VISUAL PHYSICS

SHORT NOTES

CHAPTER

Magnetic Field

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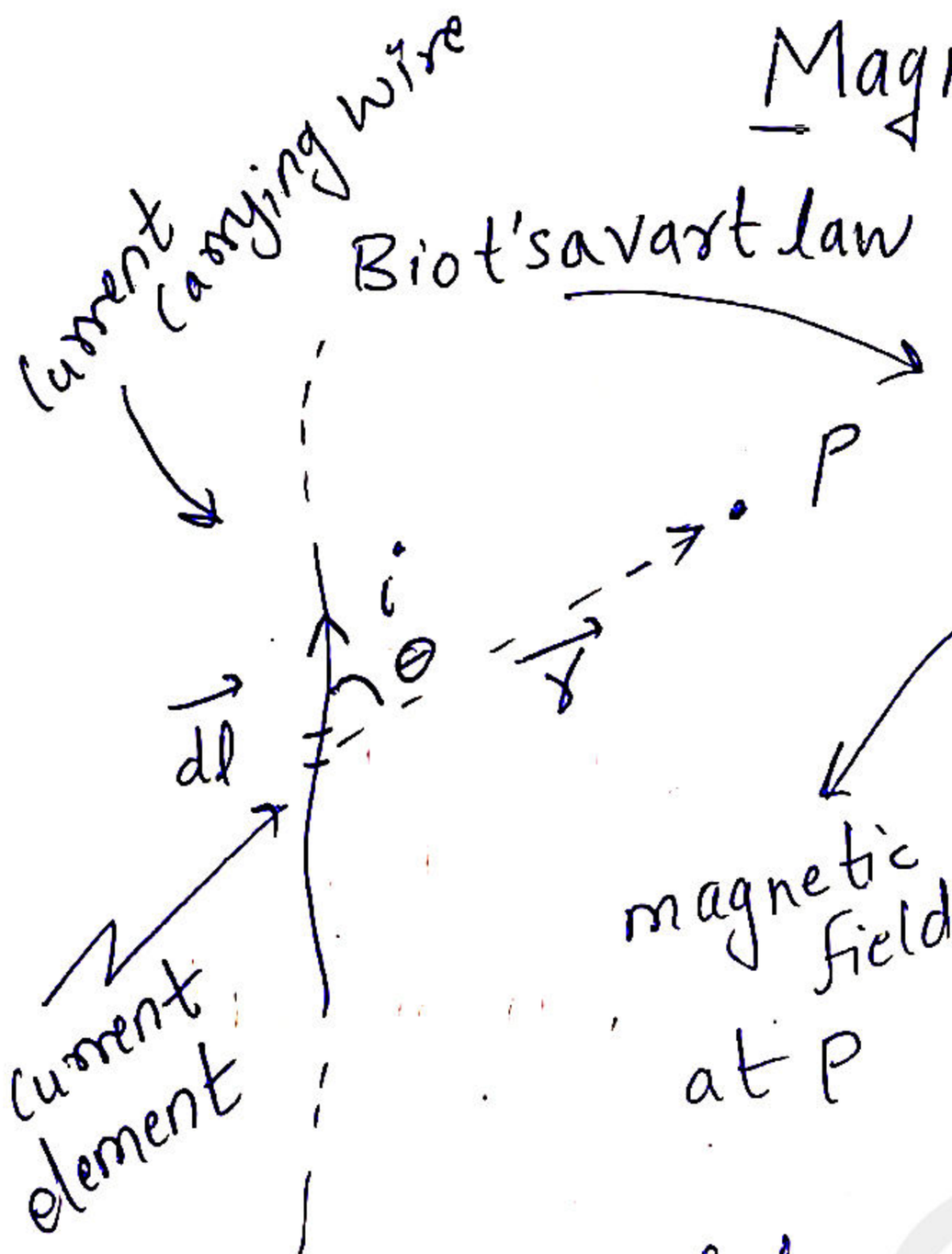
Magnetic Field

Biot's savart law



permeability of air

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dl} \times \vec{\gamma}}{\gamma^2}$$

$4\pi \times 10^{-7}\ N S^2 C^{-2}$

$\mu \rightarrow$ permeability $\rightarrow$ resistance of material against the formation of magnetic field.

if air

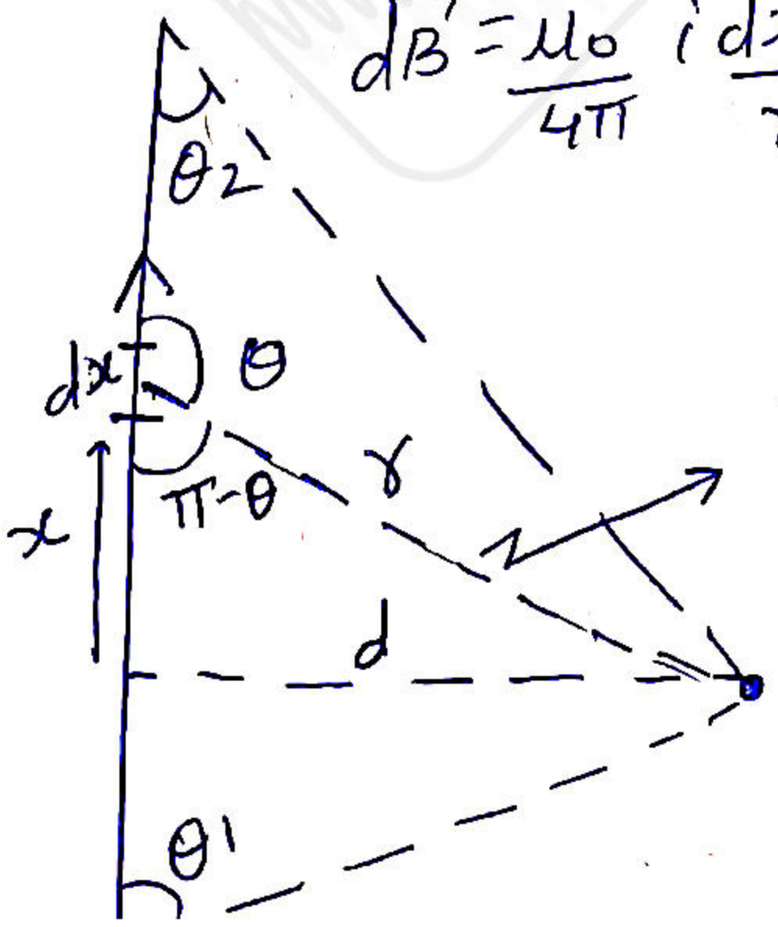
$\mu_0 = 4\pi \times 10^{-7}\ N S^2 C^{-2}$

if not air

$\mu_0 \mu_r$ $\rightarrow$ relative permeability of medium

$$\boxed{\vec{B} = \frac{\mu_0 \mu_r i}{4\pi} \frac{\vec{dl} \times \vec{\gamma}}{\gamma^2}}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dx} \times \hat{\gamma}}{\gamma^2} \qquad |d\vec{B}| = \frac{\mu_0}{4\pi} \frac{i\,dx \sin\theta}{\gamma^2}$$



