VISUAL PHYSICS

SHORT NOTES

CHAPTER

Simple Harmonic Motion

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SIMPLE HARMONIC MOTION

Periodic motion: Motion of an object that regularly returns to given position after a fixed time interval.

Oscillations $\longrightarrow$ A special type of periodic motion in which particles moves to and fro about a fixed point.

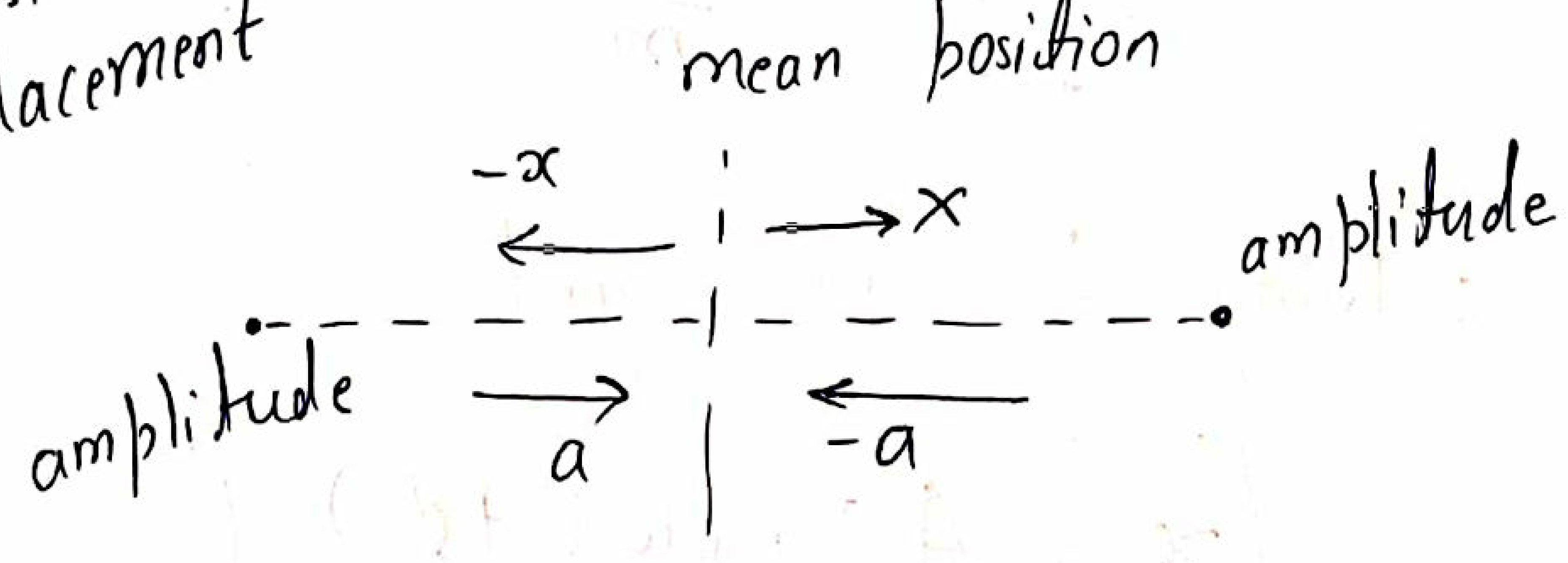
SHM (Simple Harmonic Motion): Special case of oscillation in which particle move in straight line and $\boxed{\text{acceleration} \propto \text{displacement}}$

$$\boxed{a = -\omega^2 x}$$

x → displacement

ω → angular frequency

$-$ → direction of 'a' is opposite to displacement direction



Amplitude → maximum displacement

→ Time period → Time to cover one cycle

→ frequency → $\frac{1}{T}$, ω = angular frequency = $\frac{2\pi}{T}$

EQUATION OF SHM :

$$a = -\omega^2 x$$

Necessary Condition for motion to be SHM

$$F \propto x$$

restoring force

force constant

$$F = -kx$$

→ restoring force direction is opposite to displacement

$$ma = -kx$$

$$a = \frac{-k}{m} x$$

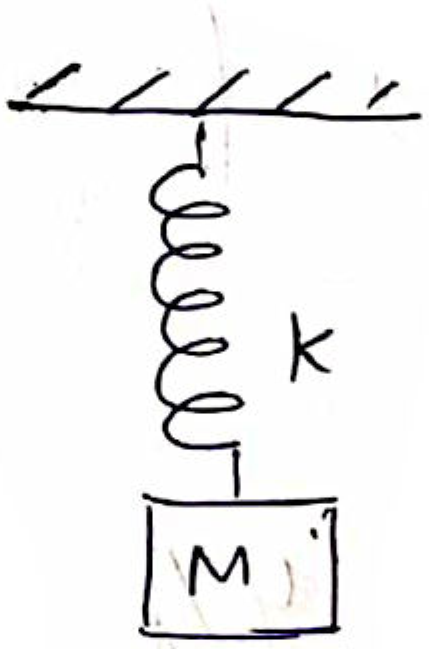
$$\frac{d^2x}{dt^2} = \frac{-k}{m} x = -\omega^2 x$$

