VISUAL PHYSICS

SHORT NOTES

CHAPTER

CoM, Momentum & Collisions

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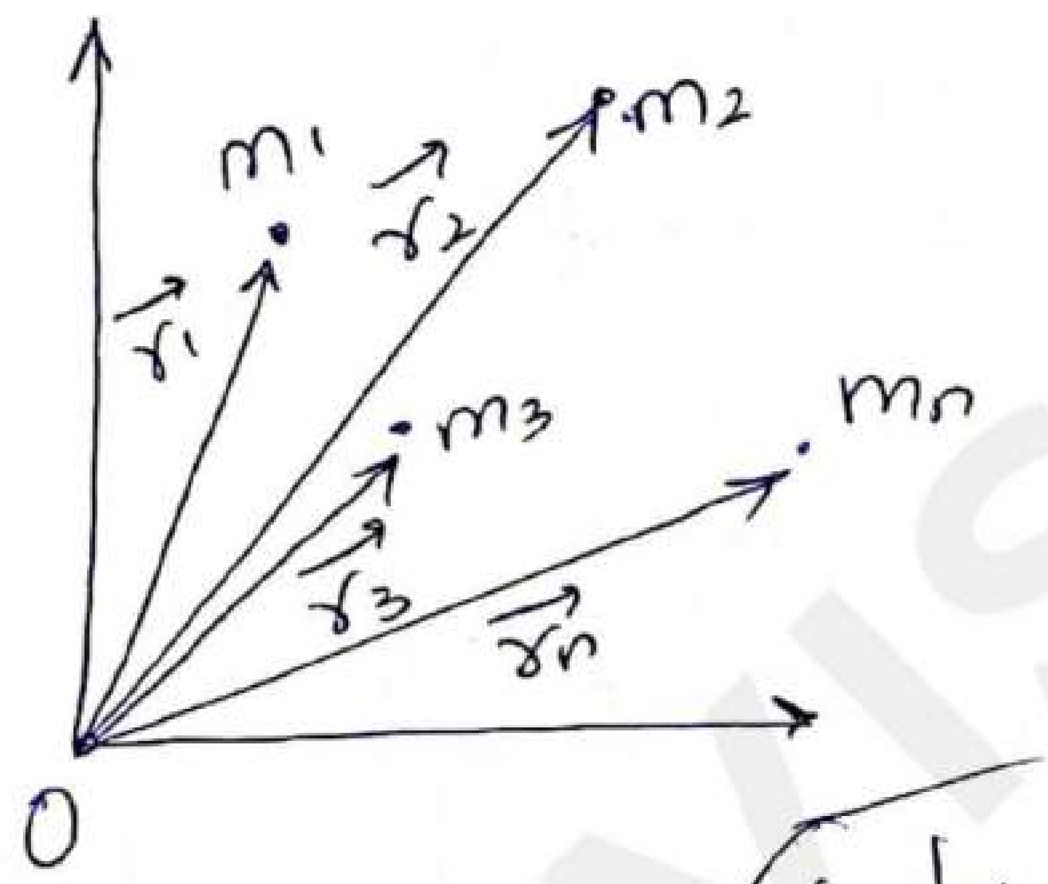
Centre of Mass, Conservation of Linear Momentum & Collision.

Centre of mass:

$\hookrightarrow$ An imaginary point where we can consider the whole mass of the body at.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots m_n\vec{r}_n}{(m_1+m_2+\cdots m_n)}$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots m_n\vec{r}_n}{M}$$

Centre of mass from given reference point

Net mass of body

In terms of components

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots m_nx_n}{M}, \quad y_{cm} = \frac{m_1y_1 + m_2y_2 \cdots}{M}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + \cdots m_nz_n}{M}$$

$$\boxed{m_1, m_2, \ldots m_n \Rightarrow \text{discrete mass}}$$

Centre of Mass of Continuous Mass

$$x_{cm} = \frac{\int x\,dm}{\int dm}, \quad y_{cm} = \frac{\int y\,dm}{\int dm}, \quad z_{cm} = \frac{\int z\,dm}{\int dm}.$$

need not to be on object

Centre of mass → Used to study the motion of body of given dimensions

As till now we used newton's law and all for point object but what about continuous object?

A point which behave as a single point mass so that we can apply all the laws of motion on it. And COM satisfy the real situation behavior

$$\Rightarrow \boxed{F_{net} = M\,\underbrace{a_{CM}}_{\text{Centre of mass Acceleration}}}$$

Centre of gravity → Where we can assume all the gravitational force to act on.

COM ⟺ Centre of Mass

COM of symmetric object lie at its centroid.

if some part is removed.

$$x_{cm} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2} \quad , \quad y_{cm} = \frac{m_1 y_1 - m_2 y_2}{m_1 - m_2}$$

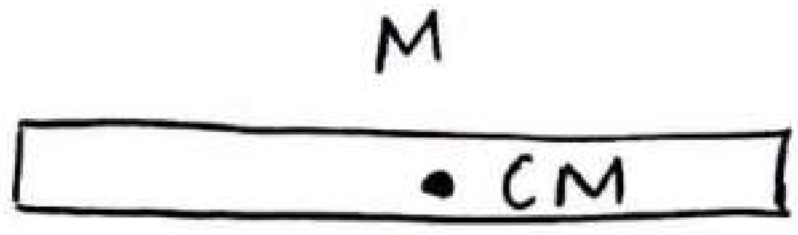
$$z_{cm} = \frac{m_1 z_1 - m_2 z_2}{m_1 - m_2}$$

