



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Rotational Dynamics



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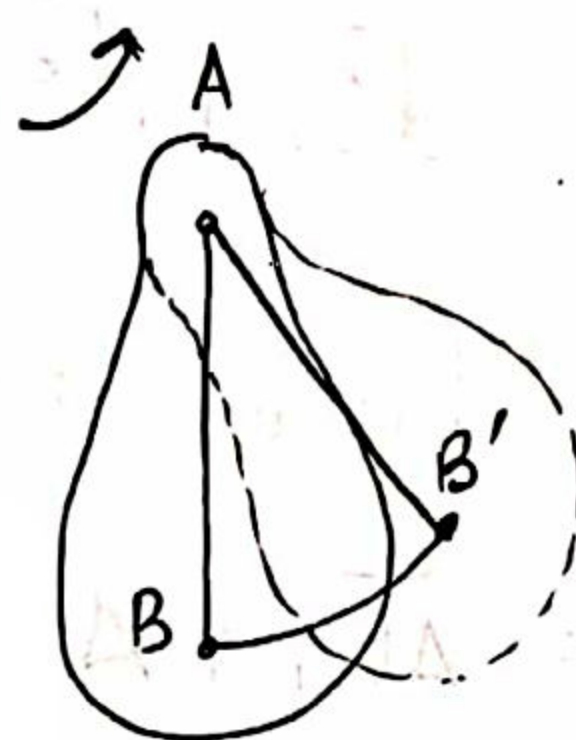
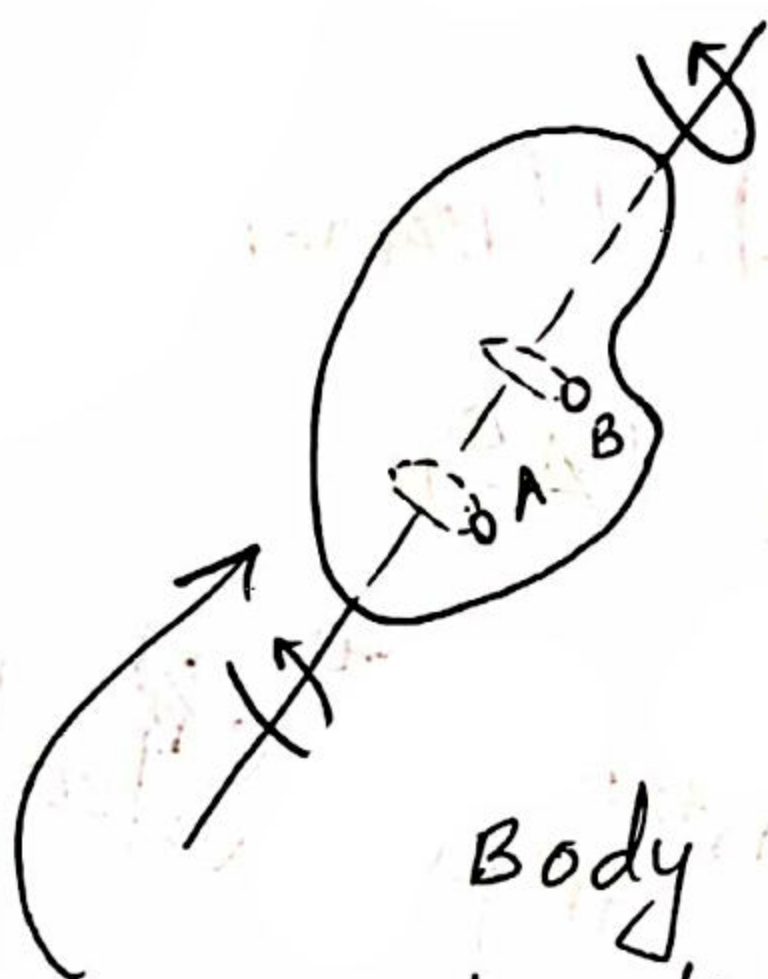
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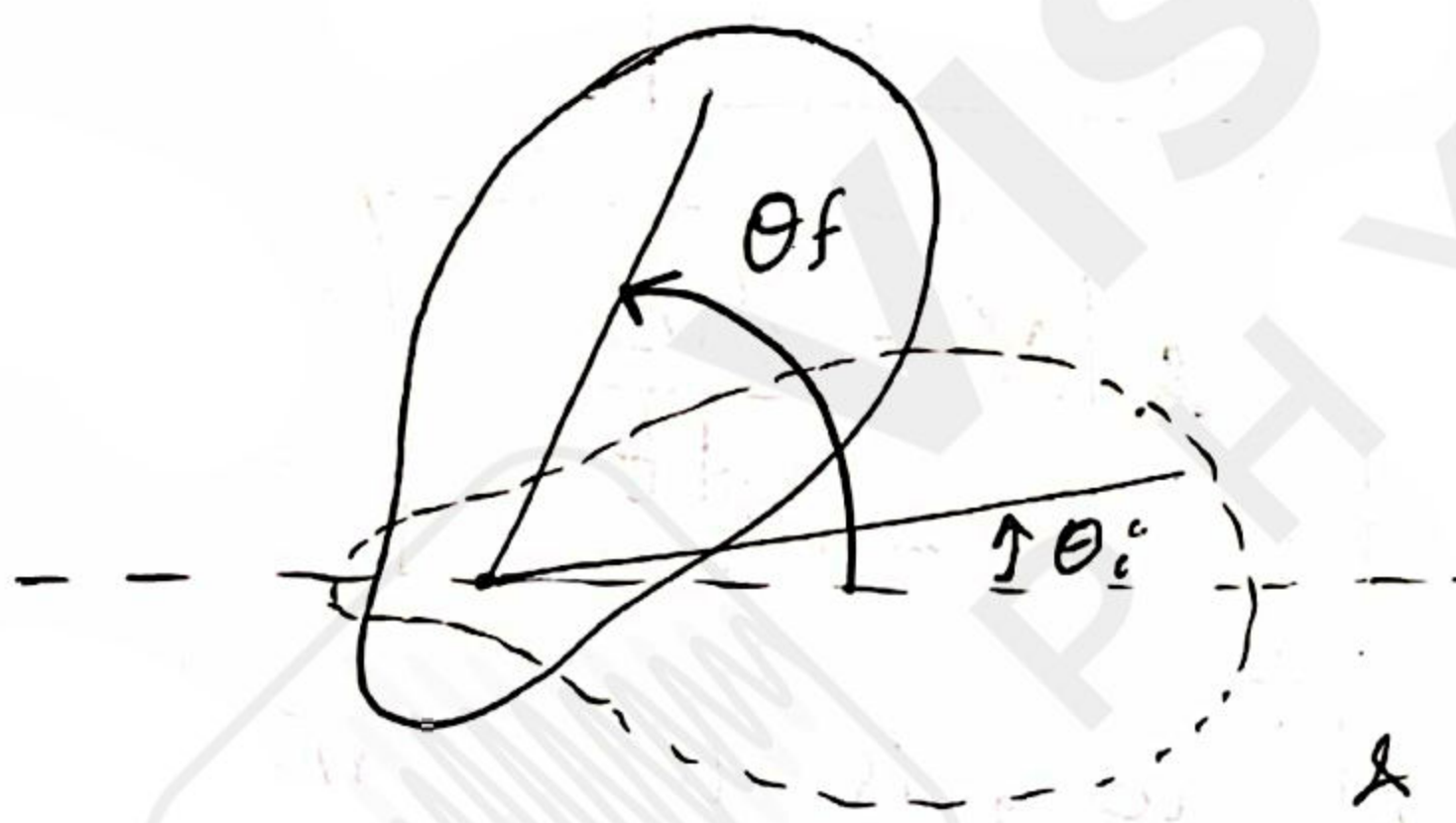
ROTATIONAL DYNAMICS



each particle move through same angle

Body is said to be in pure rotation if every particle of body moves in a circle & centre of all circles lie on a straight line.

