VISUAL PHYSICS

SHORT NOTES

CHAPTER

Elasticity

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ELASTICITY

Elasticity → property by virtue of which it regains its original configuration, when external deforming force is removed.

Plasticity → By virtue of which it does not regain its original configuration when external force is removed.

★ If a force produces a change in normal positioning of the molecules, hence changing configuration of body in length, volume etc. That force is called as deforming Force

Perfectly elastic body → Body that regain original Configuration immediately & Completely after removal of deforming Force. e.g. Quartz & phosphor bronze (nearly perfectly elastic)

Perfectly plastic body:
→ A Body which does not regain its original Configuration at all after deforming Force is removed. e.g. putty, mud (nearly perfectly plastic

$\boxed{\text{★ Nothing is perfectly elastic or perfectly plastic}}$

↳ only degree of elasticity or plasticity differs from body to body.

Stress:

$$\text{Deformation} \xrightarrow{\text{produce}} \text{Internal restoring forces}$$

$$\text{stress} \xleftarrow{\text{(Internal restoring Forces)/(Unit Area)}}$$

$$\Rightarrow \boxed{\text{Stress} = \frac{\text{Restoring Force}}{\text{Area}}}$$

★ if no permanent deformation or absence of plastic behaviour

↳ $\boxed{\text{Restoring Force} = \text{External deforming Force}}$

Hence at that case,

$$\boxed{\text{Stress} = \frac{\text{External deforming Force}}{\text{Area}}}$$

SI unit → Pa → N/m^2
↳ pascal. $[M L^{-1} T^{-2}]$

Intensity of Internal forces at a point
=
stress at point considered.

$$\Rightarrow \boxed{\text{Intensity} = \text{stress} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}}$$

★ As Internal force varies accordinly to maintain equilibrium.

STRESS

- Normal stress
  - Tensile stress
  - Compressive stress
- Volumetric stress
- Shearing or Tangential stress

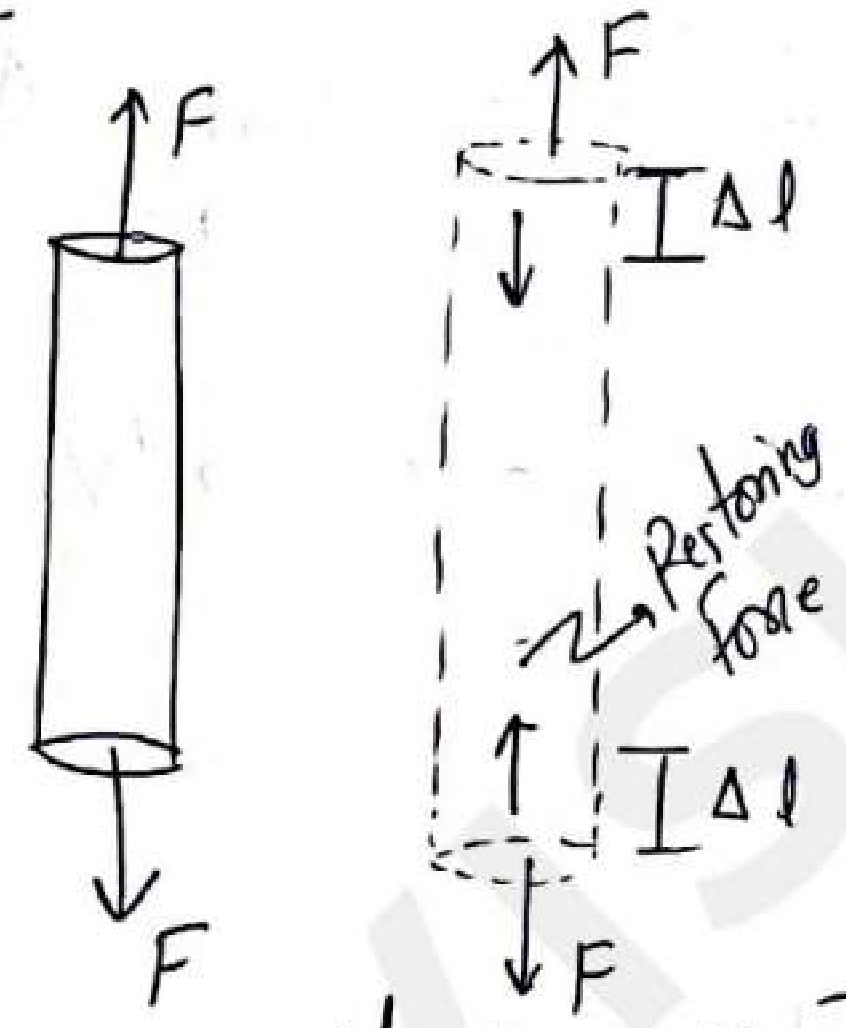
Normal stress → Internal force perpendicular (Normal) to the section considered.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$ ↳ Normal force.

(No change in volume)

Tensile stress :

$F \to$ Applied force



If increase in length in direction of applied force, (Tensile stress).

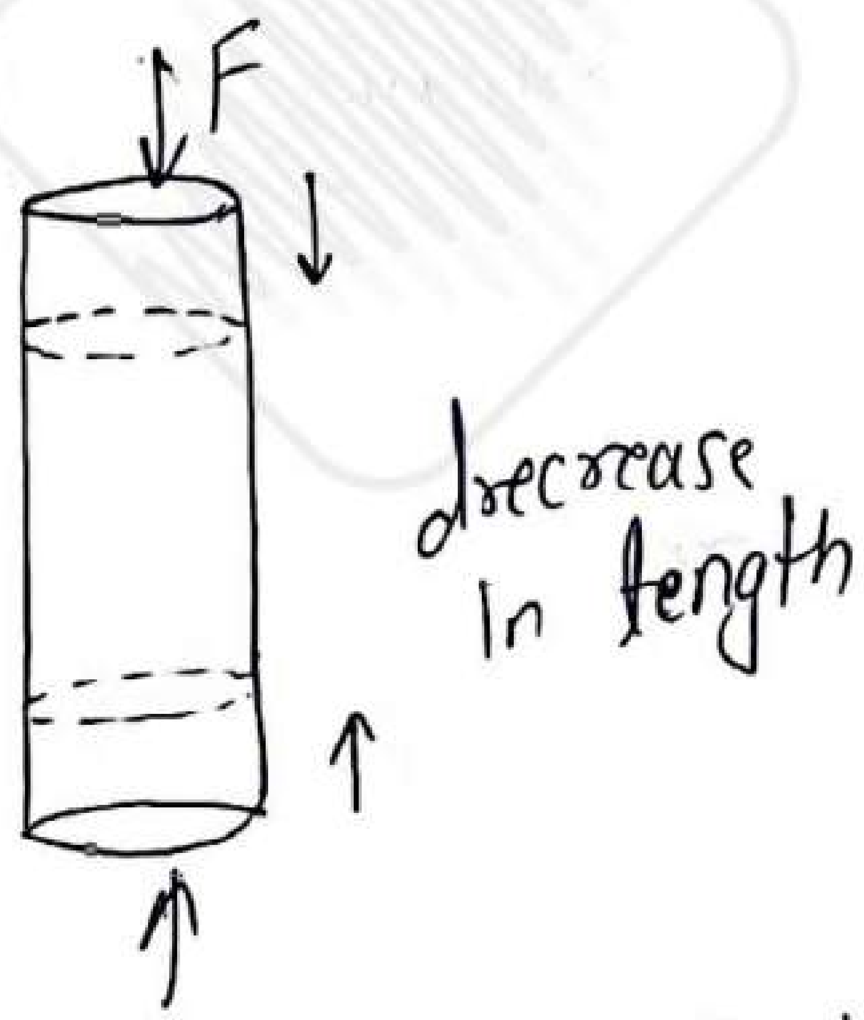
Here,

Restoring force per unit Area = Tensile stress

$[\ \sigma_t = F/A\ ]$

↳ Tensile stress

Compressive stress :