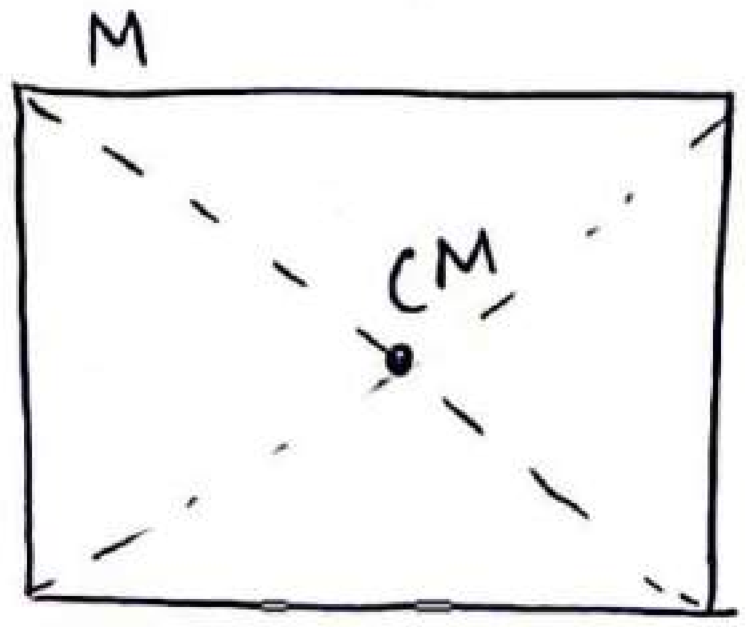
if considered uniform laminar

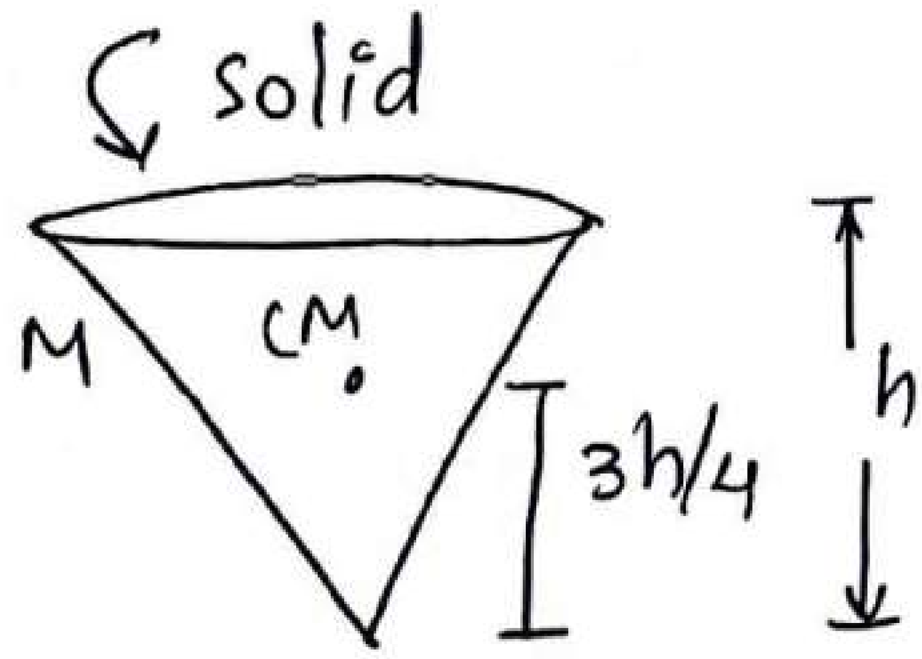
$$x_{cm} = \frac{\overset{x_1}{\cancel{m_1}} A_1 - \overset{x_2}{\cancel{m_2}} A_2}{m_1 - m_2} \quad , \quad y_{cm} = \frac{y_1 A_1 - y_2 A_2}{m_1 - m_2}$$

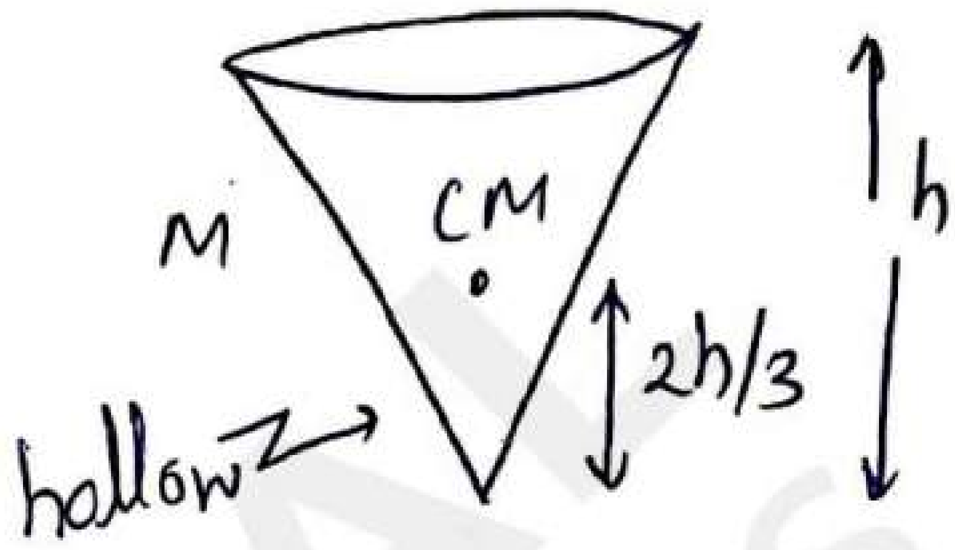
$$z_{cm} = \frac{z_1 A_1 - z_2 A_2}{m_1 - m_2}$$

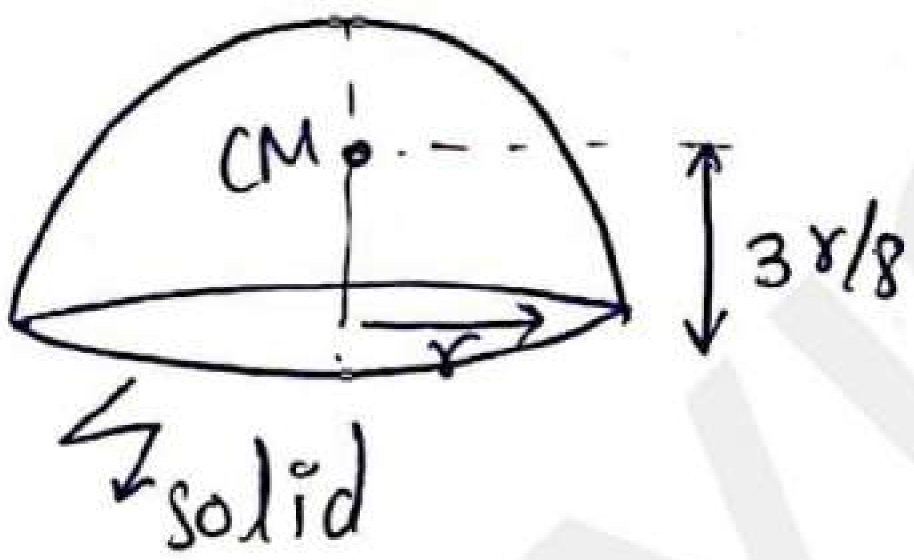
★ m_1, A_1, x_1, y_1 & z_1 are the values for the whole mass while m_2, A_2, x_2, y_2 and z_2 are the values for the mass which has been removed.

