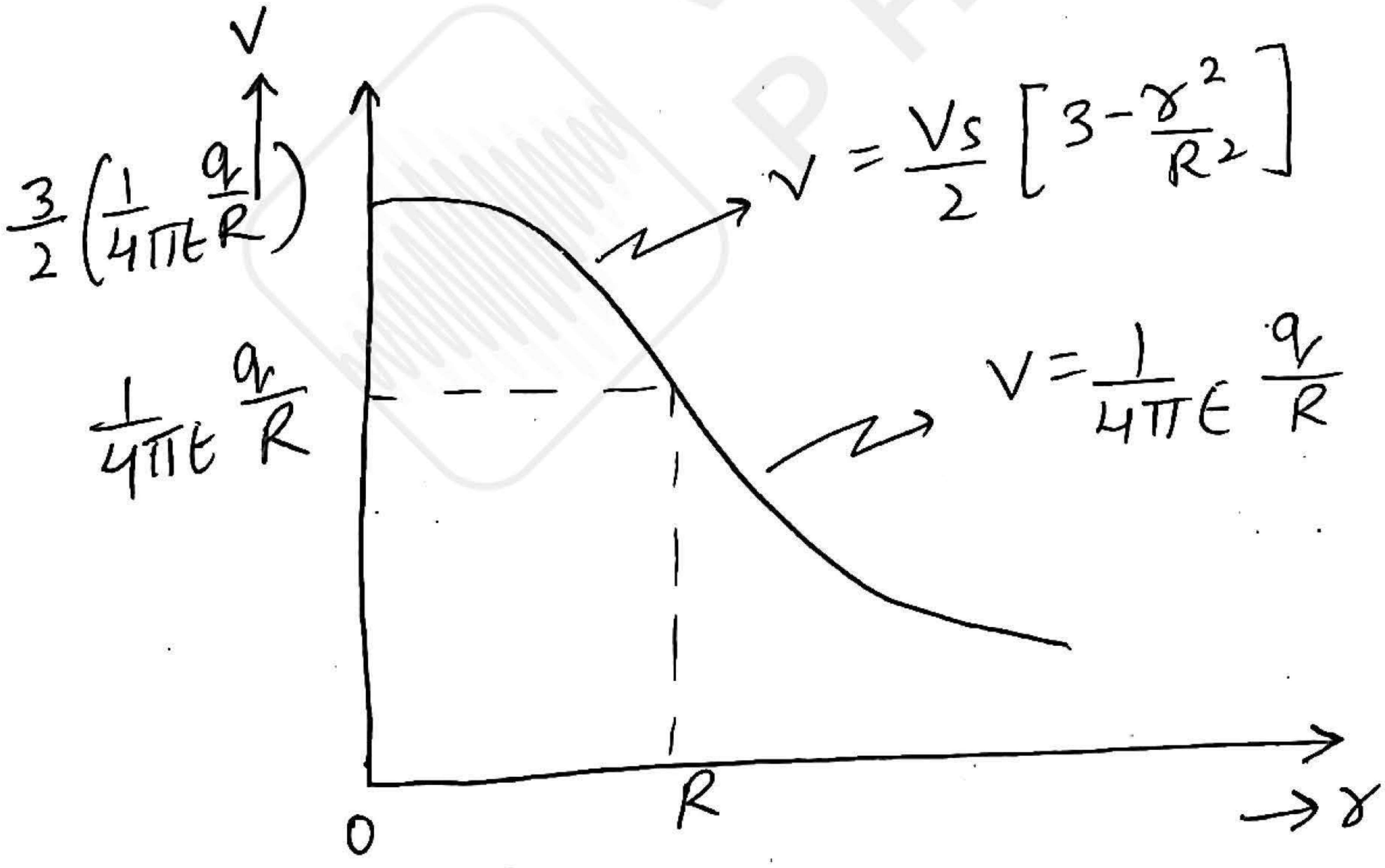
as, $V_s = \frac{1}{4\pi\epsilon}\frac{q}{R}$

So, $V = -\frac{1}{4\pi\epsilon}\frac{q}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] + V_s$