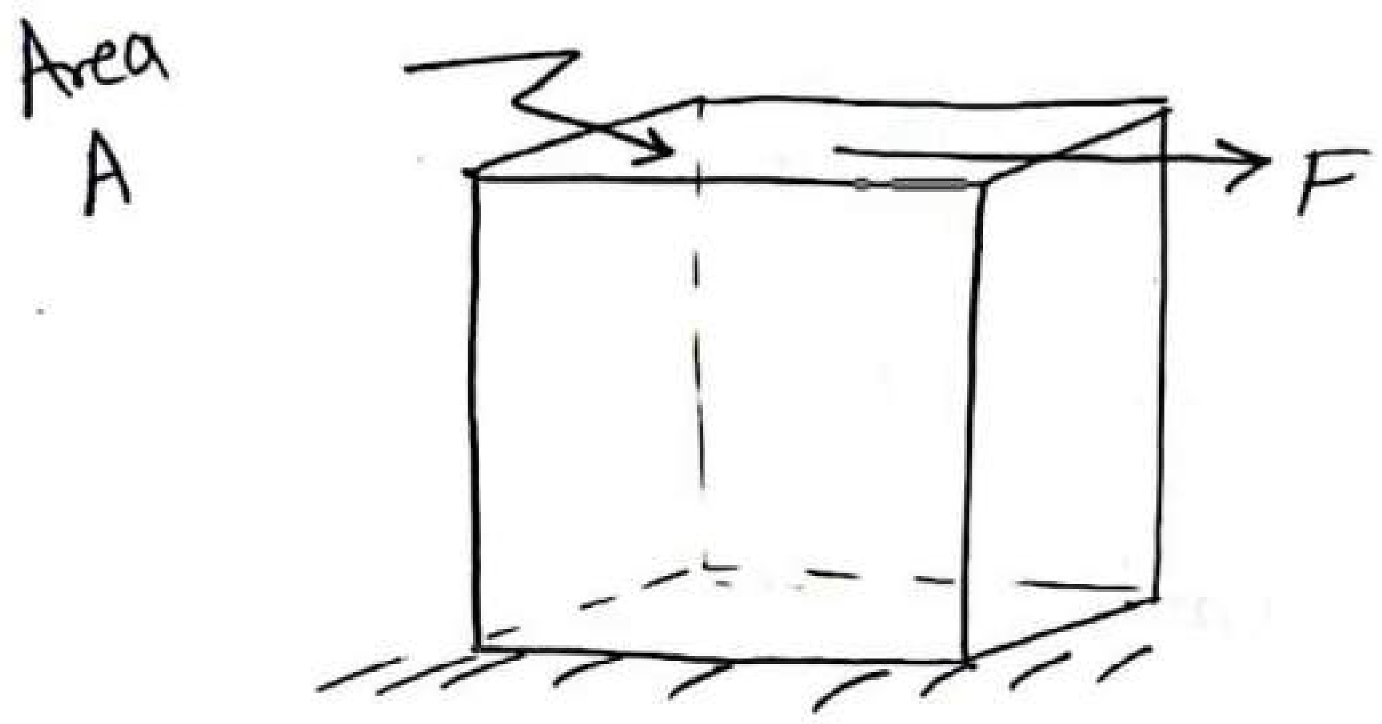


Compression under the action of two equal forces, result in generation of Compressive Internal stress.

$[\ \sigma_c = F/A\ ]$

↳ Compressive stress

Shearing stress :



★ Force acting tangentially to the surface

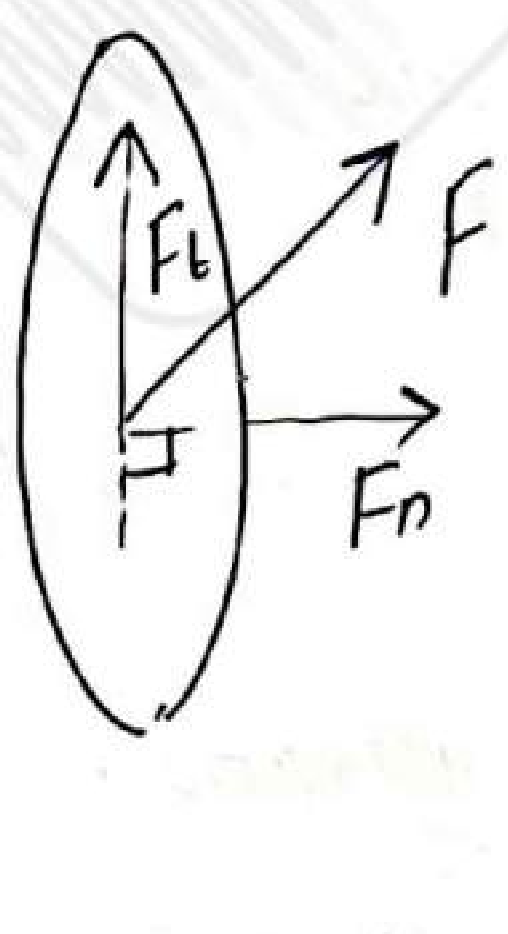
★ deformation along the force, no change in volume.

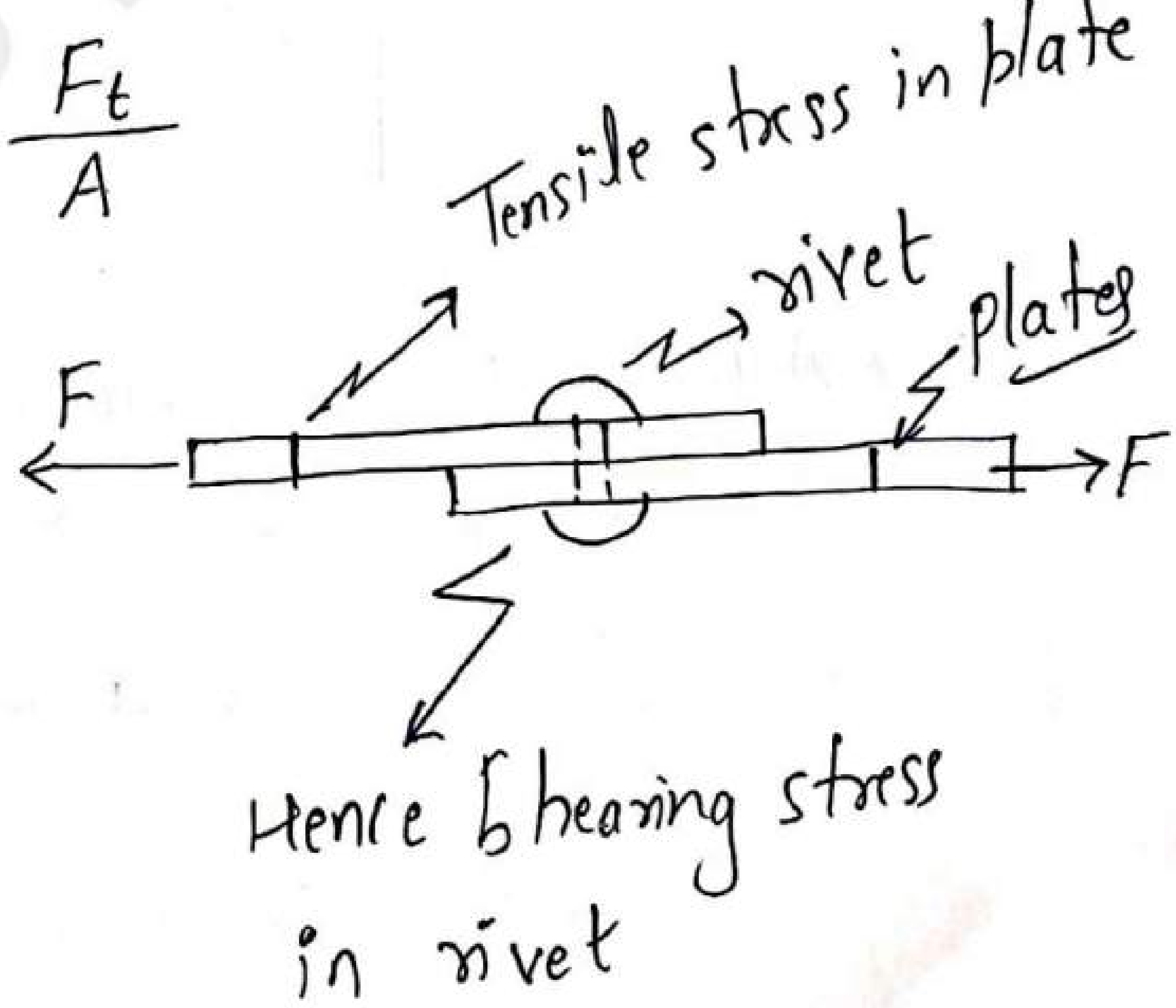
⇒ Tengential stress = $\boxed{\tau = \Delta F / A}$

