



VISUAL
PHYSICS

SHORT NOTES

CHAPTER Inductance



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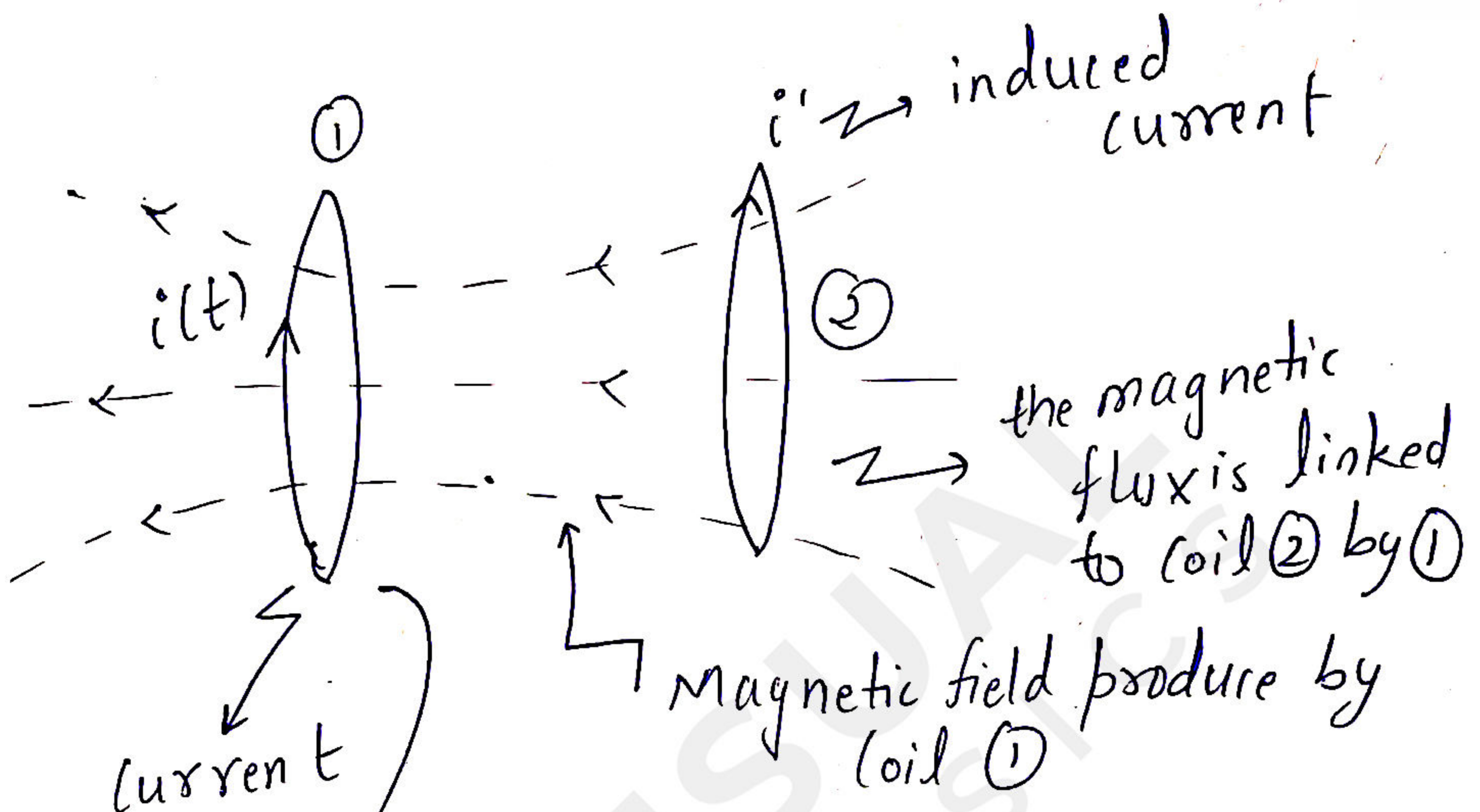


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INDUCTANCE



Now changing $i(t)$ in coil (1), changes the magnetic flux linked with (2),
Hence induced current i'

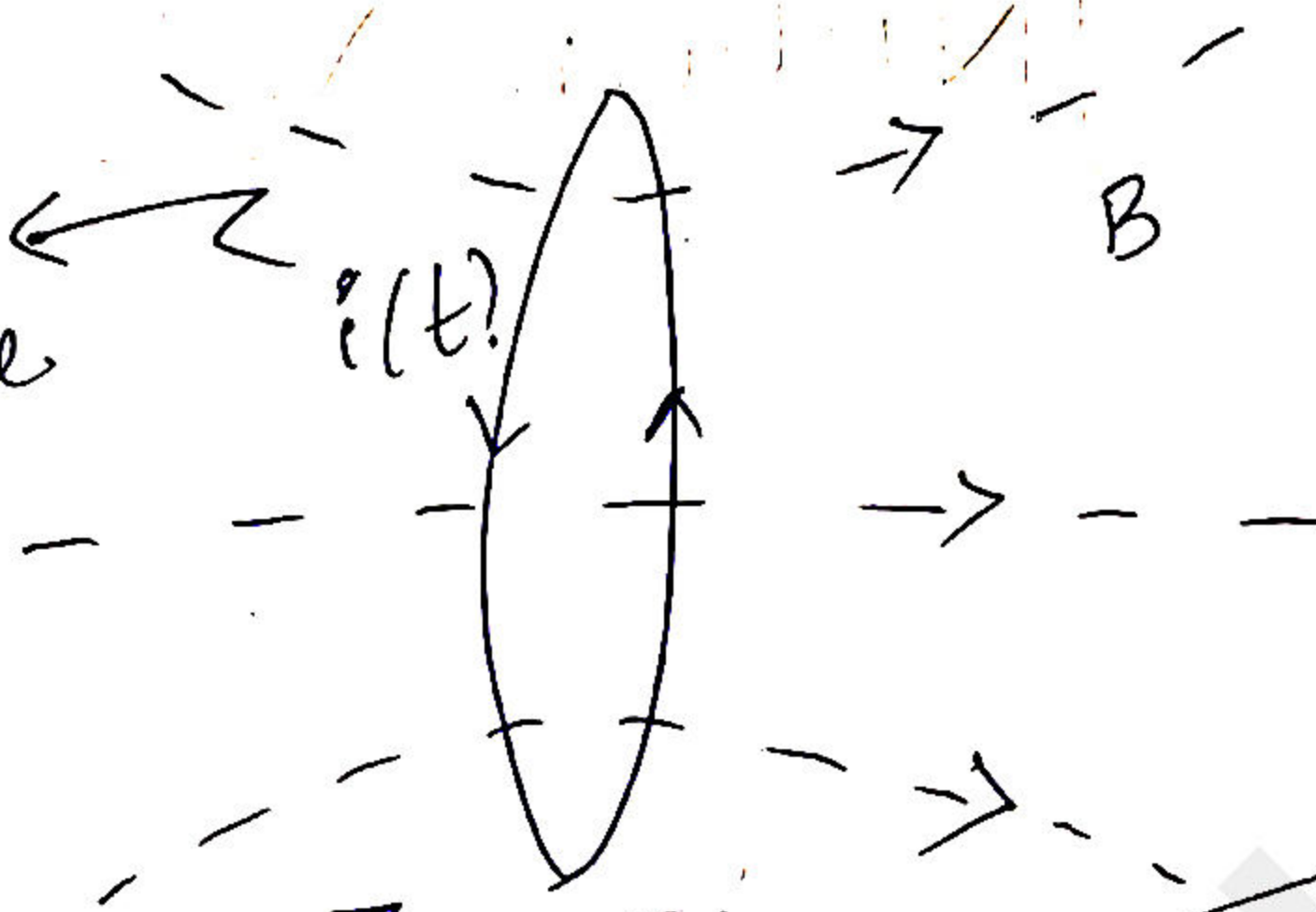
$$\text{as, } \begin{cases} B \propto i \\ \Rightarrow \phi \propto B \end{cases}$$

$$\boxed{\mathcal{E} = - \frac{d\phi}{dt}}$$

(this is called Mutual inductance)