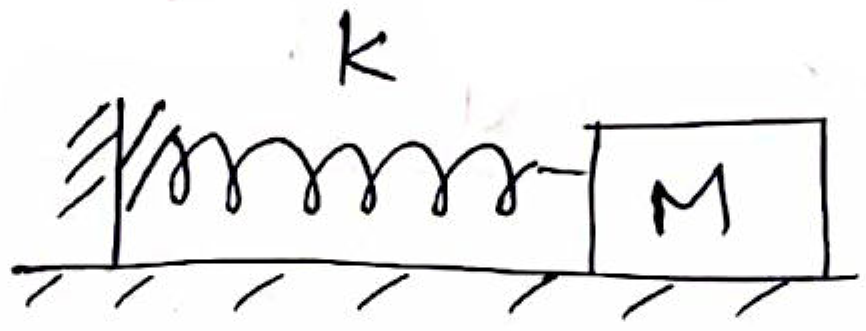
solving this we get

$$\boxed{x = A \sin(\omega t + \phi)}$$

$\phi \rightarrow$ Initial phase $\omega \rightarrow$ angular frequency

$A \rightarrow$ Amplitude

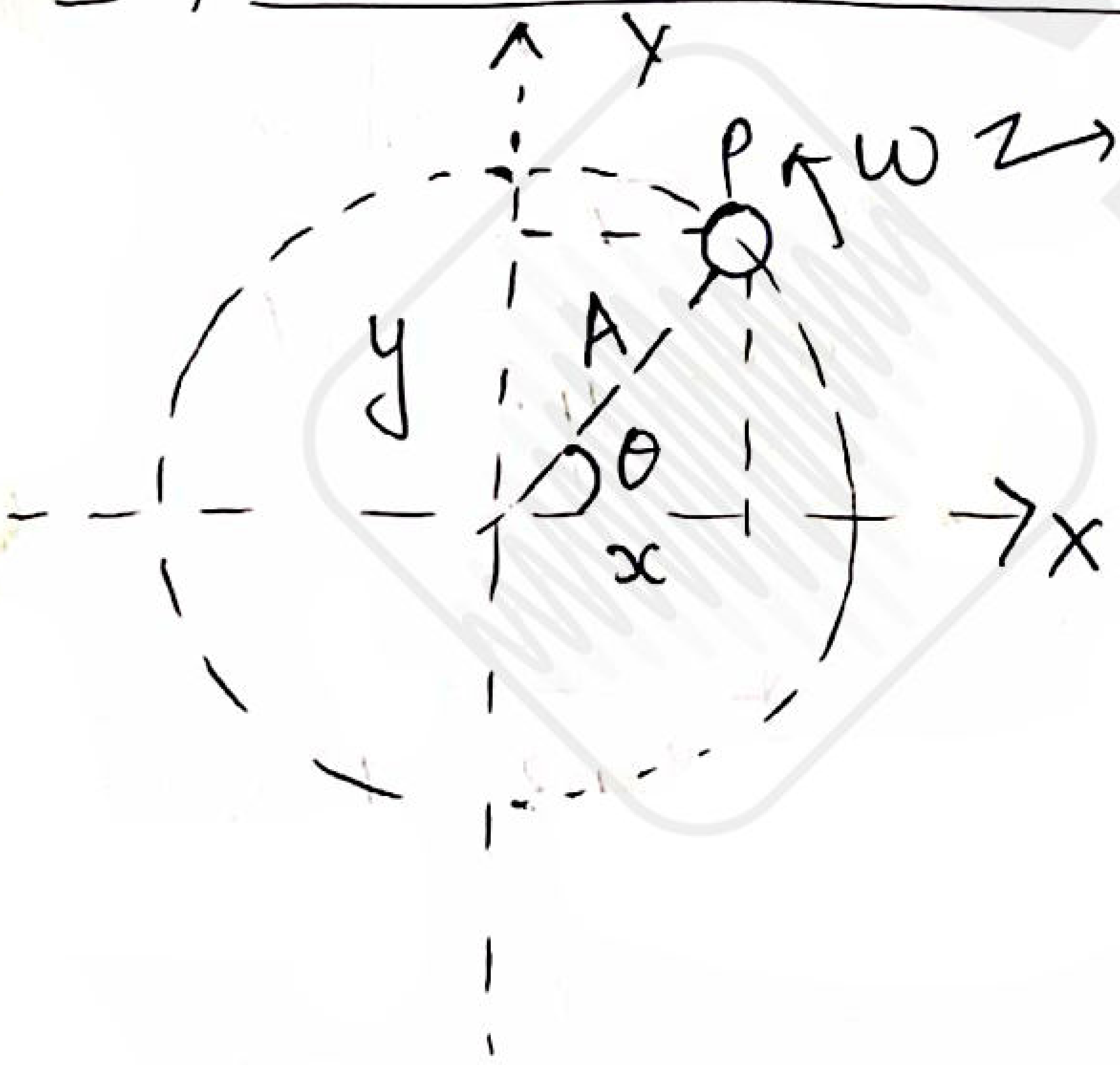




In both cases, $x = A\sin(\omega t + \phi)$

do not depend on orientation $\rightarrow$ $\boxed{\omega = \sqrt{\frac{k}{m}}}$

Simple Harmonic Motion with Uniform Circular Motion:



$\omega \rightarrow$ angular speed

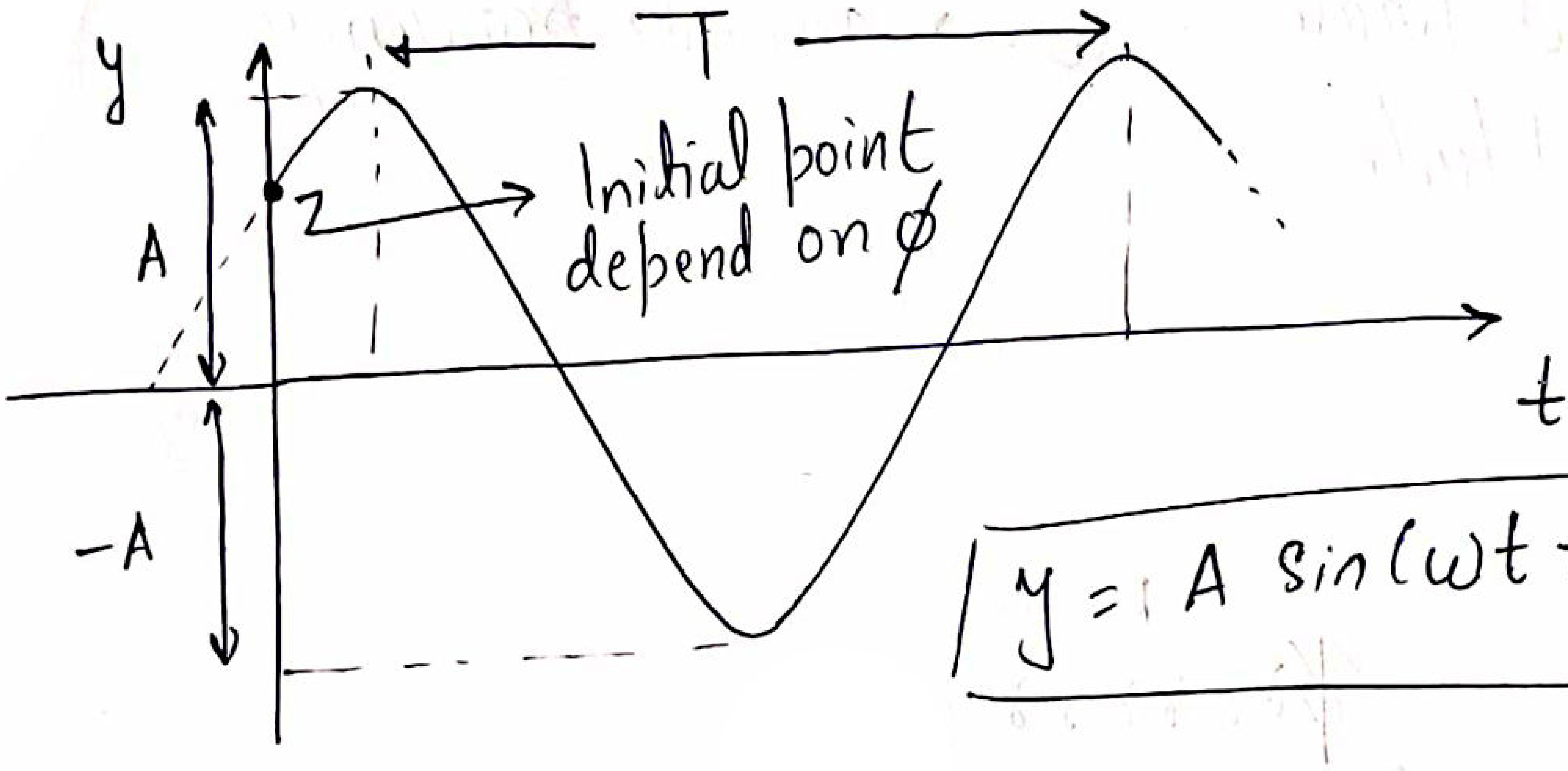
$x = A\cos\theta$ & $y = A\sin\theta$

we can say,

$$\boxed{\theta = \omega t + \underbrace{\phi}_{\text{Initial position}}}$$

$$x = A\cos(\omega t + \phi) \quad , \quad y = A\sin(\omega t + \phi)$$

Hence projection of uniform circular motion give or behave as SHM.



$$\boxed{y = A\sin(\omega t + \phi)}$$

so, $y = A\sin(\omega t + \phi)$, $\boxed{\sin(\omega t + \phi) = y/A}$

so, $v = \dfrac{dy}{dt} = \dfrac{d}{dt}\left[A\sin(\omega t + \phi)\right]$

$$v = \omega A\cos(\omega t + \phi)$$

$$\cos(\omega t + \phi) = \sqrt{1 - \sin^2(\omega t + \phi)} = \sqrt{1 - \frac{y^2}{A^2}}$$

$$\cos(\omega t + \phi) = \frac{1}{A}\sqrt{A^2 - y^2}$$

$$\boxed{v = \omega A\left[\frac{1}{A}\sqrt{A^2 - y^2}\right] = \omega\sqrt{A^2 - y^2}}$$

so, $\boxed{a = \dfrac{dv}{dt} = -A\omega^2\sin\omega t = -A\omega^2 y}$

Now, $y = A\sin(\underbrace{\omega t + \phi}_{\text{phase}})$

phase constant → when $t = 0$, phase = phase constant

$\Rightarrow t = 0$

$\Rightarrow$ phase constant $= \omega(0) + \phi$

$\Rightarrow \boxed{\text{phase constant} = \phi}$

For a given oscillation

phase, $\phi_1 = \omega t_1 + \phi$

& $\phi_2 = \omega t_2 + \phi$

$\Delta\phi = \phi_2 - \phi_1 = \omega(t_2 - t_1)$

$\omega = \frac{2\pi}{T}$ → Time Constant (Time period)

$\Delta\phi \rightarrow 2\pi$ (for e.g.)