Intensity , $\tau = \lim_{\Delta A \to 0} \Delta F / \Delta A$

So, Normal stress = $\frac{F_n}{A}$

Shearing stress = $\frac{F_t}{A}$

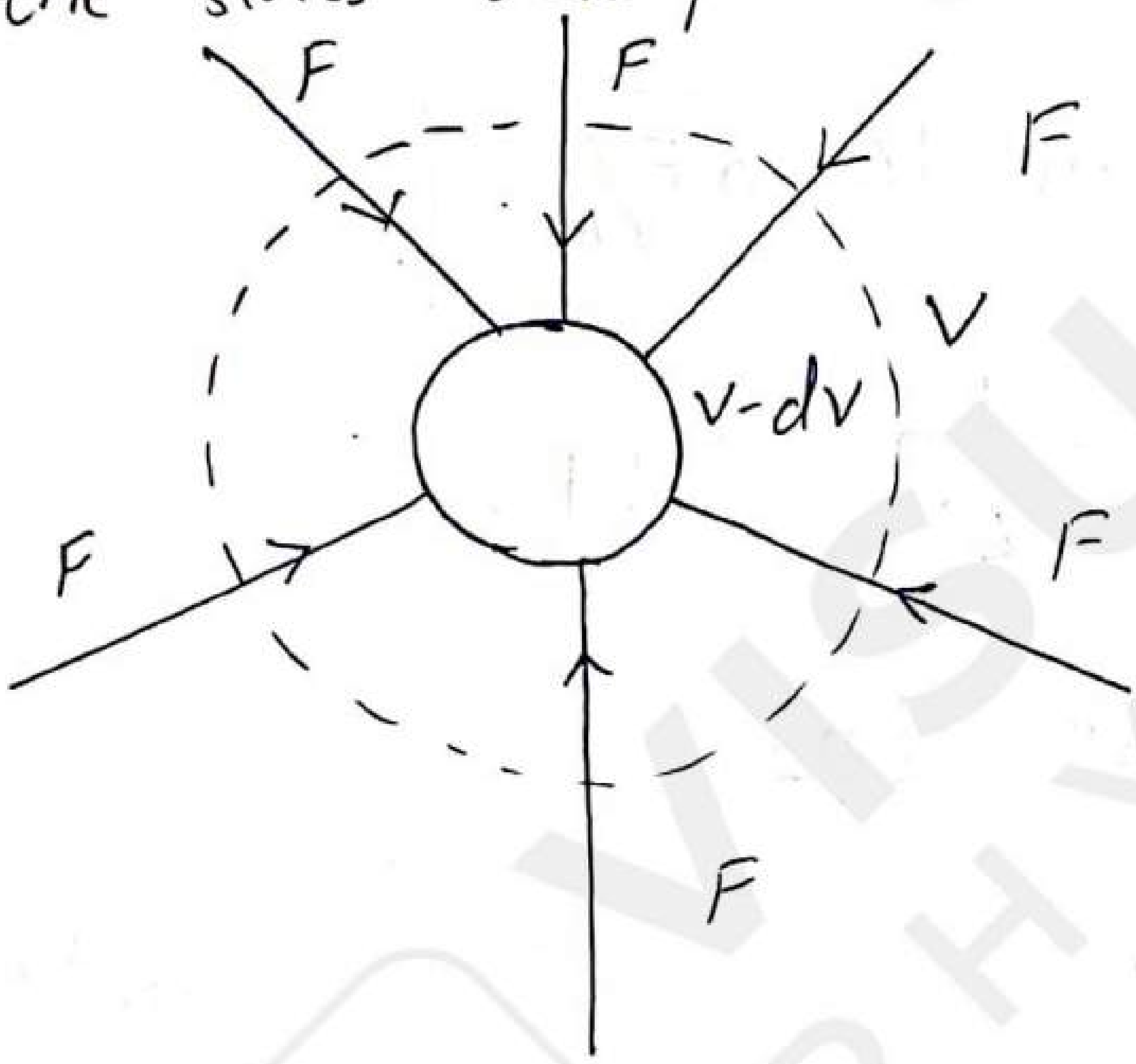




Volumetric stress:

* When solid body undergoes a change in volume without any change in its geometrical shape

* If deforming force act on body from all sides the stress develop = volumetric stress.



STRAIN: When deforming force is applied on a body, there is change in configuration.

Body is said to be strained or deformed.

$$\boxed{\text{Strain} = \frac{\text{change in Configuration}}{\text{Original Configuration}}}$$

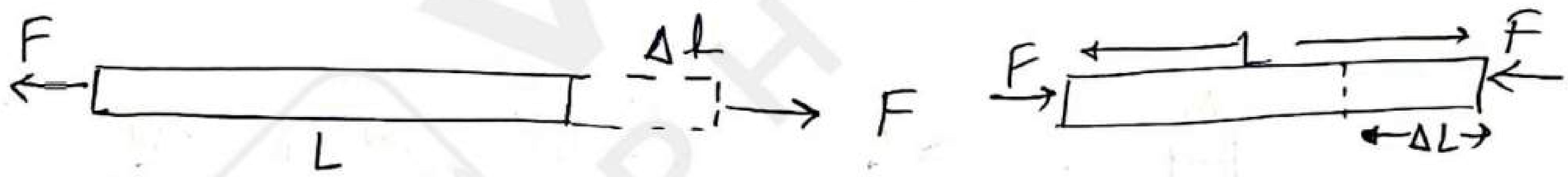
Strain

- Longitudinal strain
- Volumetric strain
- Shear strain

Longitudinal strain:

Deforming force causes a change in length of the body.

$$\varepsilon_l = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

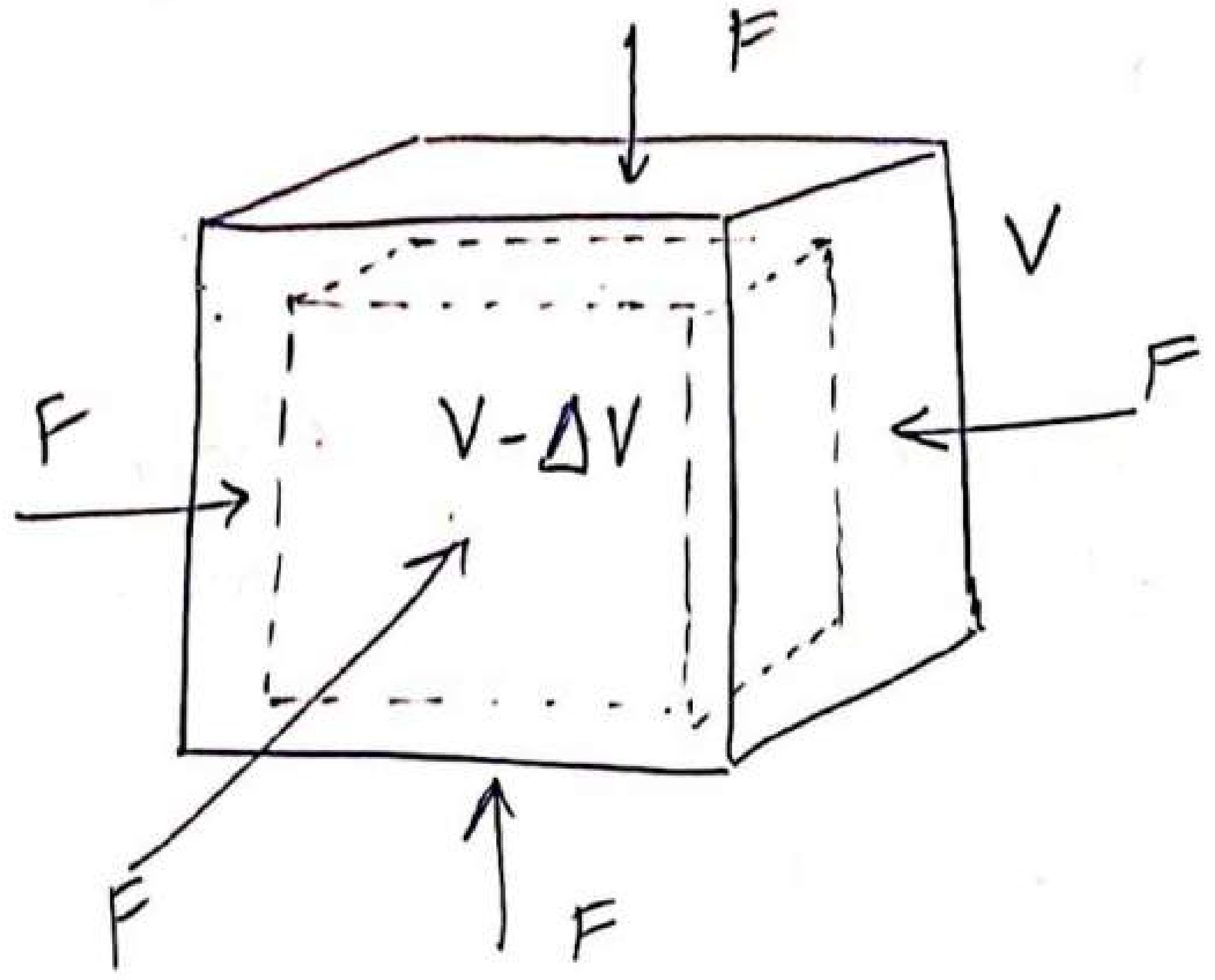


here, both Compression & extension is Considered

- → The specimen must be of uniform cross-section
- → Material must be homogeneous
- → The load must be axial.

Volumetric strain:

when deforming force produce change in volume of the body.

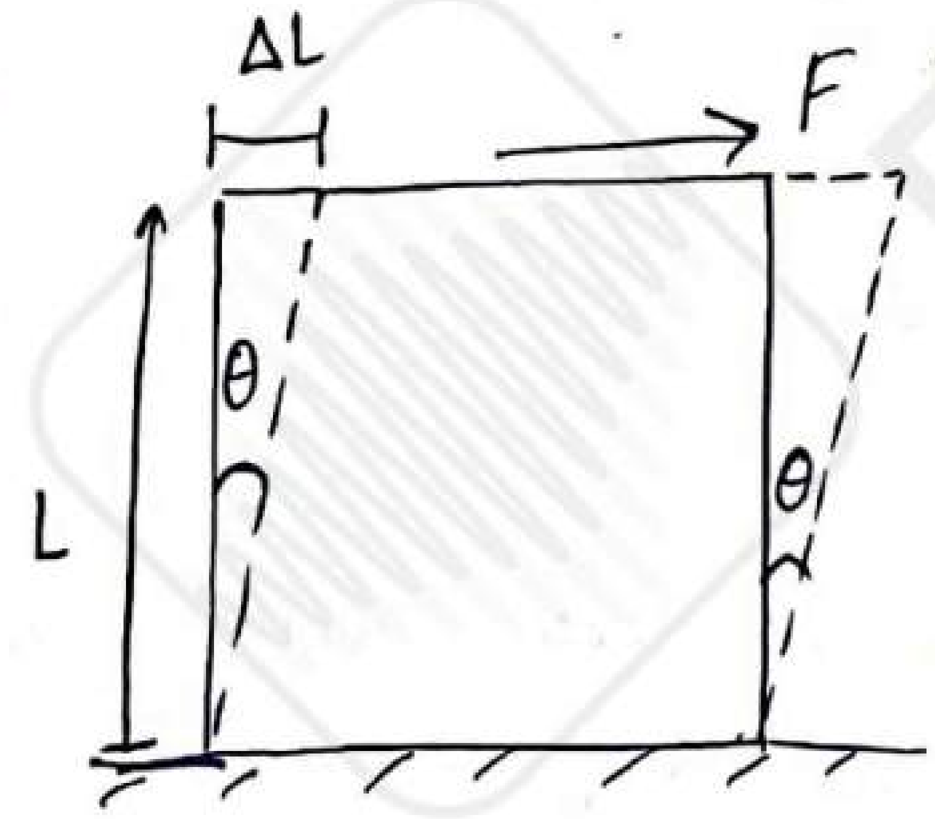


ε_v = Volumetric strain

$$= \frac{\Delta V}{V}$$

Shear strain:

Deforming force causes a change in shape of the body, without changing its volume.



shearing strain $= \theta = \frac{\Delta L}{L}$

→ Angular change between two perpendicular faces is shearing strain.