→ called Axis of Rotation



+ ↺ anticlockwise
↻
positive

- ↻ (clockwise angle direction negative)

$$\Delta\theta = \theta_f - \theta_i$$

↘ angular displacement
↘ units is radian
↘ both
↘ magnitude ↘ direction

* (for small angular displacement, it is considered as vector)

* but large angular displacement is not vector

As for small $d\theta$, it follows
cumulative law of angular displacements
i.e. $d\bar{\theta} = d\bar{\theta}_1 + d\bar{\theta}_2 = d\bar{\theta}_2 + d\bar{\theta}_1$

but for large angular displacements.

$$\bar{\Delta\theta} = \bar{\Delta\theta}_1 + \bar{\Delta\theta}_2 \neq \bar{\Delta\theta}_2 + \bar{\Delta\theta}_1$$

Hence
not considered vector.

$$\boxed{\omega_{avg} = \frac{\Delta\theta}{\Delta t}}$$

angular velocity

instantaneous angular velocity.

$$\boxed{\omega = \frac{d\theta}{dt}} \rightarrow \text{Vector}$$

angular acceleration

$$\boxed{\alpha_{avg} = \frac{\Delta\omega}{\Delta t}}$$

$$\& \boxed{\alpha = \frac{d\omega}{dt}} \rightarrow \text{Vector}$$

★ direction of ω , is using right hand thumb rule.

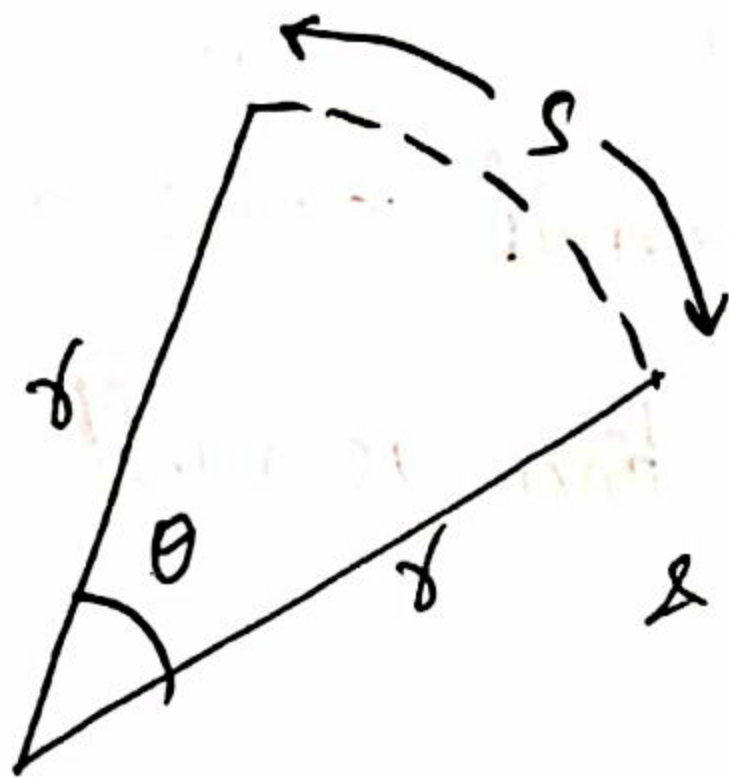
circular motion

translation motion

- i) $\omega_f = \omega_i + \alpha t$
- ii) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$
- iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

- i) $v = u + at$
- ii) $v^2 - u^2 = 2as$
- iii) $s = ut + \frac{1}{2}at^2$

Now,



$$s = r\theta$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{v = r\omega}$$

units

$$\omega \rightarrow \text{rad/s}$$

$$\alpha \rightarrow \text{rad/s}^2$$

tangential velocity

angular velocity

now

$$a = \frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

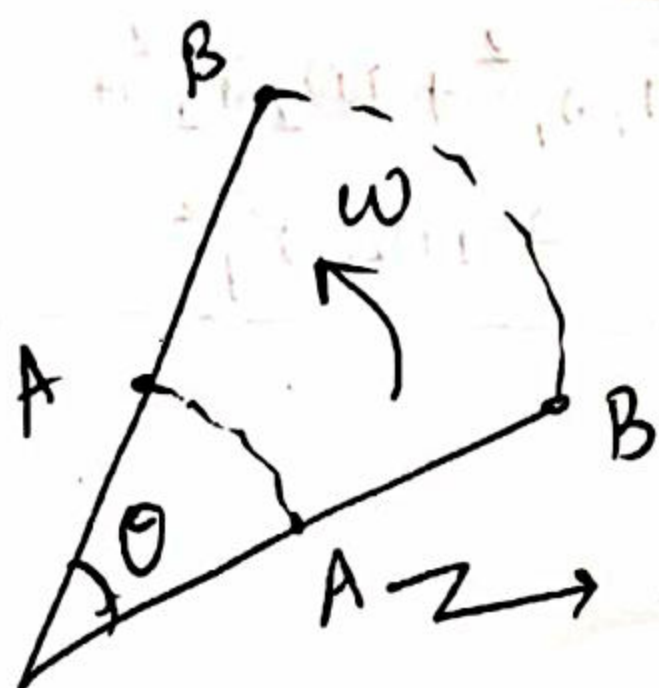
$$\Rightarrow \boxed{a = r\alpha}$$

(for $r \rightarrow \text{same}$)

tangential acceleration

angular acceleration

$\Rightarrow$ for same ω , and α if r increases
 v & a increases.