$\Rightarrow \quad \omega \Delta t = 2\pi$

$$\frac{2\pi}{T}\Delta t = 2\pi$$

or $\boxed{\Delta t = T}$

$\Rightarrow$ for 2π phase difference time difference is 'T'

ENERGY OF A BODY IN SIMPLE HARMONIC MOTION :

for SHM

$$F = -kx$$

$$dw = F dx = -kx\,dx$$

$$W = \int_0^x (-kx)dx = -\frac{1}{2}kx^2$$

$U(x) \rightarrow$ potential energy when displacement 'x'.

NOW $\quad \Delta U = U(x) - U(0) = -W$

$$\boxed{U(x) = \frac{1}{2}kx^2}$$

→ When at mean position, $x=0$, $U(x)=0$

$$U(x) = \frac{1}{2}kx^2$$

$$x = A\sin(\omega t), \quad v = \frac{dx}{dt} = A\omega\cos(\omega t)$$

$$U = \frac{1}{2}kA^2\sin^2(\omega t)$$

Now, $k = m\omega^2$, $\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow \quad U = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t)$$

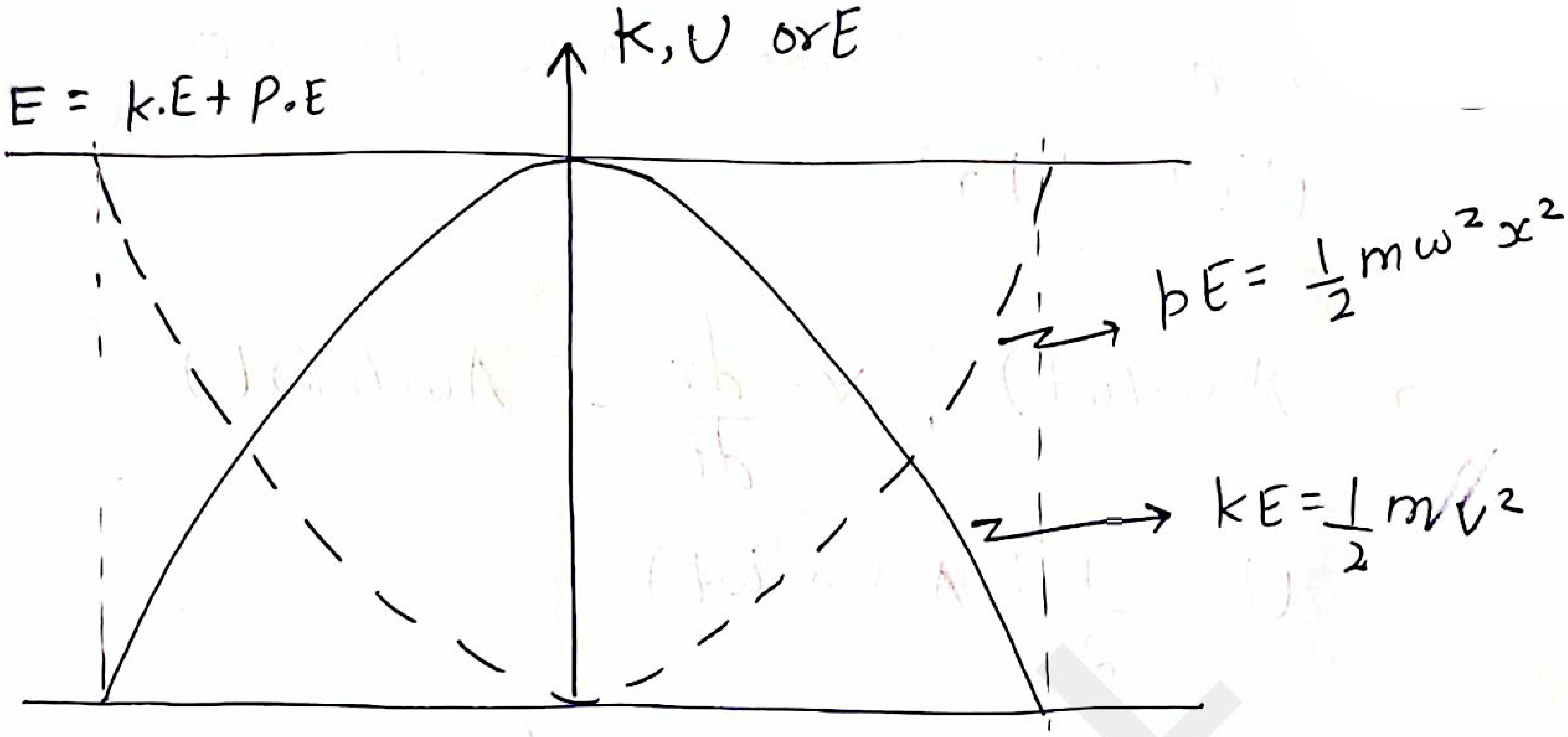
$$U = \frac{m\omega^2A^2}{4}\left[1-\cos 2\omega t\right]$$

& $$k = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t)$$

$$k = \frac{m\omega^2A^2}{4}\left(1+\sin 2\omega t\right]$$

$$E = U+k = \frac{1}{2}m\omega^2A^2\underbrace{\left[\sin^2(\omega t)+\cos^2(\omega t)\right]}_{1}$$

$$\boxed{E = \frac{1}{2}m\omega^2A^2}$$



Hence energy at time 't' is independent of 't'.
Thus the mechanical energy remains constant as Expected

Average PE & KE

$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

$$U_{average} = \frac{1}{T}\int_0^T U\,dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t+\phi)$$

$$\boxed{U_{average} = \frac{1}{4}m\omega^2 A^2}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$$KE = \frac{1}{2} m\left[\frac{d}{dt}\left[A \sin(\omega t+\phi)\right]\right]^2$$

$$= \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t+\phi)$$

$$KE_{average} = \frac{1}{T}\int_0^T \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t+\phi)\, dt$$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T \{1 + \cos 2(\omega t+\phi)\}\, dt$$

$$\boxed{K.E_{average} = \frac{1}{4} m \omega^2 A^2}$$

SUPERPOSITION OF TWO SIMPLE HARMONIC MOTION

In Same direction and of same Frequency:

$$x_1 = A_1 \sin\omega t$$
$$x_2 = A_2 \sin(\omega t+\phi)$$

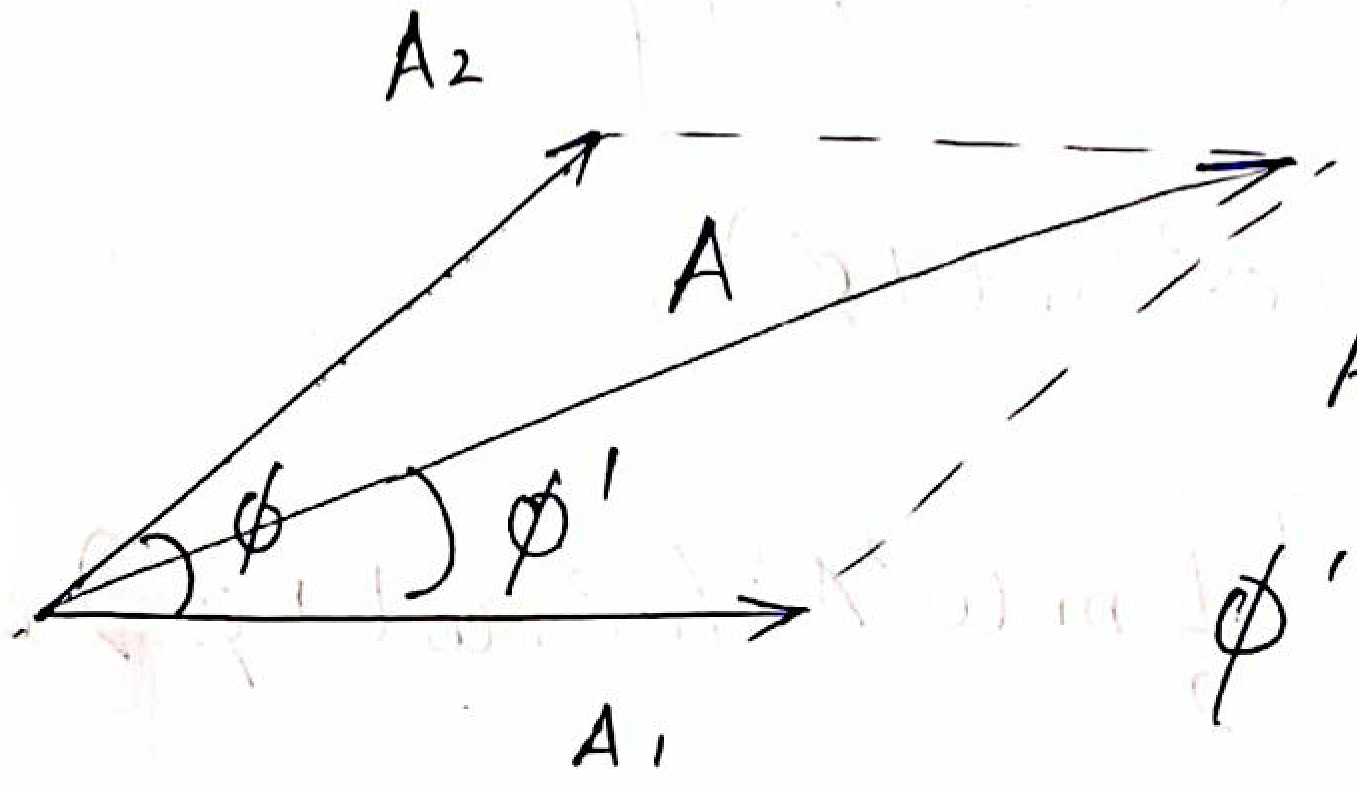
$$x = x_1 + x_2 = A_1 \sin\omega t + A_2 \sin(\omega t+\phi)$$

Resultant

$$x = A \sin(\omega t + \phi')$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\phi' = \tan^{-1}\left[\frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}\right]$$



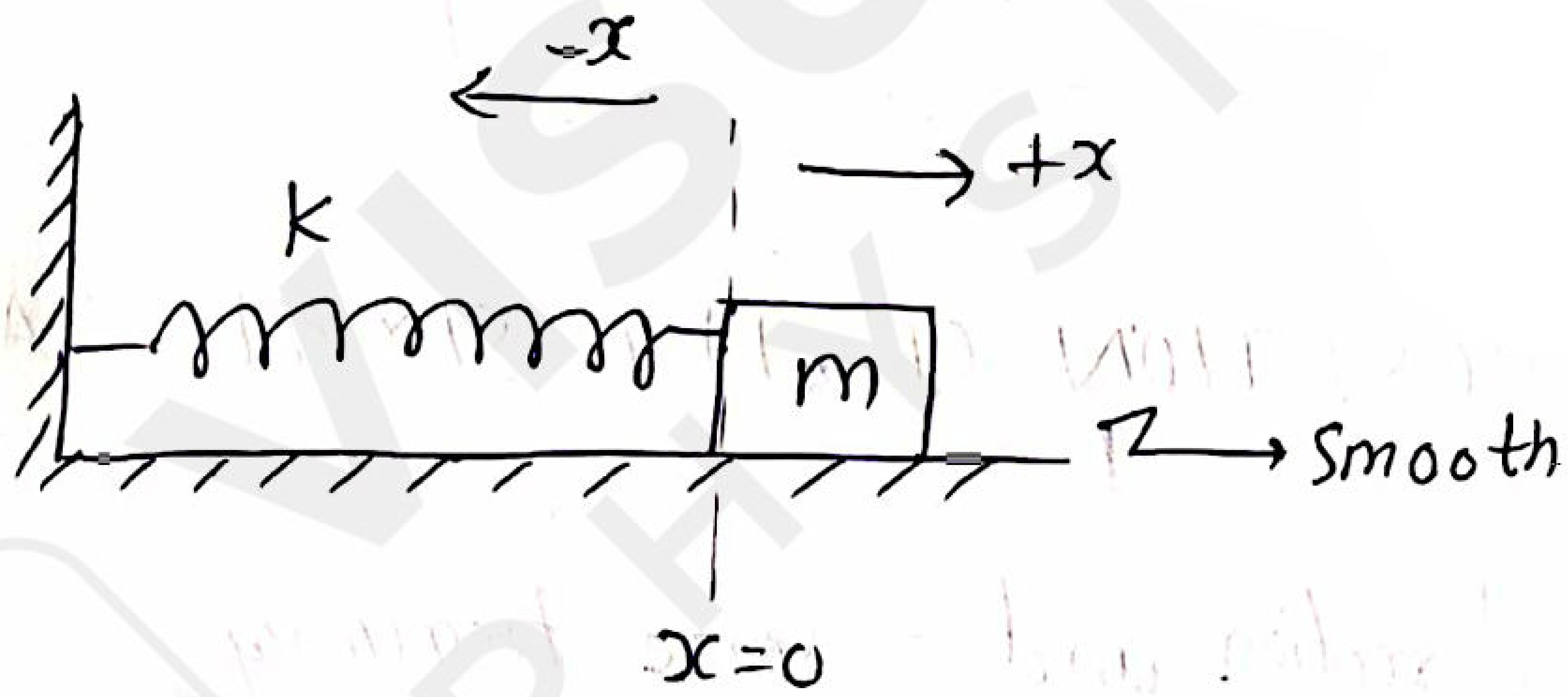
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\phi' = \tan^{-1}\left[\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right]$$

THE SPRING-MASS SYSTEM

Case 1: In Horizontal motion

Method 1:



$$F = m\frac{d^2x}{dt^2} = -kx \quad , \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\Rightarrow \quad \omega^2 = \frac{k}{m} \quad , \quad \boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\boxed{x = A\sin(\omega t + \phi)}$$

Method 2: Energy method:

Total mechanical Energy is Constant [friction = 0]

$$\Rightarrow \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \text{Constant}$$

differentiating w.r.t time

$$\frac{1}{2} m\left(2v \frac{dv}{dt}\right) + \frac{1}{2} k\left(2x \frac{dx}{dt}\right) = 0$$