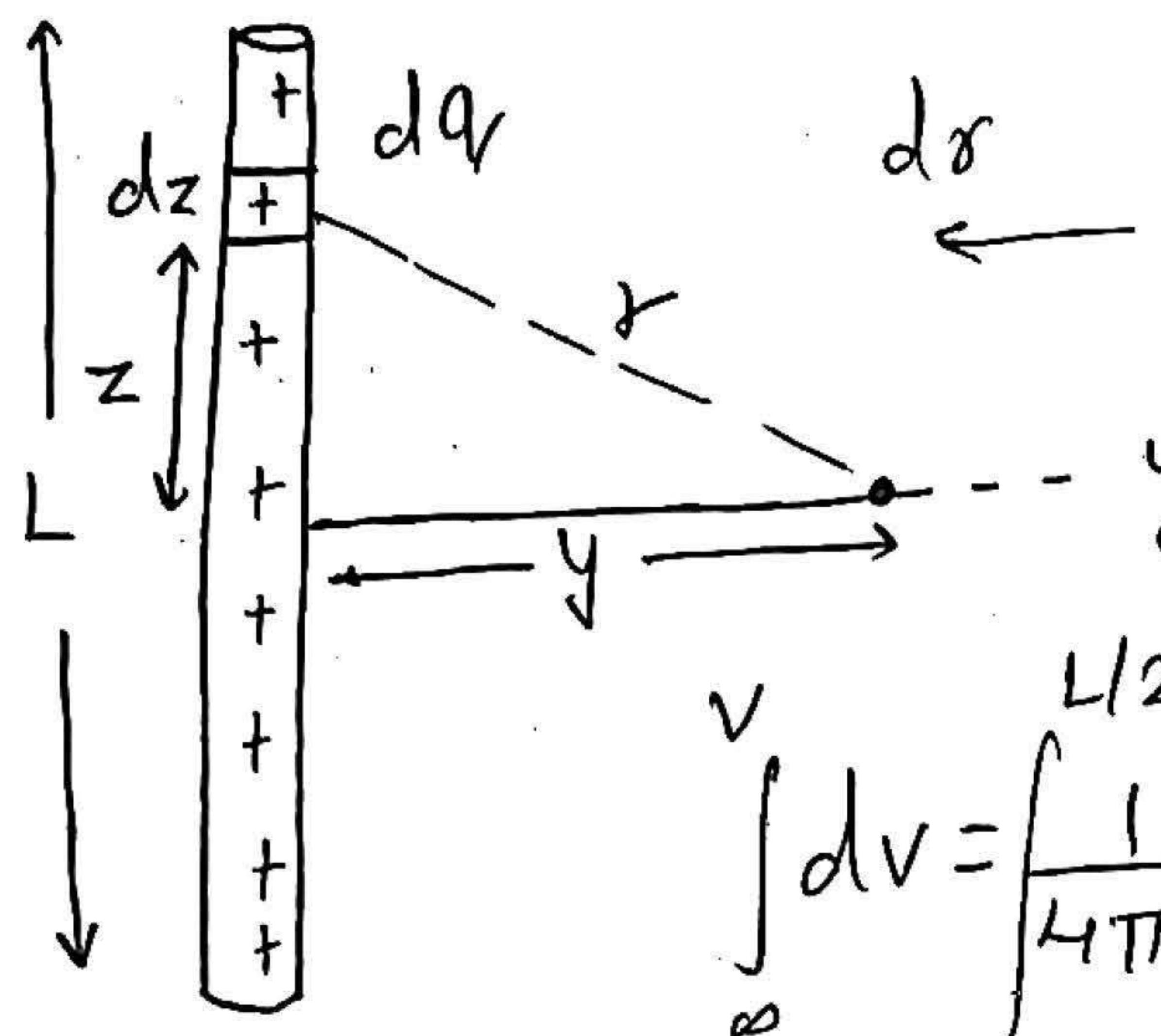
$$V = \frac{1}{4\pi\epsilon}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$$

$$\boxed{V = \frac{1}{8\pi\epsilon}\frac{q}{R}\left[3 - \frac{r^2}{R^2}\right]}$$

or $\boxed{V = \frac{V_s}{2}\left[3 - \frac{r^2}{R^2}\right]}$



A uniform line of charge:



$$dV = \frac{1}{4\pi\epsilon}\frac{dq}{r} = \frac{1}{4\pi\epsilon}\frac{dq}{\sqrt{y^2+z^2}}$$

now $dq = \lambda dz$

linear charge density

$$\int_{\infty}^{V} dV = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon}\frac{\lambda dz}{\sqrt{z^2+y^2}}$$

for linear Length L

$$V = \frac{\lambda}{4\pi\epsilon}\ln\left[\frac{L/2+\sqrt{L^2/4+y^2}}{-L/2+\sqrt{L^2/4+y^2}}\right]$$

for infinite rod

$$V_b - V_a = \int_a^b -\vec{E}\cdot\vec{dr} = \int_a^b \frac{1}{4\pi\epsilon}\frac{2\lambda}{r}\,dr$$