Current induce in coil (2), when current in other coil changes.

current
changing
with time



* Due to current in coil
there is flux linked with the coil
itself

Hence, as
 $\mathcal{E} = -\frac{d\phi}{dt}$

(so if $i(t)$ changed
in coil, there is still
change in flux)

back emf is induced
that opposes the change in
current $i(t)$

emf
induced in
itself

known as self
induction

Now, for mutual inductance.

$$\phi \propto i$$

Net flux linked.

$$\Rightarrow \boxed{\phi = M i}$$

↳ Mutual inductance

Now for N turns,

$$\phi = N \phi' \quad \begin{array}{l} \text{linked} \\ \text{flux with} \\ \text{one coil} \end{array}$$

$$\therefore \boxed{M = \frac{N \phi'}{i}}$$

For Self Inductance

Property of coil.

$$\phi \propto i$$

$$\Rightarrow \boxed{\phi = L i}$$

independent of current

↳ self Inductance.

$$\phi = N \phi' \quad \text{linked with one coil.}$$

$$\boxed{L = \frac{N \phi'}{i} = \frac{N \phi'}{i}}$$

current

↳ depends on geometrical feature only

unit of Self Inductance

$$= \frac{T m^2}{A} \rightarrow \text{Henry (H)}$$



Symbol

Net ϕ

Now, $L = \frac{N \phi'}{i}$

$$\Rightarrow \boxed{\mathcal{E} = - \frac{d\phi}{dt} = -N \frac{d\phi'}{dt}}$$

Now $N \phi' = Li'$

Opposes change

$$\Rightarrow \boxed{\mathcal{E} = -L \frac{di}{dt}}$$

$$= \ominus L \frac{di}{dt}$$

back emf

do not depend on magnitude of i

depends on change of i rate of

$\Rightarrow$ | current $\uparrow \rightarrow \mathcal{E}$ such that i decrease and vice-versa
induced emf

(polarity of $\varepsilon \rightarrow$ opposite to i)
 $\hookrightarrow$ if $i \uparrow$
 (polarity of $\varepsilon \rightarrow$ same direction of i)
 $\hookrightarrow$ if $i \downarrow$

$\rightarrow$ Inductor tries to maintain the same current through it.

Inductance of solenoid

$$B = \mu_0 n i$$

$\hookrightarrow$ Number of turns per unit length

So, $\phi' = BA = \mu_0 n i A$

flux per coil

Now net flux

$$\phi = N \phi'$$

$$\phi = N (\mu_0 n i) A$$

$n l$

$$\Rightarrow \phi = (n^2 \mu_0 i l) A$$

Now,

$$L = \frac{\text{Shared / Net flux}}{i} = \frac{\Phi}{i}$$

$$L = \frac{n^2 l \mu_0 i A}{i}$$

$$L = n^2 l \mu_0 A$$

(depends on geometry)

Now if there other medium

$$\mu = \mu_0 \mu_r$$

So

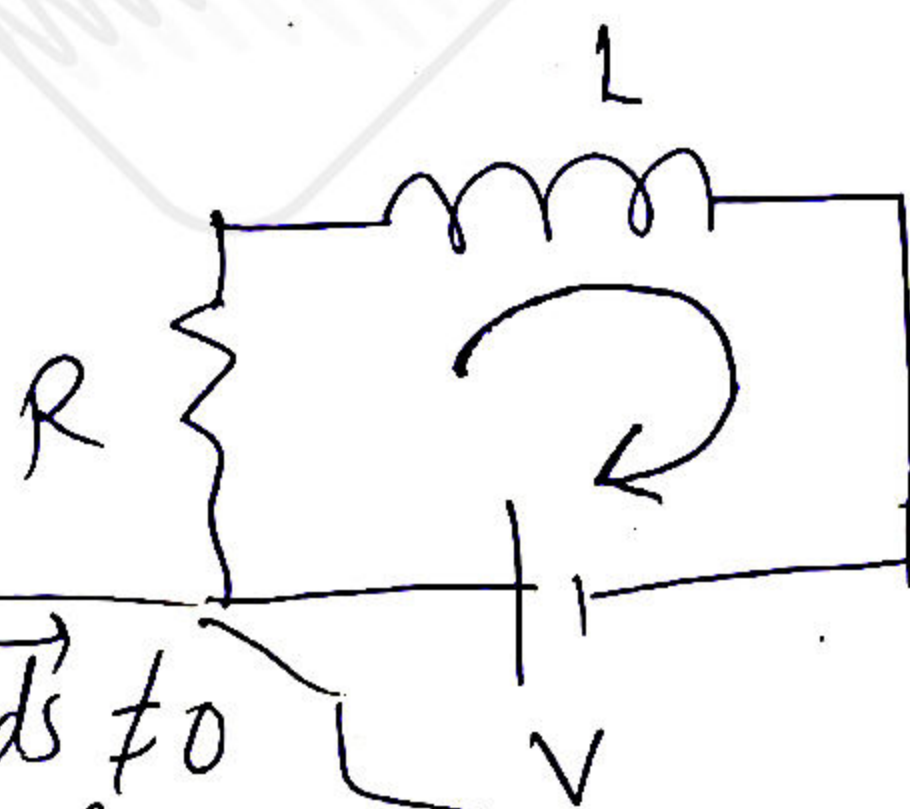
$$L = n^2 l \mu_r \mu_0 A$$

for Kirchhoff's law

Now,

$$\mathcal{E} = -L \frac{di}{dt}$$

also considered as p.d across Inductor



technically

Kirchhoff's rule cannot be applied

as $\oint \vec{E} \cdot d\vec{s} \neq 0$
 $\hookrightarrow$ for Non conservative field

as induced ' $\mathcal{E}$ ' is not conservative field

The actual Rule is faraday's path

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\phi}{dt} \rightarrow \underbrace{\mathcal{E}}_{\text{induced}}$$

$$V - iR = \mathcal{E}$$

$$\Rightarrow V - iR - \mathcal{E} = 0$$

* Same as of kirchhoff's loop law
 $\oint \vec{E} \cdot d\vec{s} = 0$

Considering the correct polarity of $\mathcal{E}$

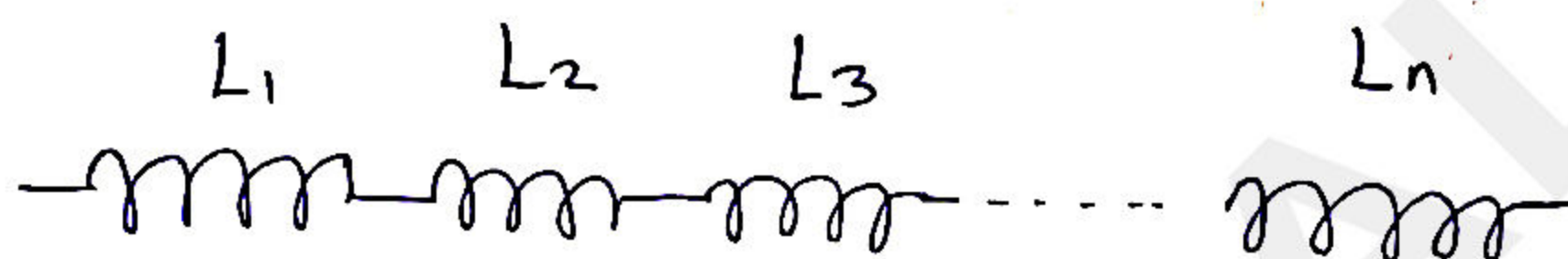
so for getting equation we use the same method as kirchhoff's law

$$V - iR - \mathcal{E} = 0$$

but technically it is farady's path method.