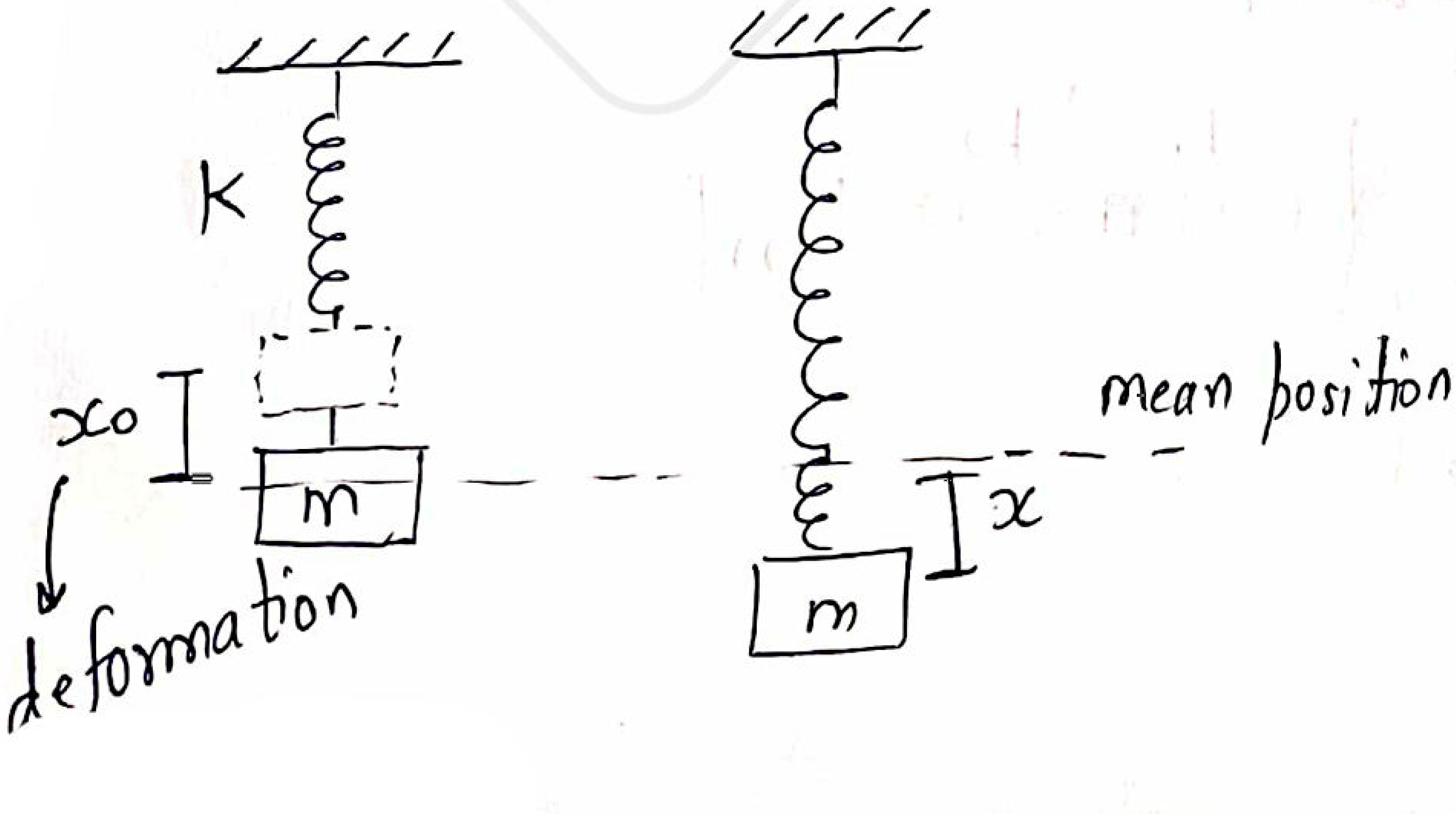
$$\frac{1}{2} m(2va) + \frac{1}{2} k(2xv) = 0$$

$$a = -\left(\frac{k}{m}\right)x \;, \Rightarrow \omega^2 = \frac{k}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \quad \Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Case II: Vertical motion

Method 1: Force



$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\omega^2 = \frac{k}{m}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Method 2: Energy Method

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x_0+x)^2 - \underbrace{mgx}_{\text{Gravitation P.E}} = \text{Constant}$$

differentiating w.r.t 't'

$$\frac{1}{2}m(2va) + \frac{1}{2}k[2(x_0+x)v] - mgv = 0$$

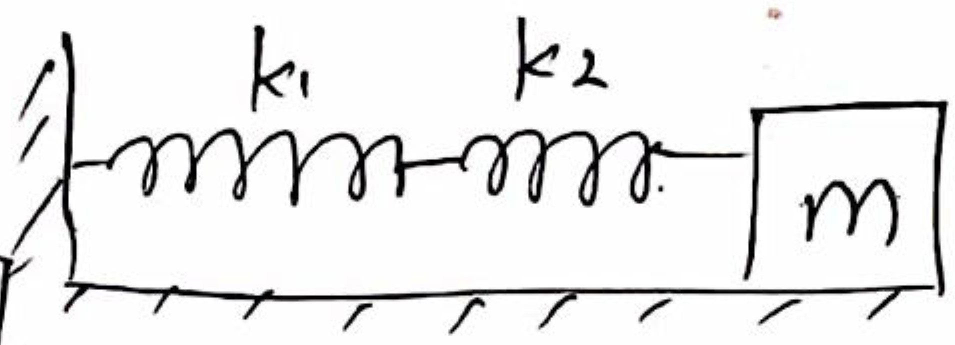
$$a = -\left(\frac{k}{m}\right)x \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

★ Time period of block - spring system depends only on mass of block & force constant of spring. (not positioning).