more v & a

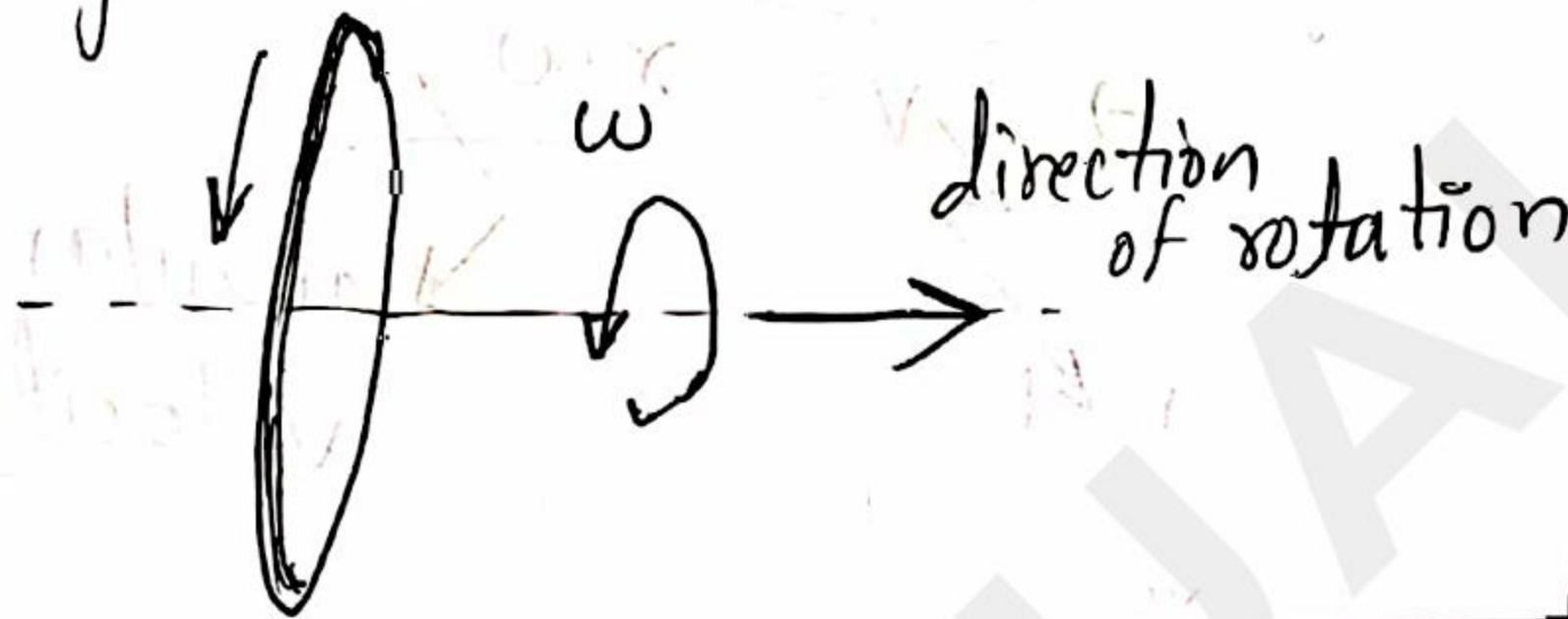
ω is same.

less v & a

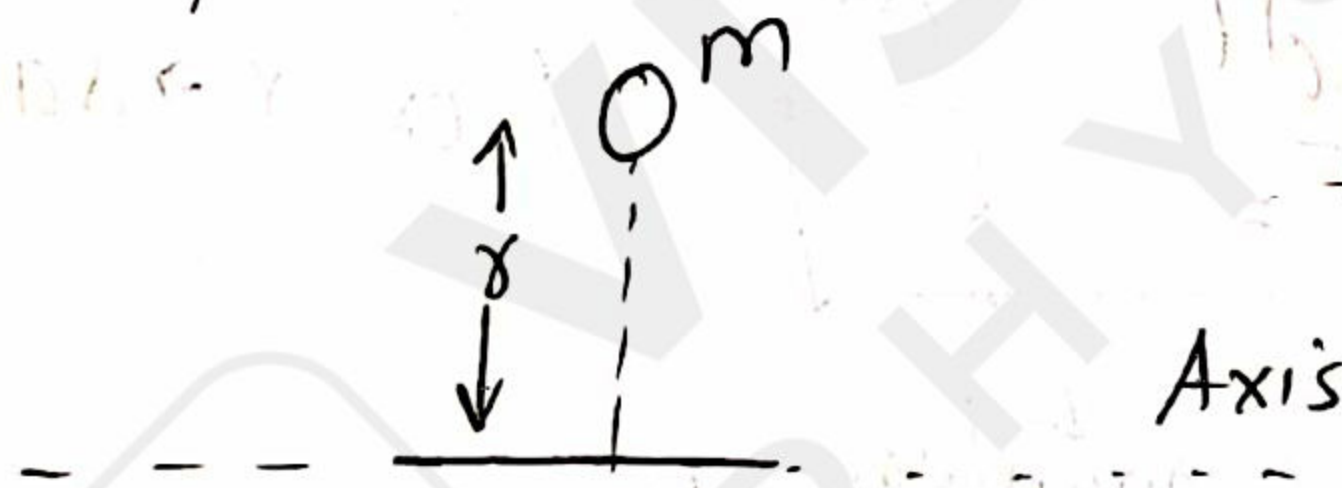
MOMENT OF INERTIA:

→ Ability of the body to resist its change in rotational state of motion.

* Rotation have both direction as well as magnitude.



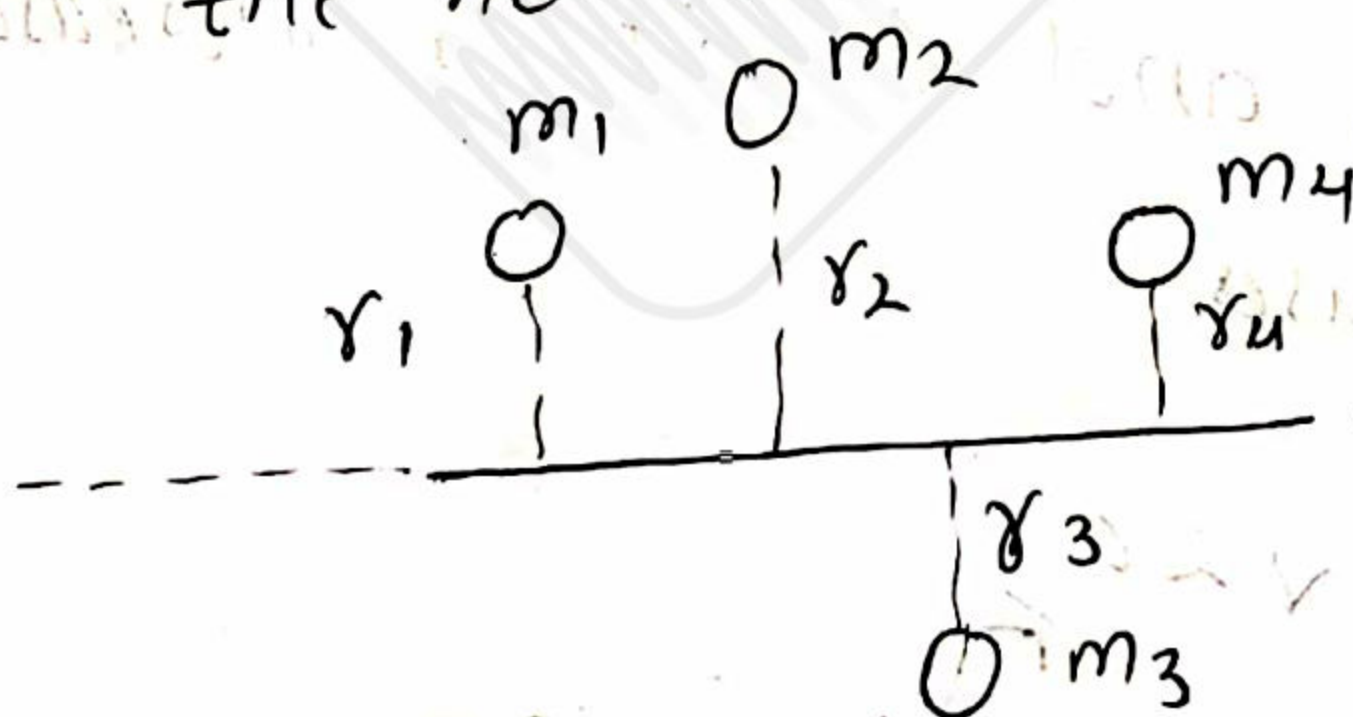
for a particle of mass 'm' ↗ moment of inertia