$$\boxed{V_b - V_a = \frac{\lambda}{2\pi\epsilon}\ln(b/a)}$$

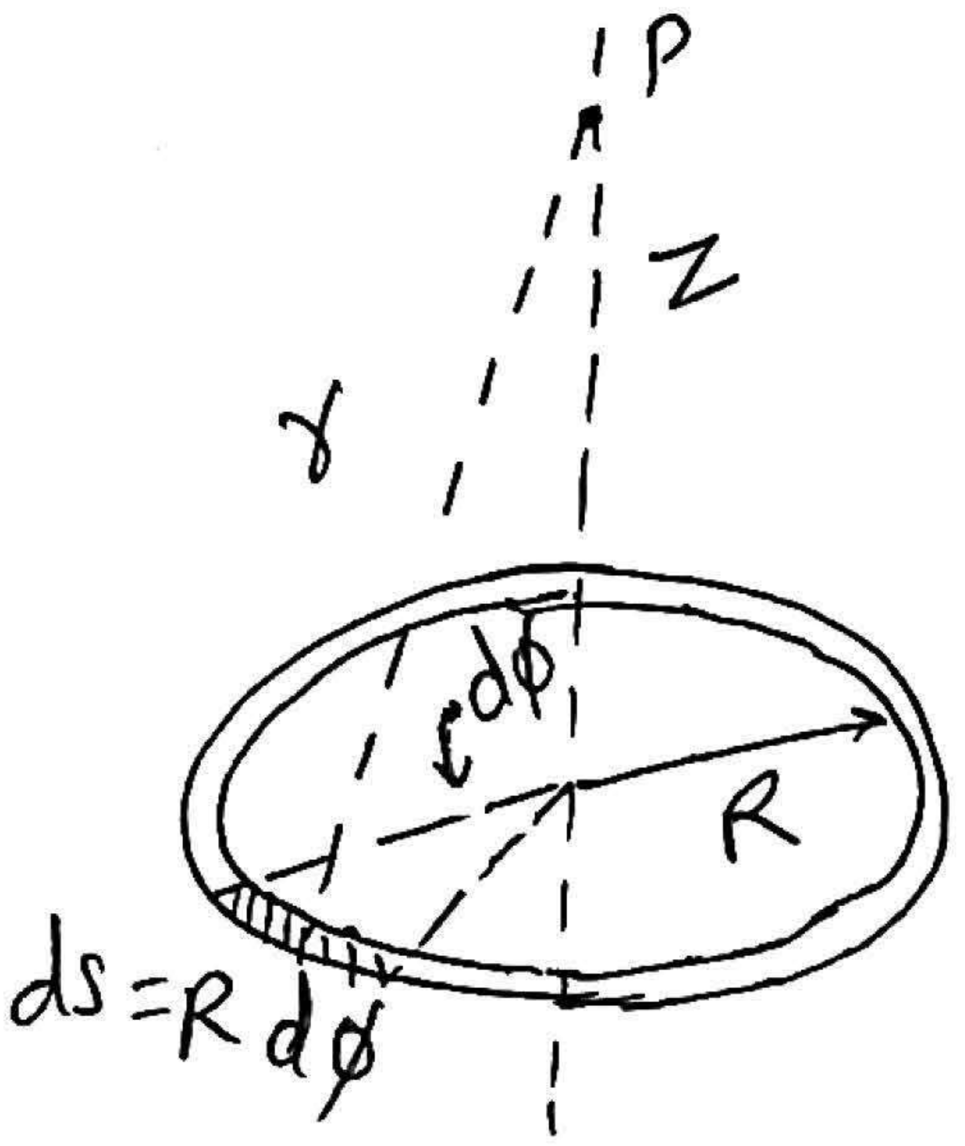
as

$\vec{E}$ →

← dr

so, $\cos\theta = -1 = \cos 180°$

A ring of charge:



$$dq = \lambda\, ds = \lambda R\, d\phi$$

$$dV = \frac{1}{4\pi\epsilon}\frac{dq}{r}$$

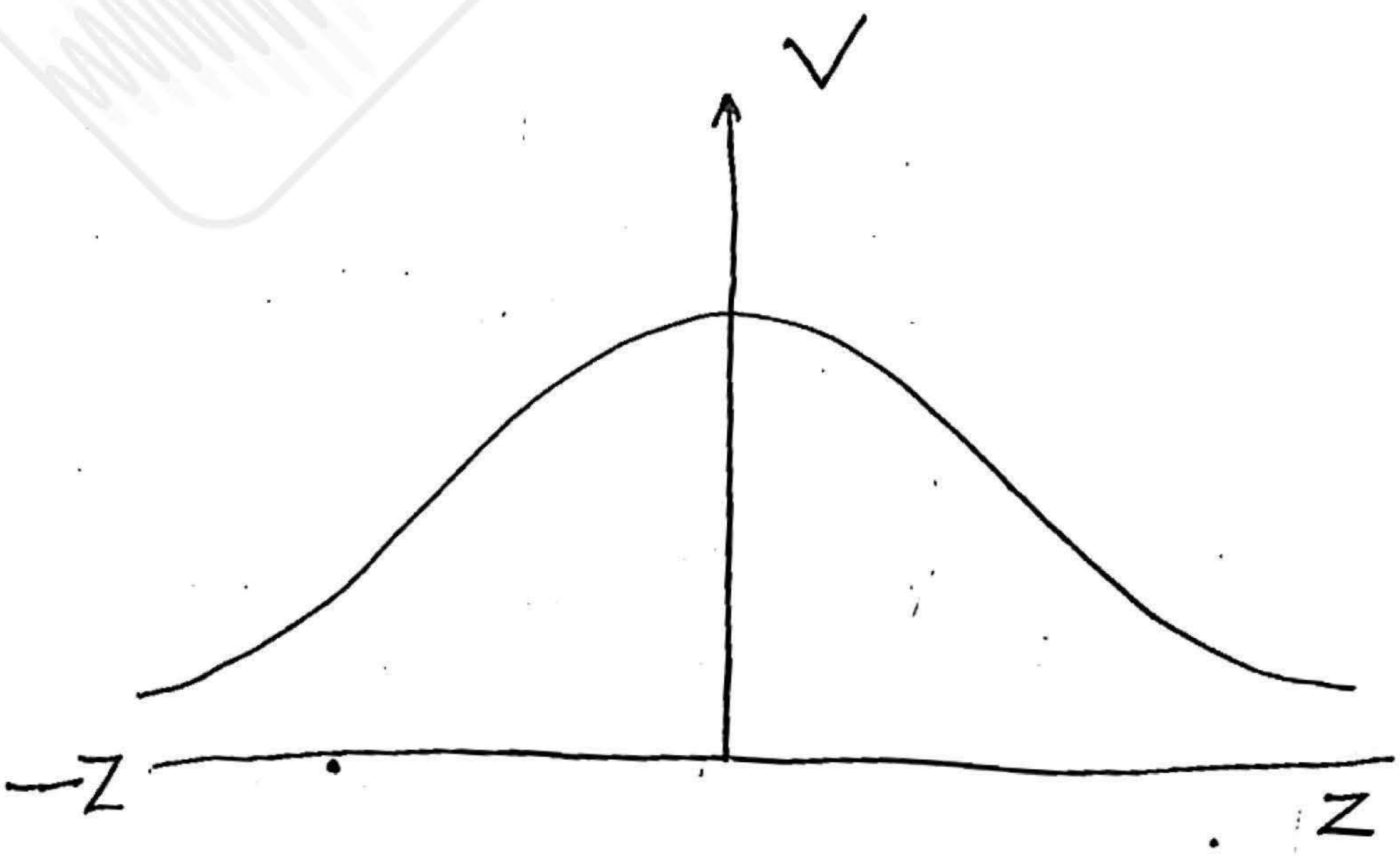
$$= \frac{1}{4\pi\epsilon}\frac{\lambda R\, d\phi}{\sqrt{R^2+z^2}}$$

$$V = \frac{1}{4\pi\epsilon}\frac{\lambda R}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\phi$$

$$V = \frac{1}{4\pi\epsilon}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}}$$

as $q = 2\pi R\lambda$, and $r = \sqrt{R^2+z^2}$

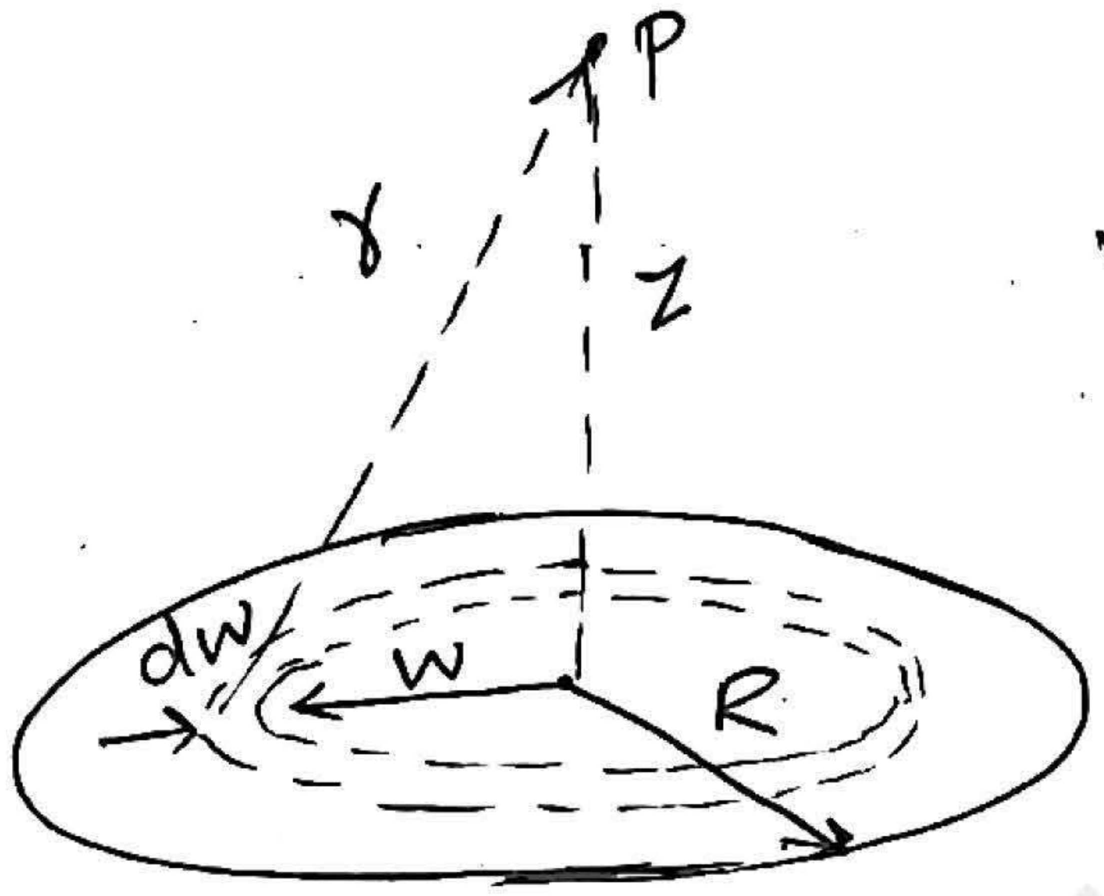
$$\boxed{V = \frac{1}{4\pi\epsilon}\frac{q}{r}}$$