so, $\omega = \sqrt{\frac{k}{m}}$

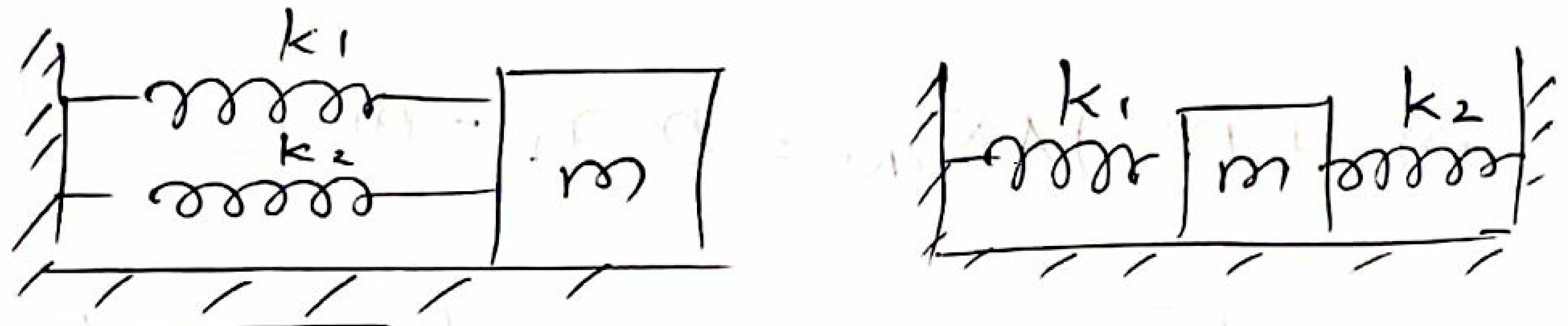
$k \rightarrow$ equivalent

so, when series



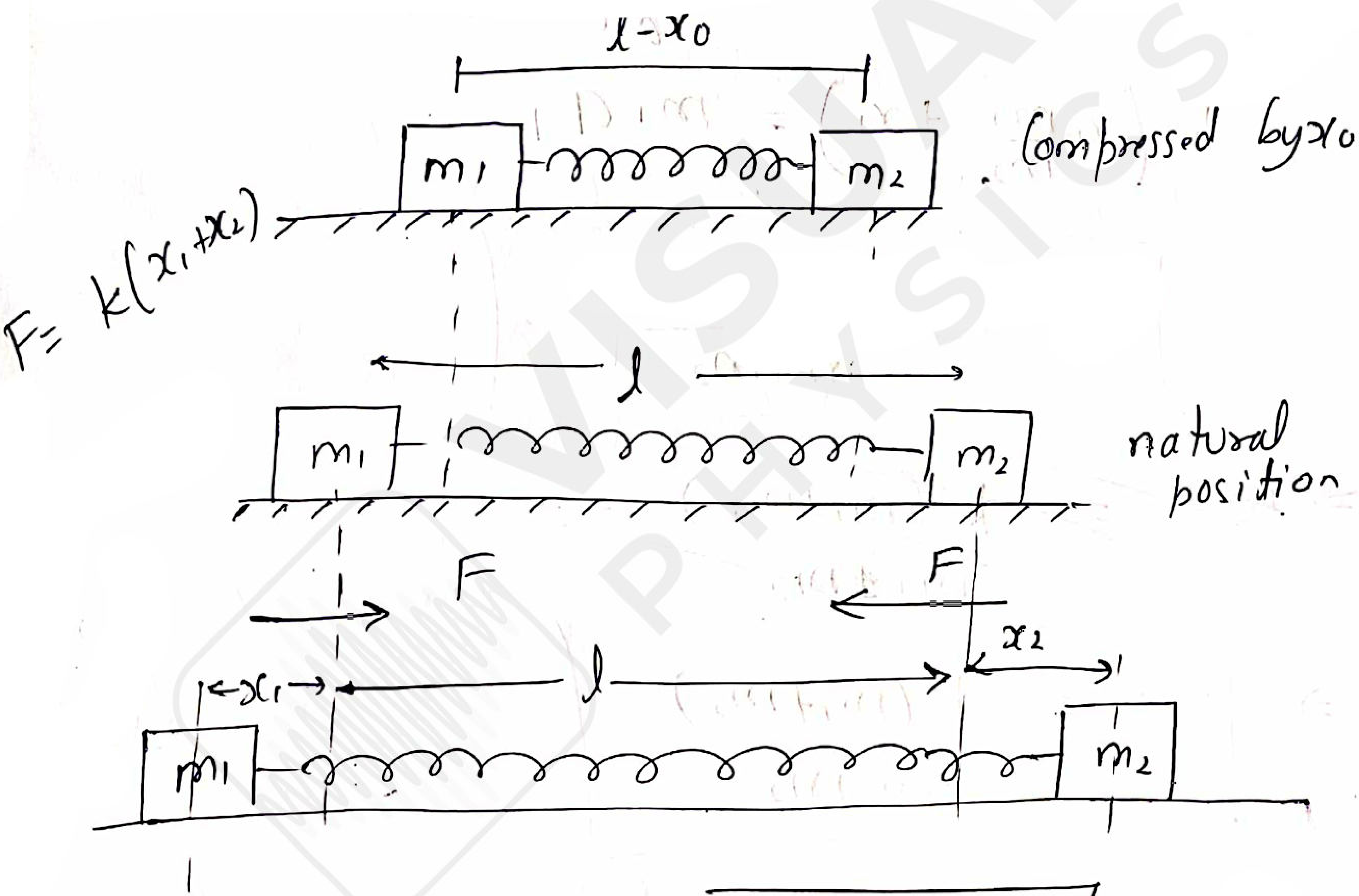
$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

parallel :



$$\boxed{k = k_1 + k_2}$$

<u>Oscillation of a two-particle system</u> <u>two-body problem</u>:



No external force, so, $\boxed{\Delta X_{cm} = 0}$

change in Centre of mass position

$\Rightarrow (m_1 + m_2)\Delta X'_{CM} = m_1 x_1 - m_2 x_2$

$\Rightarrow \boxed{m_1 x_1 = m_2 x_2}$ as $\boxed{\Delta X_{CM} = 0}$

for on first particle $k(x_1+x_2)$

$$k(x_1+x_2) = m_1 \frac{d^2x}{dt^2}$$

$$K\left(x_1 \frac{m_1}{m_2} + x_1\right) = m_1 a_1$$

$$\boxed{a_1 = \frac{k(m_1+m_2)}{m_1 m_2} x_1}$$

$$\Rightarrow \boxed{\omega^2 = \frac{k(m_1+m_2)}{m_1 m_2}}$$

$$T = 2\pi \sqrt{\frac{m_1 m_2}{k(m_1+m_2)}}$$

or for such system μ = reduce mass

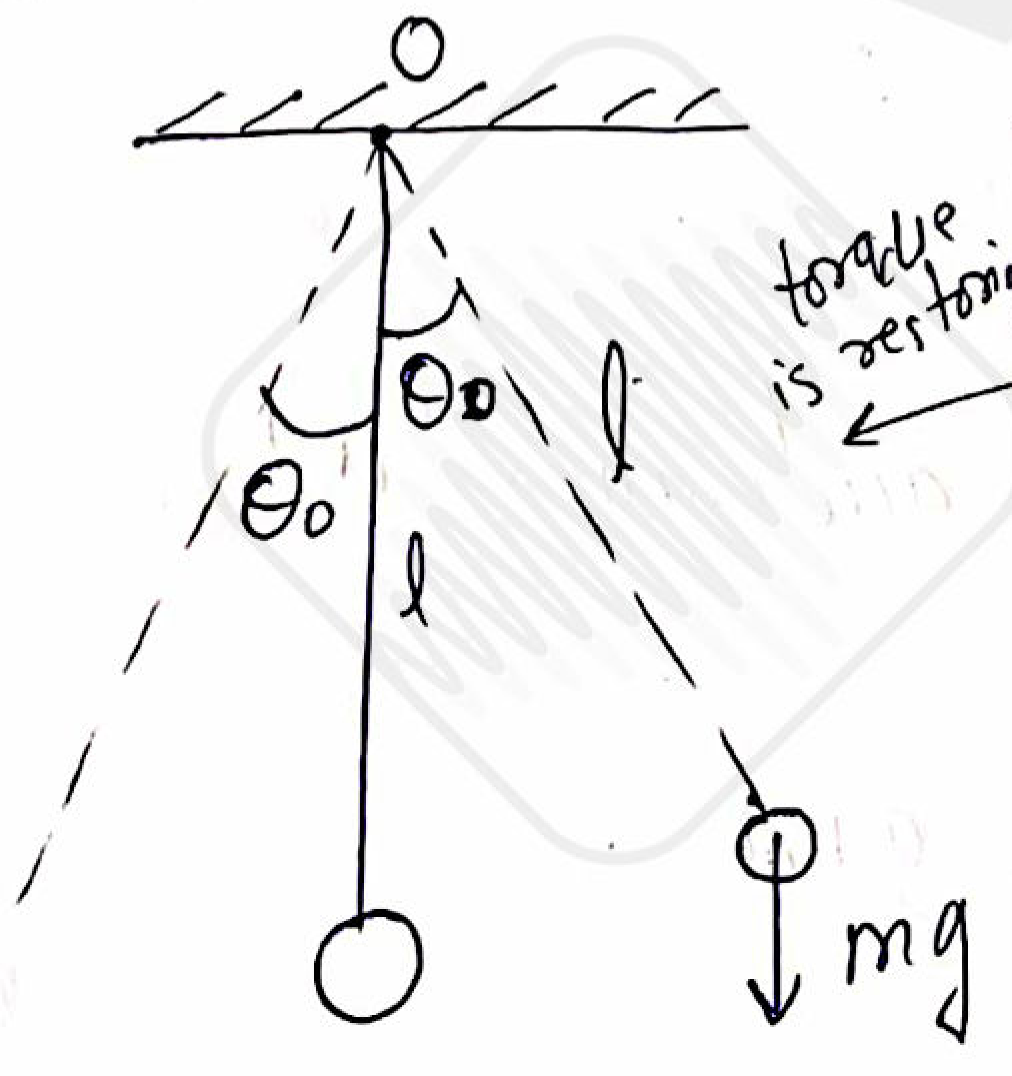
$$\boxed{\mu = \frac{m_1 m_2}{(m_1+m_2)}}$$

Hence for Connected system,

$$\boxed{T = 2\pi\sqrt{\frac{\mu}{k}} \quad , \omega^2 = \frac{k}{\mu}}$$

We use reduce mass system.