$$I = mr^2$$

so for $m_1, m_2, \dots$

the net moment of inertia of the system



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

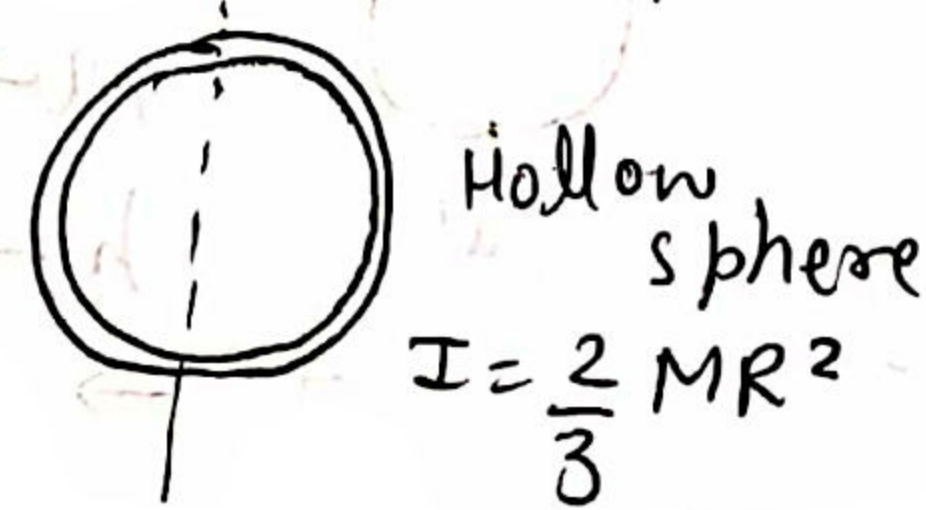
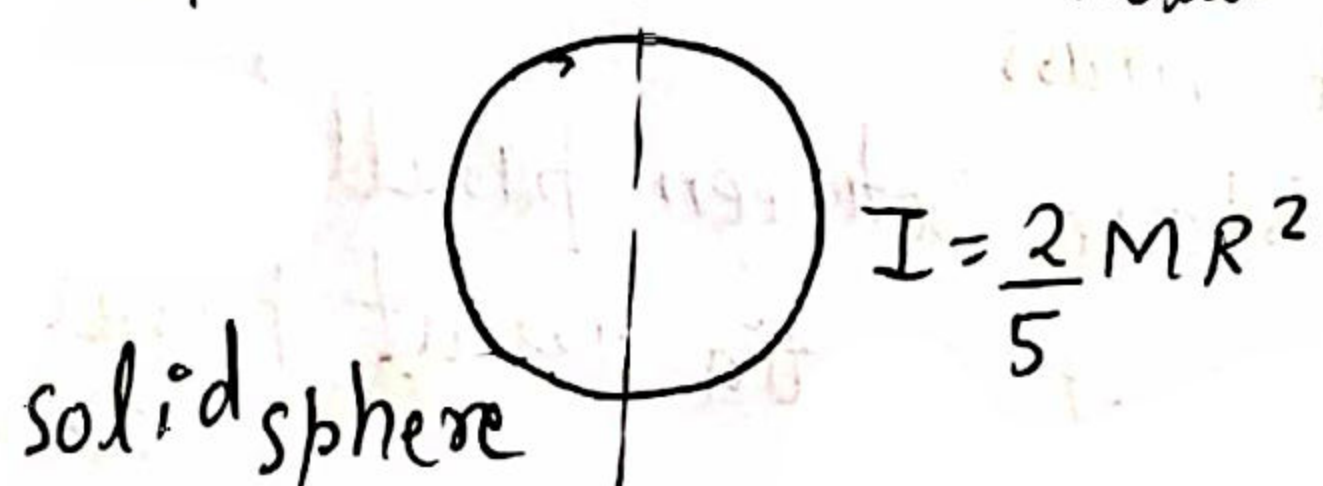
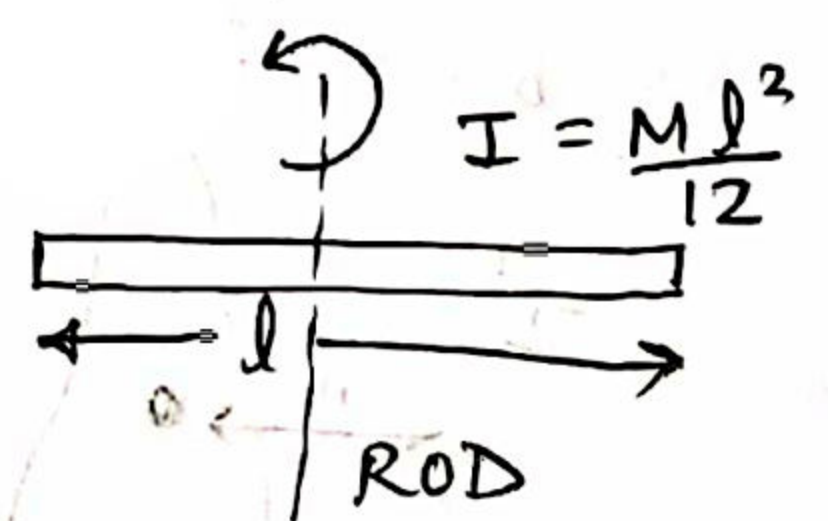
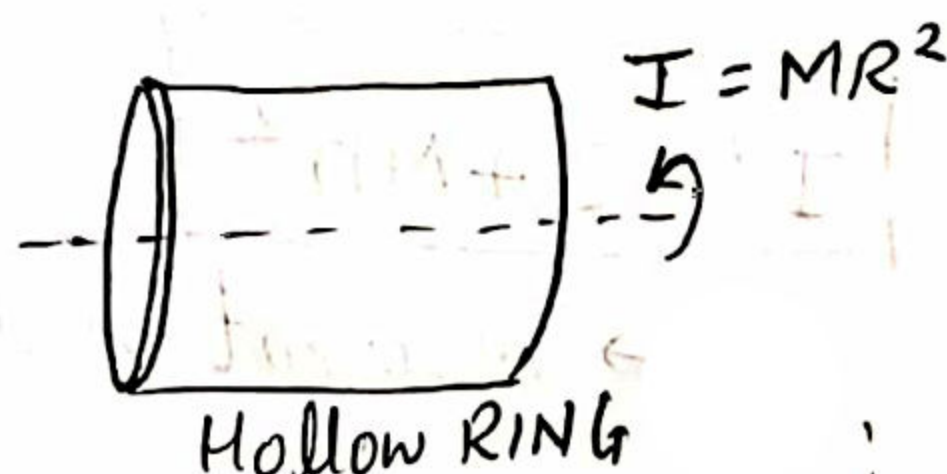
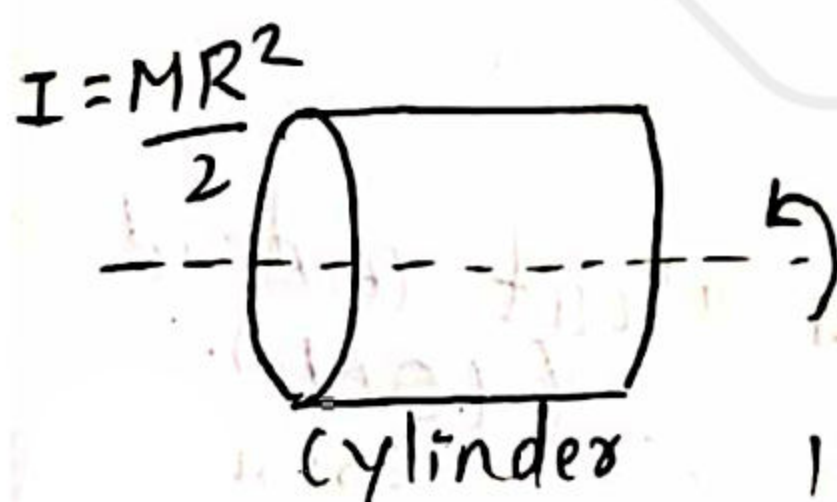
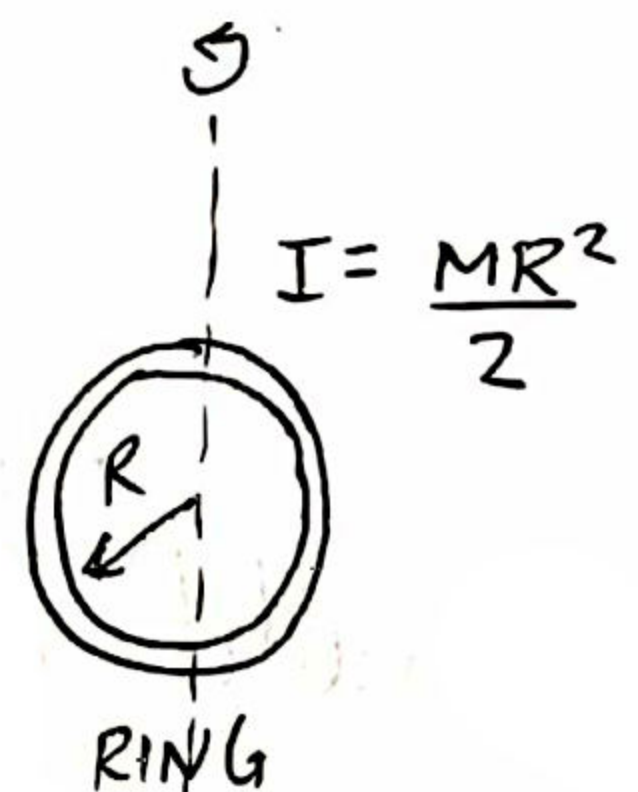
for discrete system of particles ↗