$x = -d\cot\theta$

$\Rightarrow dx = -d\left[\operatorname{cosec}^2\theta\, d\theta\right]$

$$\boxed{dx = d\operatorname{cosec}^2\theta\, d\theta}$$

$\sin\theta = \sin(\pi - \theta) = d/\gamma$

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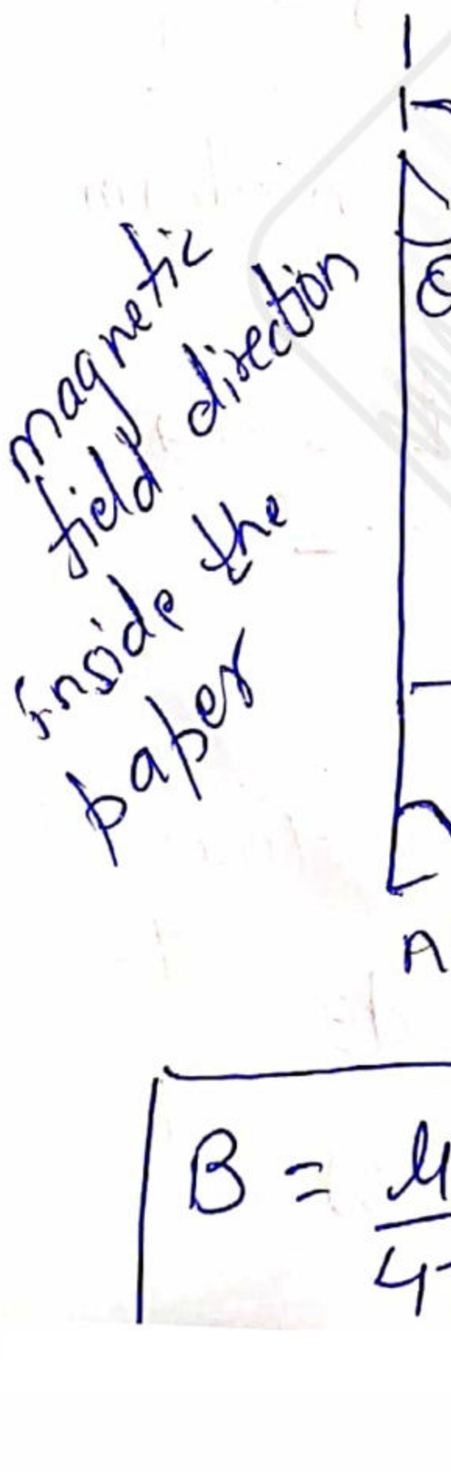
$$dB = \frac{\mu_0}{4\pi} \frac{i\,dx}{r^2} \frac{d}{r}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,d(\cot\theta\, d\theta)\, d}{d^3 \operatorname{cosec}^3\theta}$$

$$\boxed{dB = \frac{\mu_0 i}{4\pi d} \sin\theta\, d\theta} \rightarrow \text{direction using right hand thumb rule}$$

Thumb in current direction, the fingers give magnetic field direction

angle always in between current direction & position vector



$$\int dB = \frac{\mu_0 i}{4\pi d} \int_{\theta_1}^{\pi-\theta_2} \sin\theta\, d\theta$$

$$B = \frac{\mu_0 i}{4\pi d} \left[-\cos\theta\right]_{\theta_1}^{\pi-\theta_2}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i}{4\pi d} \left[\cos\theta_1 + \cos\theta_2\right]}$$

$$\boxed{B = \frac{\mu_0 i}{4\pi d} \left[\sin\alpha + \sin\beta\right]}$$

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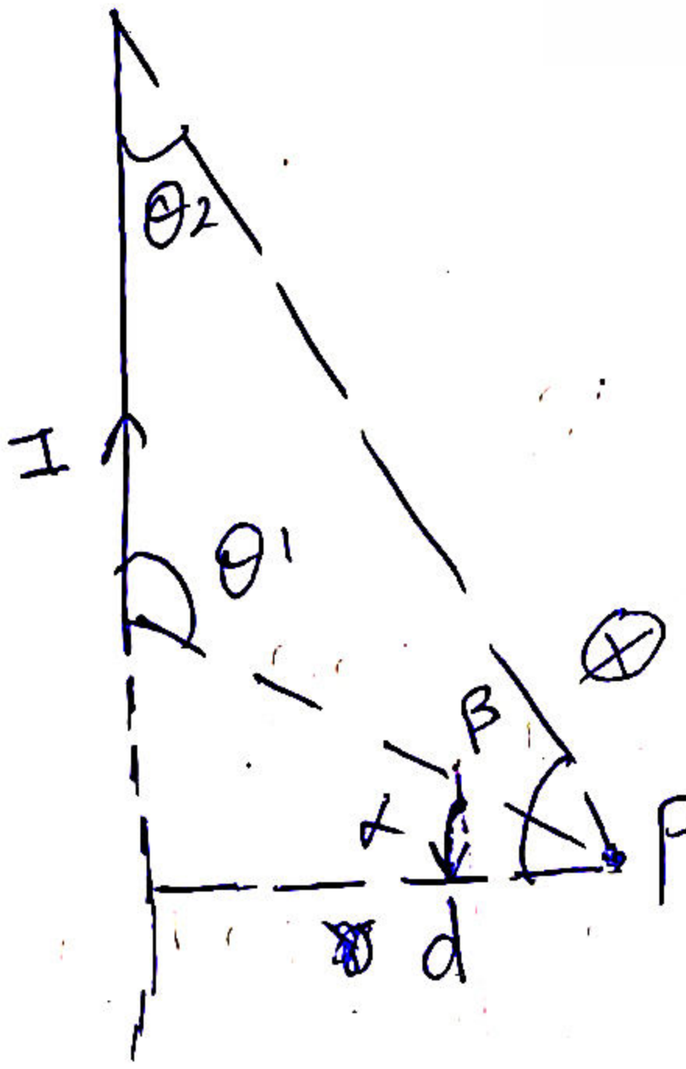
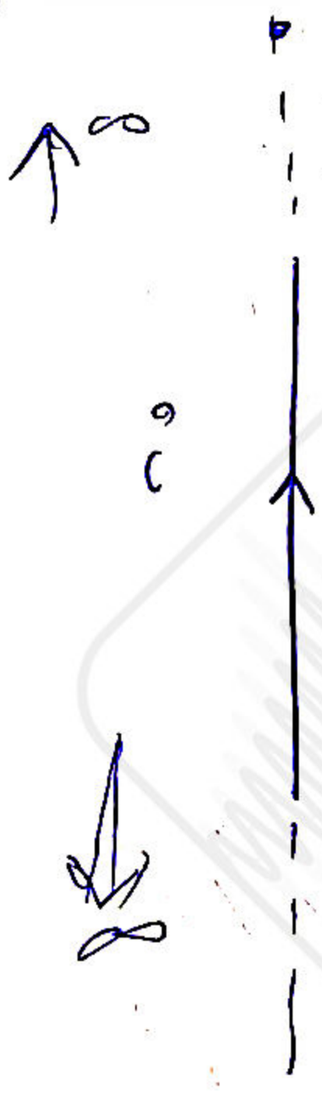


$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\beta - \sin\alpha\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\cos\theta_1 + \cos\theta_2\right]$$

still remain same

if,



$$B = \frac{\mu_0 i}{4\pi d}\left[\cos 0 + \cos 180\right]$$

$$B = 0$$

magnetic field on axis of wire and along the wire is always Zero.

Now if wire length goes to infinite

$\theta_1 \to 0,\ \theta_2 \to 0$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d}\left[\cos 0 + \cos 0\right]$$

$$B = \frac{\mu_0 i}{2\pi d}$$

$$B = \frac{\mu_0 i}{2\pi d}$$

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if the wire is semi-infinite.

$\theta_1 = 90$, $\theta_2 = 0$

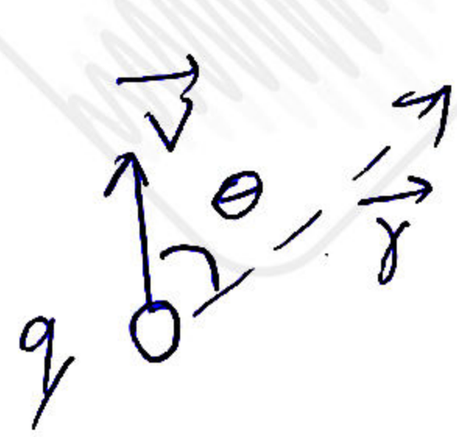
~~$B = \frac{\mu_0 i}{4\pi d}[\sin\theta + \sin$~~

$B = \frac{\mu_0 i}{4\pi d}[\cos 0 + \cos 90]$

$$\Rightarrow \boxed{B = \frac{\mu_0 i}{4\pi d}[1] = \frac{\mu_0 i}{4\pi d}}$$

magnetic field due to a moving charge:

experimentally:



$\vec{B}$ at P is:

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}}$$

$\star$ So, when $\theta \to 0$, $B = 0$

$\boxed{\vec{B}$ along the line of motion of charge $= 0$ $}$

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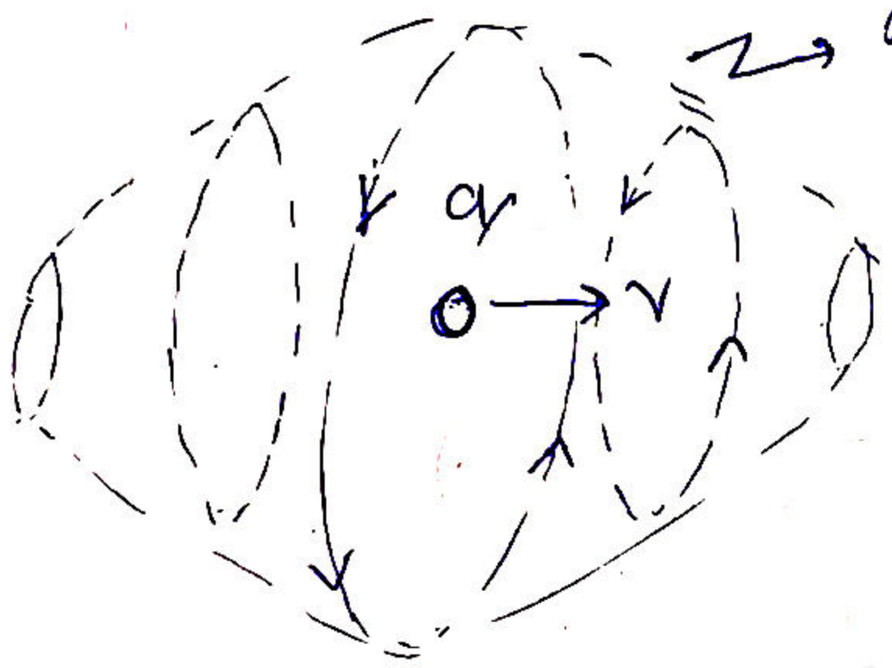
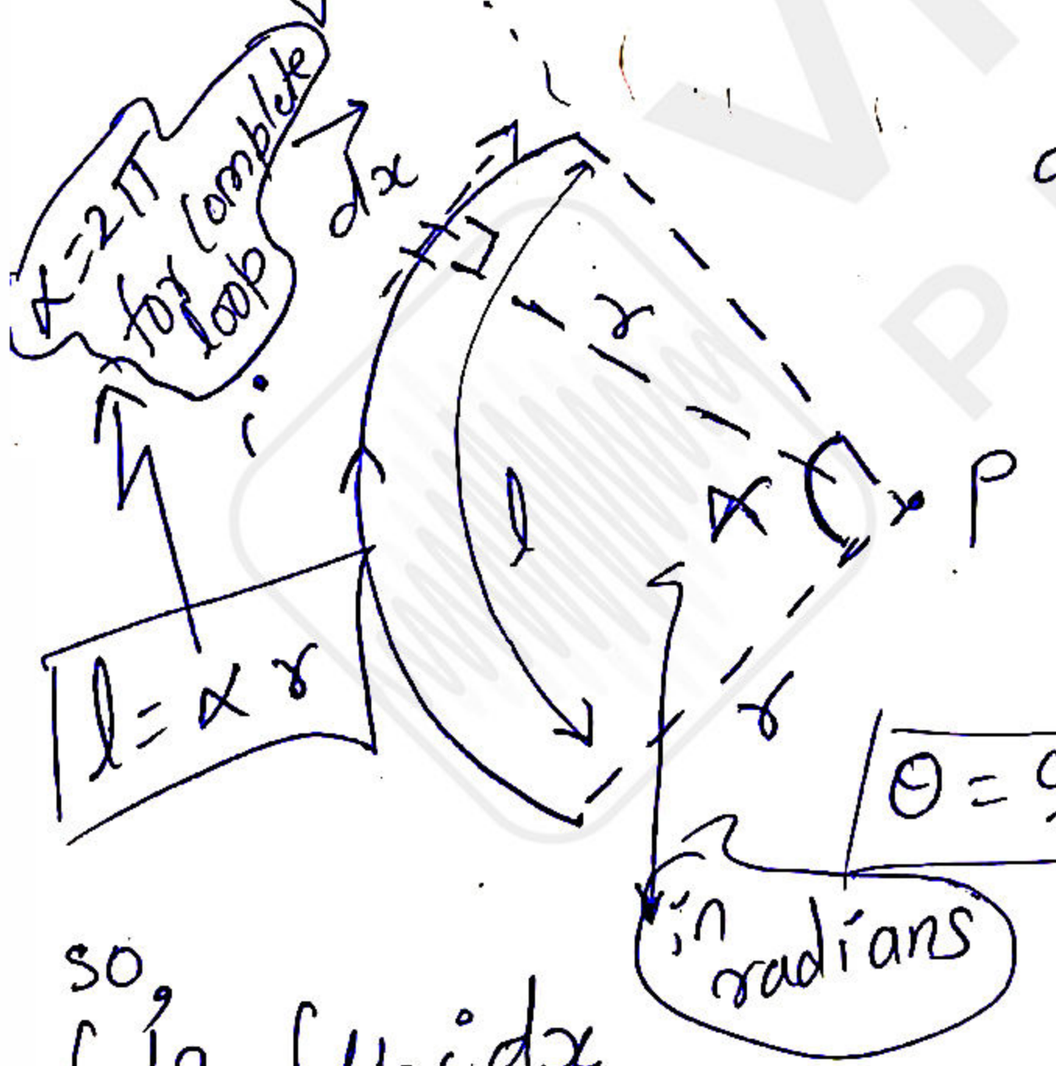
So, for a constant 'r',

$\vec{B}_{max}$ at $\theta = 90$, $\vec{B}_{min}$ at $\theta = 0$



→ magnetude of B around a moving charge q.

Magnetic field at the center of Arc



$dB = \frac{\mu_0 i\, dx \sin 90}{4\pi r^2}$

dB → magnetic field at P due to current element dx

$\theta = 90°$, as angle between current direction & $\vec{r}$ is 90° → tangent

So, $\int dB = \int \frac{\mu_0 i\, dx}{4\pi r^2}$

$B = \frac{\mu_0 i}{4\pi r^2} l$

So, $B = \frac{\mu_0 i \alpha}{4\pi r^2}$

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for circular loop, $\alpha = 2\pi$