SIMPLE PENDULUM:



torque about 'O':

torque is restoring $\boxed{\tau = -mgl\sin\theta}$ (at any position θ)

for small angle 'θ'

$\sin\theta \sim \theta$

$\Rightarrow \tau = -mgl\theta$

$$\tau = I\alpha = ml^2\left(\frac{d^2\theta}{dt^2}\right)$$

$$\Rightarrow ml^2\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta}$$

similar to $\frac{d^2x}{dt^2} = -\frac{k}{m}x$

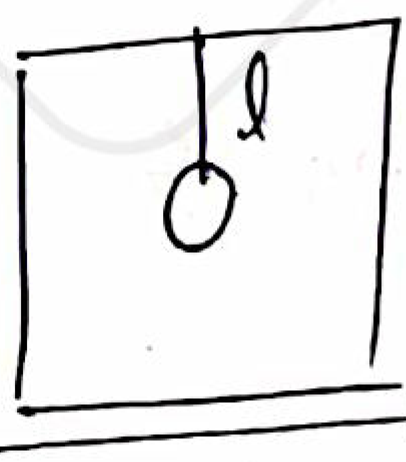
$$\rightarrow \boxed{\omega^2 = \frac{g}{l}} \quad \text{so, } \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Independent on mass of the bob.

$$\boxed{T = 2\pi\sqrt{\frac{l}{g_{eq}}}}$$

equivalent acceleration

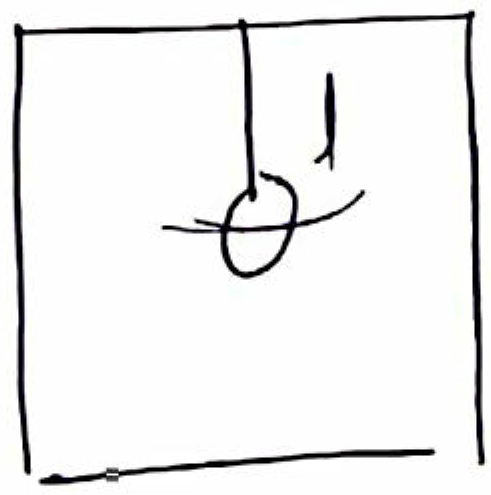
1. When pendulum going up by acceleration 'a'



$\uparrow a \quad g_{eq} = g + a$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g+a}}}$$

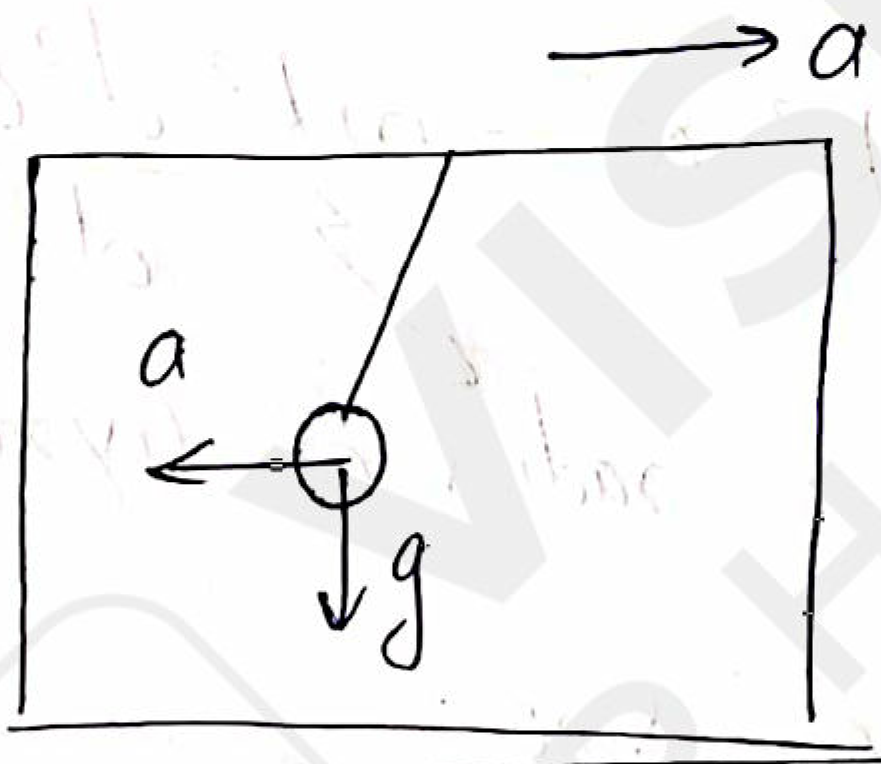
2. When moving down with acceleration 'a'



$\downarrow a \quad g_{eq} = g - a$

So, $T = 2\pi\sqrt{\dfrac{l}{g-a}}$

3. for horizontal acceleration :



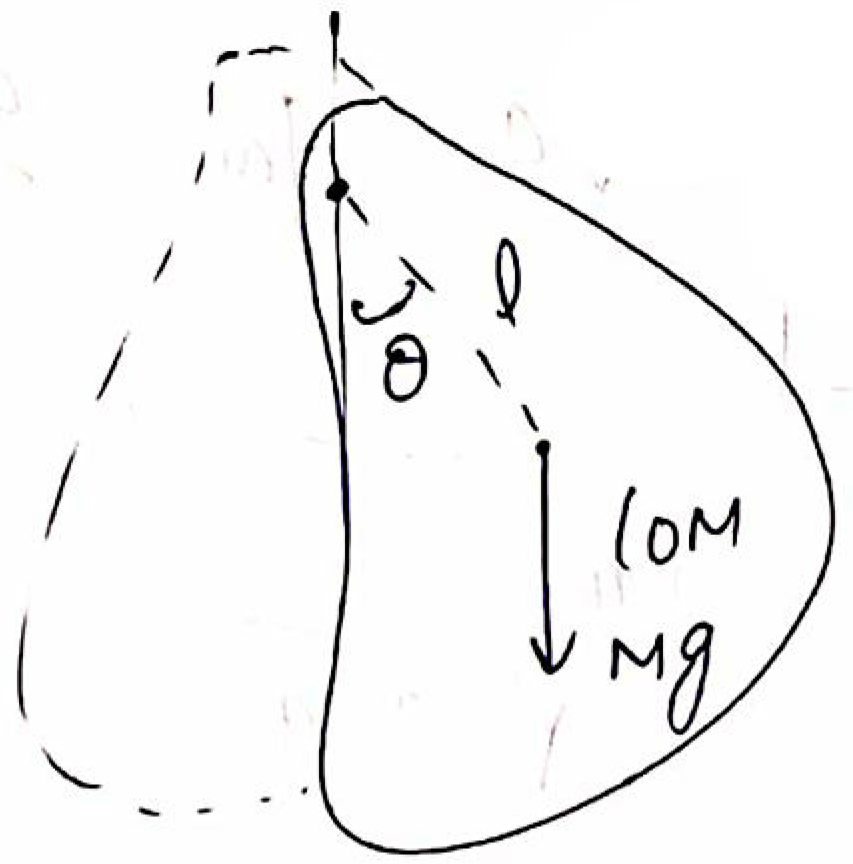
$g_{eq} = \sqrt{a^2+g^2}$

$$\boxed{T = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2+a^2}}}}$$

PHYSICAL PENDULUM (COMPOUND PENDULUM)

Any rigid body suspended from a fixed support constitutes a physical pendulum.