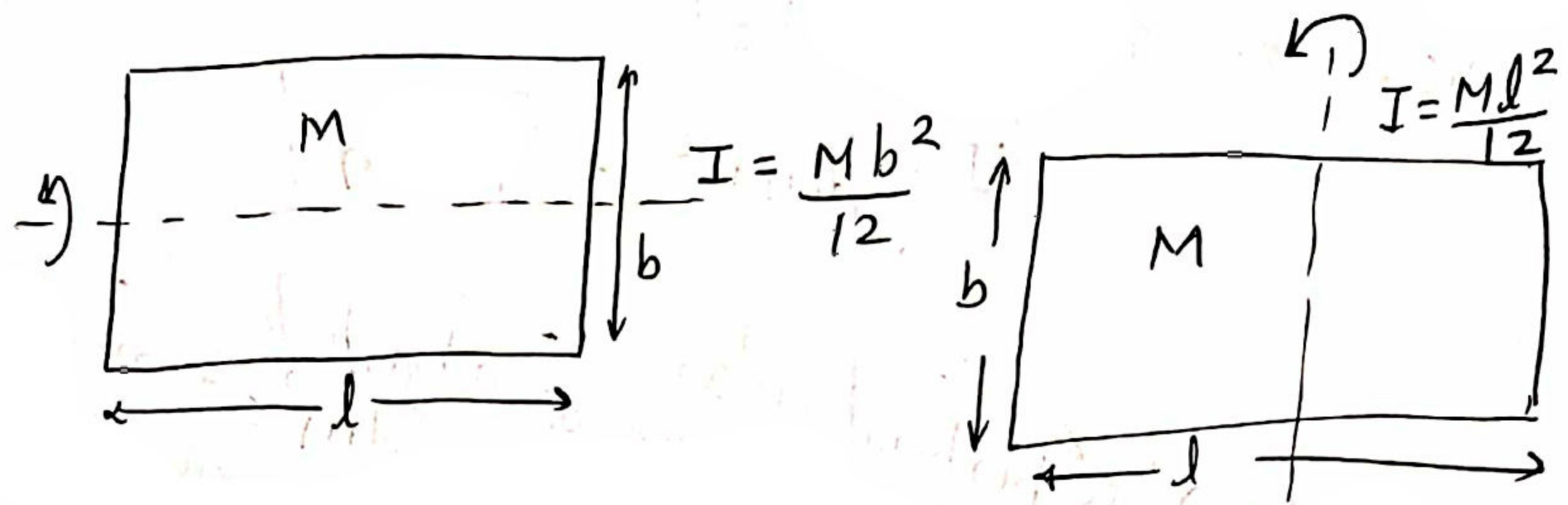
so, for continuous body

$$I = \int dm r^2 = \int r^2 dm$$

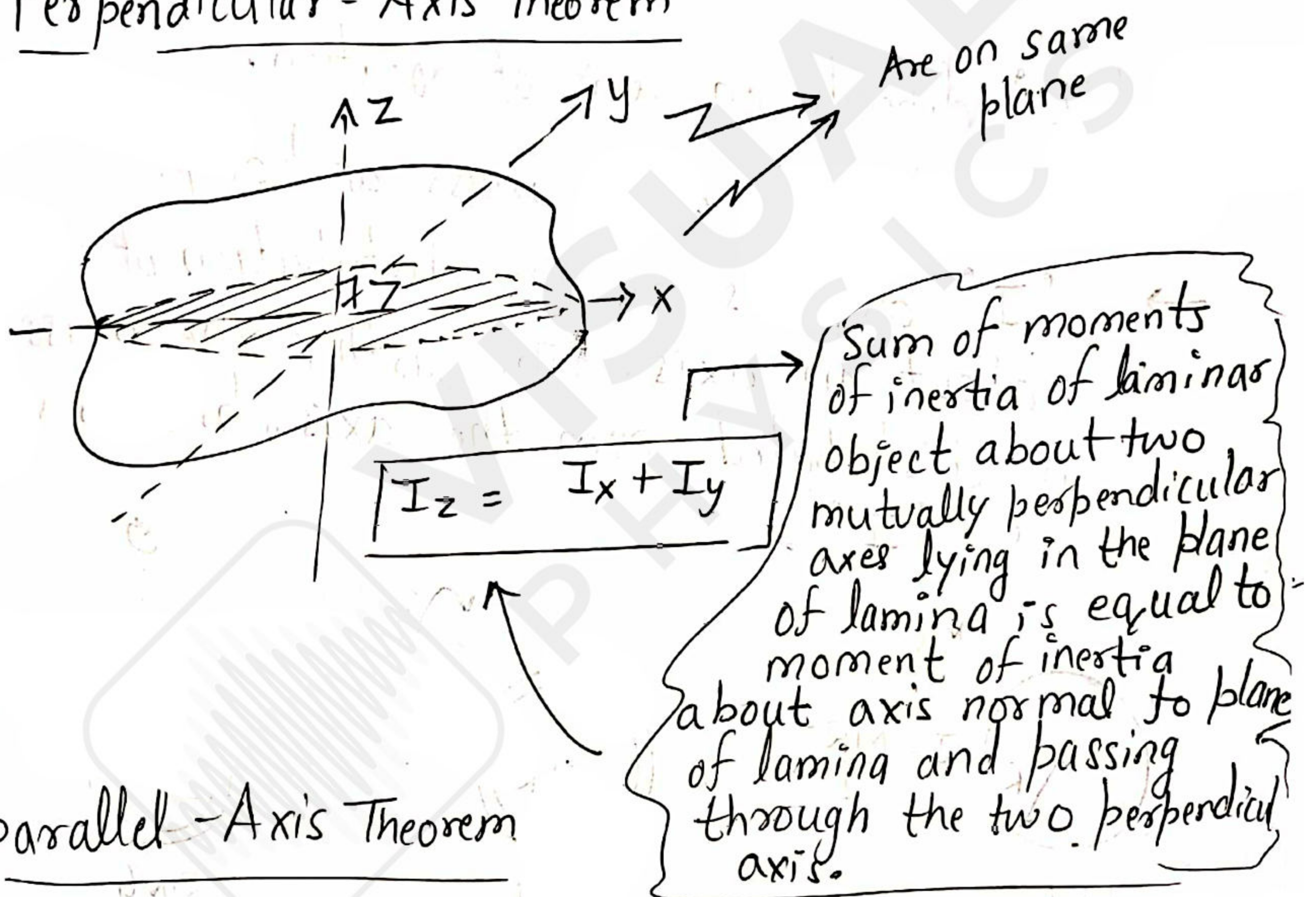
here, 'r' is perpendicular distance of a particle of mass 'dm' of rigid body from axis of rotation,

- 'I' depends on axis of rotation.
- 'I' depends on mass of body.
- 'I' depends on the distribution of mass about an axis. The farther the mass is distributed from the axis, higher will be 'I'.

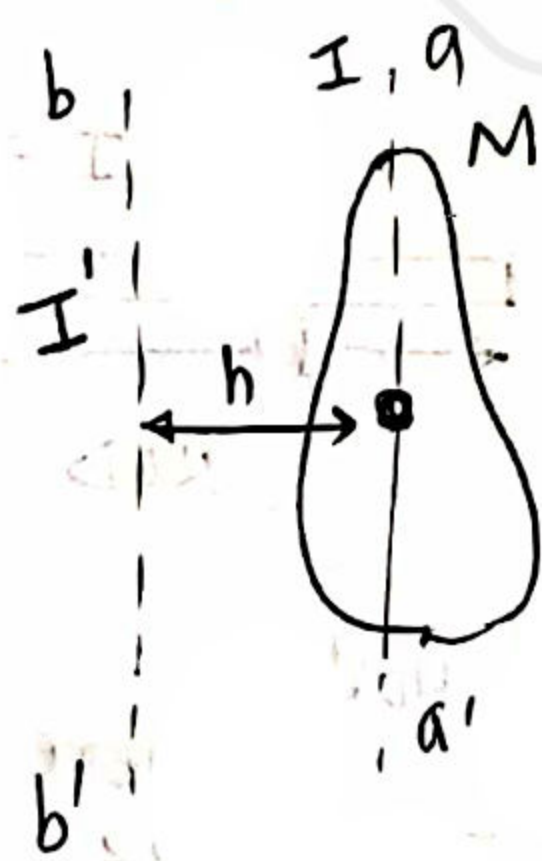




Perpendicular - Axis Theorem



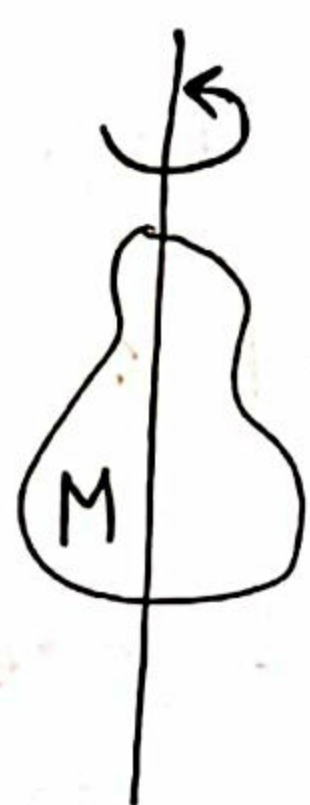
