VISUAL PHYSICS

SHORT NOTES

CHAPTER

Gravitation

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GRAVITATION

Force of attraction between any two bodies of the universe is known as Gravitation.

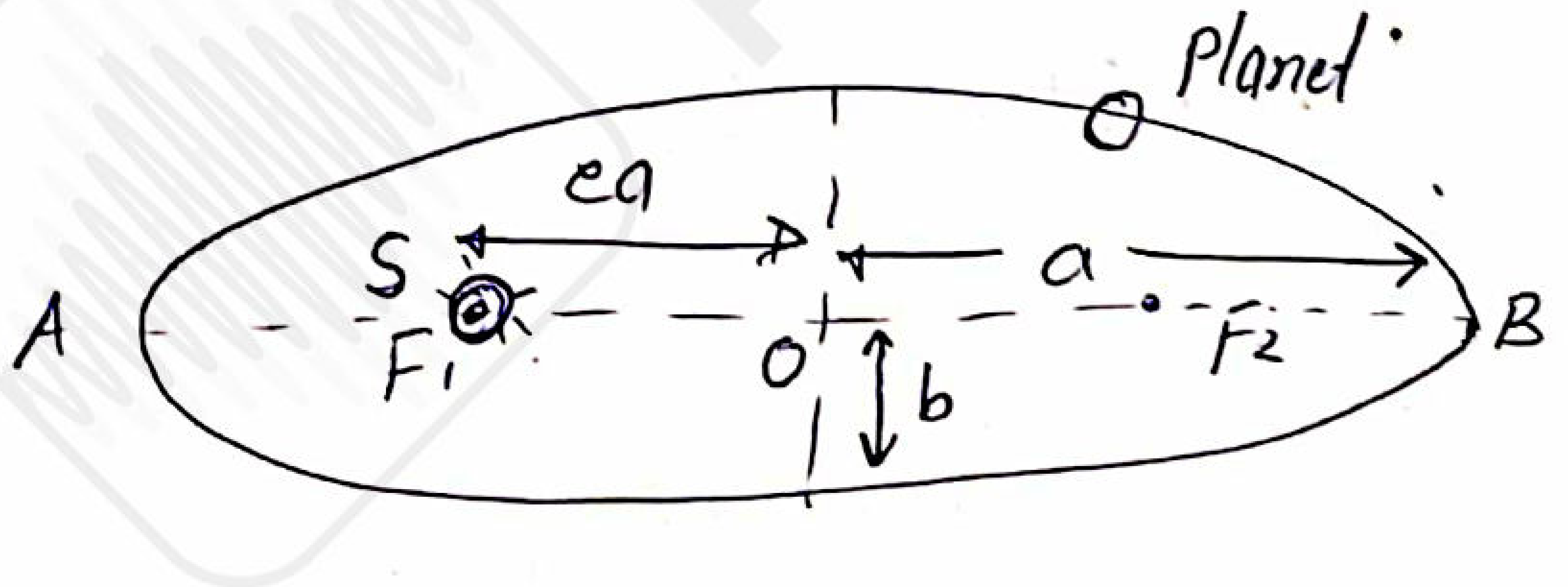
↓

Weakest force among the four fundamental forces of nature.

KEPLER'S LAWS [for heavenly body revolving around planet/star in an orbit i.e. natural satellite.]