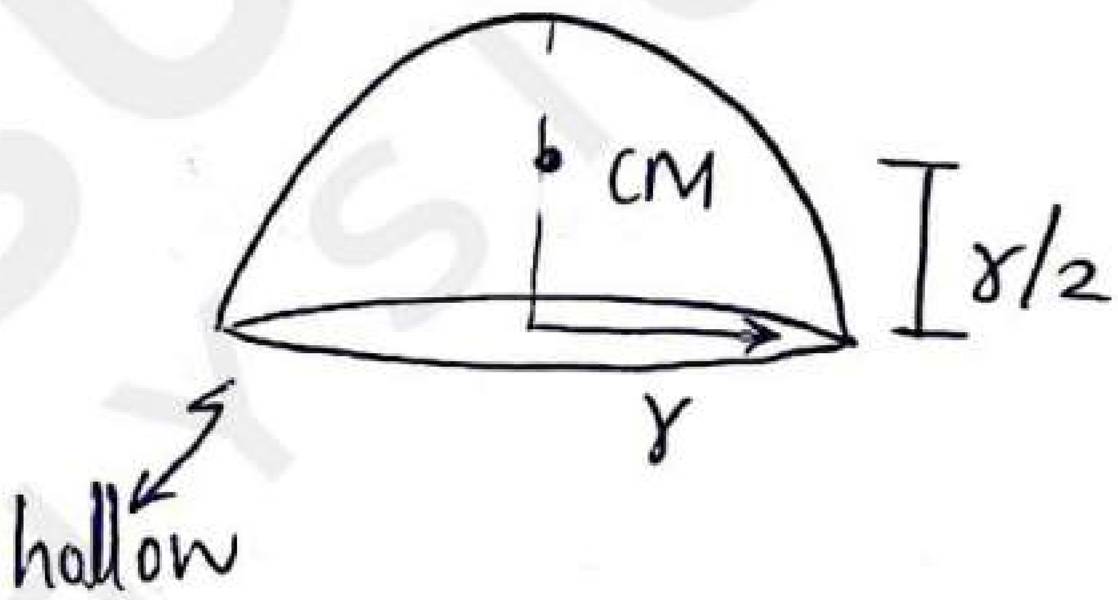












Motion of COM:

$$\vec{V}_{CM} = \frac{m_1 \frac{d\vec{r}_1}{dt} + \cdots m_n \frac{d\vec{r}_n}{dt}}{m_1 + m_2 + \cdots m_n}$$

$$\vec{V}_x = \frac{m_1 \vec{v}_{x1} + m_2 \vec{v}_{x2} + \cdots}{m_1 + m_2 + \cdots m_n}, \quad \vec{V}_y = \frac{m_1 \vec{V}_{y1} + \cdots m_n \vec{V}_{yn}}{m_1 + \cdots m_n}$$

$$\vec{V}_z = \frac{m_1 \vec{v}_{z1} + \cdots m_n \vec{V}_{zn}}{m_1 + \cdots m_n}$$

$$\boxed{\vec{P}_{total} = [M_{total}]\,\vec{V}_{CM}}$$

Acceleration of centre of Mass:

$$\vec{a}_{CM} = \frac{d}{dt}\vec{V}_{CM} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{V}_2}{dt} + \dots m_n\frac{d\vec{v}_n}{dt}}{M}$$

$$\vec{a}_{CM} = \frac{\vec{F}_{ext}}{M} = \frac{\text{Net external force}}{M}$$

So if there is no net external force, $\vec{a}_{CM} = 0$

★ if internal forces are there the discrete particle will move under it's influence, but $\vec{a}_{CM} = 0$

Now, the COM will act as individual particle at which we can apply all the formulas of Newton's law.

Momentum:

for a given particle $\rightarrow$ Velocity $\vec{v}$

$\rightarrow$ MASS $\rightarrow$ M

momentum, $\boxed{\vec{p} = M\vec{v}}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\rightarrow$ change of momentum of system