'B' at centre of loop

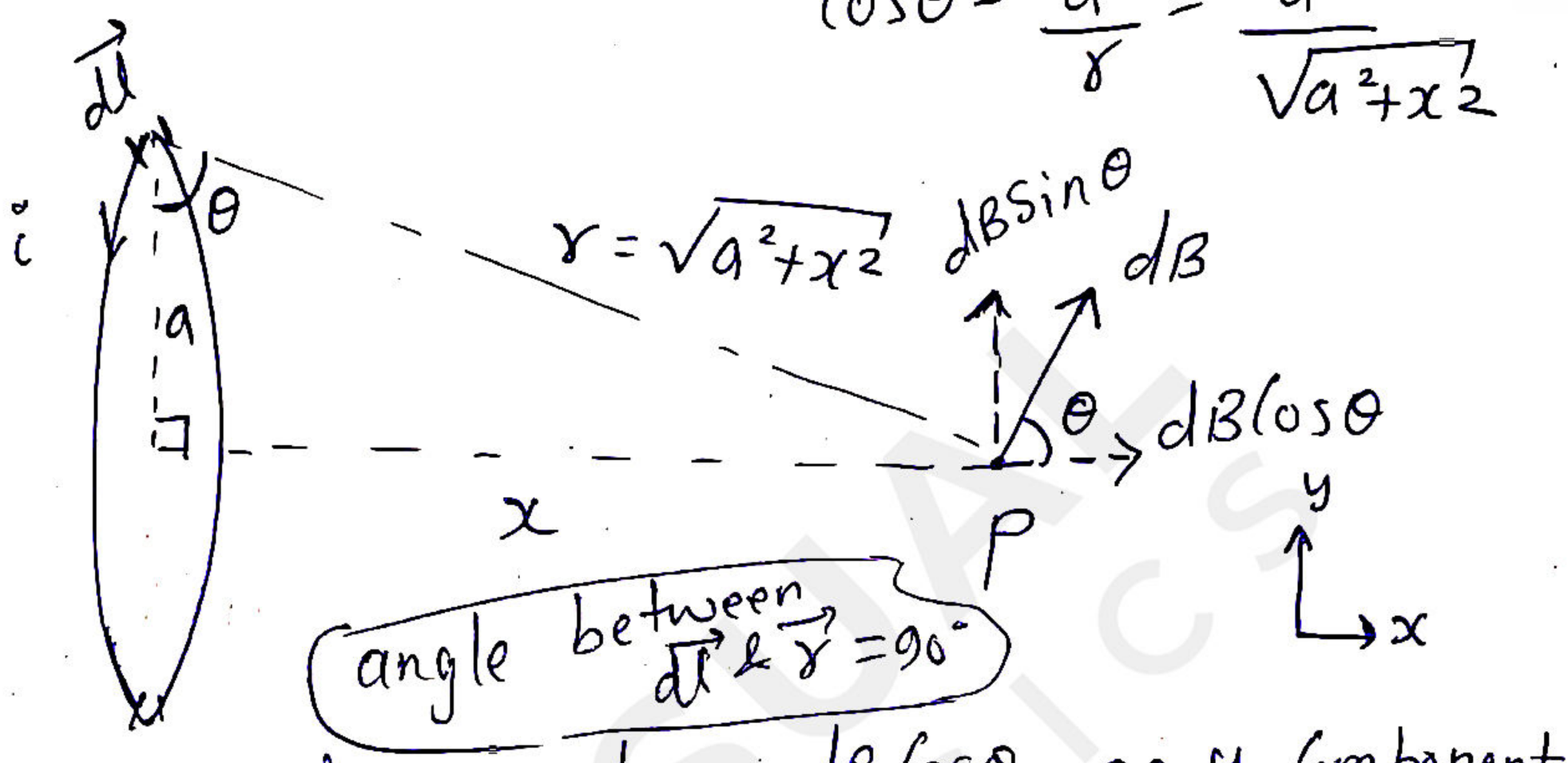
$$= \frac{\mu_0 i (2\pi)}{4\pi r^2}$$

$$\boxed{B = \frac{\mu_0 i}{2r}}$$

for 'n' loop, $\alpha = n(2\pi)$

so $$\boxed{B = \frac{n\mu_0 i}{2r}}$$

Magnetic field of a circular current loop at axis :



$$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$$

* as, we only consider $dB\cos\theta$, as y component for any current element will be cancel out by opposite current element (diametrical)

so, $$dB_{net} = dB\cos\theta = \frac{\mu_0 i\, dl \sin 90 \cos\theta}{4\pi r^2}$$

$$\Rightarrow B_{net} = \frac{\mu_0 i}{4\pi} a \int \frac{dl}{r^3}$$

$$\boxed{B_{net} = \frac{\mu_0 i a^2}{2(x^2+a^2)^{3/2}}}$$ in x-directia

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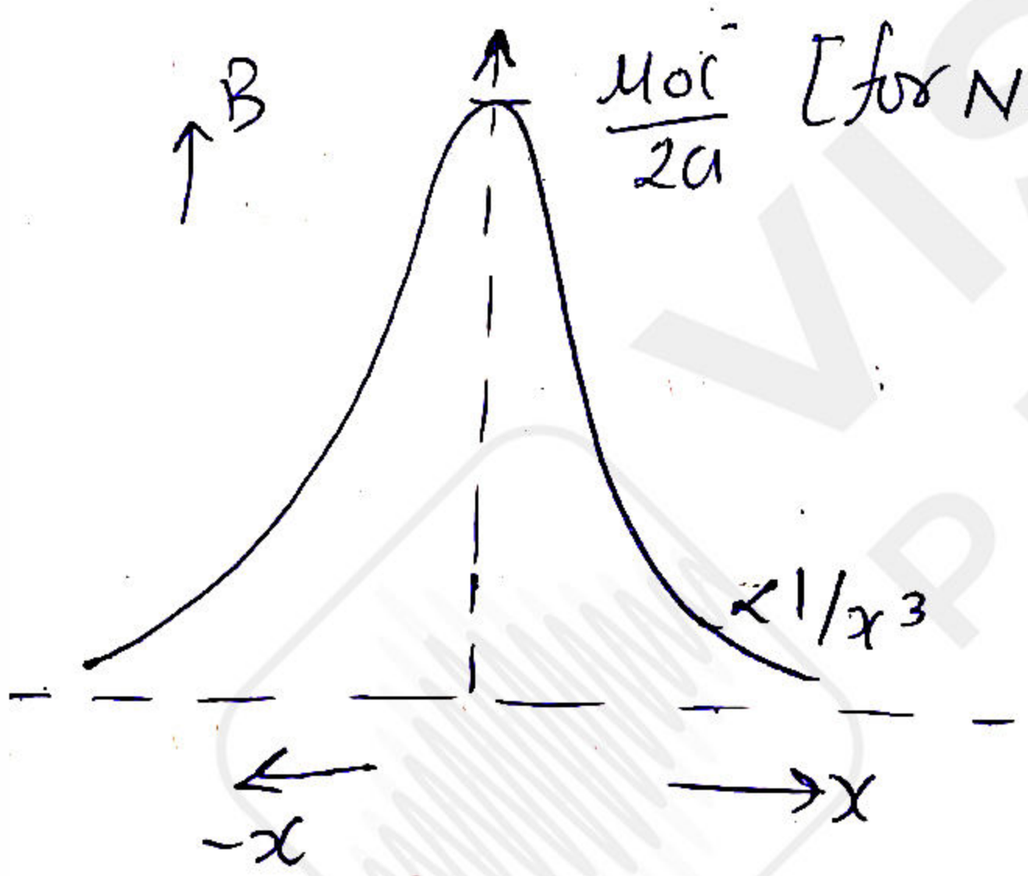
When $x >> a$

$\Rightarrow$ $\boxed{B_{net} = \frac{\mu_0 I a^2}{2x^3}}$ $\Rightarrow B \propto \frac{1}{x^3}$

at $x = 0$

$\boxed{B_{net} = \frac{\mu_0 i}{2a}} = B_{max}$

if N turns $B_{net} = \frac{N \mu_0 i a^2}{2(x^2+a^2)^{3/2}}$



As (M = magnetic dipole moment)

$M = i \times Area$

$= i \times 2\pi a^2$

$B_{net} = \frac{B_{net} \times 2\pi}{2\pi}$

$\Rightarrow B_{net} = \frac{N \mu_0 \cdot i a^2 \times 2\pi}{4\pi (x^2+a^2)^{3/2}}$