→ shearing strain never accompanied by volume change.

★ A change in shape/size i.e. dimensions need not necessarily imply strain.
e.g. When body is heated to expand, but strain is zero.

→ Unless & until internal elastic forces operates to bring the body to original state, no strain exists

ELASTIC LIMIT:

↳ Upper limit of deforming force up to which, if deforming force is removed, the body regain its original form completely.

→ beyond this deforming force, body loses its property of elasticity & gets permanently deformed

Hooke's Law:

it is applicable up to proportional limit.

According to this:

stress produce ∝ strain

$$\boxed{\text{stress} = (\text{Modulus of Elasticity}) \times \text{strain}}$$

Modulus of Elasticity

- Young's modulus of elasticity (Y)
- Bulk modulus (k)
- Modulus of Rigidity (η)

Young's Modulus: → Normal stress is acting

so, $$\boxed{Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}}$$

→ greater the force ⇒ larger the deformation.

$$Y = \frac{F/A}{\Delta L/L}$$

$$\Rightarrow F = \left(\frac{YA}{L}\right)\Delta L$$

as $\Delta L \propto \frac{1}{A}$, $\Delta L \propto \frac{1}{Y}$

→ thicker rod produces less deformation and a stiffer material produces less deformation.

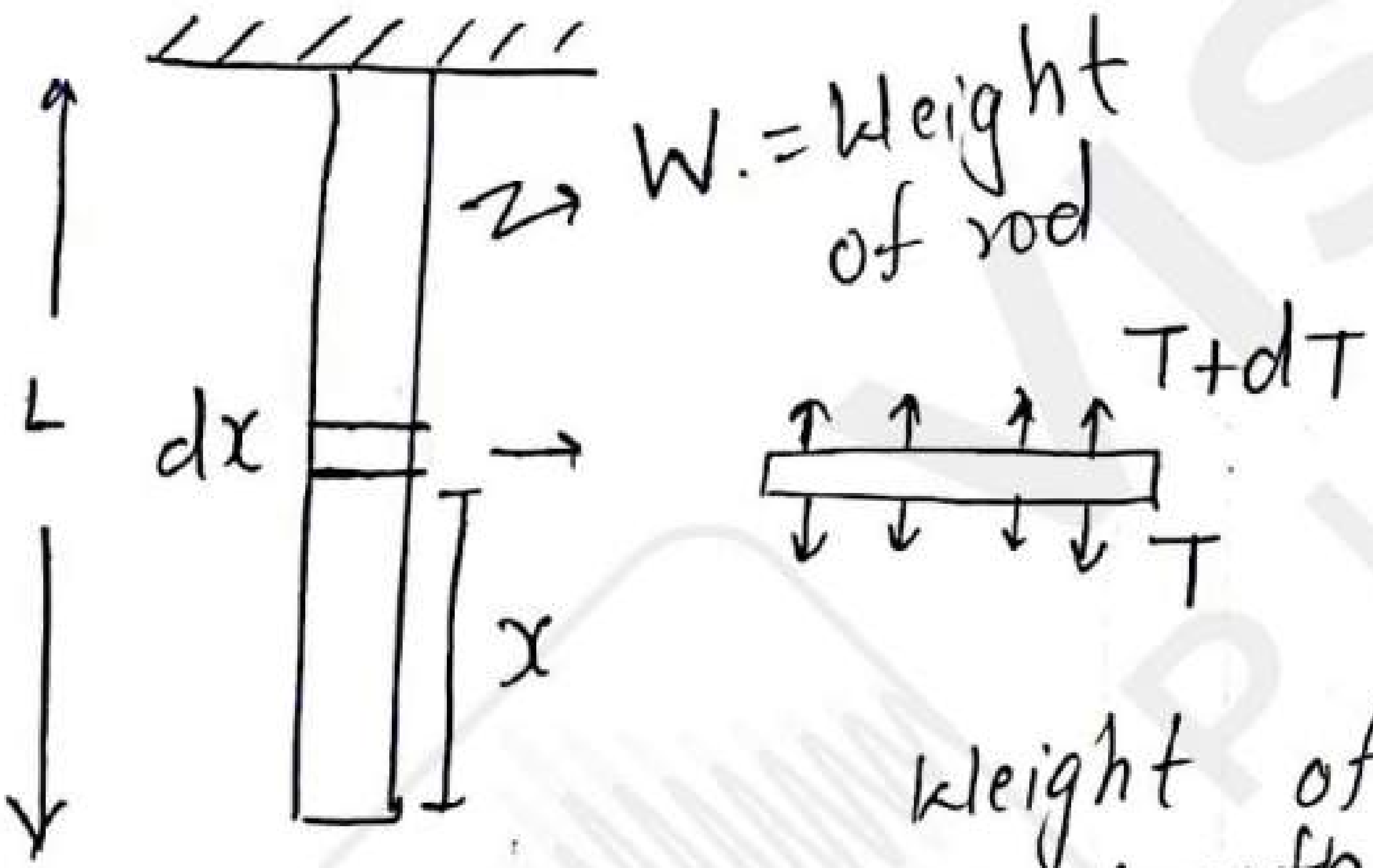
→ As $\boxed{F \propto \Delta L}$

so, $k = \frac{YA}{L}$, $\boxed{F = K\Delta L}$

↳ stiffness Constant

Hence, uniform cross section may be considered as an elastic spring $\boxed{K = YA/L}$

→ Longer the rod, lesser the stiffness and thicker the rod.



Weight of Length $x = \left(\frac{W}{L}x\right)$

$$\text{Stress} = \frac{W}{A} = \frac{Wx}{AL}$$

$$\frac{dL}{L} = \frac{\text{stress}}{Y}$$

so, for 'dx' part, change in length be $d\delta$.

$$\frac{d\delta}{dx} = \frac{Wx}{ALY}$$

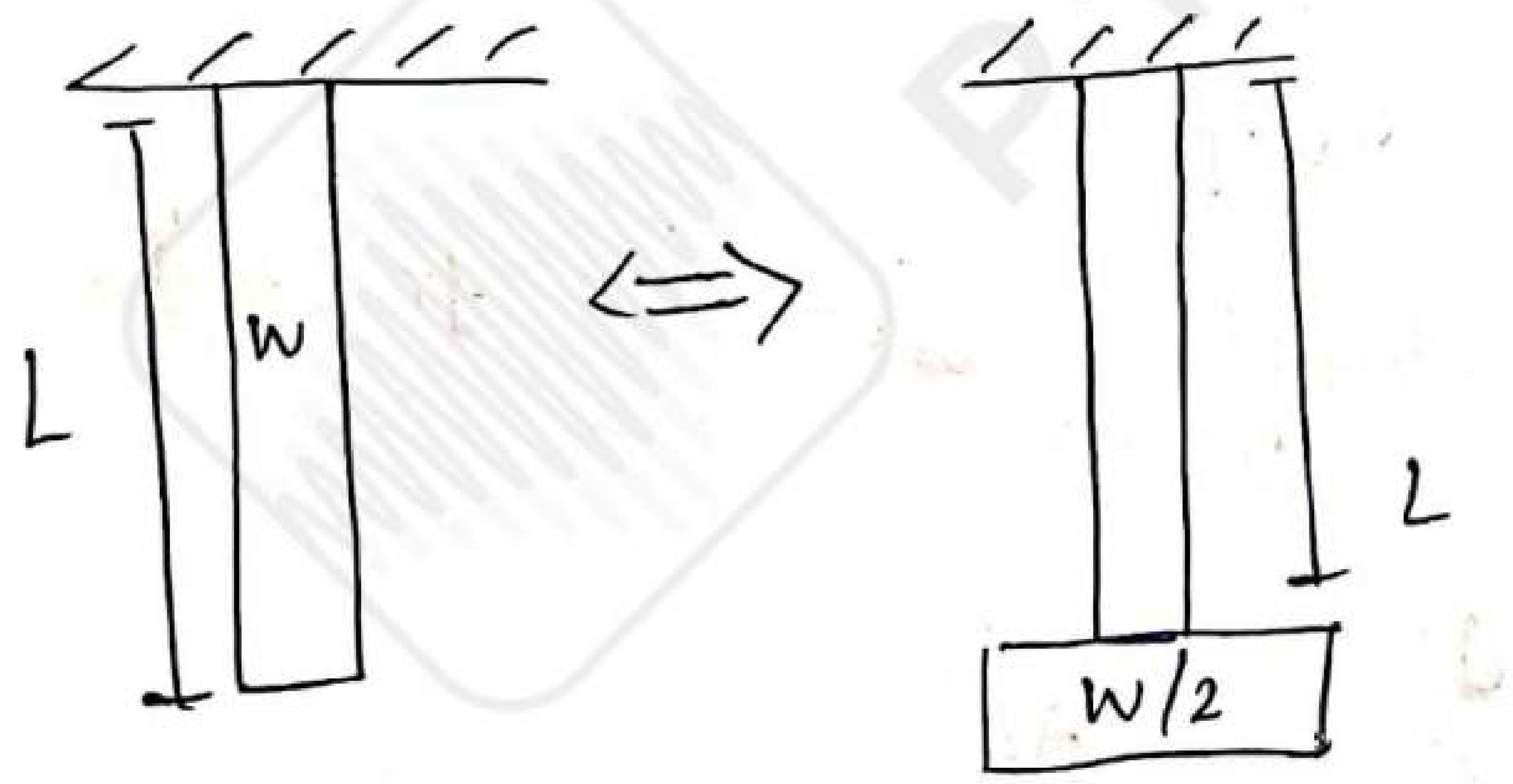
$$\int d\delta = \int \frac{W}{YAL}\, x\, dx$$