Parallel - Axis Theorem



$$I' = I + Mh^2$$

$I \rightarrow$ moment of inertia about centroid (C.O.M).
 $M \rightarrow$ net mass
 $h \rightarrow$ distance between parallel axis
 $I' \rightarrow$ moment of inertia about parallel axis

RADIUS OF GYRATION:



$k \rightarrow$ radius of gyration.

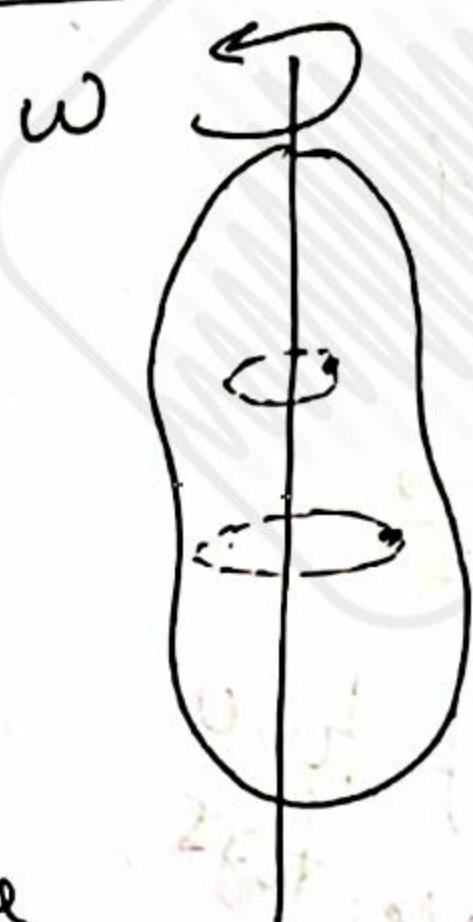
$$I = Mk^2 = \int r^2 dm$$

$\rightarrow$ radius of gyration (k) of a body about an axis is effective distance from axis where whole mass can be assumed to be concentrated so that moment of inertia remains the same.