external force

$m_1\vec{a_1} + m_2\vec{a_2} = 0$ $\rightarrow$ Net external force $= 0$

$m_1 \frac{d}{dt}\vec{v_1} + m_2 \frac{d\vec{v_2}}{dt} = 0$

$\frac{d}{dt}[m_1\vec{v_1} + m_2\vec{v_2}] = 0$

$\frac{d}{dt}[\underbrace{\vec{P_1} + \vec{P_2}}_{p_{total}}] = 0$

$p_{total} \rightarrow$ total momentum

$\Rightarrow \quad \vec{P}_{tot} = \text{Constant}$

$$\boxed{\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}}$$

$$\boxed{\vec{\Delta P} = \vec{p}_f - \vec{p}_i}$$

$p = mv$

as kinetic energy $= \frac{1}{2} mv^2$

$$K.E = \frac{1}{2} m \left[\frac{p}{m}\right]^2$$

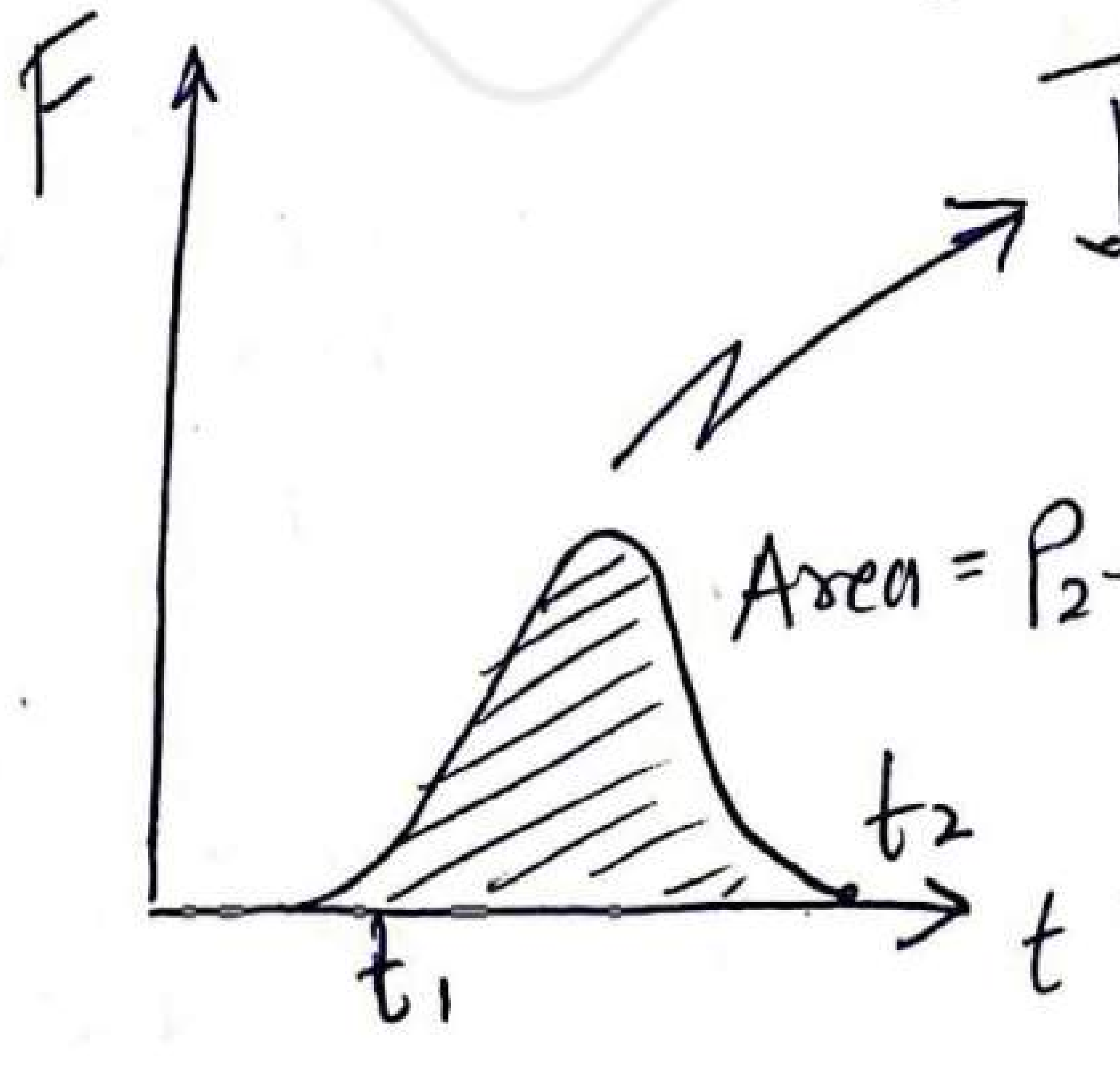
$$\boxed{K.E = \frac{1}{2} \frac{p^2}{m}}$$

Impulse:

Impulse of force F acting on body for time $(t_2 - t_1)$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}\, dt = \int_{t_1}^{t_2} m \frac{d\vec{v}}{dt} dt = \int_{v_1}^{v_2} m\, d\vec{v}$$

$$\vec{J} = \text{change of momentum} = \overrightarrow{\Delta p}$$



$\vec{J} = \int \vec{F}\, dt$

Area Under F - t graph gives Impulse = change In Momentum

Impulsive force → A force of relatively higher magnitude and acting for relatively shorter time.

$\boxed{F_{ext} = 0}$

COLLISION:

→ momentum is always conserved in any type

→ Collision is an isolated event in which a strong force acts between two or more bodies for a short time.

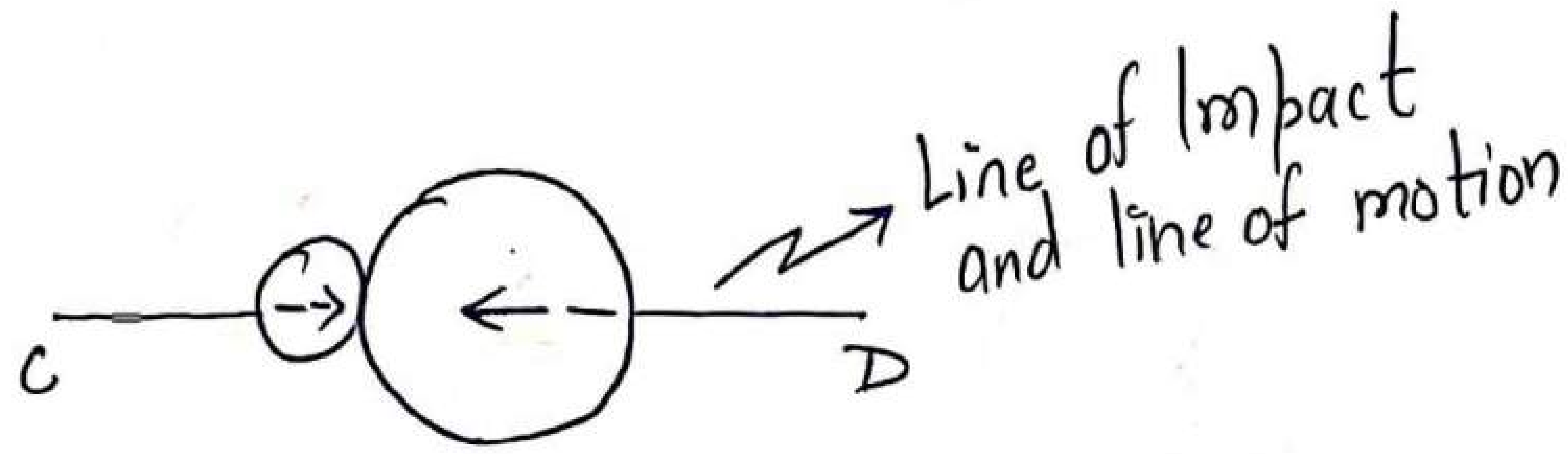
→ In collision particle may or may not come in physical contact

→ time of collision Δt is negligible.