* by considering $\mathcal{E}$ as p.d across inductor

Series parallel combination :



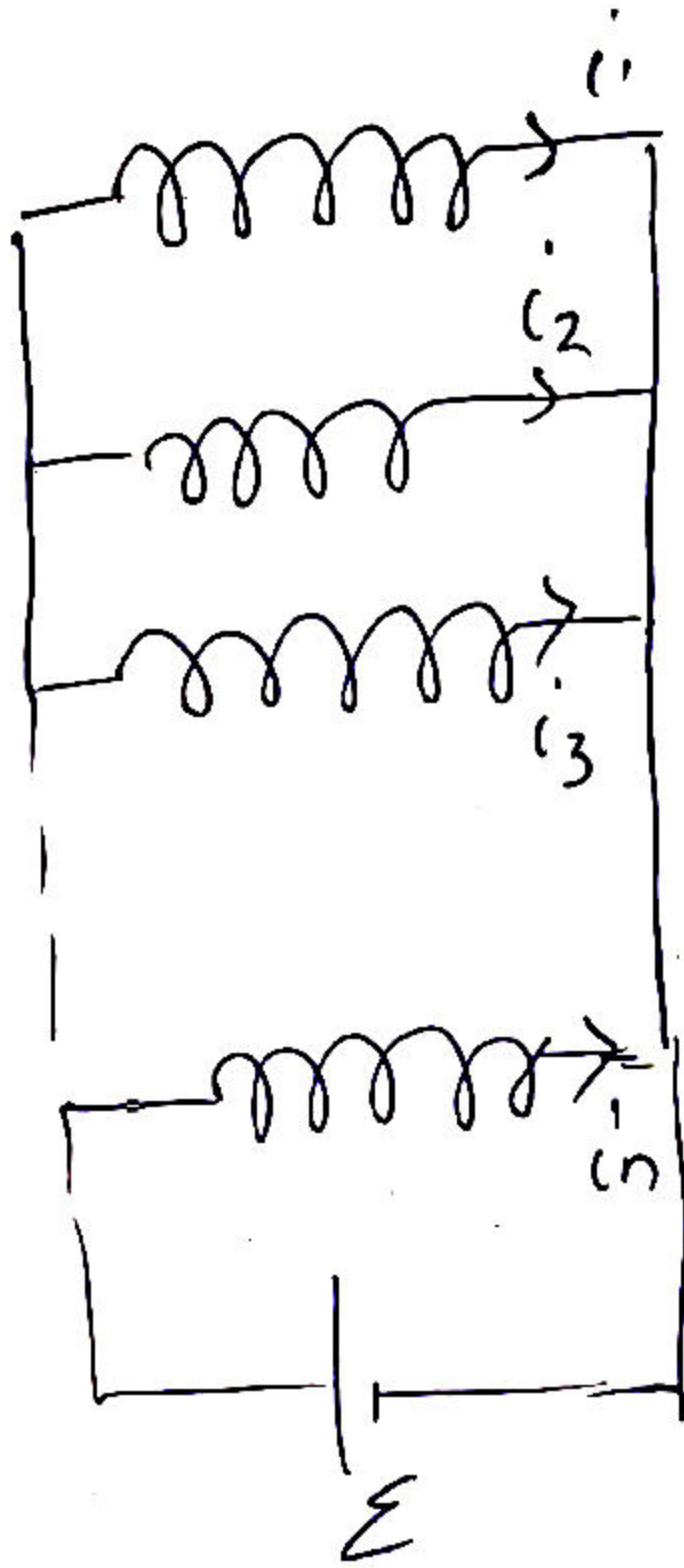
$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \dots + \mathcal{E}_n$$

$$\mathcal{E} = -L_1 \frac{di}{dt} + (-L_2 \frac{di}{dt}) + \dots + (-L_n \frac{di}{dt})$$

$$\Rightarrow \mathcal{E} = -(L_1 + L_2 + \dots + L_n) \frac{di}{dt}$$

$$\Rightarrow \boxed{L_{\text{net}} = L_1 + L_2 + L_3 + \dots + L_n}$$

$$\Rightarrow \boxed{\mathcal{E} = -L_{\text{net}} \frac{di}{dt}}$$



$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \dots + \mathcal{E}_n$$

$$\Rightarrow i = i_1 + i_2 + \dots + i_n$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \dots + \frac{di_n}{dt}$$

$$\Rightarrow -\frac{\mathcal{E}}{L_{eq}} = -\left[\frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2} + \dots + \frac{\mathcal{E}}{L_n} \right]$$

$$\Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

Mutual Inductance :

$\phi_{21} \rightarrow$ flux in coil 2 due to ~~the~~ current in coil 1

$$\phi_{21} \propto i_1$$

$$\phi_{21} = M_{2,1} i_1$$

Mutual Inductance of the second coil wrt to first coil

Now if second coil have 'N' turns

$$\Rightarrow \boxed{N_2 \phi_{2,1} = M_{2,1} i_1}$$

$$N_2 \frac{d\phi_{2,1}}{dt} = M_{2,1} \frac{di_1}{dt}$$

$$\Rightarrow \mathcal{E}_2 = -N_2 \frac{d\phi_{2,1}}{dt}$$

$$\mathcal{E}_2 = -M_{2,1} \frac{di_1}{dt} \Rightarrow \boxed{M_{2,1} = \frac{N_2 \phi_{2,1}}{i_1}}$$