A CHAGED DISK:

$dA = 2\pi w\, dw$

$dv = \frac{1}{4\pi\epsilon} \frac{dq}{\sqrt{w^2+z^2}}$



now

$dq = 2\pi \sigma w\, dw$

$dV = \frac{1}{4\pi\epsilon} \frac{2\pi \sigma w\, dw}{\sqrt{w^2+z^2}}$

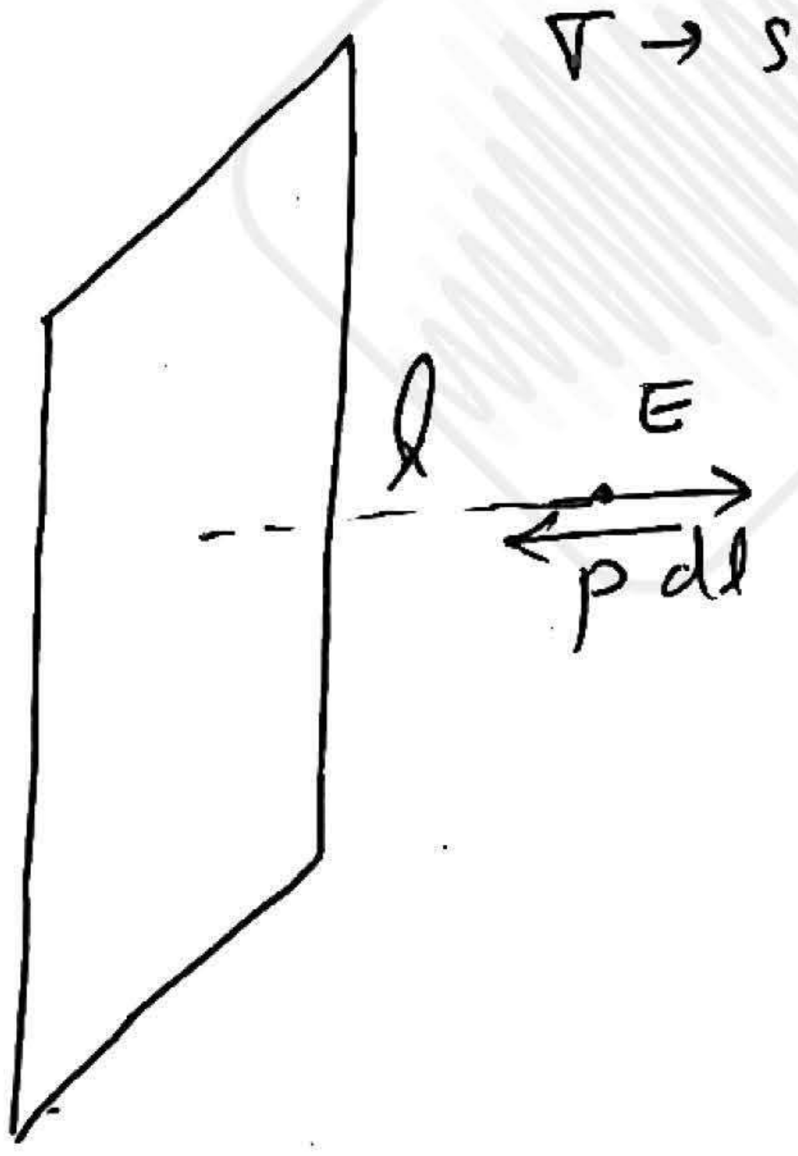
$V = \frac{\sigma}{2\epsilon} \int_0^R \frac{w\, dw}{\sqrt{w^2+z^2}}$