I. LAW of orbit :

Each planet moves around the sun in an elliptical orbit, with sun at one foci

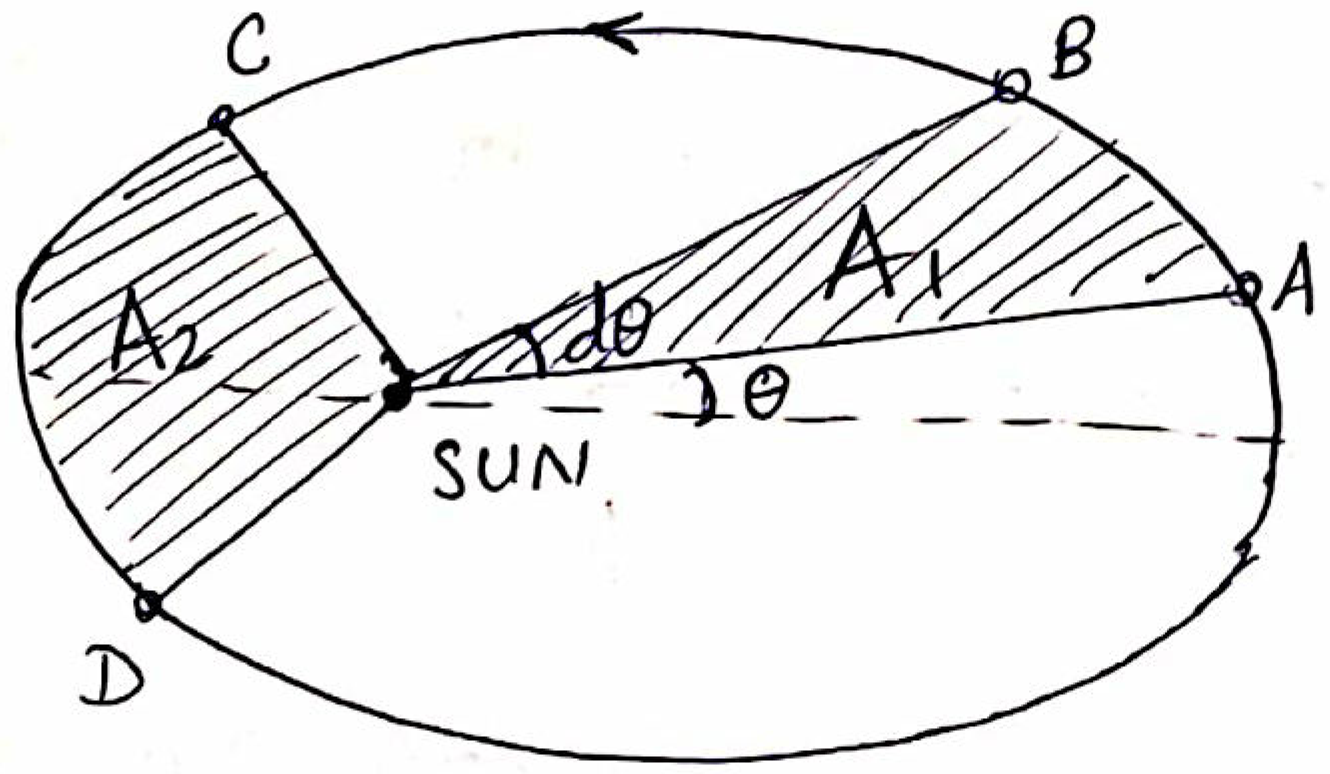


$F_1, F_2 \rightarrow$ foci

$e = \frac{SO}{a}$, $SO = ea$

→ Greatest distance (BS) of the planet from sun is called "Apogee"

II. LAW OF Areas :



for a time 't' ; $\boxed{A_1 = A_2}$

In a given by equal Areas are covered that means, planet is moving faster at 'C' as compared with 'A'.

$$\boxed{\text{Areal velocity} = \frac{\text{Area swept}}{\text{Time}} = \text{Constant}}$$

$$\frac{\frac{1}{2} r(r d\theta)}{dt} = \text{Constant}$$

$$\boxed{\frac{1}{2} r^2 (\omega) = \text{Constant}}$$

if we multiply, m both side
↳ mass of planet

$$\Rightarrow \underline{m r^2} \omega = \text{Constant}$$

$$\boxed{I \omega = \text{angular moment} = \text{Constant}}$$

III Law of periods:

$$\boxed{T^2 \propto a^3}$$

T → Time period of revolution of planet

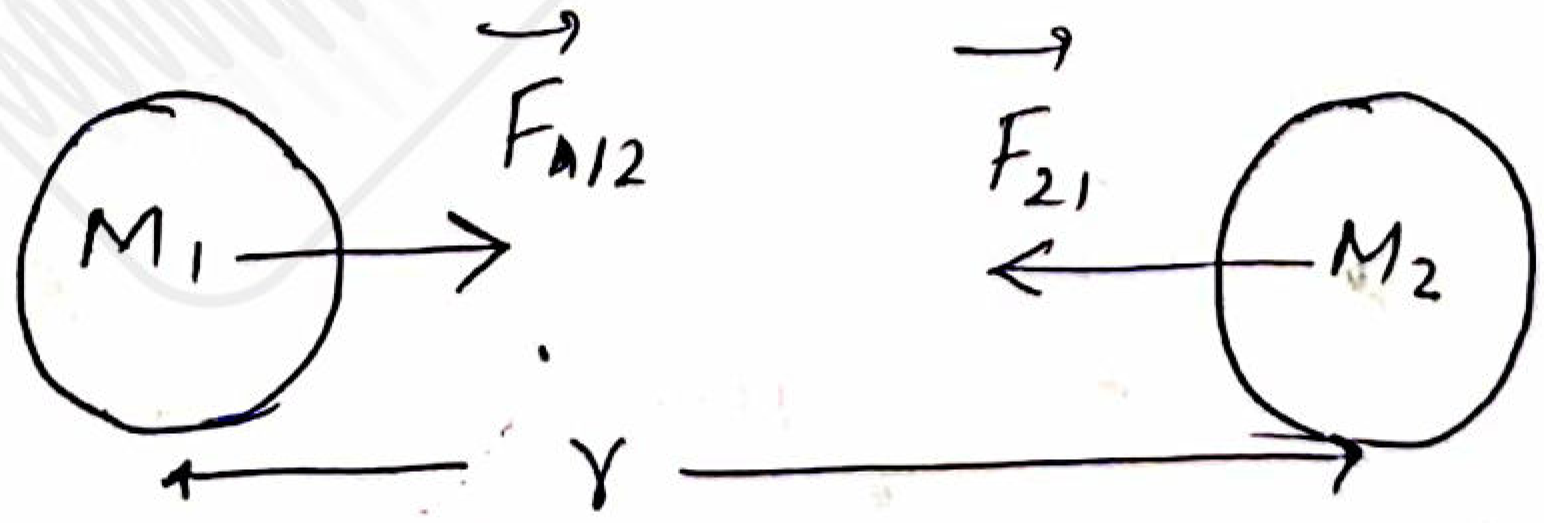
a → Semi-major axis of ellipse.

$$\Rightarrow \boxed{T^2 = k r^3}$$

r → radius (∵ Considering orbit to be circular)

★ Kepler's laws are applicable not only for solar system but also equally well for satellites orbiting around a planet.

NEWTON'S LAW OF GRAVITATION:



M_1, M_2 → two point masses

F_{12} → force on 1 due to 2

F_{21} → Force on 2 due to 1

$$|\vec{F_{12}}| = |\vec{F_{21}}| \rightarrow \text{Newton's third law}$$

$$\vec{F_{12}} = -\vec{F_{21}}$$

$\rightarrow$ opposite in direction.

$$\boxed{F \propto \frac{m_1 m_2}{r^2}} \rightarrow \text{Newton's Gravitation law}$$