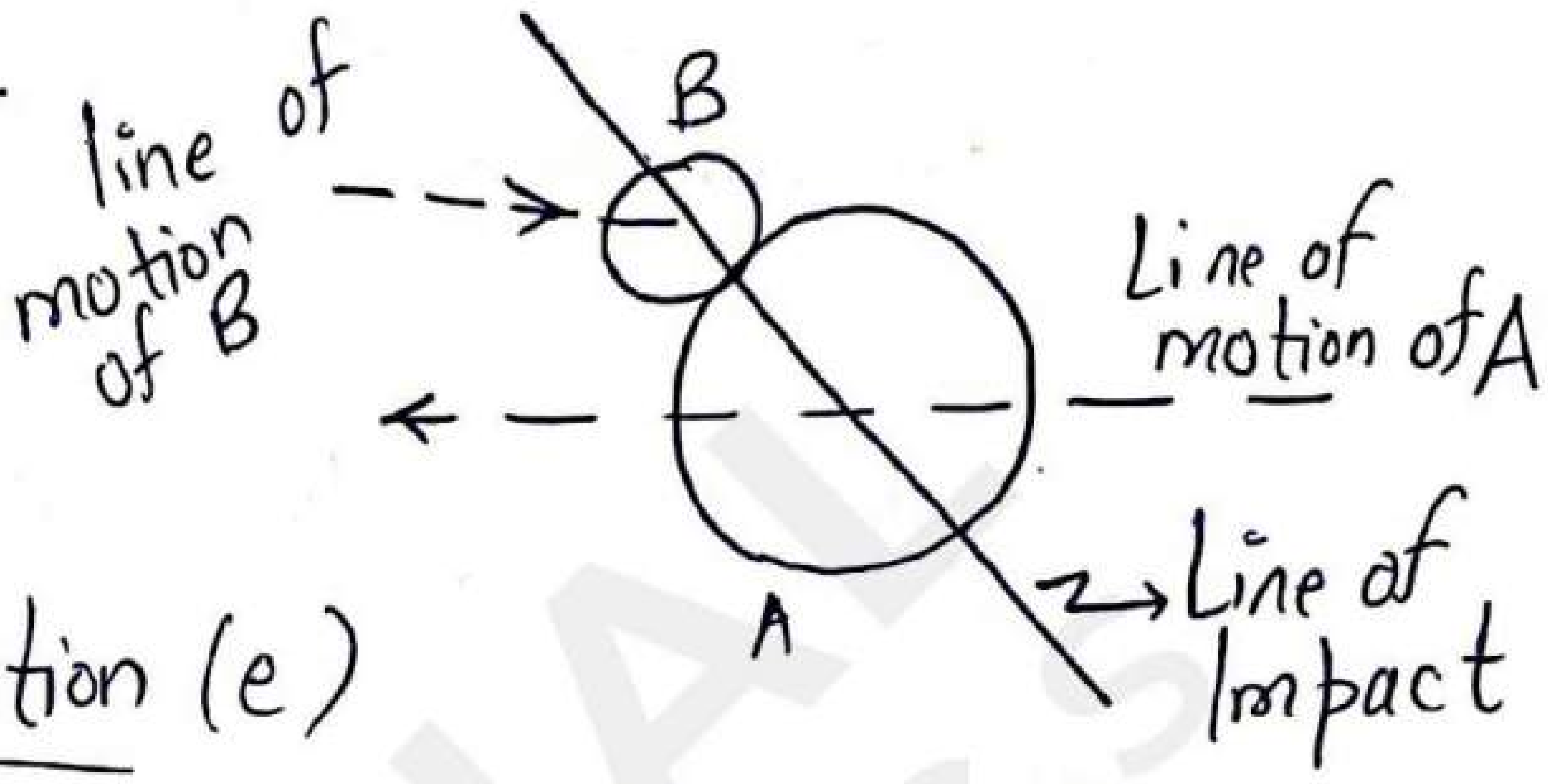
line of impact: the line passing through the common normal to the surfaces in contact during the impact is called line of impact.

Classification of Collisions

- on basic of line of Impact
  - Head-on Collision
  - oblique Collision
- on the basis of energy
  - elastic (no mechanical energy lost)
  - Inelastic (energy lost as heat)