i.e. for solid sphere, $I = \frac{2}{5} MR^2 = Mk^2$

$$\Rightarrow k = \sqrt{\frac{2}{5}} R$$

Rotational KE:



as per particle

$$d(k.E) = \frac{1}{2} (dm) v^2$$

$$\Rightarrow \int d(k.E) = k.E = \int \frac{1}{2} dm (\omega r)^2$$

$$\Rightarrow k.E = \frac{1}{2} \int dm (\omega^2 r^2)$$

$$= \frac{1}{2} \omega^2 \left(\int dm r^2 \right)$$

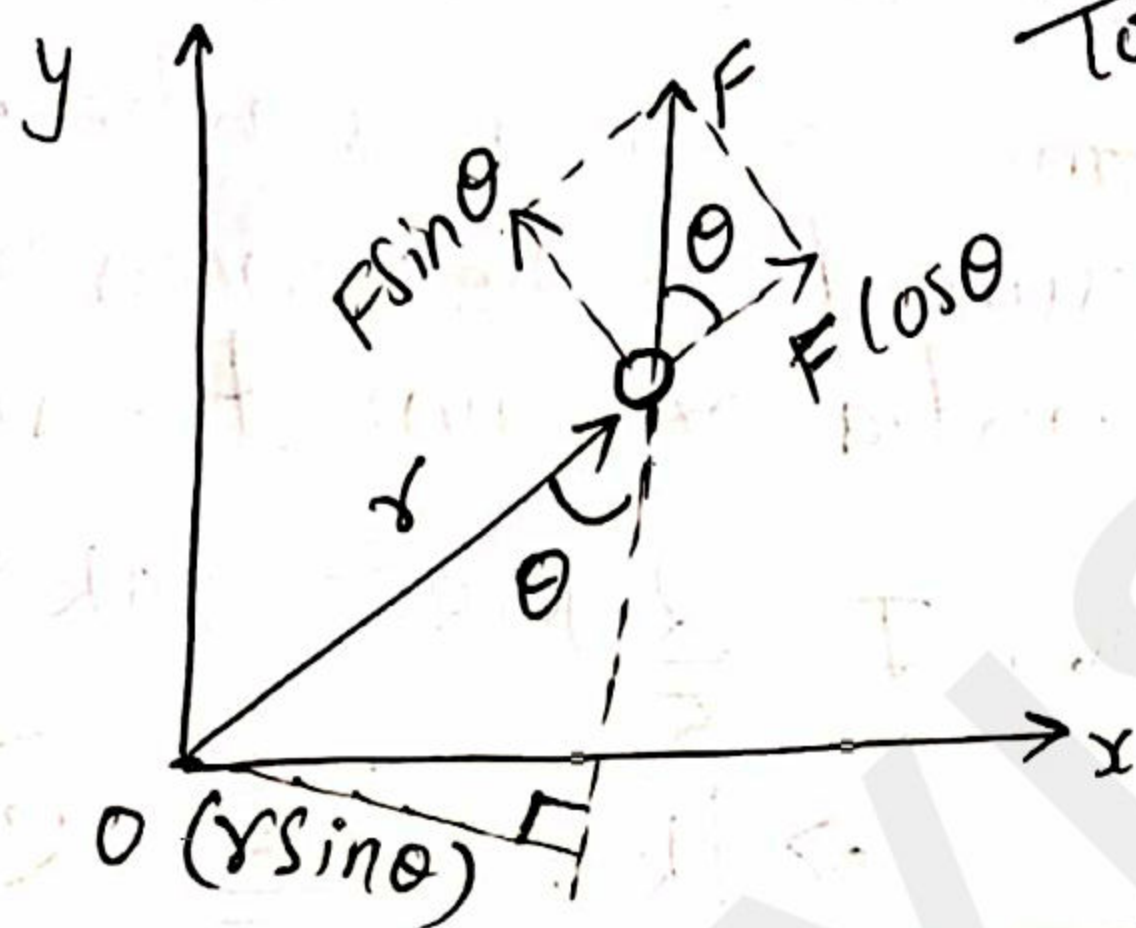
moment of inertia

$\rightarrow I$

$$k.E = \frac{1}{2} I \omega^2$$

for pure rotation kinetic energy

TORQUE: $\rightarrow$ Rotational effect of force is torque.



Torque about O

torque cause angular acceleration

$$\vec{\tau} = \vec{r} \times \vec{F}$$

from position (axis) of rotation

$$\Rightarrow |\vec{\tau}| = \tau = r(F \sin \theta) = r F_{\perp} \quad \text{(Force component perpendicular to } \vec{r} \text{)}$$

$$\tau = (r \sin \theta) F = r_{\perp} F$$

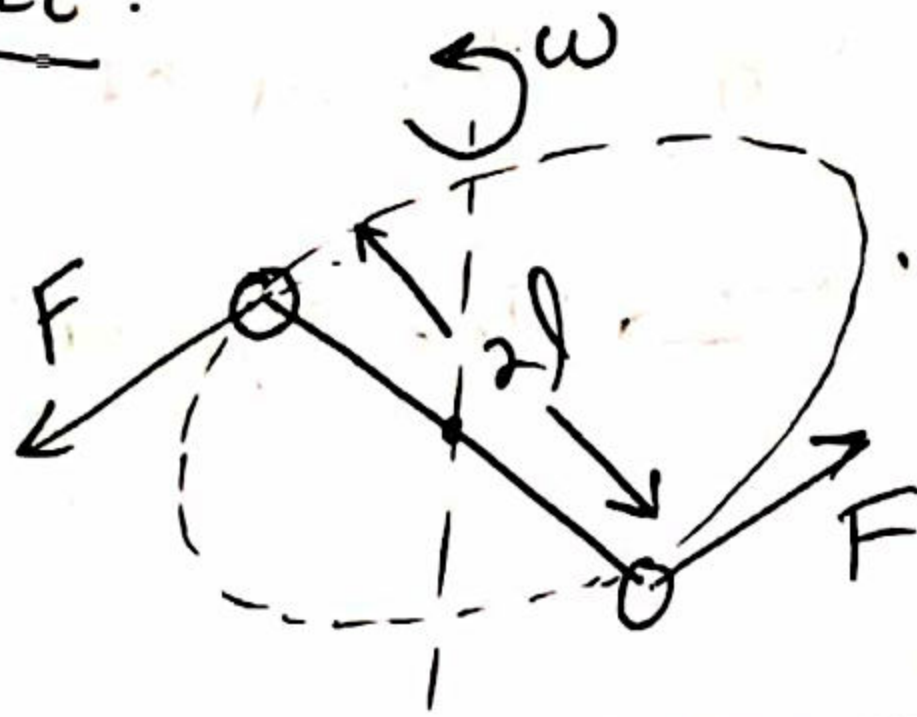
means from ' $r_{\perp}$ ' to F

For direction of torque $\vec{\tau} = \vec{r} \times \vec{F}$

cross product of two vectors

'Right hand rule'

COUPLE:



Two equal and opposite forces whose line of action do not coincide make a couple.