$$\Rightarrow \quad \delta = \frac{W}{YAL}\left[\frac{x^2}{2}\right]_0^L$$

$$\boxed{\delta = \frac{WL}{2AY}}$$

OR if We consider rod massless, but having $(W/2)$ weight at end, so extension produce would be same

$$\boxed{\delta = \frac{(W/2)\, L}{AY}}$$



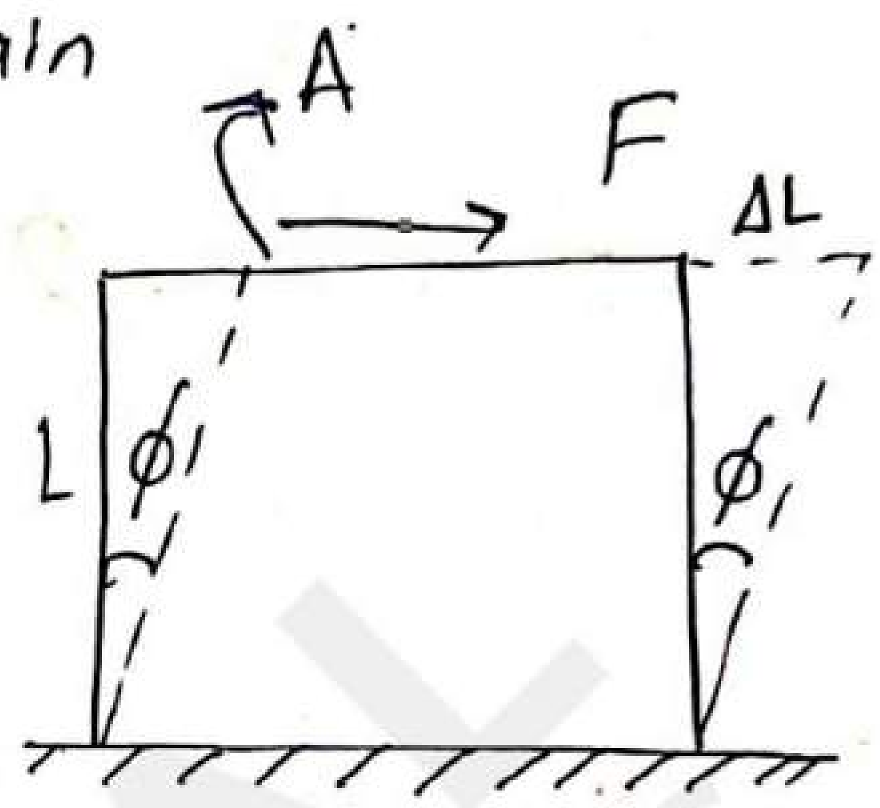
Modulus of Rigidity:

Tangential stress $= F/A$

And shearing strain $= \phi = \frac{\Delta L}{L}$

$$\eta = \frac{\text{Tangential stress}}{\text{shearing strain}}$$

$$\eta = \frac{F}{A\phi}$$



BULK MODULUS:

$p \rightarrow$ Uniform compression stress or expansion stress

$\Delta V \rightarrow$ change in Volume

$V \rightarrow$ Initial volume.

$$P \propto \frac{\Delta V}{V}, \quad P = B\left(\frac{\Delta V}{V}\right)$$

$$\boxed{B = \frac{PV}{\Delta V}}$$

for Compression $\boxed{B = \frac{-PV}{\Delta V}}$

* Negative sign shows increase in pressure (P) cause decrease in volume (ΔV)

Compressibility:

↳ Reciprocal of bulk modulus of elasticity is called Compressibility.

$B_{solids} > B_{liquid} > B_{gases}$.

$$\boxed{k = 1/B}$$

↳ Compressibility

↳ Ability to get compressed.