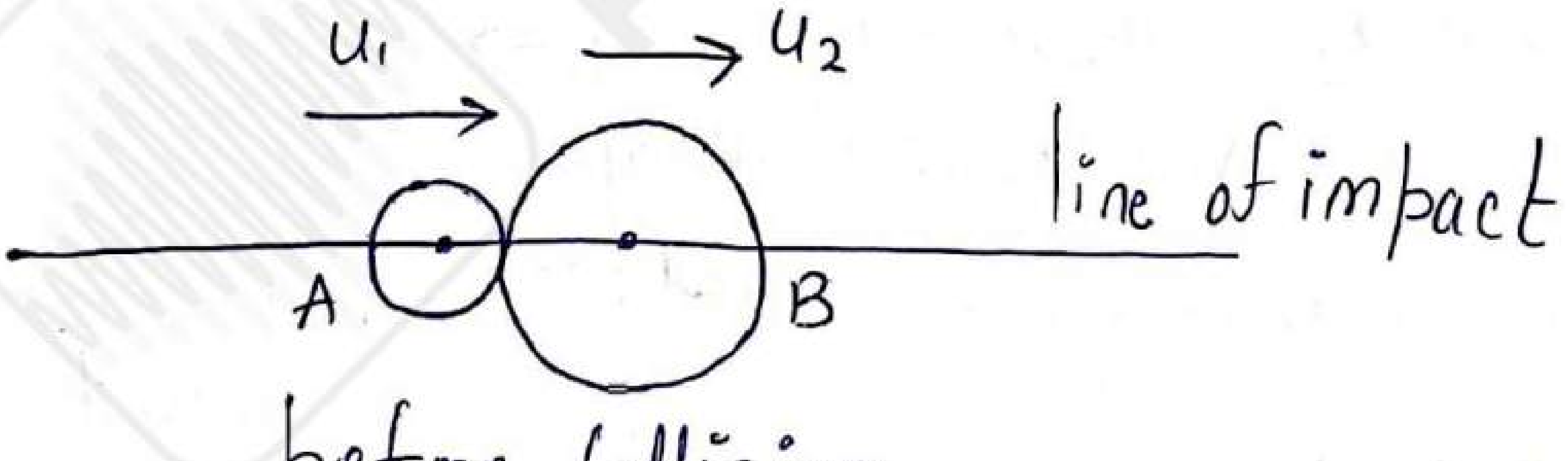
Head-on Collision



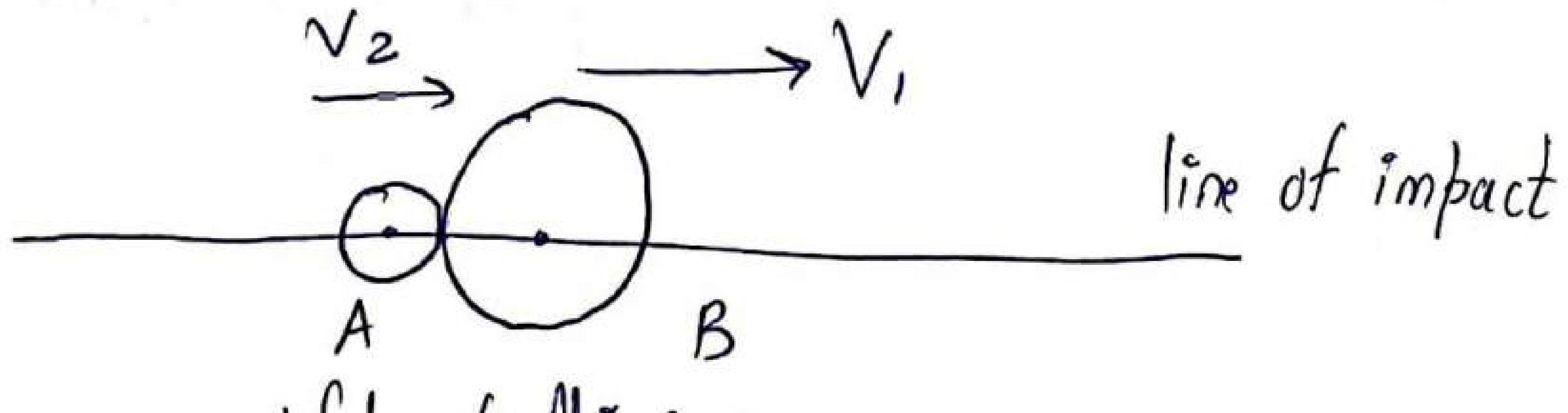
Coefficient of Restitution (e)

oblique collision

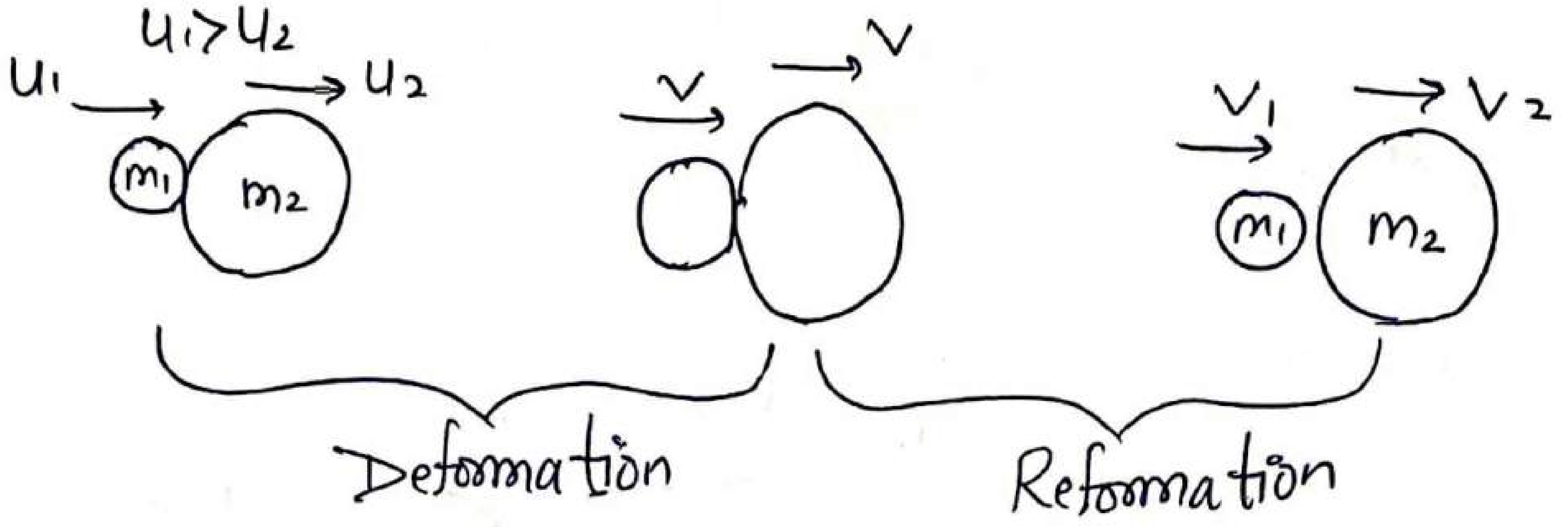
$$\boxed{e = \frac{\text{Velocity of Separation along line of Impact}}{\text{Velocity of apprach along line of Impact}}}$$



before Collision



After Collision



$\because F_{ext} = 0$, Hence net momentum is conserved

if $\underline{e = 1}$ → perfect elastic collision

$\underline{e = 0}$ → perfect Inelastic collision

perfect elastic collision	perfect Inelastic collision
→ Velocity of separation = Velocity of approach	→ Velocity of separation = 0
→ kinetic energy is conserved	→ kinetic energy is not conserved
→ $e = 1$	→ $e = 0$
→ Momentum conserved	→ momentum conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$e = \frac{\text{velocity of separation}}{\text{Velocity of approach}}$$