$$\boxed{F = \frac{G M_1 M_2}{r^2}}$$

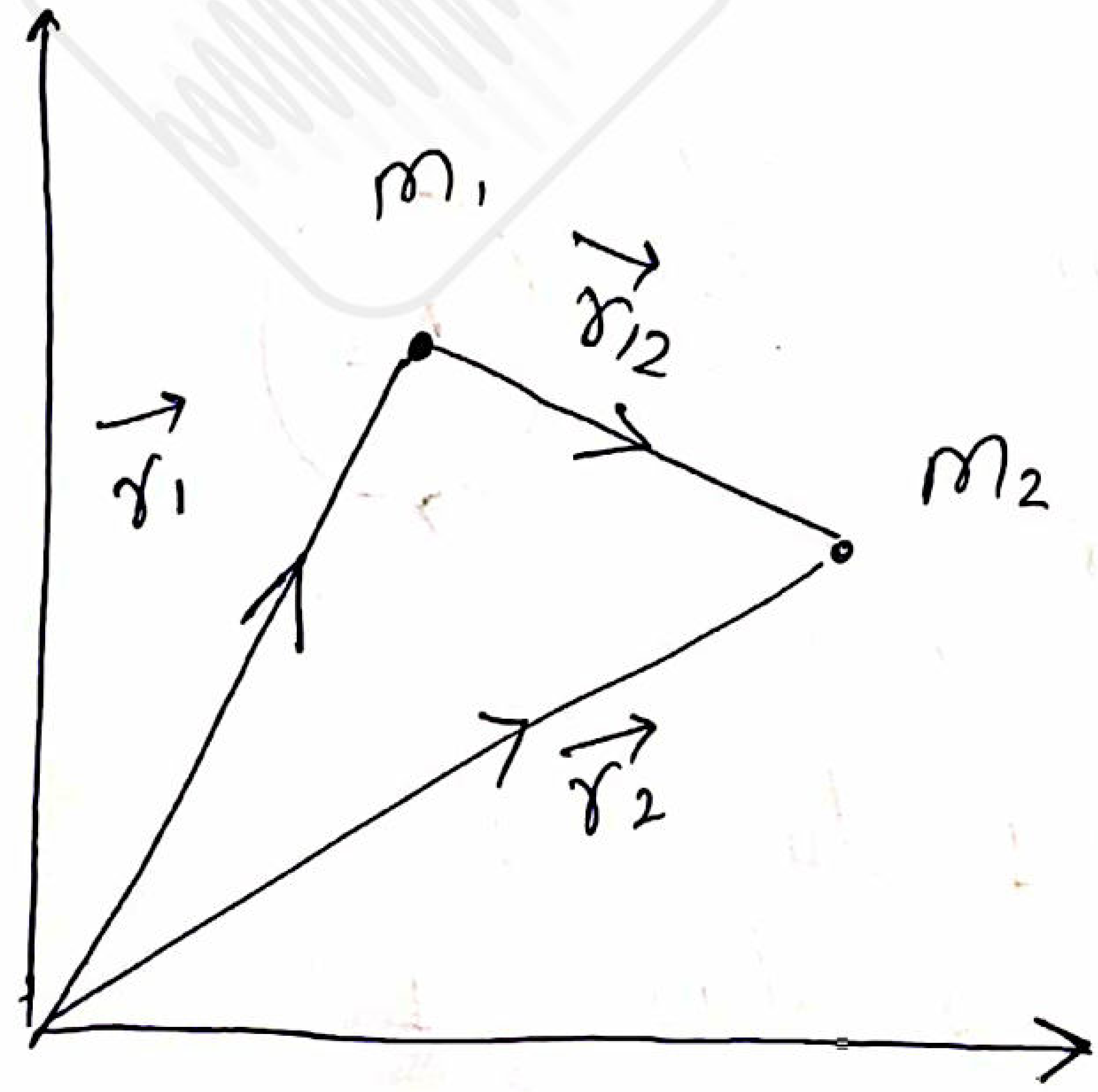
G → Universal gravitational constant.

$$G = 6.67 \times 10^{-11} \ Nm^2/kg^2$$

→ directly proportional to masses product

→ Inversly proportional to square of distance between the point masses.

Vector form



$$\vec{F_{12}} = \frac{G m_1 m_2}{(\vec{r_{12}})^2} \hat{r}_{12}$$

$$= \frac{G m_1 m_2}{(r_{12})^2} \frac{\vec{r_{12}}}{(r_{12})}$$

$$= \frac{G m_1 m_2}{(r_{12})^3} \vec{r_{12}}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\& \quad \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