$V = \frac{\sigma}{2\epsilon} \left(\sqrt{R^2+z^2} - |z|\right)$

So, $R \to \infty$, $V \to \infty$ so can't define absolute potential

Infinite charge plane

$\sigma \to$ surface charge density



$E = \frac{\sigma}{2\epsilon}$

V

so, $\int_\infty dv = -\int_\infty^l \vec{E} \cdot \vec{dl}$

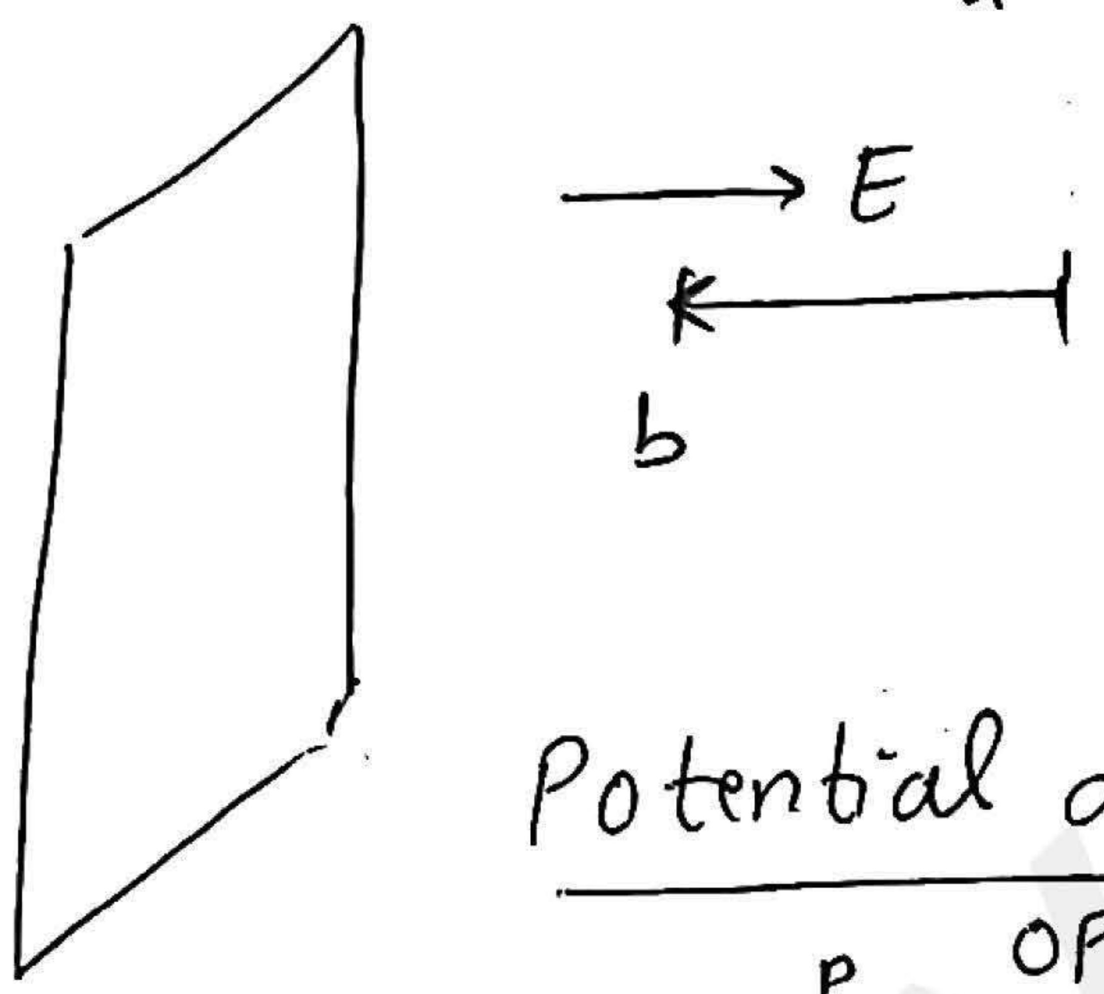
$V - 0 = \int_\infty^l \frac{\sigma}{2\epsilon} dl = \frac{\sigma}{2\epsilon} [l - \infty]$

V absolute not define

but we can define potential difference between two points.