So $\boxed{B_{net} = \frac{\mu_0}{4\pi} \frac{2M}{(x^2+a^2)^{3/2}}}$

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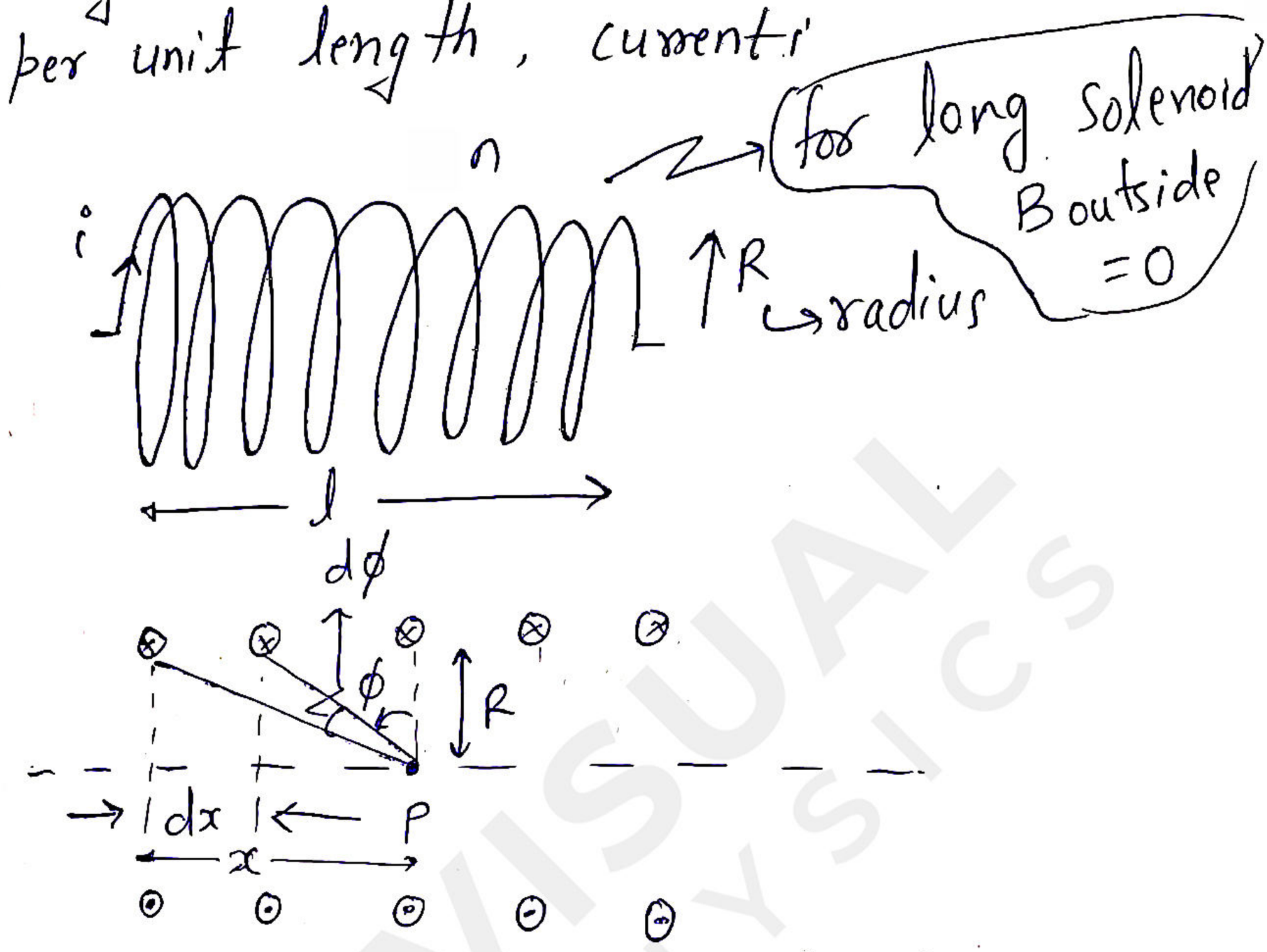
Magnetic field due to solenoid, N turns per unit length, current i



n = number of turns per unit length

$N = n\,dx$, $x = R\tan\phi$, $dx = R\sec^2\phi\,d\phi$

turns in dx length

$$dB = \frac{\mu_0 n\,dx \times i R^2}{2(R^2+R^2\tan^2\phi)^{3/2}}$$

for loop

$$B = \int dB = \int_{\alpha}^{\beta} \frac{\mu_0 n\,dx \times i R^2}{2(R^2+R^2\tan^2\phi)^{3/2}}$$

$$\boxed{B = \frac{\mu_0 n i}{2}[\sin\alpha + \sin\beta]}$$

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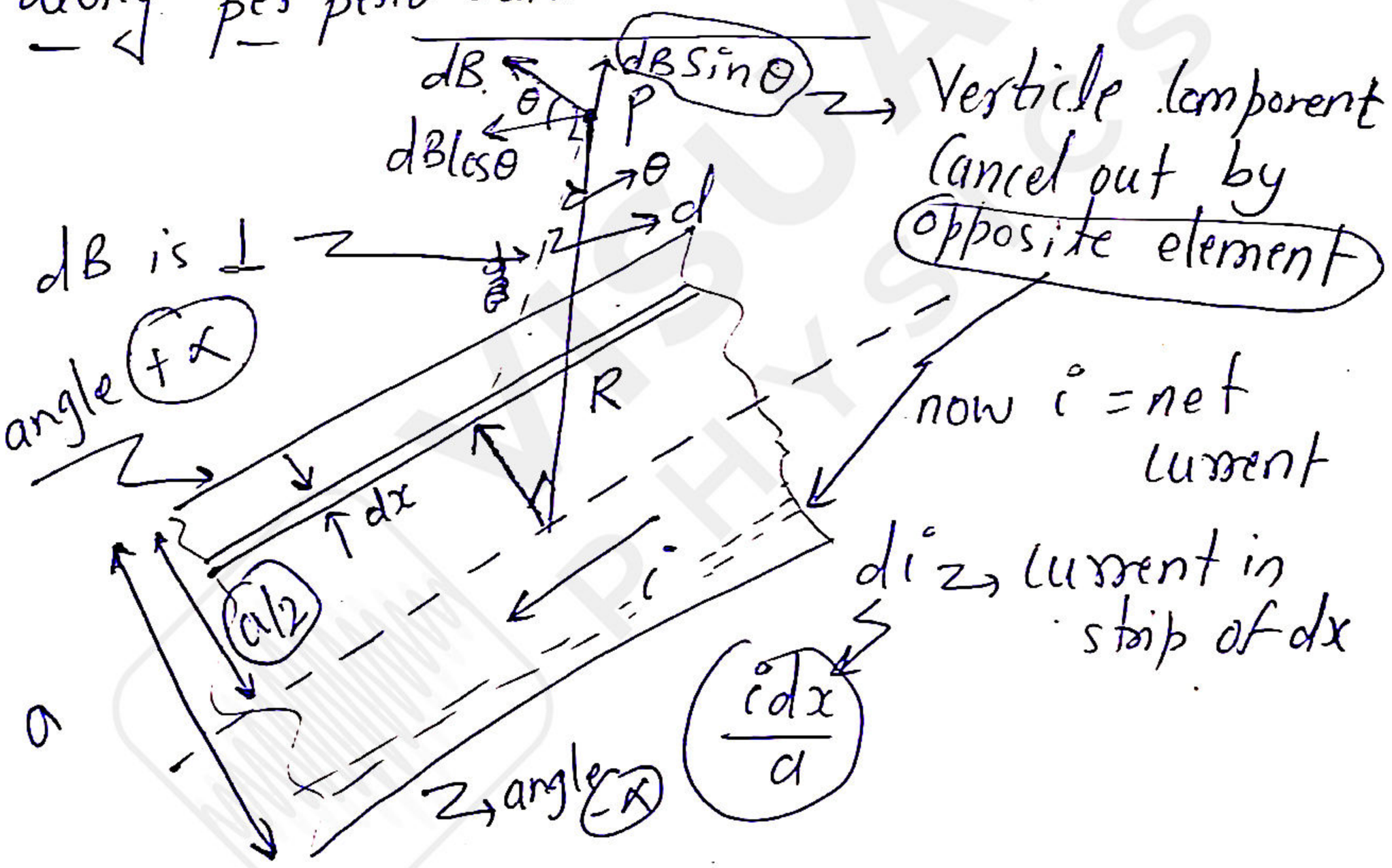
for long, $\alpha = 90^\circ = \beta = 90^\circ$

$\Rightarrow$ $\boxed{B_{net} = \mu_0 n i}$

Magnetic field $\vec{B}$ at point P, at a distance R from centre of a Flat strip, width 'a', along perpendicular Bisector:



$$\begin{cases} dB = \dfrac{\mu_0 di}{2\pi d} = \dfrac{\mu_0}{2\pi}\dfrac{i(dx/a)}{R\sec\theta} \\ dB_{net} = \int dB\cos\theta \end{cases}$$

act as current wire

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$$B_{net} = \int \frac{\mu_0 i (dx/a)}{2\pi R \sec\theta} \cos\theta$$

$$= \frac{\mu_0 i}{2\pi a R} \int \frac{dx}{\sec^2\theta}$$

$x = R\tan\theta, \quad dx = R\sec^2\theta\, d\theta$

$$B_{net} = \frac{\mu_0 i}{2\pi a R} \int_{-\alpha}^{+\alpha} d\theta$$

$$\boxed{B_{net} = \frac{\mu_0 i}{2\pi a} \alpha = \frac{\mu_0 i}{\pi a} \tan^{-1}\left(\frac{a}{2R}\right)}$$