$$|r_{12}| = r_{12} = |r_{21}| = r_{21}$$

So, $$\vec{F}_{12} = \frac{Gm_1m_2}{(r_{12})^3}(\vec{r}_2 - \vec{r}_1)$$

$$\vec{F}_{21} = \frac{Gm_1m_2}{(r_{21})^3}(\vec{r}_1 - \vec{r}_2)$$

$$\Rightarrow \boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

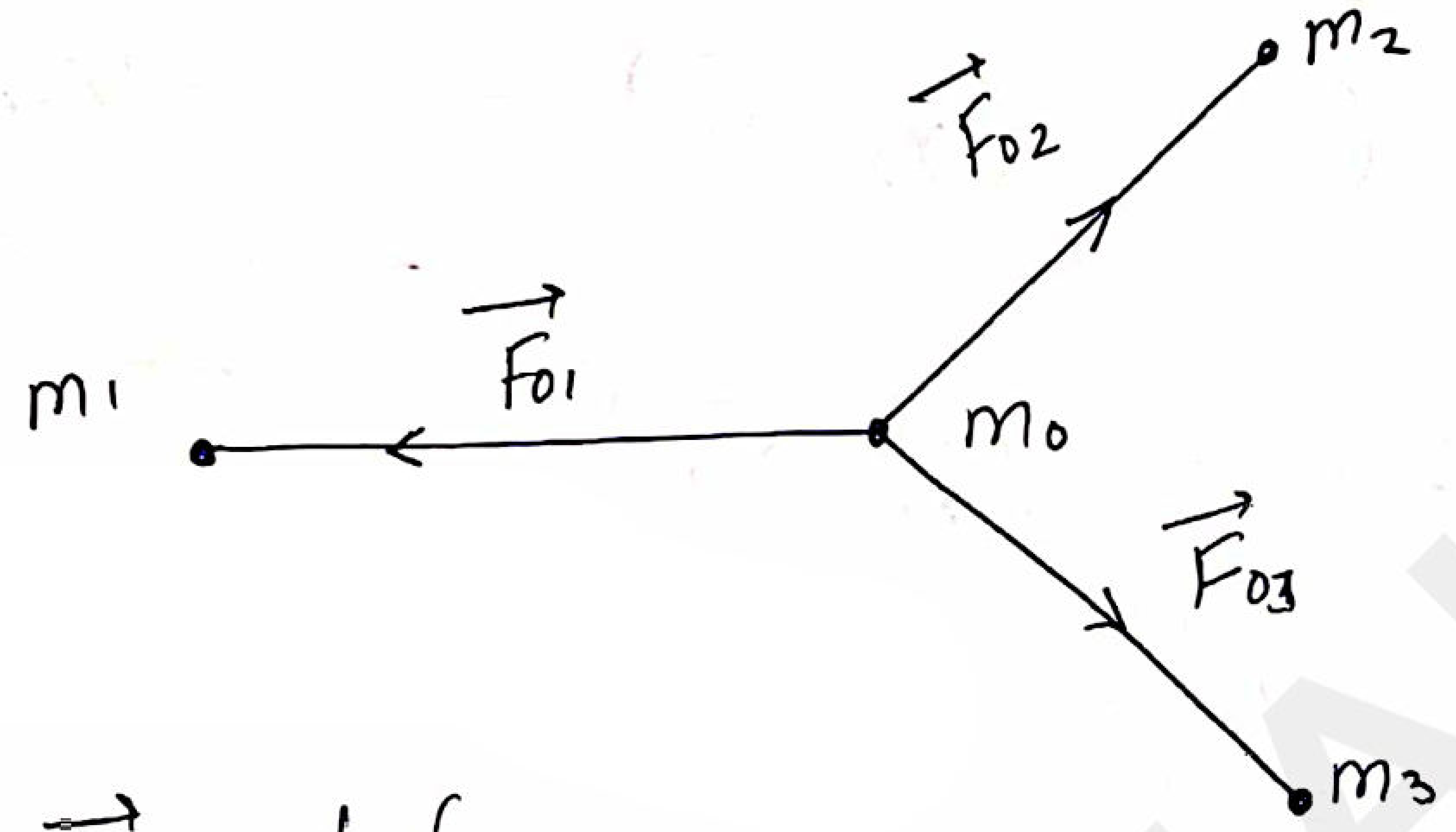
→ Gravitational force is independent of nature of intervening medium.

→ Gravitational force is independent of nature and size of bodies till their masses remain same and distance between their centres is fixed.

→ Gravitational force acts along the line joining the centres of two bodies

→ Gravitational force is conservative in nature

Principle of Superposition of Gravitation:



$\vec{F}$ = net force

$$\boxed{\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n}}$$

$$\boxed{\vec{F} = G m_0 \left[\frac{m_1 \hat{r}_{01}}{|\vec{r}_{01}|^2} + \frac{m_2 \hat{r}_{02}}{|\vec{r}_{02}|^2} + \dots \right]}$$

GRAVITATIONAL FIELD:

Region around a mass in which another mass can be influenced ~~us~~ is field around the mass.