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{(1+e)m_2}{(m_1+m_2)}u_2$$

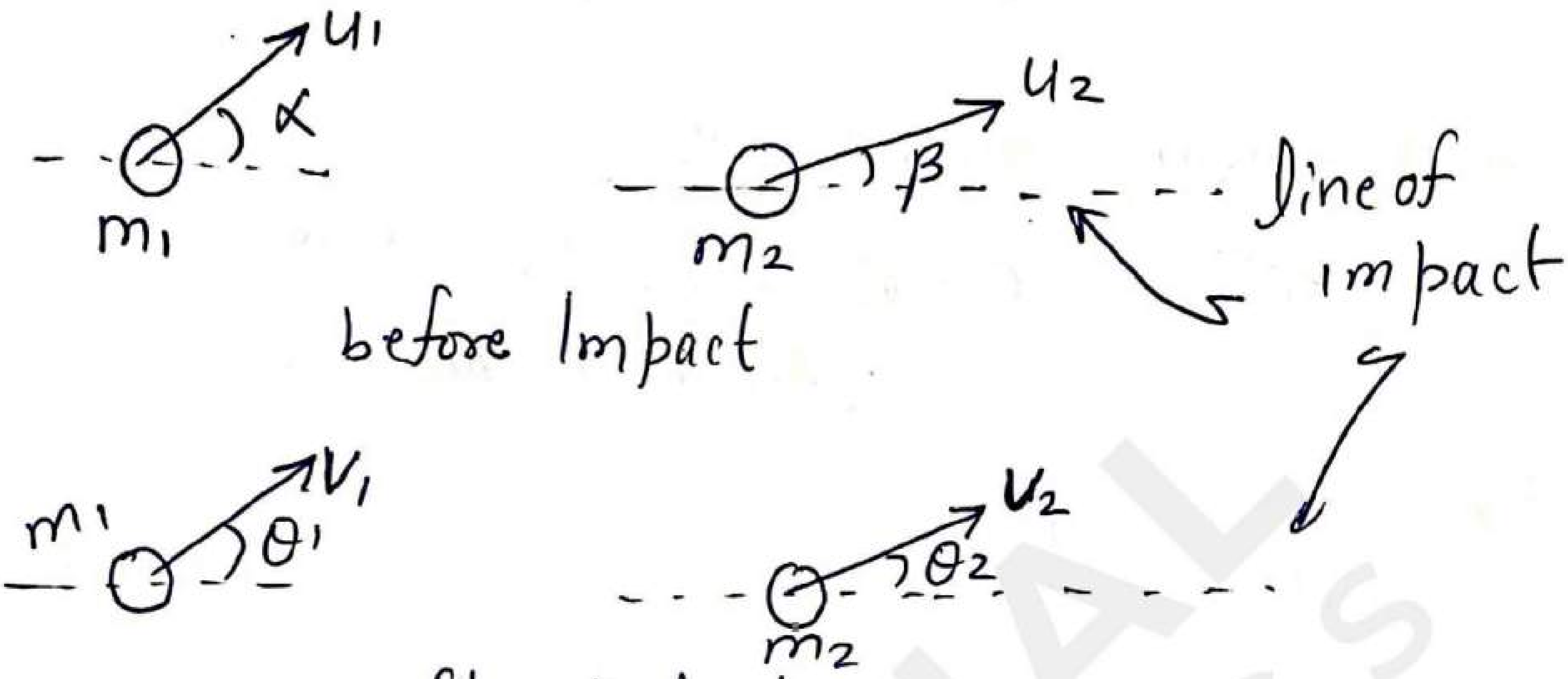
$$v_2 = \left(\frac{(1+e)m_1}{(m_1+m_2)}\right)u_1 + \frac{(m_2 - em_1)u_2}{(m_2+m_1}$$

For elastic collision, $e = 1$

$$\boxed{v_1 = \frac{(m_1 - m_2)u_1}{(m_1+m_2)} + \frac{2m_2u_2}{(m_1+m_2)}}$$

$$\boxed{v_2 = \frac{2m_1u_1}{(m_1+m_2)} + \frac{(m_2 - m_1)u_2}{(m_1+m_2)}}$$

Collision in Two-Dimensions (oblique)



after impact

Momentum Is Conserved:

so In x-direction:

$$m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

In y-direction:

$$m_1 u_1 \sin\alpha + m_2 u_2 \sin\beta = m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2$$

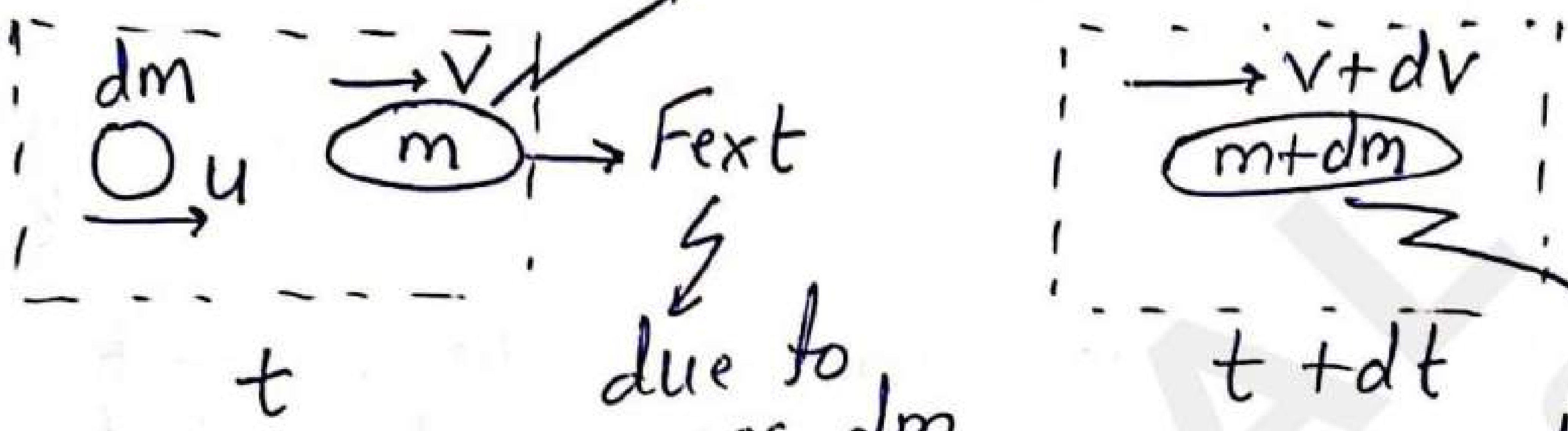
$$e = \frac{v_2 \cos\theta_2 - v_1 \cos\theta_1}{u_1 \cos\alpha - u_2 \cos\beta}$$