Similarly if coil 2 is carrying current

$\phi_{1,2} \rightarrow$ Flux in coil 1 due to current in coil 2

always $N_1 \phi_{1,2} \propto i_2$

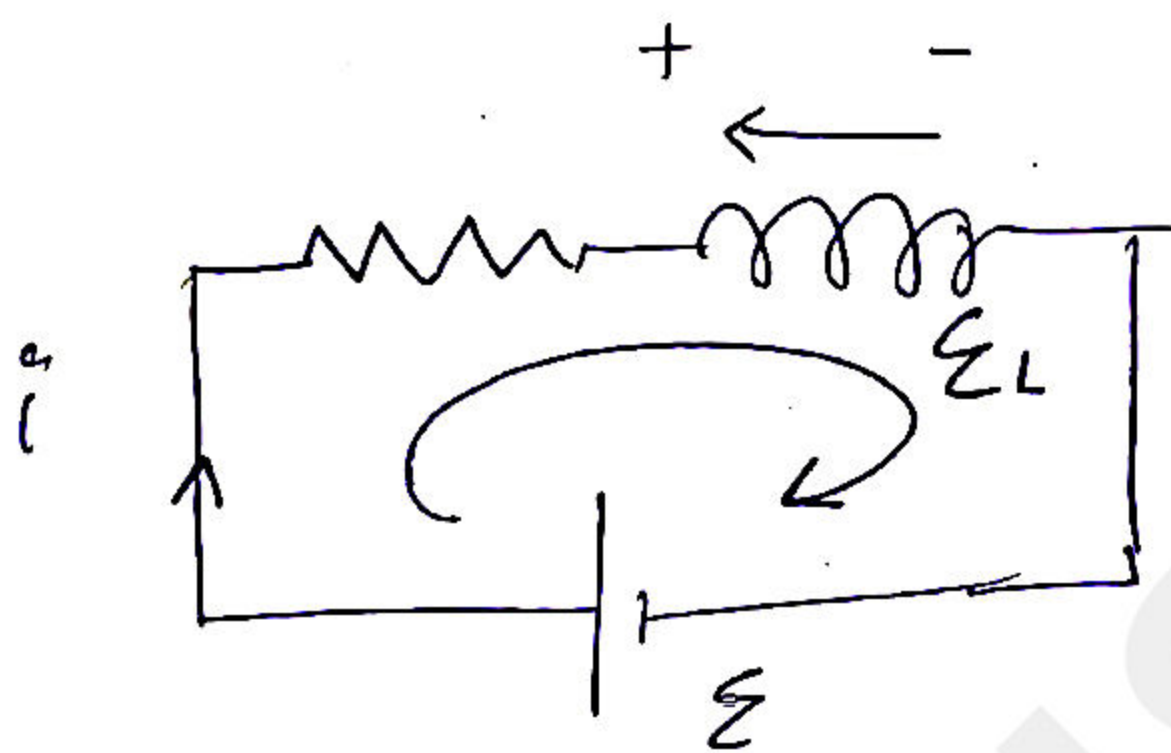
$$\Rightarrow N_1 \phi_{1,2} = M_{1,2} i_2$$

$$\star \boxed{M_{2,1} = M_{1,2} = M}$$

Mutual Inductance between coil

Mutual Inductance, of the first coil w.r.t second coil

$$\boxed{\mathcal{E}_1 = -M \frac{di_2}{dt}}, \quad \boxed{\mathcal{E}_2 = -M \frac{di_1}{dt}}$$



Inductor does not allow a sudden change of current

Series R-L circuit

$$\mathcal{E} - iR - \mathcal{E}_L = 0$$

$$\text{So, } \mathcal{E} - iR - L \frac{di}{dt} = 0$$

$$\Rightarrow L \frac{di}{dt} = \mathcal{E} - iR$$

$$\int_0^i \frac{di}{\mathcal{E} - iR} = \int_0^t \frac{dt}{L}$$

$$\left. \frac{\ln(\mathcal{E} - iR)}{-R} \right|_0^i = \frac{t}{L}$$

$$\ln\left(\frac{\varepsilon - iR}{\varepsilon}\right) = -\frac{R}{L}t$$

$$1 - \frac{R}{\varepsilon}i = e^{-(R/L)t}$$

$$\Rightarrow \frac{R}{\varepsilon}i = 1 - e^{-(R/L)t}$$

$$i = \frac{\varepsilon}{R} [1 - e^{-(R/L)t}]$$

$$t = 0$$

$$i_0 = \frac{\varepsilon}{R} (1 - e^0)$$

$$\Rightarrow i_0 = 0$$

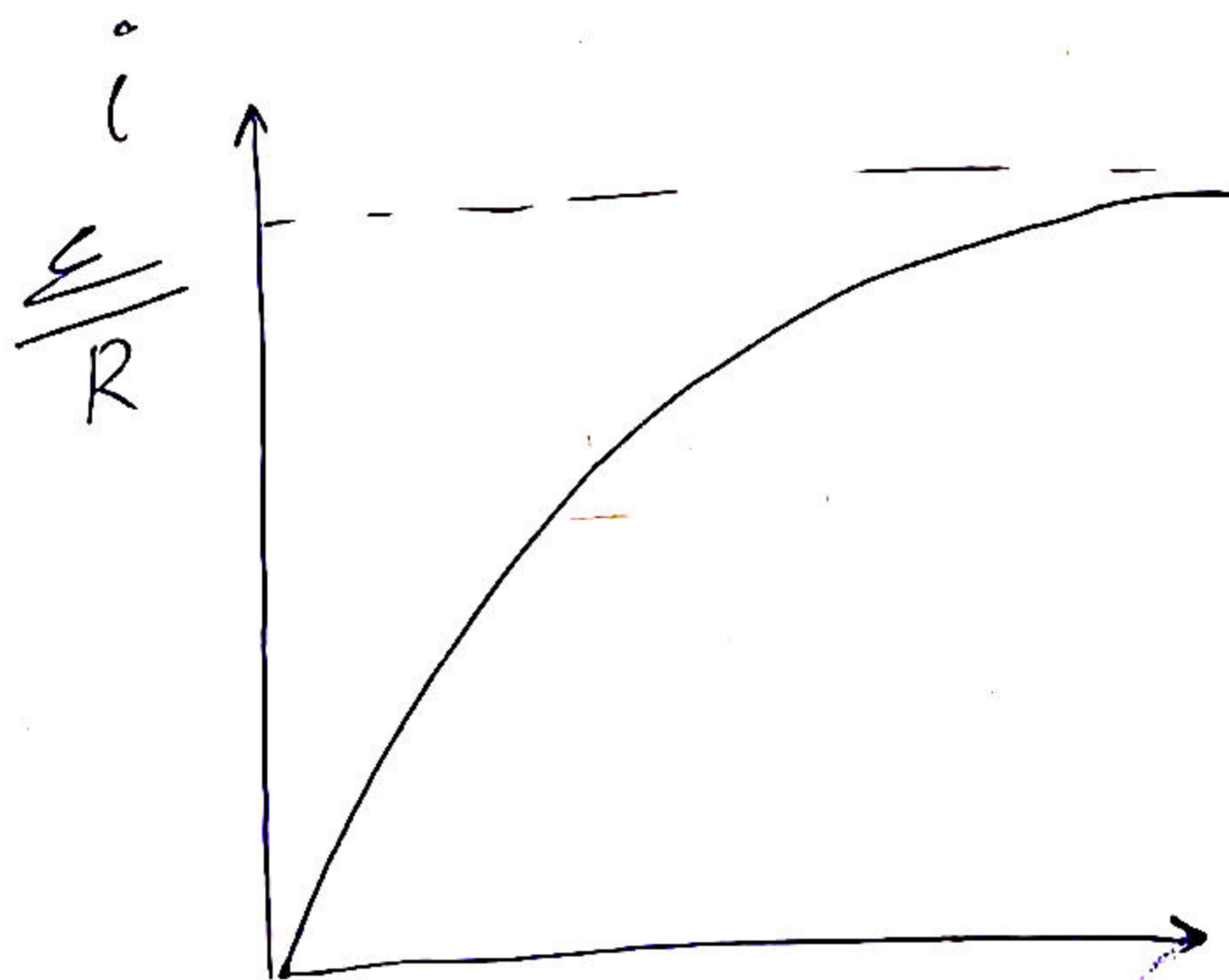
Just when battery is connected there is no current in circuit