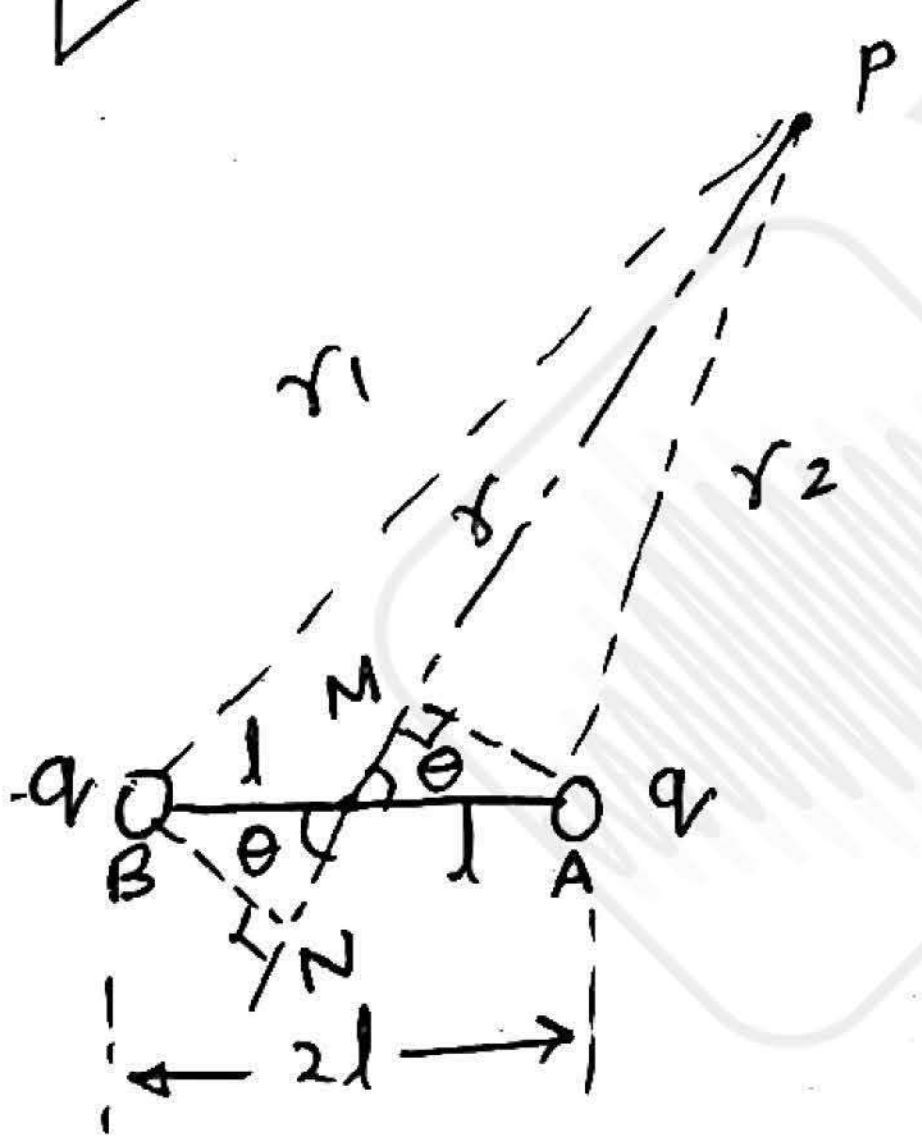
$$\int_a^b dV = -\int_a^b \vec{E}\cdot\vec{dl}$$

$$V_b - V_a = \int_a^b E\,dl\cos 180^\circ = \frac{\sigma}{2\epsilon}\int_a^b dl$$



$$\Rightarrow \boxed{V_b - V_a = \frac{\sigma}{2\epsilon}[b-a]}$$

Potential due to an electric dipole:



$OP = r$, $\angle AOP = \theta$

$BP = r_1$, $AP = r_2$

$r_1 \approx PN = r + l\cos\theta$

$r_2 = AP = MP = r - l\cos\theta$

$$V = \frac{-q}{4\pi\epsilon r_1} + \frac{q}{4\pi\epsilon r_2} = \frac{q}{4\pi\epsilon}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = \frac{q}{4\pi\epsilon}\left[\frac{r + l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{1}{4\pi\epsilon}\,\frac{q(2l\cos\theta)}{(r^2 - l^2\cos^2\theta)}$$

$\because p = q(2l)$

$$\boxed{V = \frac{1}{4\pi\epsilon}\,\frac{p\cos\theta}{(r^2 - l^2\cos^2\theta)}}$$