Ampere's law

$$\boxed{\oint_{\text{closed loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}}$$

closed loop integeral of magnetic field
= μ_0 time the net current enclosed inside the loop

→ current into paper plane → -ve
→ current out of paper → +ve
→ counterclockwise → positive

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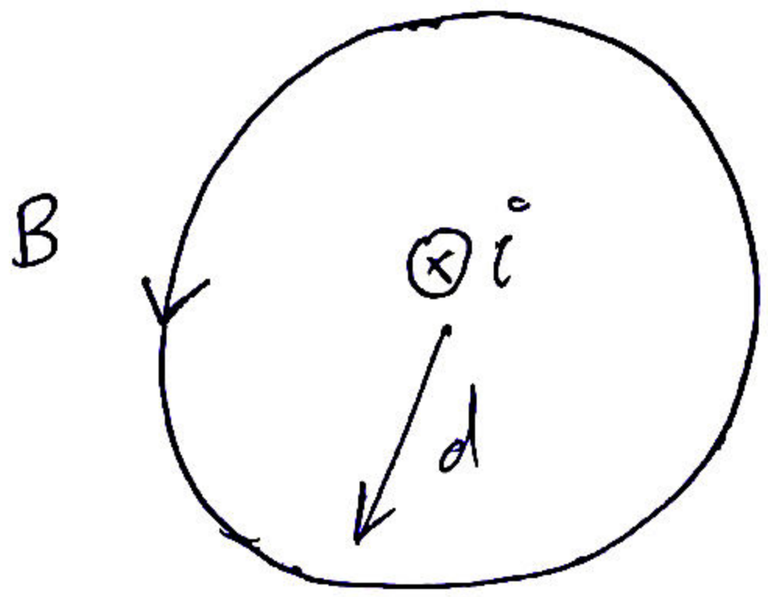
Field of a long, straight current carrying Conductor:



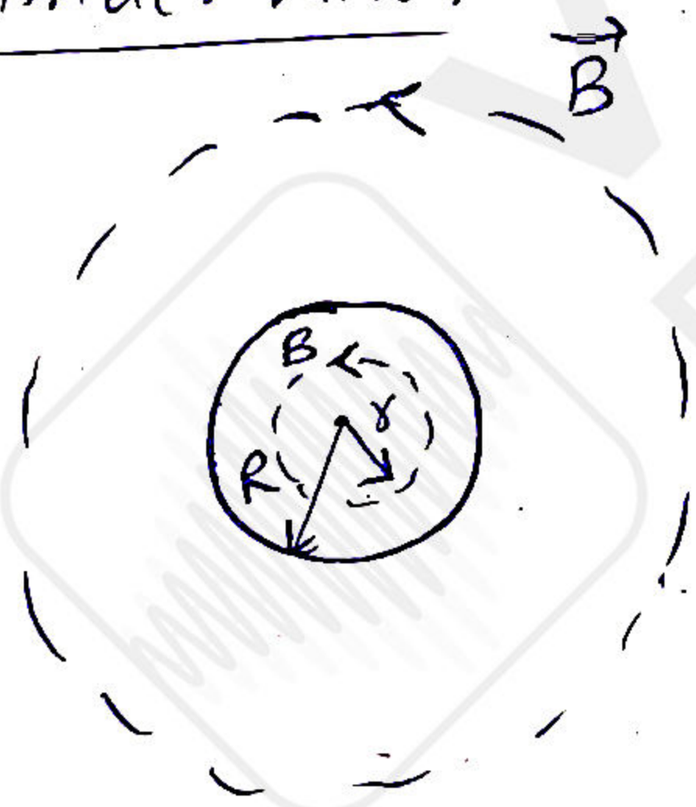
from ampere's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i$$

$$\Rightarrow B \times 2\pi d = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

Magnetic field Inside and outside a cylinder wire:



$$\pi R^2 \longrightarrow i$$

$$\Rightarrow \pi \gamma^2 \rightarrow i_1 = \frac{i \times \pi \gamma^2}{\pi R^2}$$

$$\Rightarrow \boxed{i_1 = i\left(\frac{\gamma^2}{R^2}\right)}$$

for $\gamma < R$ ↘ inside

$$\Rightarrow \oint \vec{B} \cdot \vec{dl} = \mu_0 i_1$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i \gamma^2}{R^2 \times 2\pi\gamma} = \frac{\mu_0 i \gamma}{2\pi R^2}} \quad \boxed{B \propto \gamma}$$

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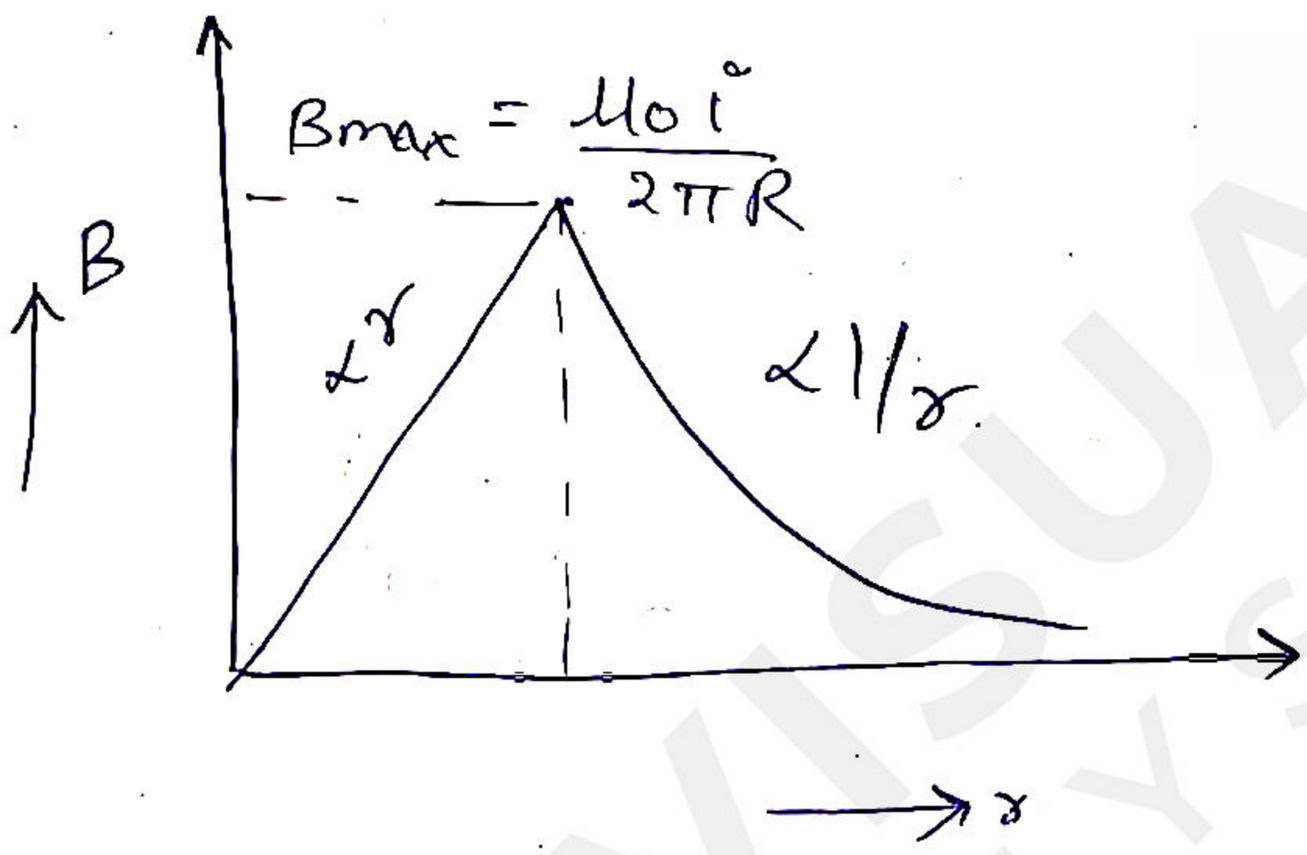
outside, means $r \geq R$