$$t \rightarrow \infty$$

$$i_\infty = \frac{\varepsilon}{R} (1 - e^{-\infty})$$

$$\Rightarrow i_\infty = \frac{\varepsilon}{R}$$

When, there is no inductor

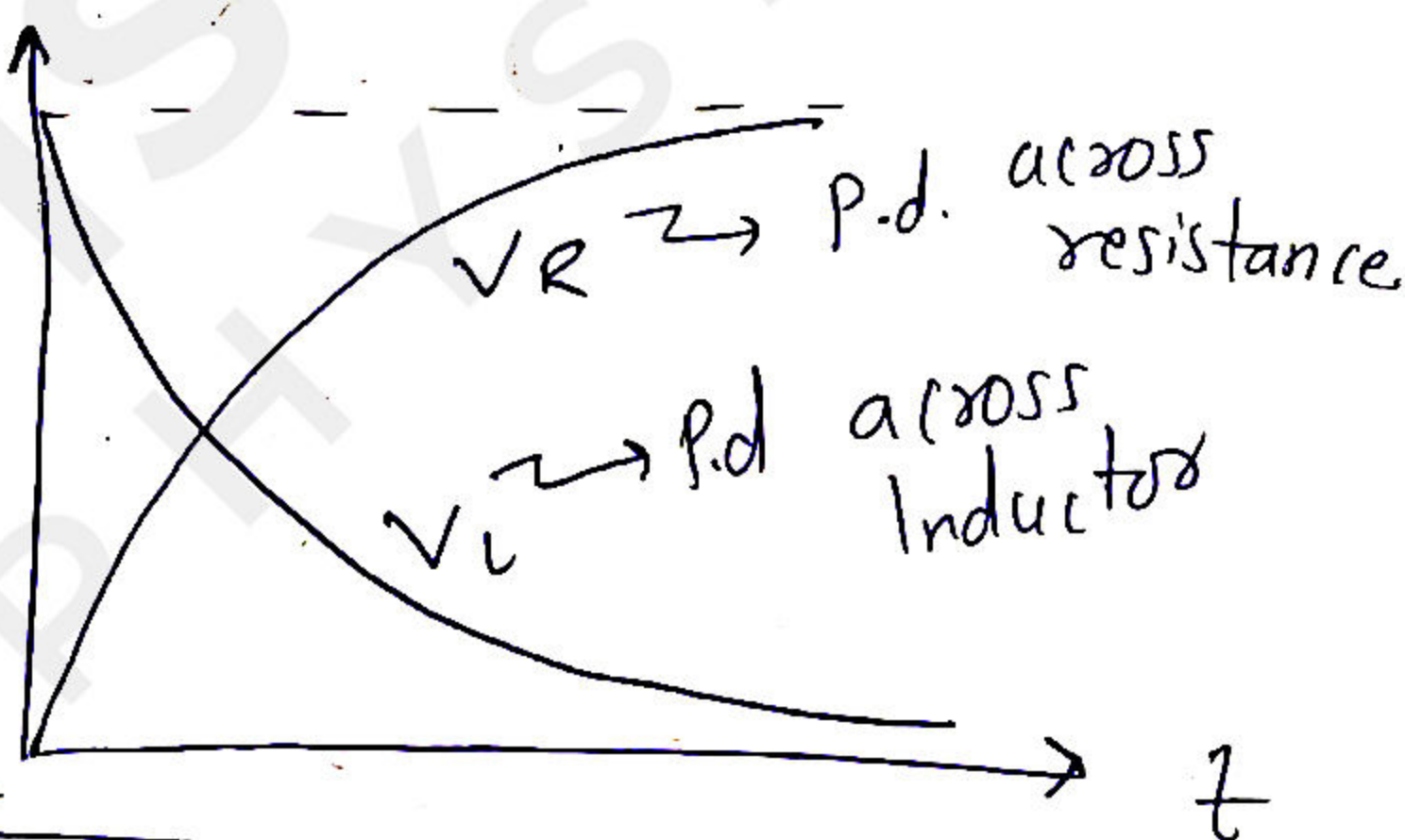


if $i_0 \rightarrow 0$
 $\Rightarrow$ p.d across Inductor
 $= -\varepsilon$

when $i \rightarrow 0$
 $\star$ means induce Inductor
 is zero

$\star$ So, (when Inductor is connected it acts as open circuit) means $\frac{di}{dt} = 0$

after τ $i \rightarrow 63\%$ of i_{max}



$$i = \frac{\varepsilon}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t} \right)$$

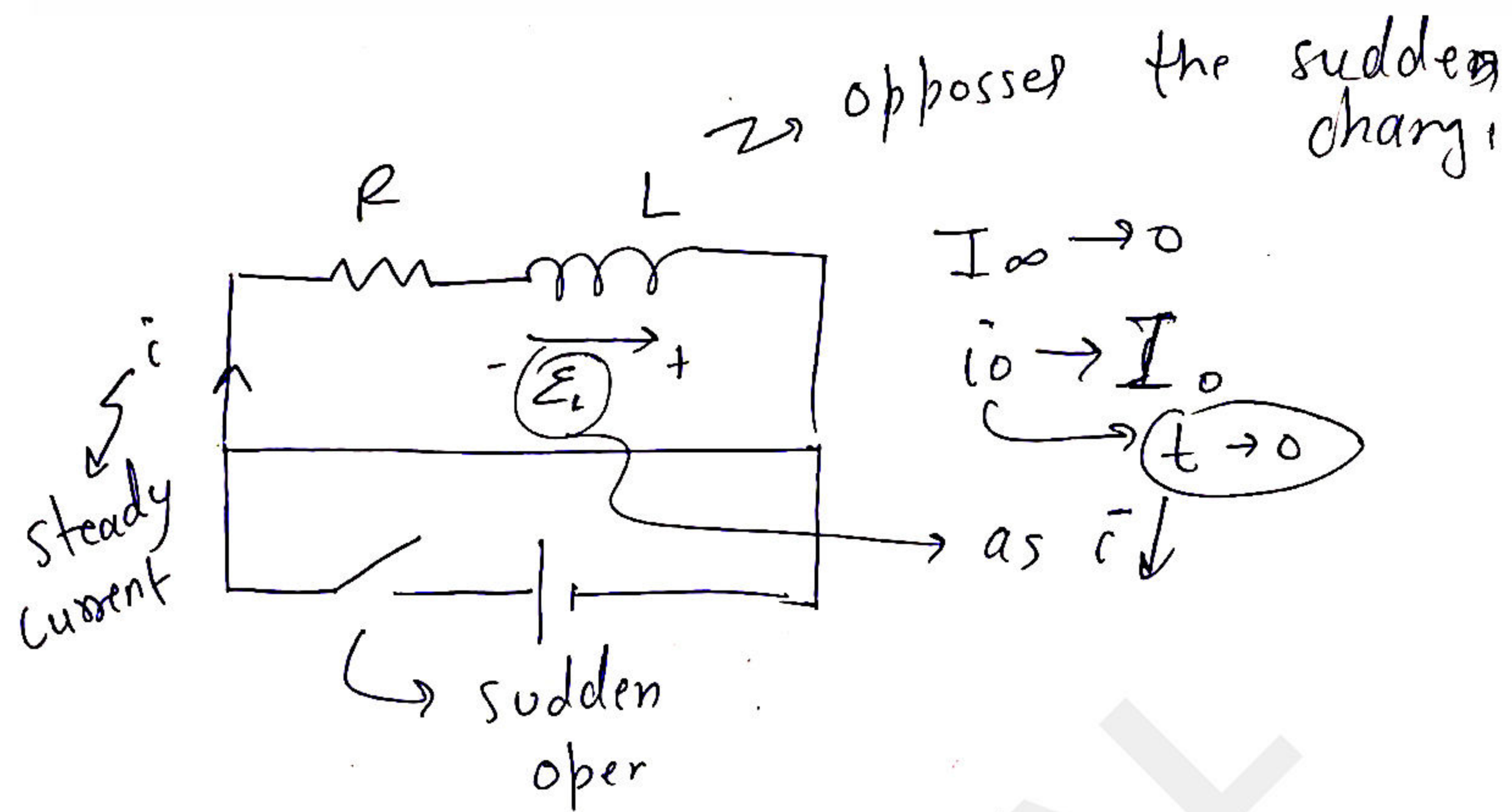
Now, $\frac{L}{R} \rightarrow \tau$ Inductive time constant