$l \rightarrow$ distance b/w point of suspension & center of mass



$$\tau = -mgl\sin\theta$$

$$\tau = I\alpha = mk^2\alpha = mk^2\frac{d^2\theta}{dt}$$

radius of Gyration

for small angle, $\theta \sim \sin\theta$

$$\Rightarrow \quad mk^2\frac{d^2\theta}{dt^2} = -mgl\theta$$

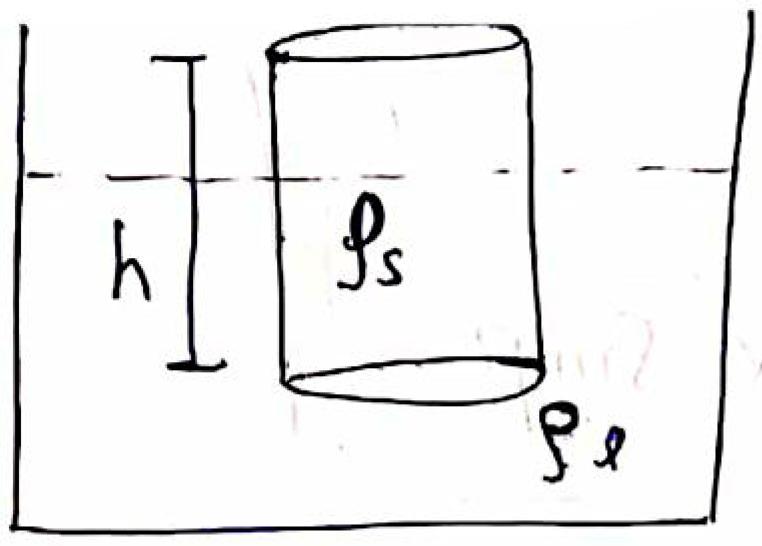
$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{gl}{k^2}\theta$$

$I = mk^2$

$$\boxed{\omega^2 = \frac{gl}{k^2}} \qquad T = 2\pi\sqrt{\frac{k^2}{gl}}$$

or $\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$

OSCILLATION OF FLOATING BODY:



$\rho_l > \rho_s$

now at equillibrium, $W = B$

Weight   Buoyancy.

now when displaced by x from mean position net Buoyant force act upwards.

Net force $= -\rho_l A x g$

displacement from mean position

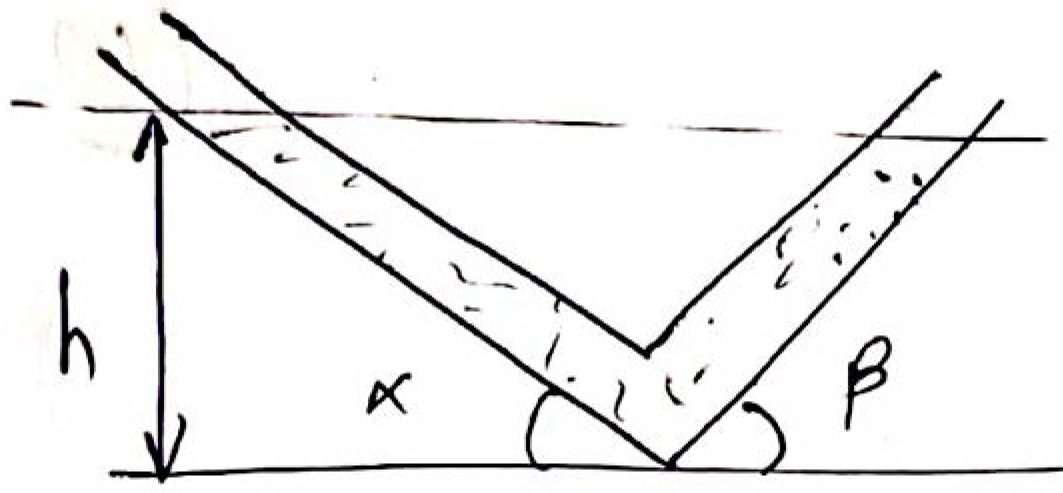
$\Rightarrow \quad ma = -\rho_l A x g$

$$\rho_s A h \frac{d^2x}{dt^2} = -\rho_l A g \cdot x$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{\rho_l g}{\rho_s h} x$$

$$\boxed{\omega^2 = \frac{\rho_l g}{\rho_s h}}$$

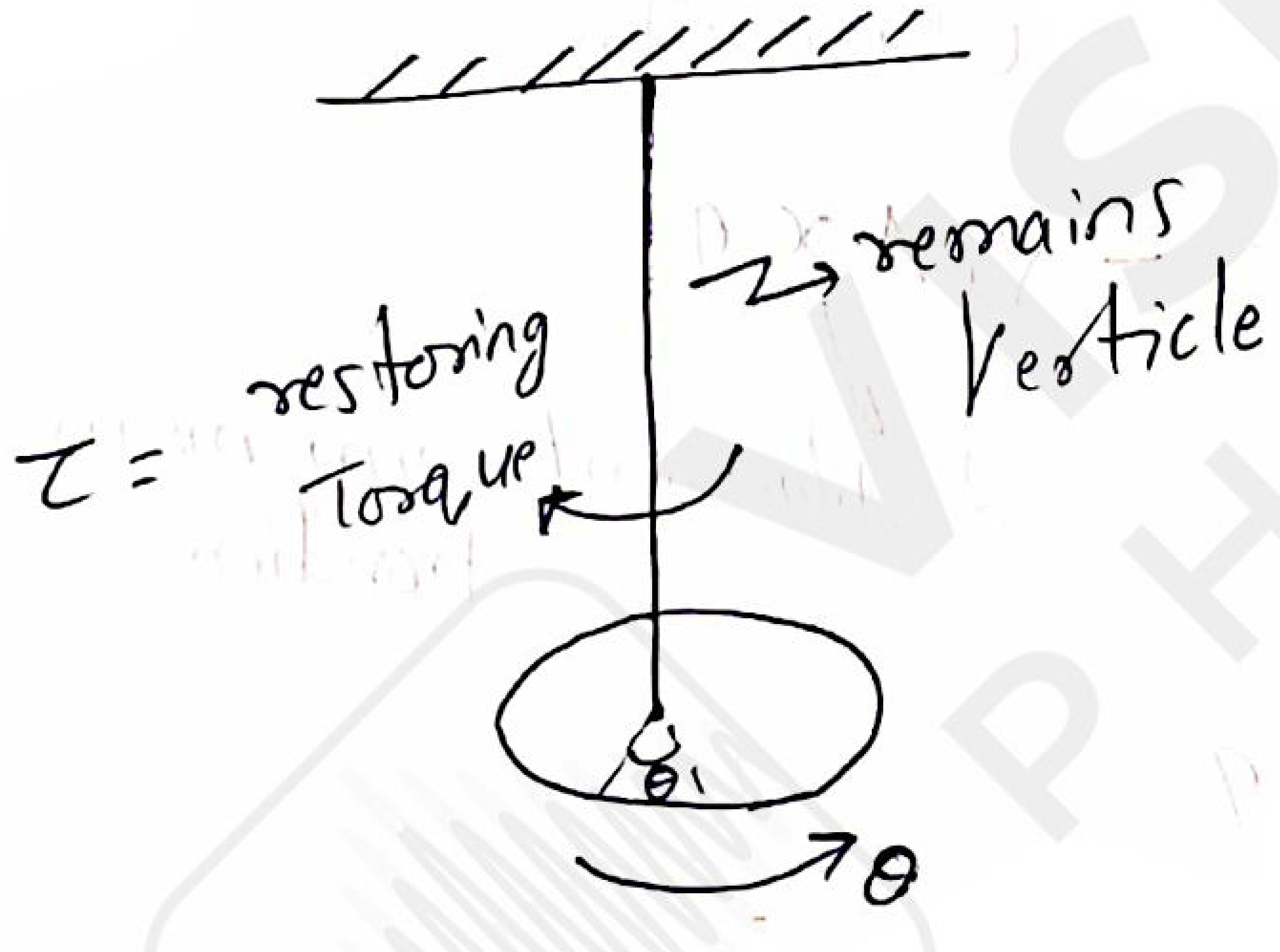
$$T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{\rho_s h}{\rho_l g}}}$$



$$\boxed{\omega = \sqrt{\frac{g \sin\alpha \sin\beta}{h}}}$$

$$\boxed{T = 2\pi \sqrt{\frac{h}{g \sin\alpha \sin\beta}}}$$

Torsional pendulum :



$$\boxed{\tau = -k\theta}$$

k → Torsional Constant of thread or wire.

$$\Rightarrow I\alpha = -k\theta$$

$$\Rightarrow \boxed{\alpha = -\frac{k}{I}\theta}$$

$$\omega = \sqrt{\frac{k}{I}}, \quad T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{k}}}$$