if $m_1 = m_2$

$\boxed{\theta_1 + \theta_2 = 90^\circ}$ → Angle between the objects after collision

Variable mass system :

system is only mass 'm'



due to mass dm

System mass increased or say changed

$$(F_{ext})dt = (m+dm)(v+dv) - mv - dm\,u$$

Impulse on system

$dm\,dv$ → negligible

$$\Rightarrow (F_{ext})dt = m\,dv - (u-v)\,dm$$

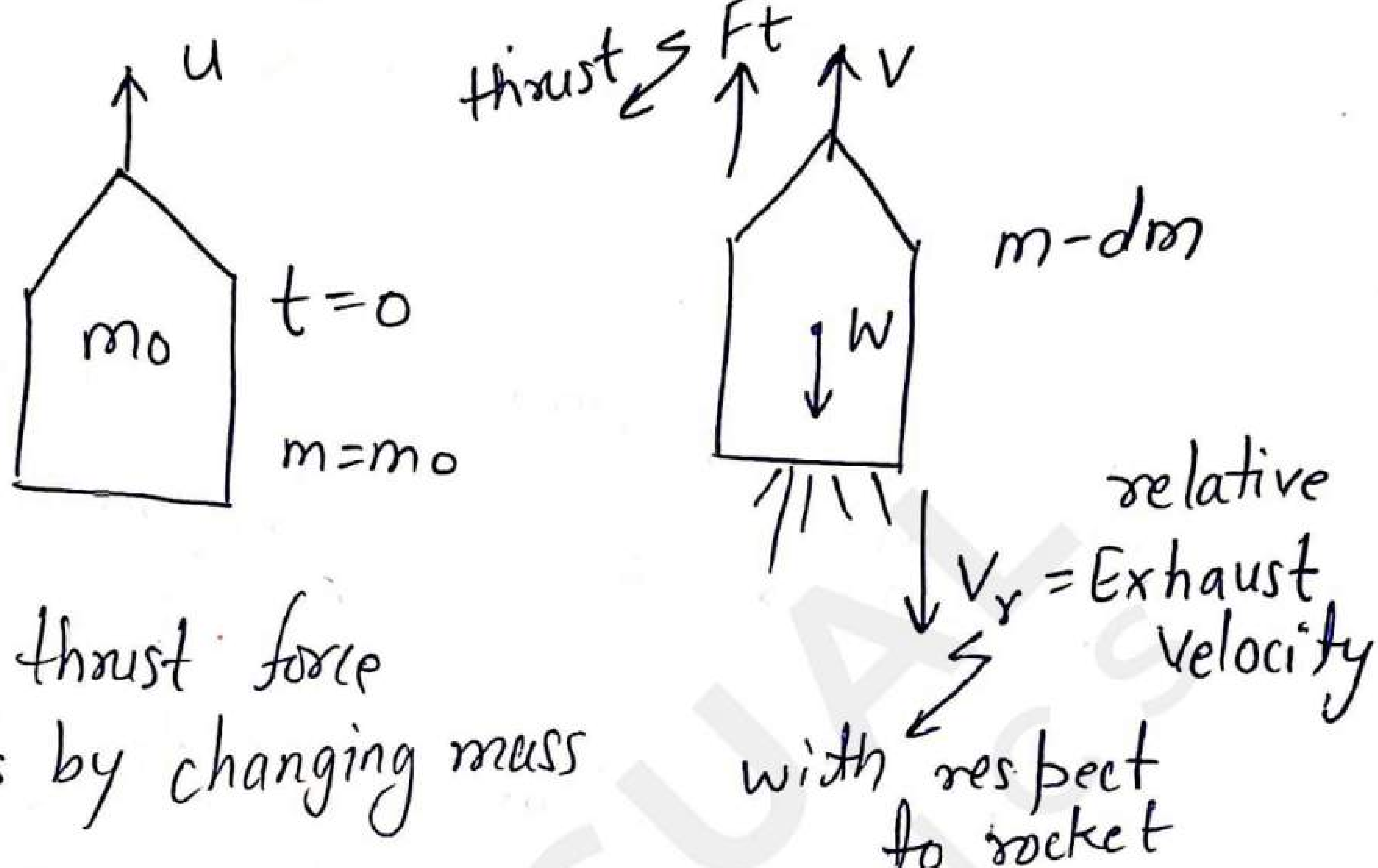
$V_{rel} = u - v$

relative velocity

thrust [force due to change of mass]

$$\boxed{F_{ext} + \left(V_{rel}\frac{dm}{dt}\right) = m\frac{dv}{dt}}$$

Rocket propulsion



now thrust force is by changing mass

$$F_t = v_r\left(\frac{dm}{dt}\right)$$

now net force $F_{net} = F_t - W$

$$\Rightarrow F_{net} = V_r\left(\frac{dm}{dt}\right) - mg$$

$$a = \frac{F_{net}}{m} = \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{V_r}{m}\left(\frac{dm}{dt}\right) - g$$

$$\int_u^v dv = V_r\int_{m_0}^{m}\frac{dm}{m} - g\int dt$$

$$v - u = -v_r \ln\left(\frac{m_0}{m}\right) - gt$$

$$\Rightarrow \boxed{v = u - gt - v_r \ln\left(\frac{m_0}{m}\right)}$$