unit
 $[T]$

charge on capacitor

$$q = q_0 \left(1 - e^{-\frac{t}{RC}} \right)$$

for RC circuit



$$\Rightarrow \mathcal{E}_L = -L \frac{di}{dt}$$

$$\Rightarrow -Ri + \left(-L \frac{di}{dt}\right) = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L} dt$$

$$\Rightarrow i = I_0 e^{-(R/L)t}$$

$t \rightarrow \tau$
 $i \rightarrow 37\% \text{ of } I_0$

$$\left[\frac{L}{R} \rightarrow \tau_L \right]$$

Time Constant

P.d of R

$$\mathcal{E}_R = \left(I_0 e^{-(R/L)t} \right) R$$

$$\mathcal{E}_L = -L \frac{di}{dt}$$

$$\mathcal{E}_L = (I_0 R) e^{-(R/L)t}$$

if we suddenly disconnect from circuit

$$\rightarrow \frac{di}{dt} \rightarrow \infty$$

means,

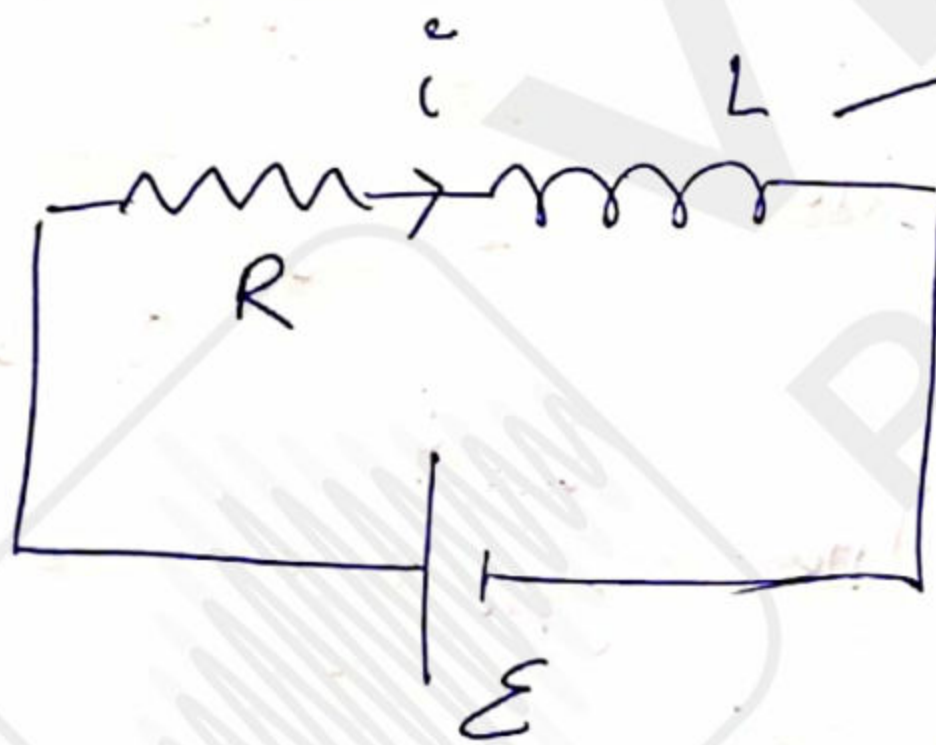
$$\mathcal{E}_L = L \frac{di}{dt} \Rightarrow \infty$$

sparking

hence infinite back emf.

damage to Inductor etc.

Energy in RL Circuit:



it has magnetic field

so store energy in form of magnetic field

$$\left(\mathcal{E} - iR - L \frac{di}{dt} = 0 \right) \times i$$

$$\Rightarrow i\mathcal{E} - i^2R - L i \frac{di}{dt} = 0$$

Power delivered by Battery

Power dissipated in Resistor

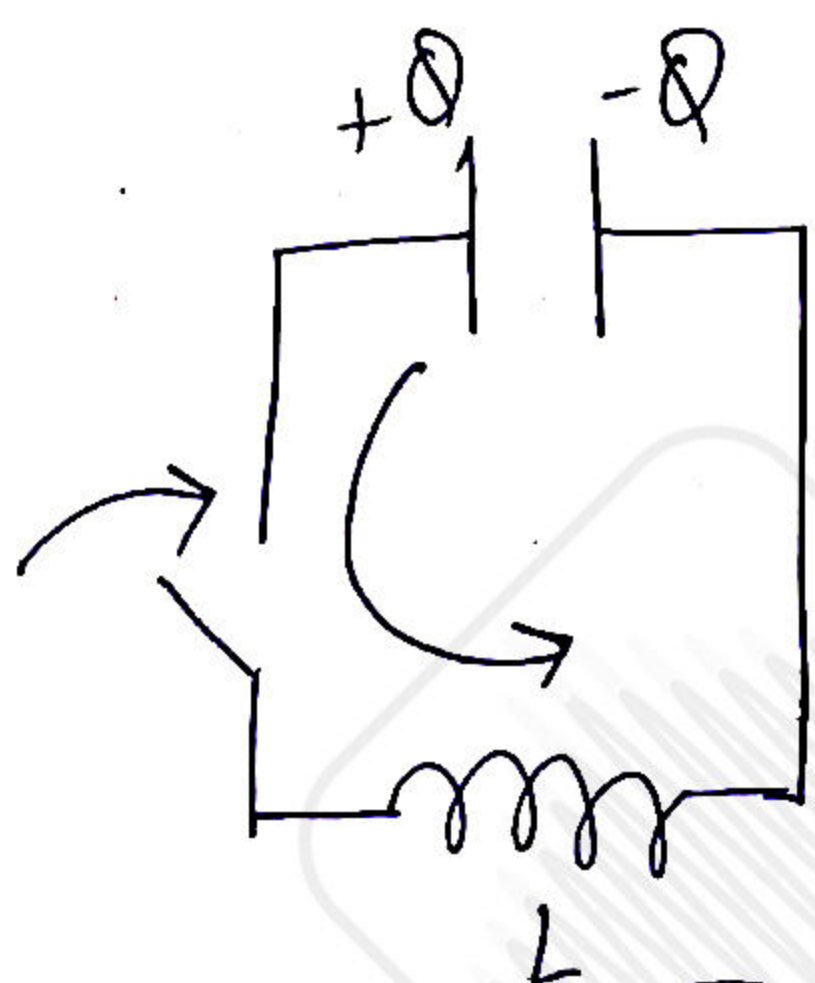
$$\frac{dU_L}{dt} = L i \frac{di}{dt}$$

$$U_L = \frac{Li^2}{2}$$

similar for the Capacitor, in form of Electric field

$$U_c = \frac{1}{2} \frac{Q^2}{C}$$

LC Circuit-1



→ so sudden release of charge → current

Inductor opposes sudden change.