The magnitude at a point 'P' around Mass 'M' is given by:

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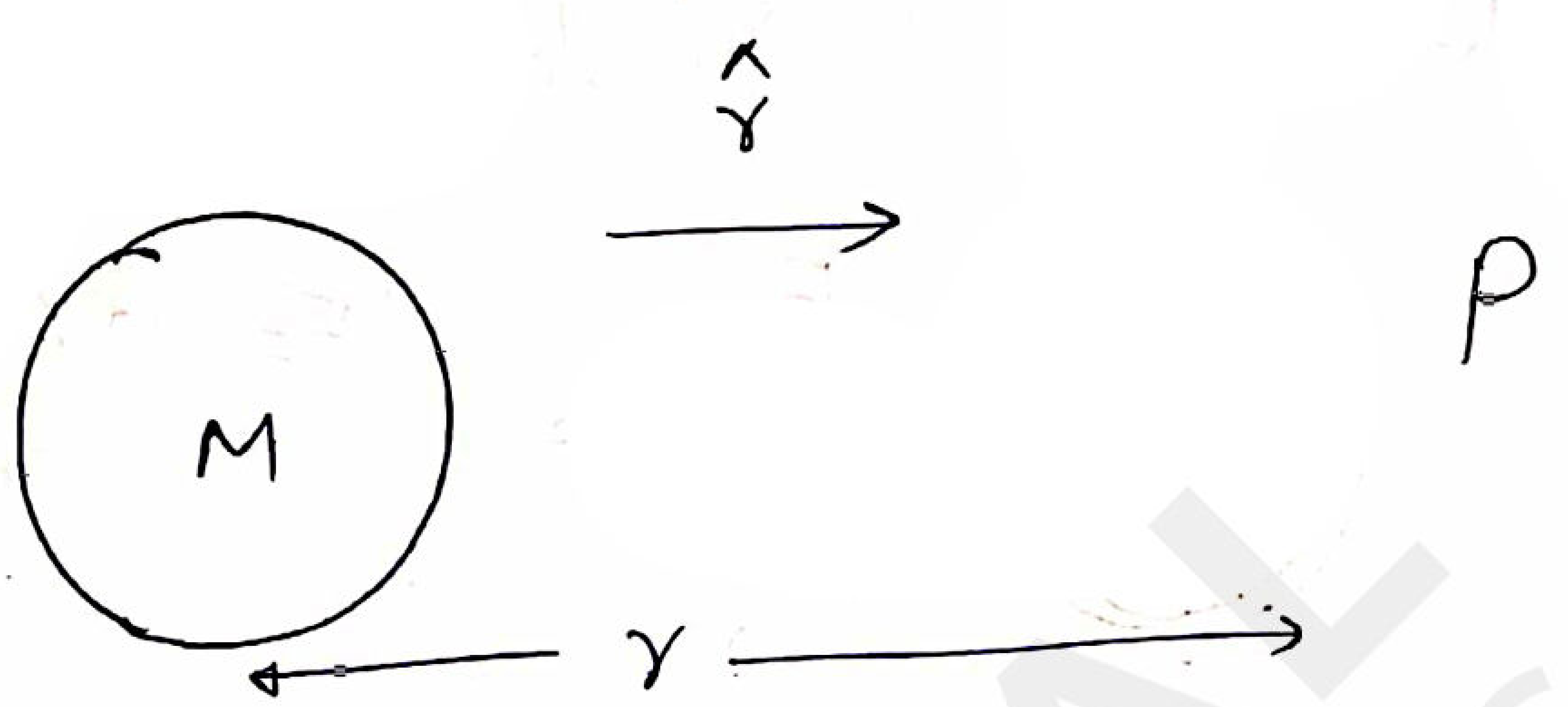
$$\boxed{\vec{E} = \frac{GM}{r^2}\hat{r}}$$