And $|\vec{\tau}| = \tau = 2lF$

LAWS OF ROTATION:

τ about the axis of rotation = I about axis of rotation $\times \alpha$

$\alpha \rightarrow$ angular acceleration about axis of rotation

$\Rightarrow \boxed{\tau = I \alpha} \Rightarrow \boxed{\vec{\tau} = I \vec{\alpha}}$

$|\vec{\tau}| = \tau = r F = r (m a) \times r$
 $= r^2 m \left(\frac{a}{r} \right)$
 (external torque)

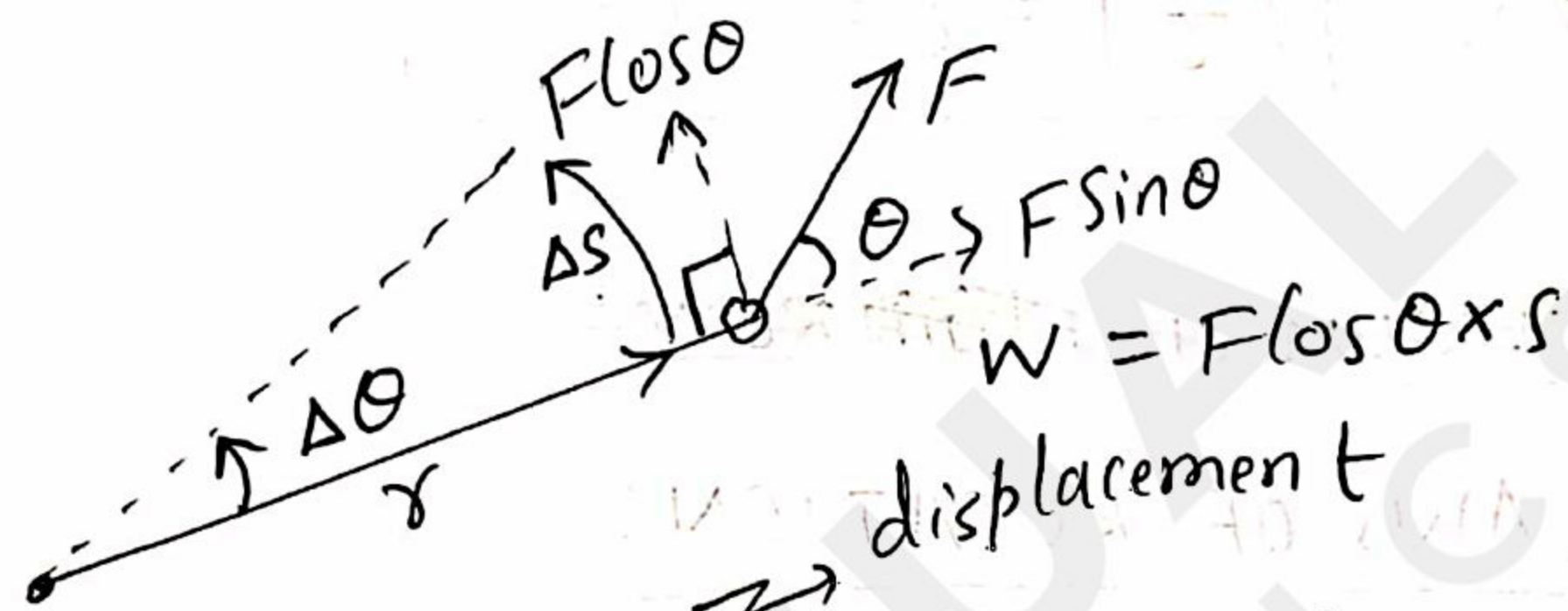
$\boxed{\tau = I \alpha}$

Similar to $\boxed{F = m a}$

(derived for particle)

Valid for
any body

Work - Energy Theorem !



Work done = $(F_{\perp}) \Delta s = (F_{\perp}) r \Delta \theta$

now $F_{\perp} r = \tau = \text{Torque}$.

$\Rightarrow dw = F_{\perp} r d\theta = \tau d\theta$

$\Rightarrow \boxed{\int dw = W = \int \tau d\theta}$

external Torque

Network

$$\Delta K.E = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

change in kinetic Energy

for single particle

$$W = \Delta K.E = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\int \tau d\theta = \frac{1}{2} m (\omega r_f)^2 - \frac{1}{2} m (\omega r_i)^2$$

$$\int \tau d\theta = \frac{1}{2} m r^2 (\omega_f^2 - \omega_i^2)$$

$$\boxed{\int \tau d\theta = \frac{1}{2} I (\omega_f^2 - \omega_i^2)}$$

As for single particle $I = m r^2$

but applicable for any body.

Hence in general, for rotational dynamics.

scalar $\star \rightarrow \boxed{W = \int \tau d\theta = \frac{1}{2} I (\omega_f^2 - \omega_i^2) = \Delta K.E}$

As Power = $P = \frac{dW}{dt}$

Now $dW = \tau d\theta$

$$\Rightarrow P = \frac{d}{dt} (\tau d\theta) = \tau \frac{d\theta}{dt}$$

$\Rightarrow \boxed{P = \tau \omega}$ $\rightarrow$ rotational speed.

scalar $\rightarrow$

similar to

$$\boxed{P = FV}$$

Angular Momentum:

for a moving particle about a point is defined as,

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$$\text{As Torque} = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\Rightarrow \text{if } \vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$$

external
Torque

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow$$

$$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$$

$$\text{now, } \vec{L} = \vec{r} \times m\vec{v}$$

$$\text{as } \boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

$$\Rightarrow \vec{L} = \vec{r} \times (m \vec{\omega} \times \vec{r})$$

for system of particles.

$$\vec{L} = \sum (m_i r_i^2) \vec{\omega}$$

Vector
quantity

$$\boxed{\vec{L} = I \vec{\omega}}$$

$$\Rightarrow \boxed{|\vec{L}| = I \omega}$$

applicable
for any
system of
particles
& rigid
body

As,

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\text{if } \vec{\tau} = 0 = \frac{d\vec{L}}{dt}$$

Means change
in angular
momentum
is zero

if external $\Rightarrow$
Torque is zero
than, angular
momentum of the
system is not zero.

$$d\vec{L} = 0$$

change in angular
momentum is
zero.

$$\Rightarrow \int \phi$$

$$L_f - L_i = 0$$

$L_f \rightarrow$ final angular momentum

$L_i \rightarrow$ initial angular momentum

$$L_f = L_i$$

$\Rightarrow$ angular momentum
of rotational system
remains same.

$$\text{or } I_f \omega_f = I_i \omega_i$$

$I_f \rightarrow$ final ~~an~~ moment of Inertia

$I_i \rightarrow$ initial moment of Inertia

$\omega_i \rightarrow$ initial angular velocity

$\omega_f \rightarrow$ final angular velocity