So capacitor is not instantly discharged but takes a finite time.

So after some time polarity of capacitor reversed

it keeps on repeating

so there would be electric oscillation
 $\rightarrow$ LC oscillation.

as energy is not lost $\rightarrow$ as $(R=0)$

$\rightarrow$ Energy oscillates in Electrical energy in capacitor

magnetic energy in inductor

$\leftarrow$ to

at some time t ,

charge $\rightarrow q$

current $\rightarrow i$

$q \downarrow \Rightarrow i \uparrow$

$$i = -\frac{dq}{dt}$$

so,
$$-L \frac{di}{dt} + \frac{q}{C} = 0$$

so
$$-L \frac{d^2 i}{dt^2} + \frac{1}{C} \frac{dq}{dt} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0$$

$\Downarrow$
SHM equation

$\nearrow$ max current

$$\Rightarrow i = i_0 \sin(\omega t + \phi)$$

so,
$$\omega = \frac{1}{\sqrt{LC}}$$

$t=0, i=0 \Rightarrow \phi=0$

$$i = i_0 \sin \omega t$$

$$\text{as, } -L \frac{di}{dt} + \frac{q}{C} = 0$$

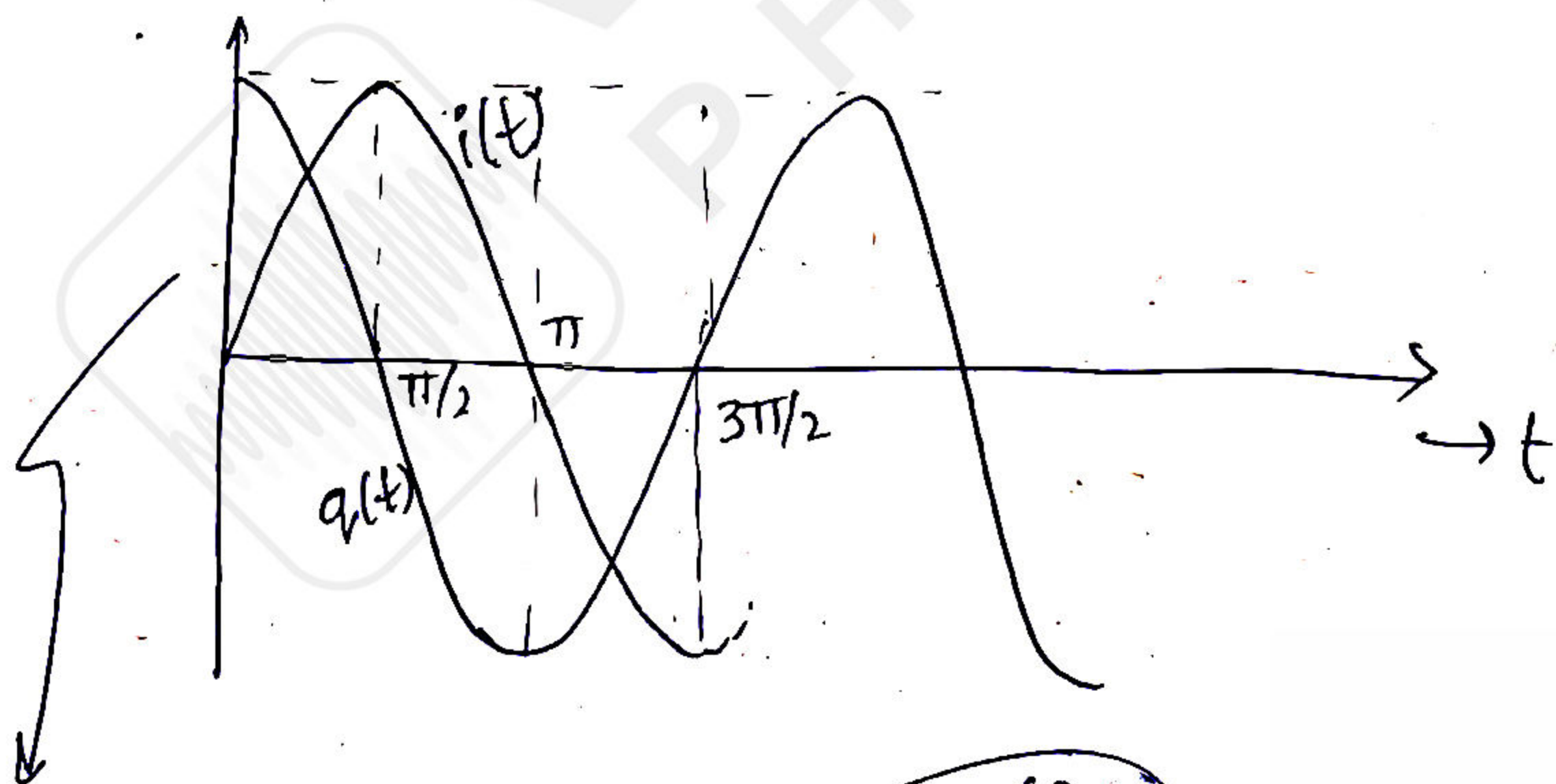
$$-L \left[-\frac{d^2 q}{dt^2} \right] + \frac{q}{C} = 0$$

$$q = Q_0 \sin(\omega t + \phi)$$

$$t = 0, q = Q_0 \Rightarrow \phi = \pi/2$$

$$\Rightarrow \boxed{q = Q_0 \cos \omega t}$$

So, charge and current in LC oscillates by $\pi/2$ phase difference.