$i_{enclosed} = i$

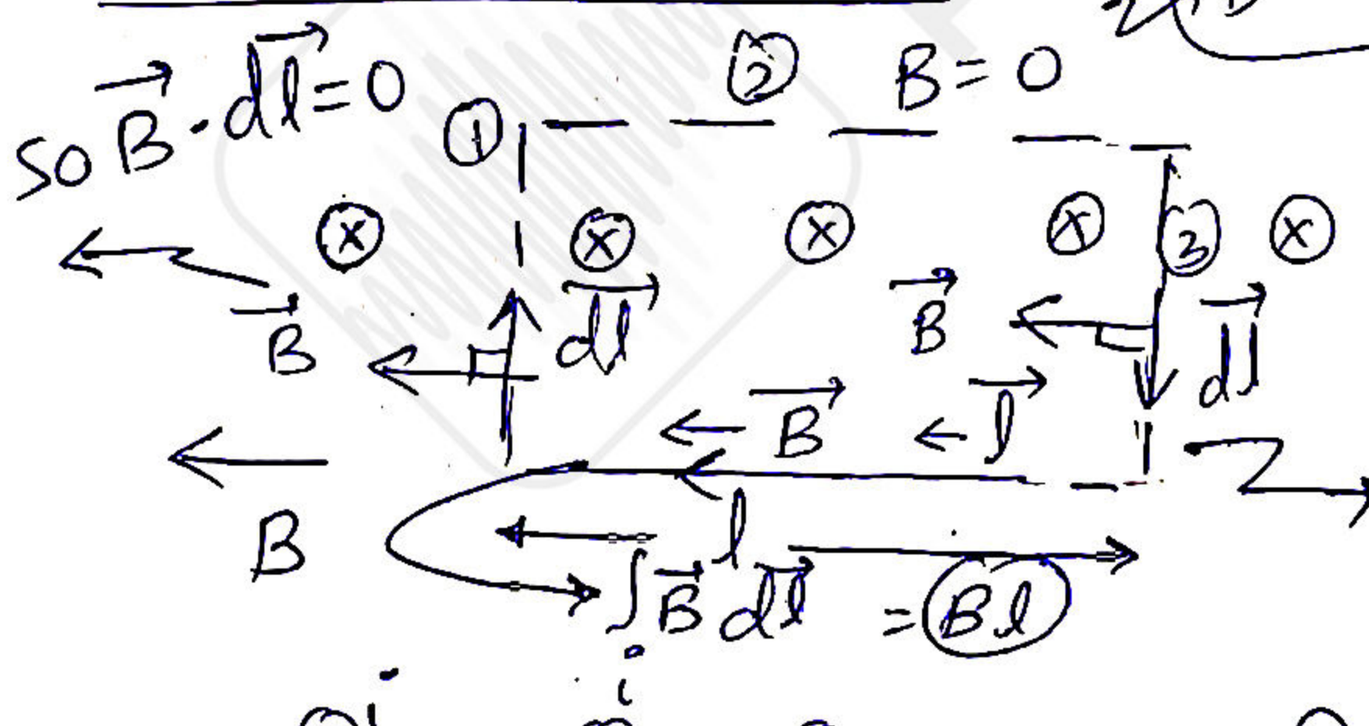
$\Rightarrow B_{net} = B = \frac{\mu_0 i}{2\pi r}$, $B \propto \frac{1}{r}$



Field for Solenoid

$\oint \vec{B} \cdot d\vec{l} = 0$

So $\vec{B} \cdot d\vec{l} = 0$



n = number of turns per unit length

net current = $(nl)i$

as $\int_1^2 \vec{B} d\vec{l} = \int_2^1 \vec{B} d\vec{l}$

$= -\oint \vec{B} \cdot d\vec{l} = 0$

So,

$\oint \vec{B} \cdot d\vec{l} = Bl = \mu_0 (nl) i$

$\boxed{B = \mu_0 n i}$

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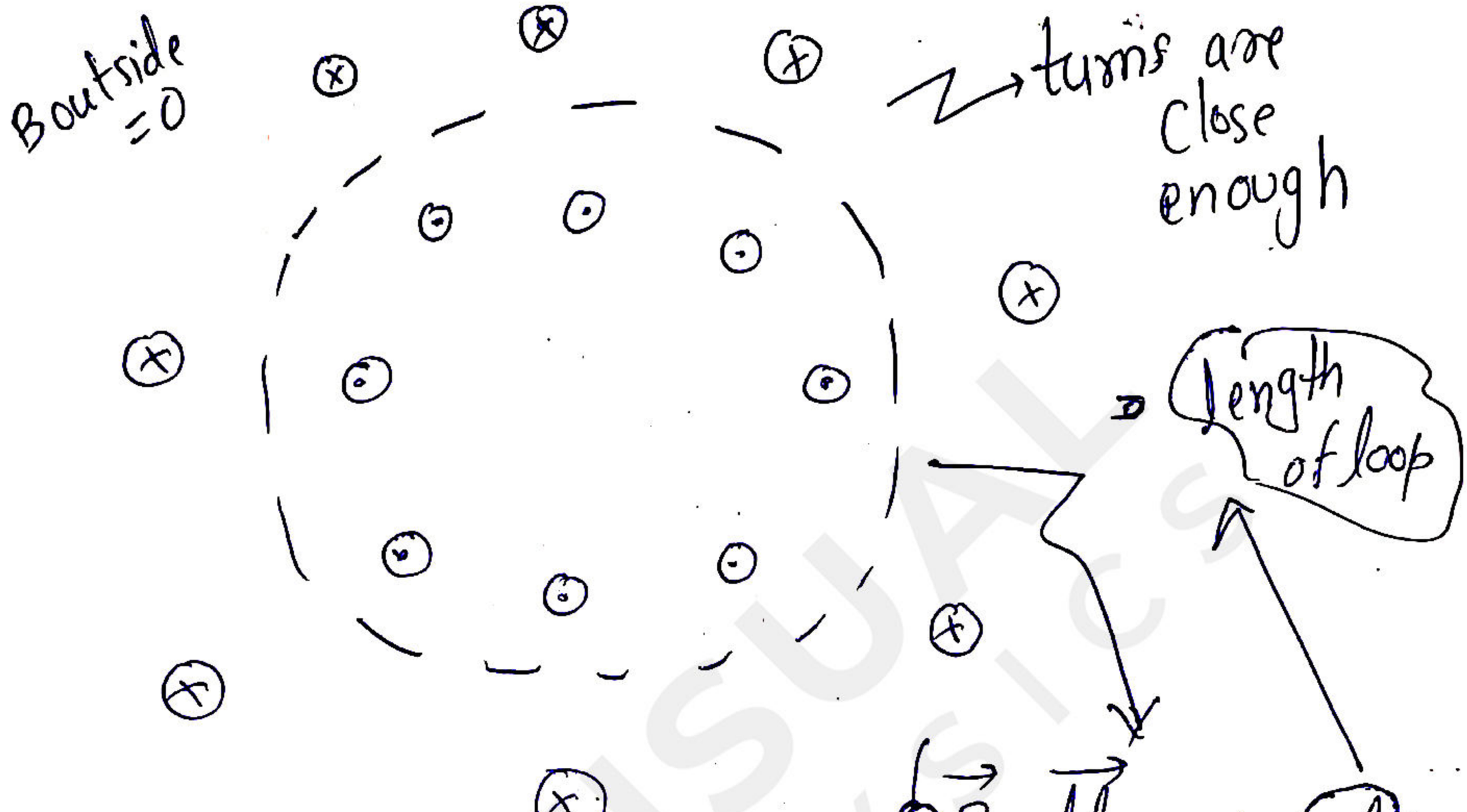
Similarly for toroid.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (nl) i$$