Analogy of rod as spring

As for spring, $F = kx$

$x \rightarrow$ Extension

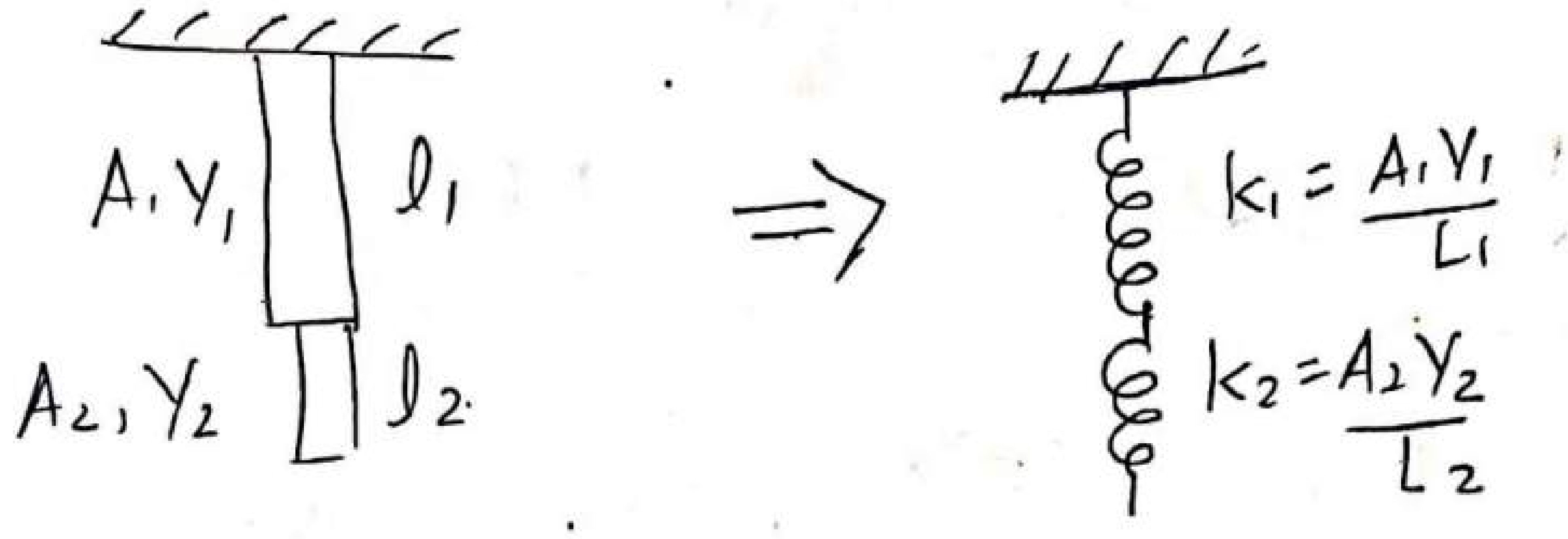
similarly rod,

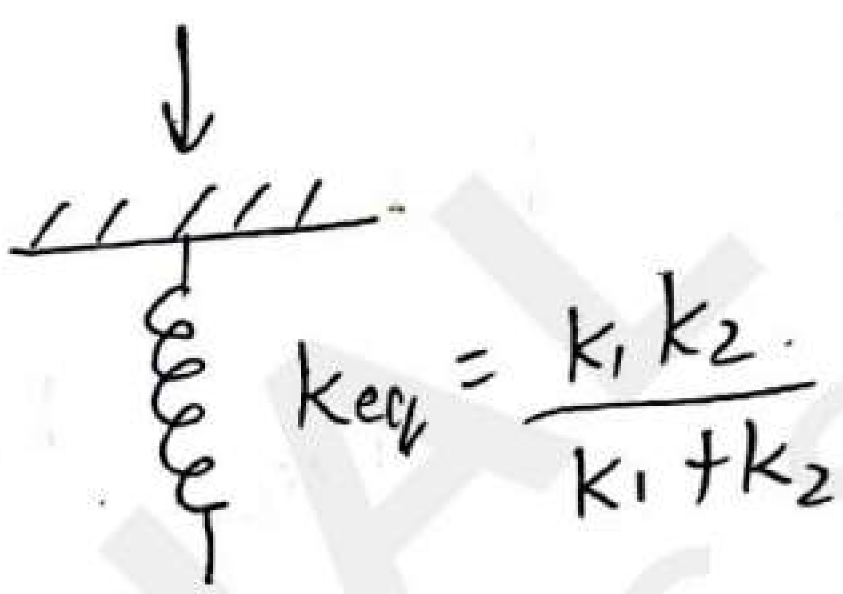
$$F = \left(\frac{AY}{L}\right) x$$

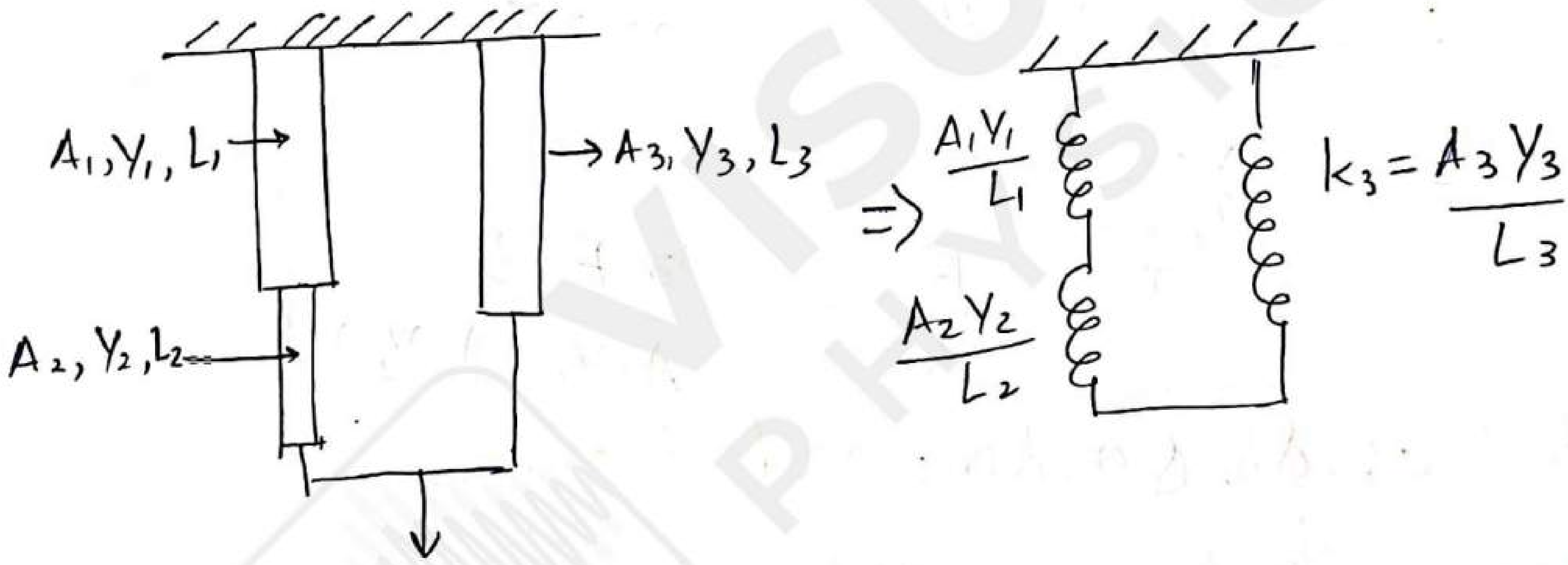
$x \rightarrow$ change in length

so, $\left(k = \frac{AY}{L}\right)$

$$\boxed{F = kx}$$





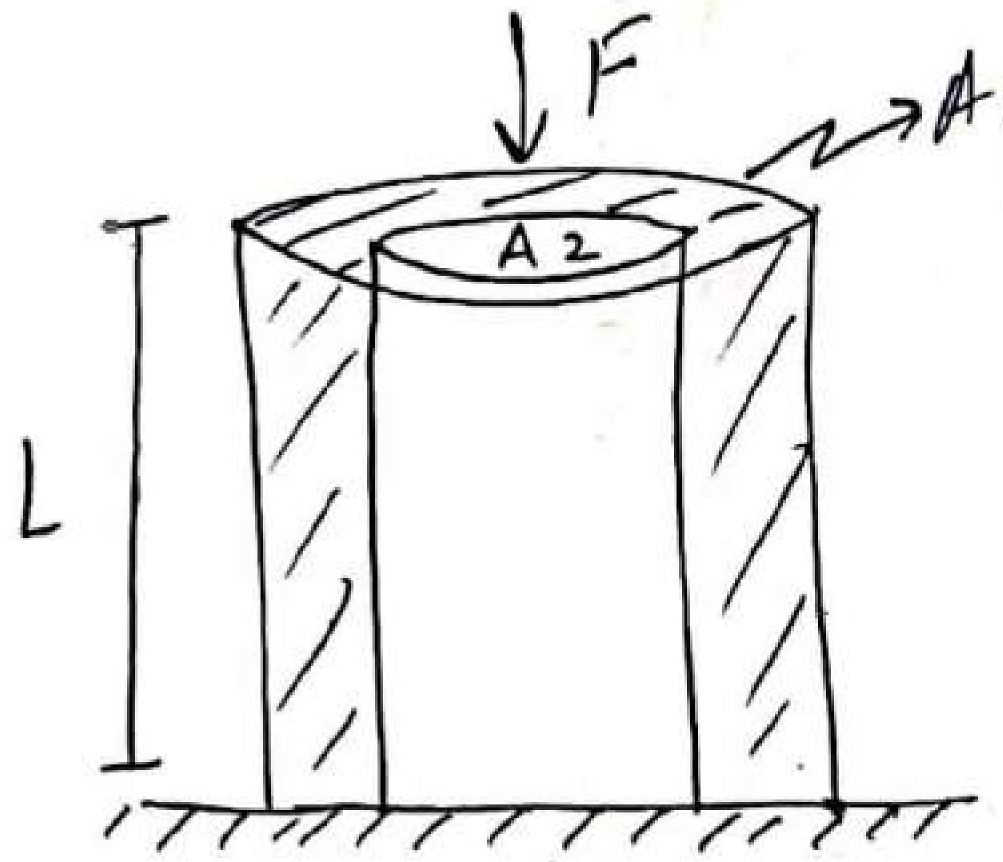


As, $k = \frac{YA}{L}$ $\Rightarrow$ $\boxed{kL = YA = \text{Constant}}$ for Constant 'A'

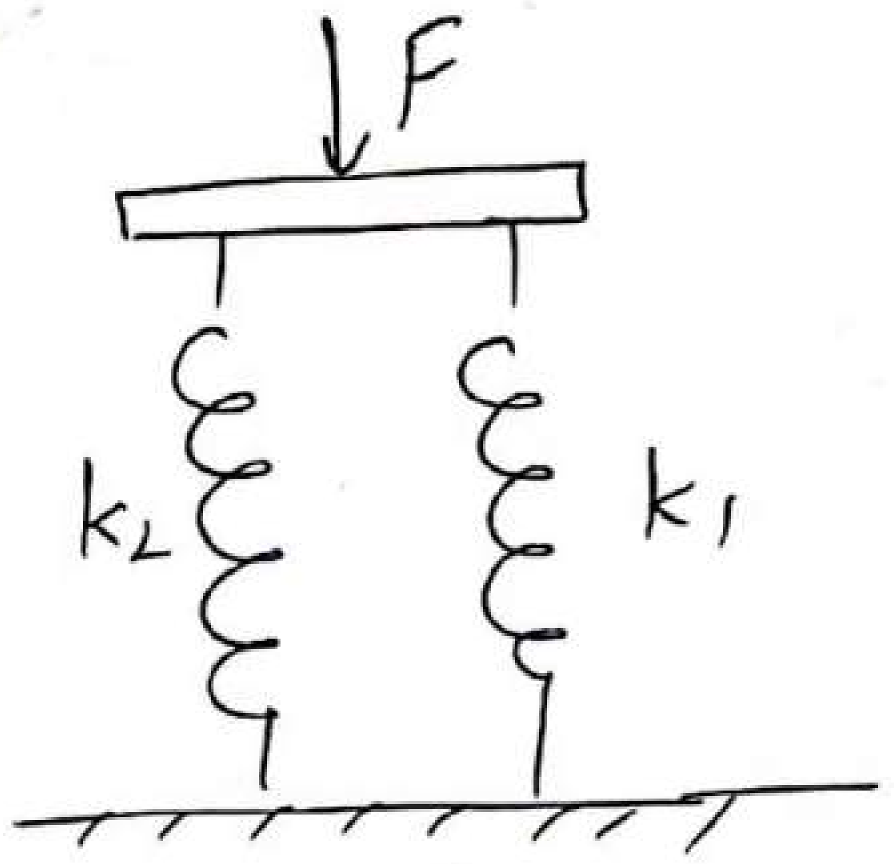
Hence, if we cut the rod,

$$\boxed{k_1 L_1 = k_2 L_2}$$

BARS OF COMPOSITE SECTION :