when $i \rightarrow i_{\text{max}} \rightarrow q = 0$ and vice-versa

Now at any instant

$$V_C + V_L = \text{Constant} = E$$

so, when $V_C = 0$, $q = 0$

$\rightarrow V_L = E$; max energy in Inductor

$$\text{Now, } (V_C)_{\max} = \frac{Q_0^2}{2C}$$

$$\text{and } (V_L)_{\max} = \frac{1}{2} L I_{\max}^2$$

$$\Rightarrow (V_C)_{\max} = (V_L)_{\max}$$

$$\Rightarrow I_{\max}^2 = \frac{Q_0^2}{LC}$$

$$\text{as } \omega = \frac{1}{\sqrt{LC}}$$

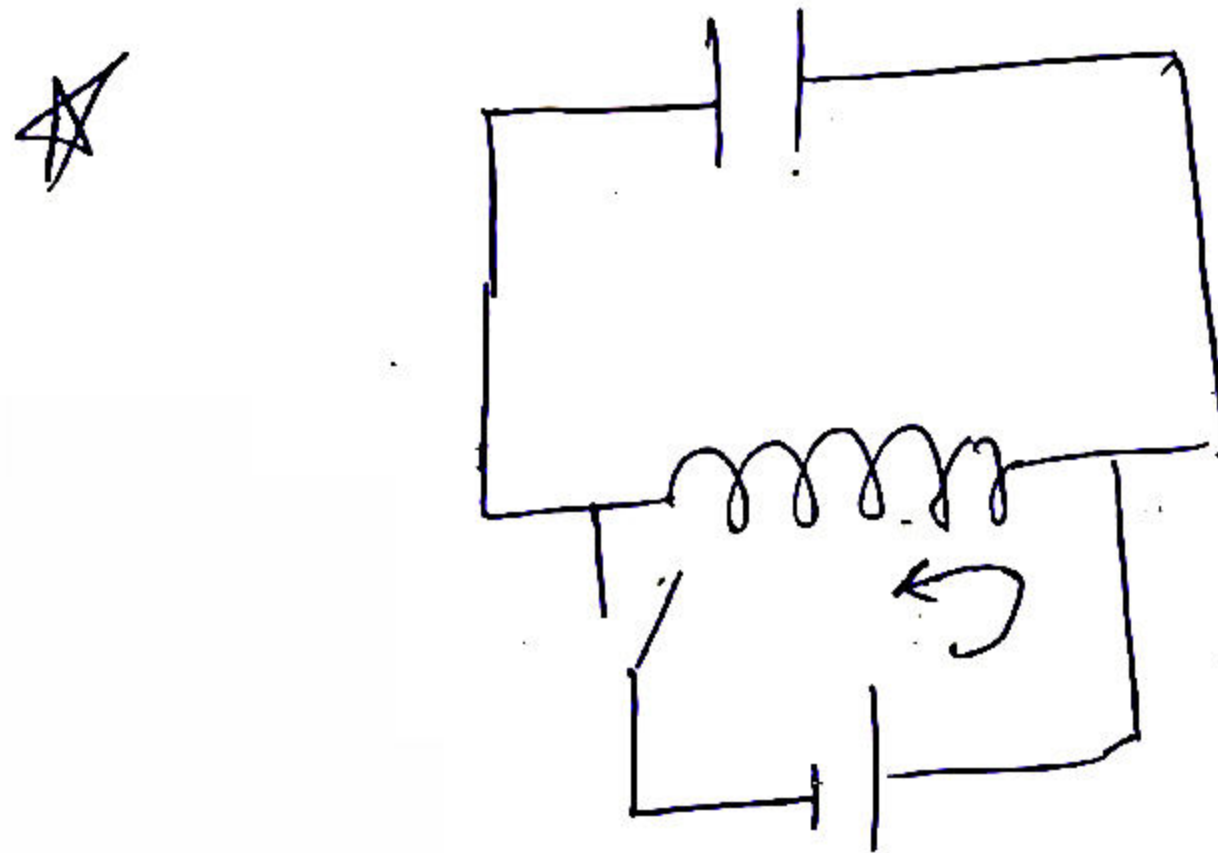
$$\Rightarrow \boxed{I_{\max} = I_0 = \omega Q_0}$$

★

when $q \downarrow \rightarrow i \uparrow$

and $q \uparrow \rightarrow i \downarrow$

if $t=0$, $q=0$ and current was there in inductor



$$\begin{cases} i(t) = I_0 \cos \omega t \\ q(t) = \left(\frac{I_0}{\omega} \right) \sin \omega t \end{cases}$$

just add by phase of $\pi/2$

$t=0, i=0, q=Q_0$	$t=0, i=I_0, q=0$
$i(t) = \omega Q_0 \sin \omega t$ $q(t) = Q_0 \cos \omega t$	$i(t) = I_0 \cos \omega t$ $q(t) = \left(\frac{I_0}{\omega} \right) \sin \omega t$

$V_c(t) = \frac{q(t)}{C} = \frac{Q_0}{C} \cos \omega t$
Varies Sinusoidally

$V_L(t) = L \frac{di}{dt}$
 $= L \frac{Q_0}{LC} \cos \omega t$

$V_L = L \omega^2 Q_0 \cos \omega t$

Varies Sinusoidally

Now,

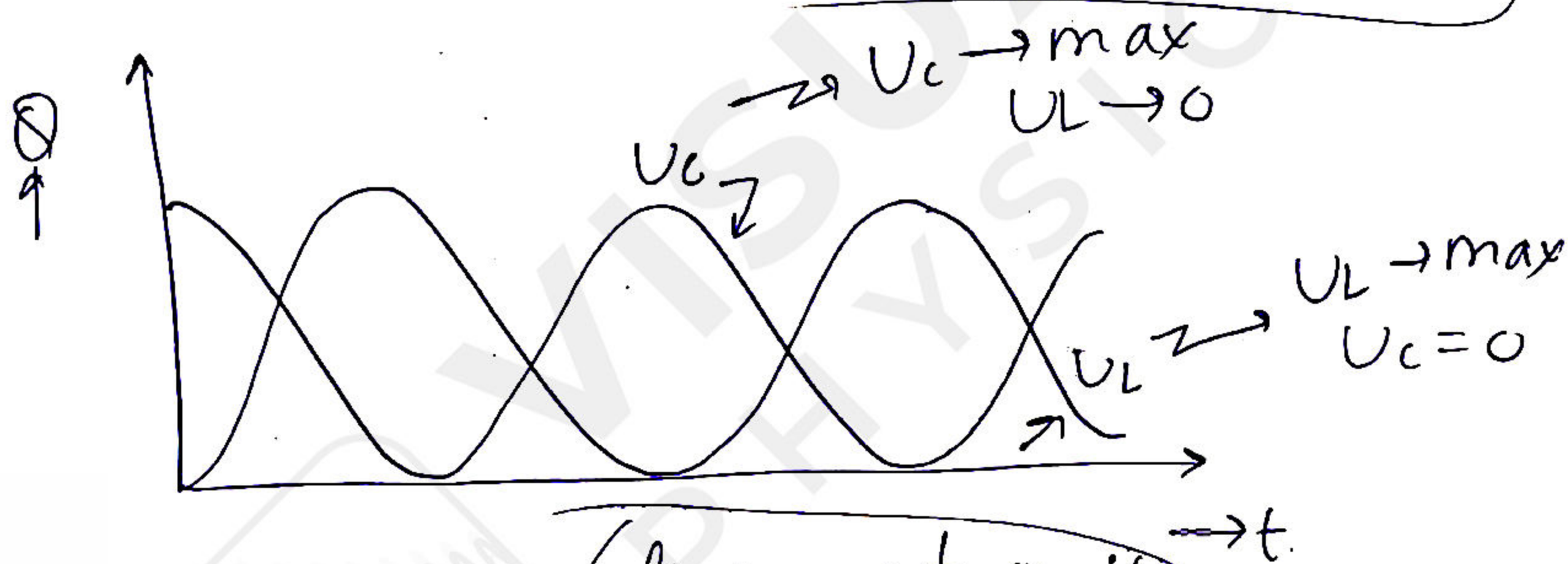
$$U_C = \frac{q^2}{2C}, \quad U_L = \frac{1}{2} L i^2$$

$$\Rightarrow U_C = \frac{Q_0^2}{2C} \cos^2 \omega t$$

$$U_L = \frac{1}{2} L I_0^2 \sin^2 \omega t$$

OR

$$U_L = \frac{Q_0^2}{2C} \sin^2 \omega t$$



(always at any t)

Now,

$$U = U_C + U_L = \text{Constant}$$

$$U = \frac{Q_0^2}{2C} [\cos^2 \omega t + \sin^2 \omega t]$$

$$U = \frac{Q_0^2}{2C}$$

Similar to spring-block
 Energy $\rightarrow$ (between potential and kinetic)