→ if $l << r$

$$V = \frac{1}{4\pi\epsilon}\frac{P\cos\theta}{r^2}$$

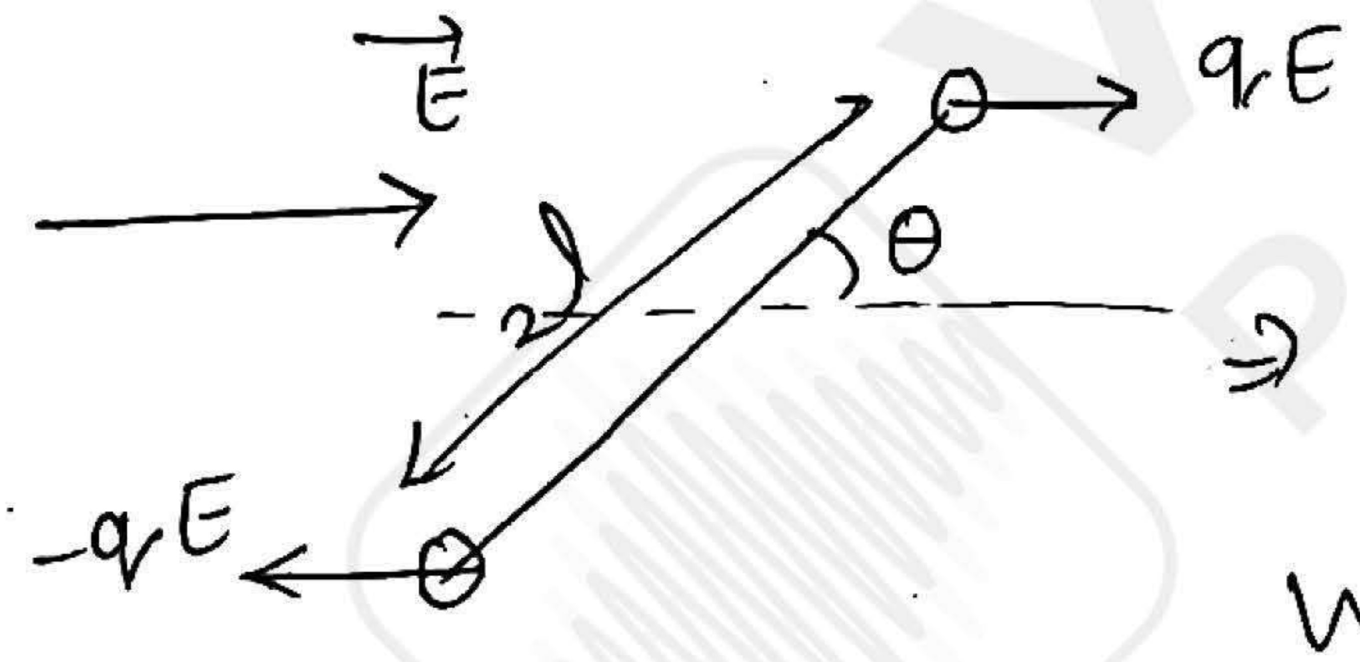
→ $\theta = 0°$

$$V = \frac{1}{4\pi\epsilon}\frac{P}{(r^2 - l^2)}$$

→ $\theta = 90°$, $\boxed{V = 0}$

Work done in rotating Electric dipole in a uniform Electric Field.

as, $dW = \text{Torque} \times d\theta$



$$\text{Torque} = qE(2l\sin\theta)$$
$$= PE\sin\theta$$

$$\Rightarrow dW = PE\sin\theta\, d\theta$$

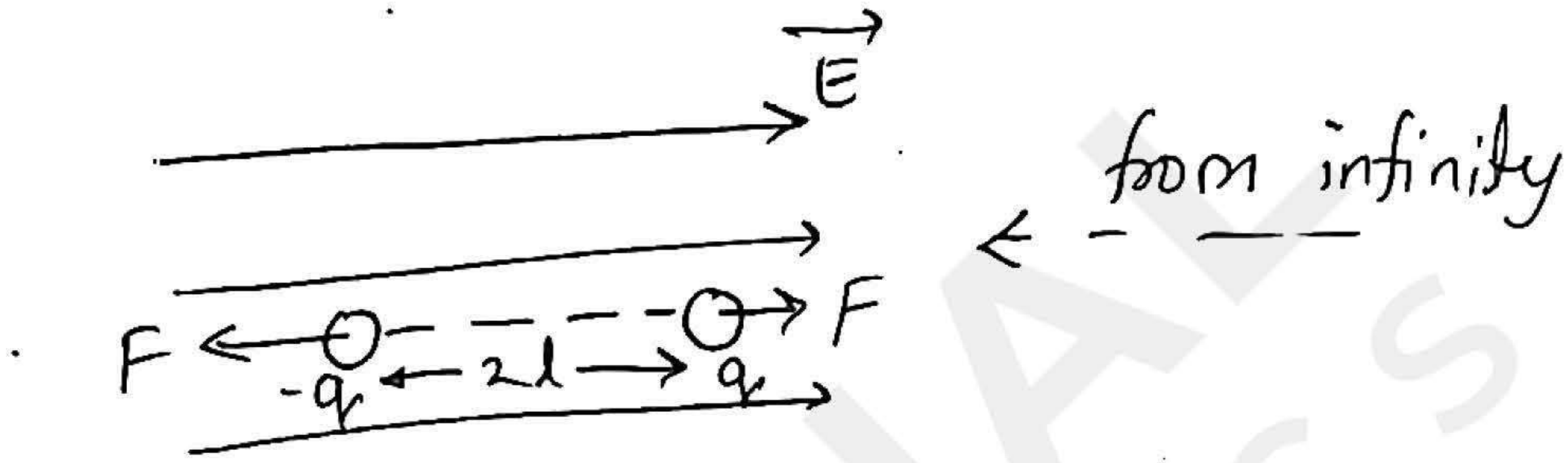
$$W = \int_0^\theta PE\sin\theta\, d\theta =$$
$$= PE(-\cos\theta)_0^\theta$$

$$\Rightarrow W = PE[-\cos\theta + \cos 0)$$

$$\boxed{W = PE(1 - \cos\theta)}$$

Potential Energy of an electric dipole in a uniform Electric field:

→ Work done in bringing an electric dipole from infinity to some point inside the field. is equal to the potential energy of an electric dipole.



Work in bringing dipole = Force on charge $(-q)$ × extra distance moved

$$\Rightarrow \boxed{U = -PE}$$

(orientation is $\theta = 0$)

Now in rotating from $\theta = 0 \rightarrow \theta$

$$\text{Work} = PE(1-\cos\theta)$$

So net potential at θ

$$U_0 = U + W = -PE + PE(1-\cos\theta)$$

$$U_0 = -PE\cos\theta$$

$$\boxed{U_0 = -\vec{P}\cdot\vec{E}}$$

so work in $\theta_i \rightarrow \theta_f$

$W = U_f - U_i$

$$\boxed{W = -PE(\cos\theta_i - \cos\theta_i)}$$