$\Leftrightarrow$



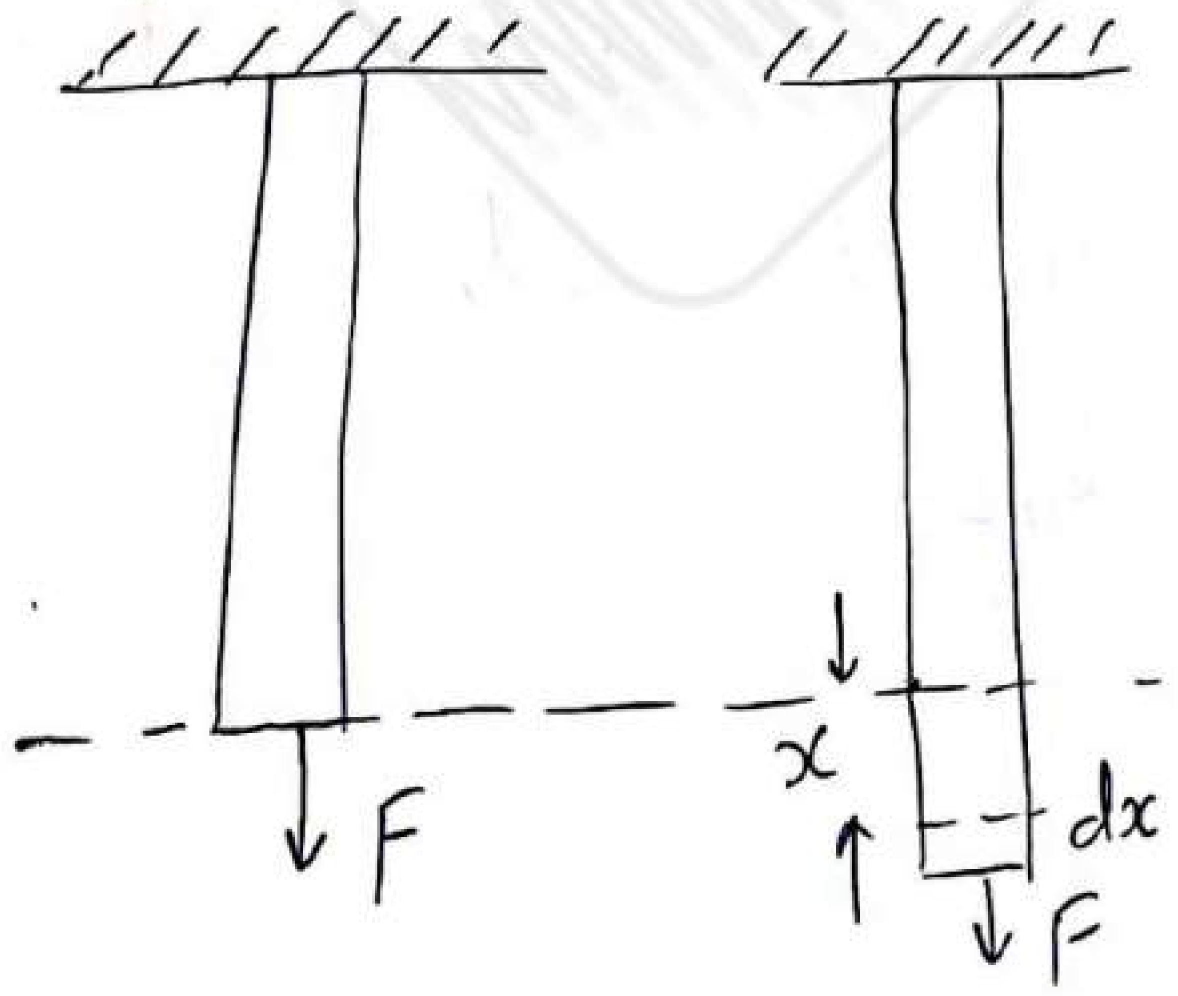
$$k_{eq_{\parallel}} = k_1 + k_2 = \frac{A_1 Y_1}{L} + \frac{A_2 Y_2}{L}$$

change in length is same $= x$

$\Rightarrow$

$$\boxed{x = \frac{F}{k} = \frac{F L}{(A_1 Y_1 + A_2 Y_2)}}$$

Energy stored in deformed body :



$$F = \left(\frac{YA}{L}\right) x$$

$$\int dw = \int F\,dx$$

$$w = \frac{YA}{L} \int_0^{\delta} x\,dx$$

$$\boxed{w = \frac{1}{2} \frac{YA}{L} \delta^2}$$

$\delta \rightarrow$ Net extension / Compression

Energy stored = $W = U = \frac{1}{2}\frac{YA}{L}\delta^2$

Similar result if considered spring analogy

$$U = \frac{1}{2} k_{eq}\delta^2$$

$$K = \frac{YA}{L} \Rightarrow \boxed{U = \frac{1}{2}\left(\frac{YA}{L}\right)\delta^2}$$

Now, $U = \frac{1}{2}\frac{YA}{L}\delta^2$

$$\delta = \frac{\text{stress}}{Y} = \frac{FL}{AY}$$

$$\Rightarrow U = \frac{1}{2}\frac{YA}{L}\left(\frac{FL}{AY}\right)^2 = \frac{1}{2}\frac{F^2}{AY}L = \frac{1}{2}F\delta$$

$$U = \frac{1}{2}\frac{F}{A} \times \frac{F}{AY} AL$$

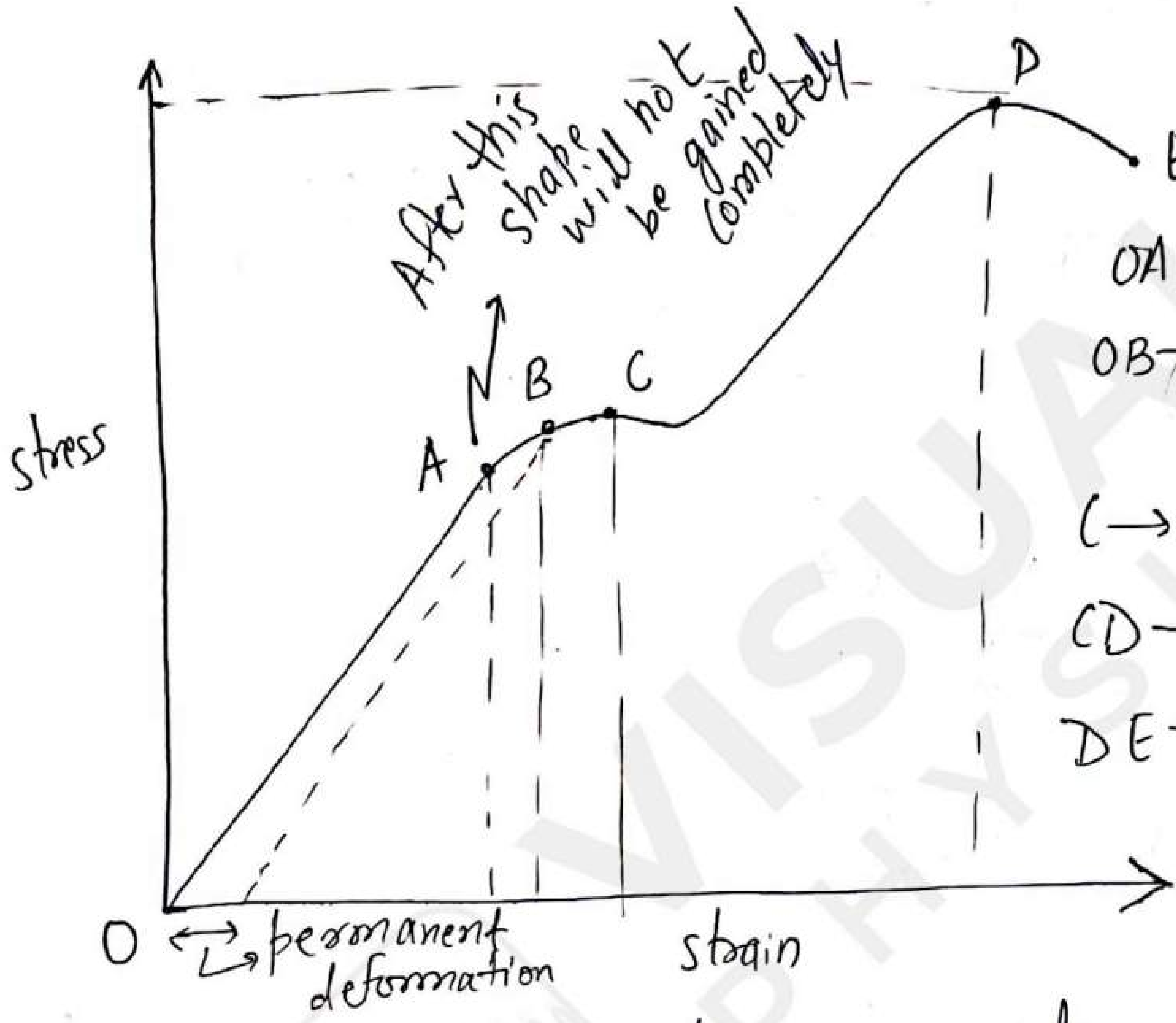
$$\Rightarrow U = \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{Volume}$$

or $U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{Volume}$

$$\boxed{\text{strain Energy density} = \frac{U}{\text{Volume}} = u = \frac{1}{2} \times Y \times (\text{strain})^2}$$

$$\boxed{u = \frac{1}{2} \times \text{stress} \times \text{strain}}$$

↳ Area Under stress strain Curve give Energy density



OA → Elastic limit

OB → Limit of propor-tionality

C → Yield point

CD → plastic behavior

DE → fracture

D → Ultimate point

→ In yield region, strain 15-20 times those take place up to proportional limit.