length of Ampere loop $= l$

$nl = N$ = net number of turns in toroid

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = Bl = \mu_0 n l i$$

$$\Rightarrow \boxed{B = \mu_0 n i}$$

no. of turns per unit length

Same as of solenoid

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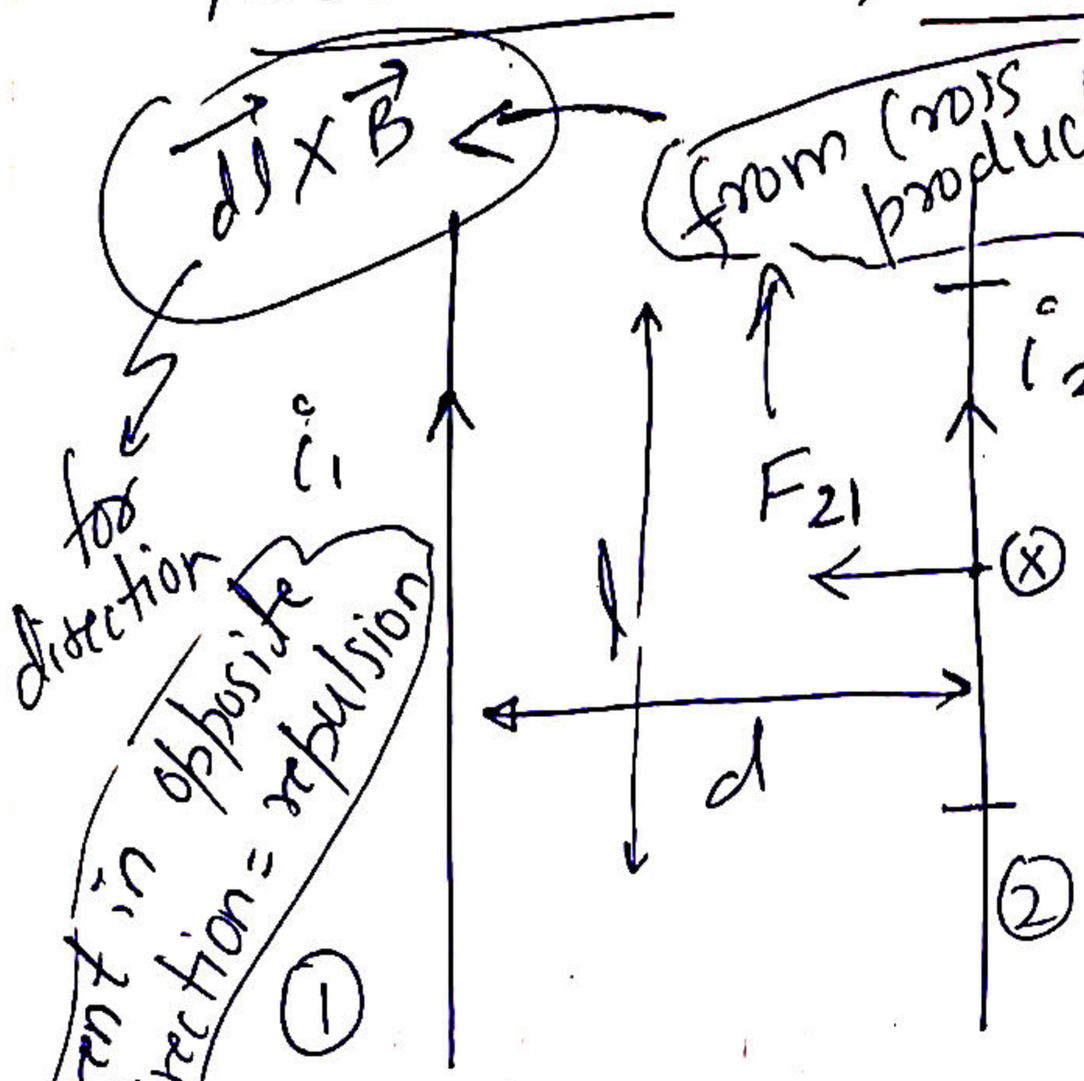
Force between parallel wires:

$\vec{dl} \times \vec{B}$ ← (from cross product we found direction)



for direction

(current in opposite direction = repulsion)

→ magnetic field by wire ① at wire ②.

$B = \frac{\mu_0 i_1}{2\pi d}$

So if current in same direction = attraction

So, force of wire ② = (current of wire ②) × length × magnetic field of wire ①.

$\Rightarrow$ F_{21} = force on ② by ① = $i_2 \times l \times \frac{\mu_0 i_1}{2\pi d}$

So force per unit length

$\Rightarrow$ $\boxed{f_{21} = \frac{F_{21}}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}}$

Similarly

f_{12} = force per unit length on ① by ②

$\boxed{f_{12} = \frac{\mu_0 i_1 i_2}{2\pi d}}$

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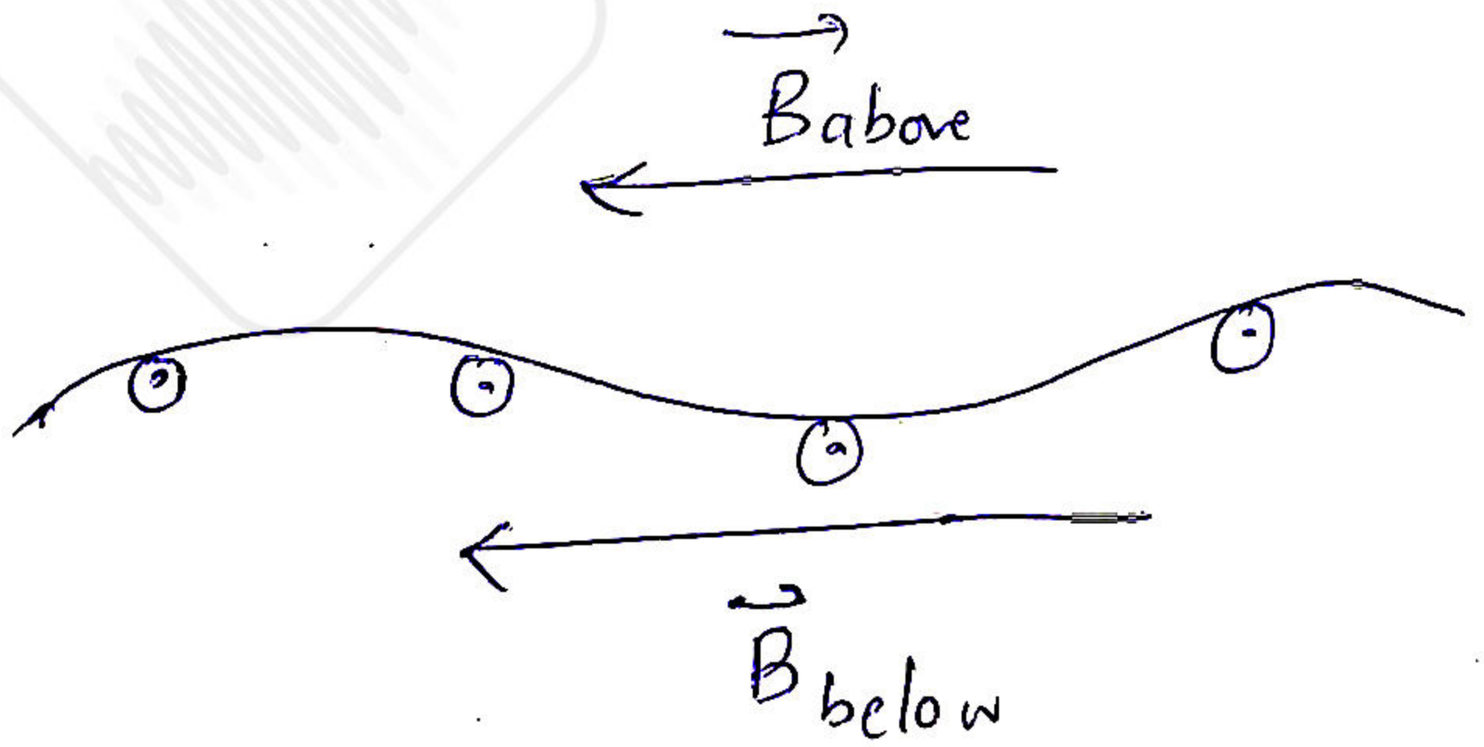
Magnetic pressure:

↳ if we have a surface current distibution

Surface current density = ∇

So, magnetic pressure = P_B

$$P_B = \nabla \left[\frac{1}{2}(B_{above} + B_{below})\right]$$

- P_B → magretic Pressure
- ∇ → surface current density
- $\frac{1}{2}(B_{above} + B_{below})$ → Average of magretic field on either side