→ Yield point → close to elastic limit for most purposes the two may be taken as one

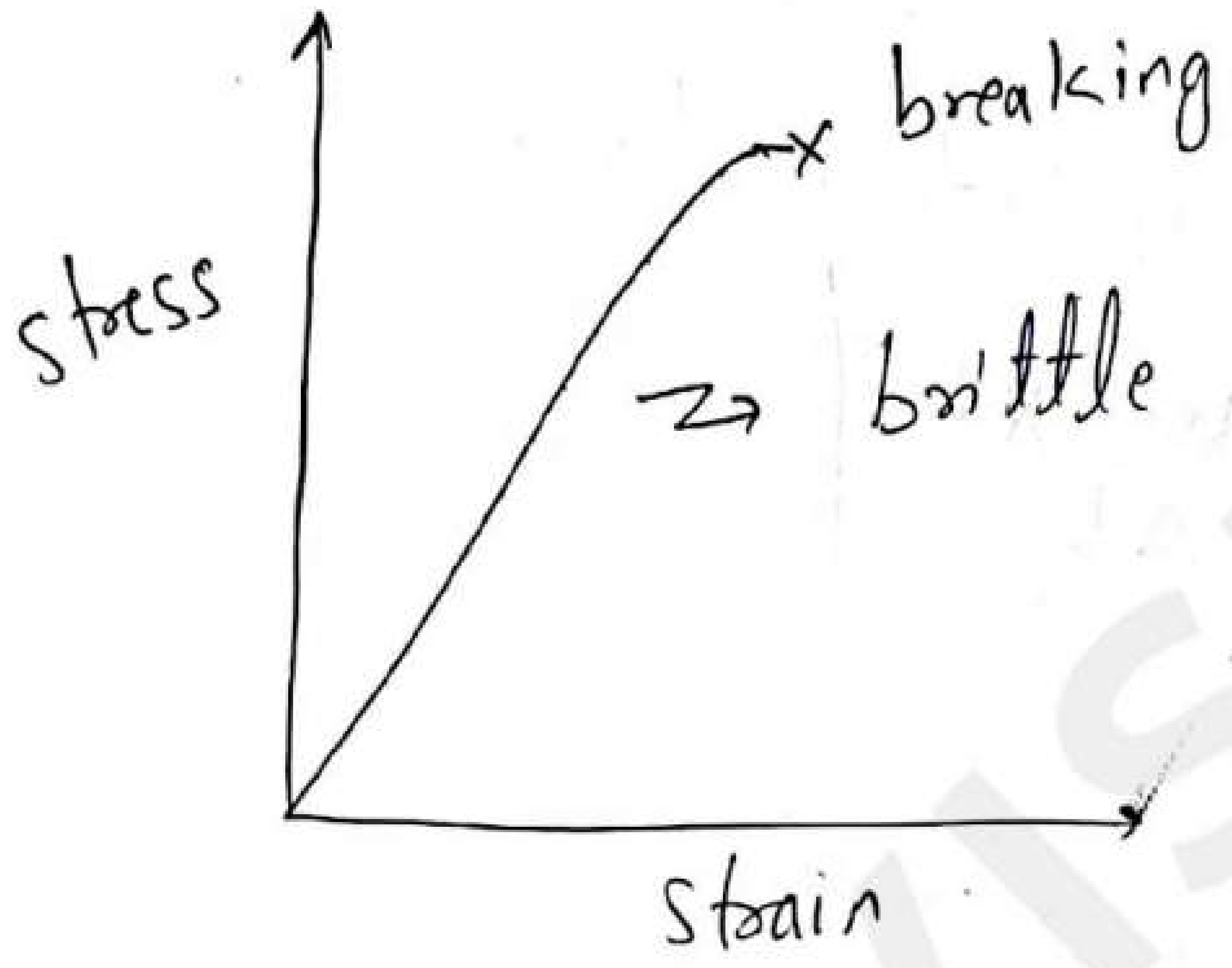
Working stress is lower than breaking stress

$$\boxed{\text{factor of safety} = \text{breaking stress}/\text{working stress}}$$

→ Steeper curve indicates a stiffer material.

→ long graph parallel to the strain axis; indicates ductile material.

→ Absence of yield point or plastic zone refer to brittle material.

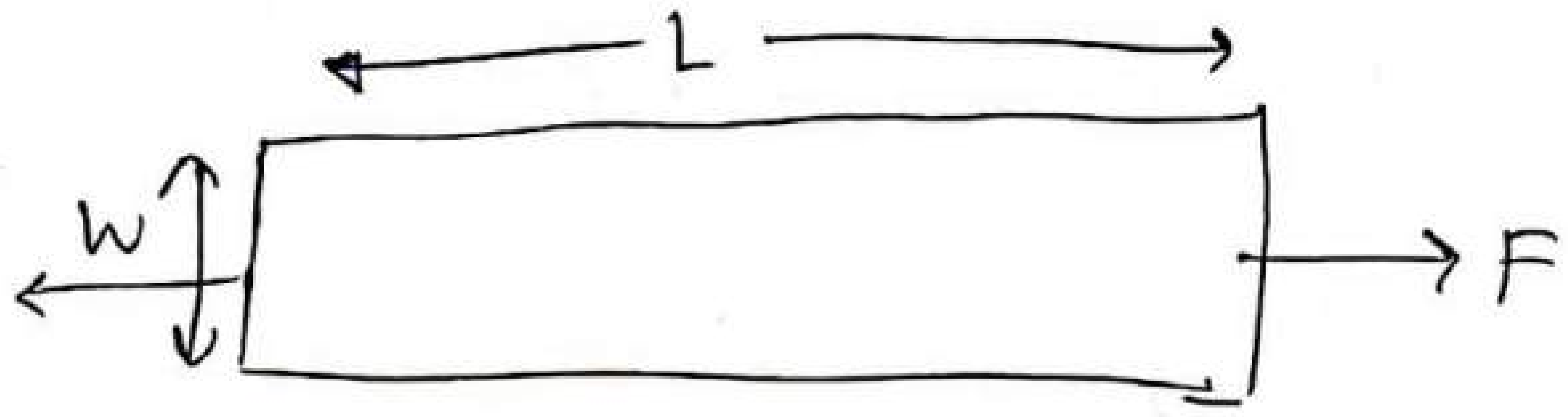


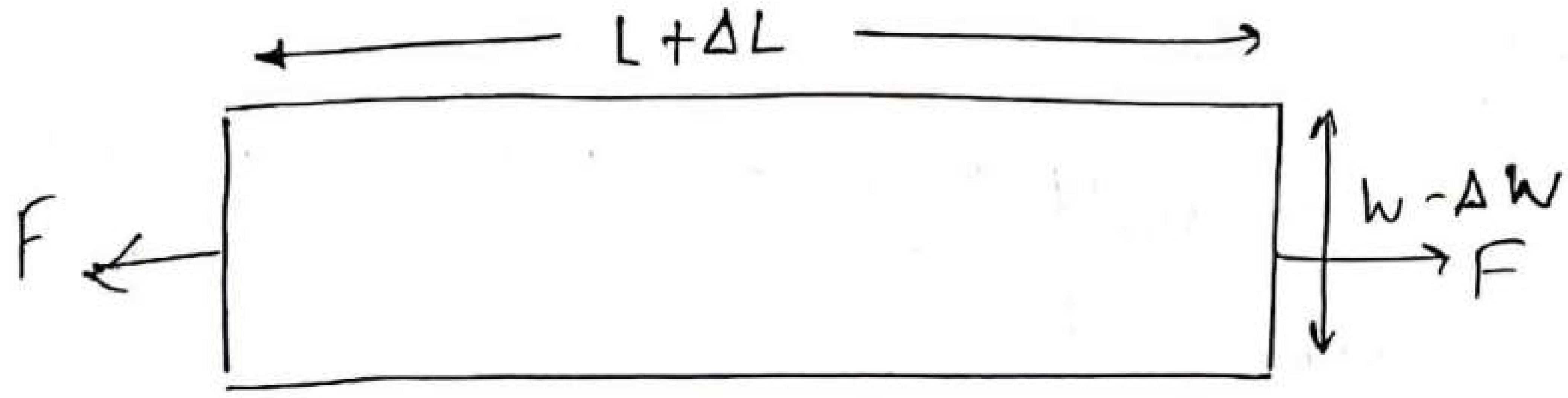
more Y → more elastic material

→ More the area under stress vs strain curve Implies more <u>tougher material</u>. Means more energy per unit volume is required to break the material.

<u>Poisson's Ratio</u>:

As during streching & compression.
As volume remains constant.
⇒ Change in longitudinal causes change in Lateral dimension.





poission's Ratio $= \dfrac{\text{lateral strain}}{\text{longitudnal strain}}$

$$\boxed{\sigma = -\frac{\Delta W}{W\,\Delta L} \times L}$$

poission's Ratio

→ property of material.

→ does not depend on size & shape of material.

Usualy σ is positive.

→ means if $L\uparrow$, $W\downarrow$

But for some material

$\sigma \rightarrow$ negative

→ means if $L\uparrow$, $W\uparrow$

e.g. polymers foams