so for mass m_1

$$\vec{F} = \frac{GMm_1}{r^2}\hat{r}$$

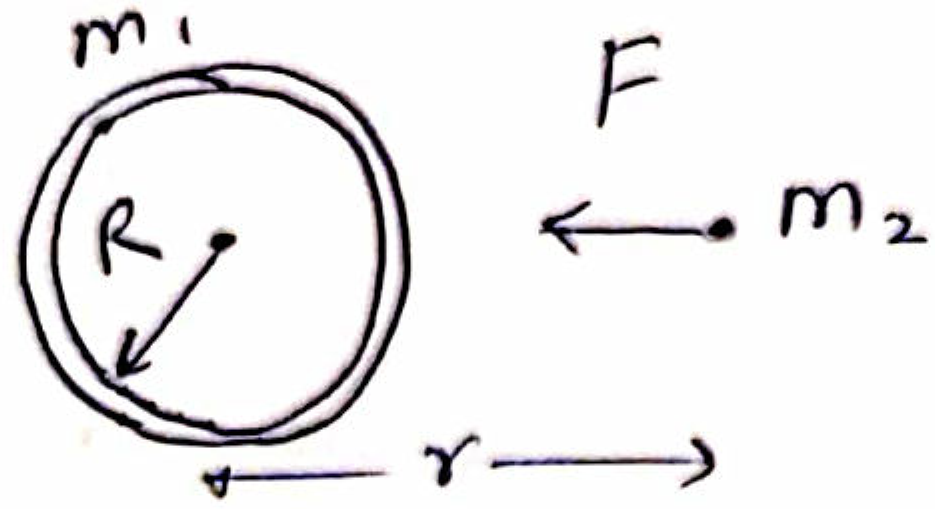
so, $\boxed{\vec{E} = \frac{\vec{F}}{m_1}} \Rightarrow \boxed{\vec{E} = \frac{GM}{r^2}\hat{r}}$

If gravitational field is due to earth at point 'P' lies on surface of earth then, $r = R$

$\Rightarrow |E| = \frac{GM}{R^2} = g$ (acceleration due to gravity at surface of earth)

Force of attraction between hollow spherical shell of uniform density and a point mass

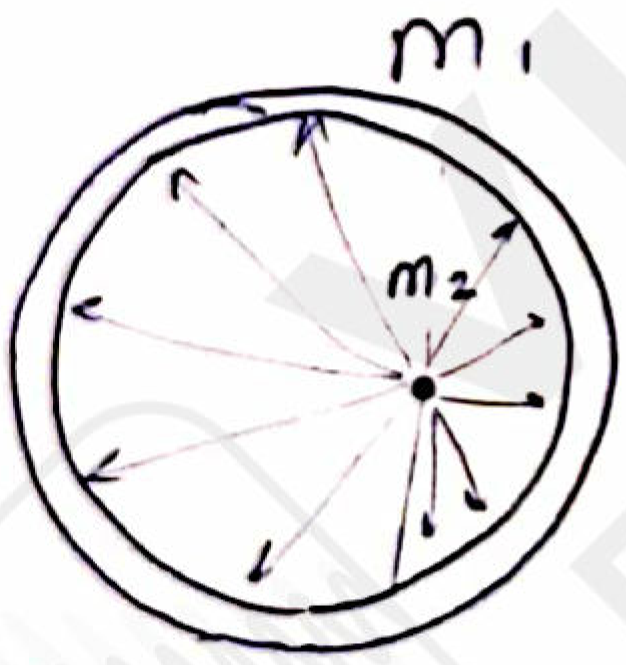
Case I : point mass is outside the shell.



$$\boxed{F = \frac{G m_1 m}{r^2}}$$

$r \rightarrow$ from Centre of mass

Case II : point mass inside the shell.



$$\boxed{F = 0 = \text{Force on } m_2}$$

$\rightarrow$ Gravitational force on a point mass situated inside spherical shell of uniform density will be zero.

★ **GRAVITY** $\rightarrow$ force exerted by earth towards its centre on a body lying on or near to surface of earth.

★ Gravitation is force of attraction acting between any two bodies of universe. Gravity is earth's gravitational pull on body lying on near the surface of earth.

Acceleration due to Gravity:

Acceleration due to gravity is defined as force of gravity acting on a unit mass of the body placed near the surface of the earth.

$$g = \frac{F}{m} = \frac{GMm}{R^2 m}$$

$$\boxed{g = \frac{GM}{R^2}}$$

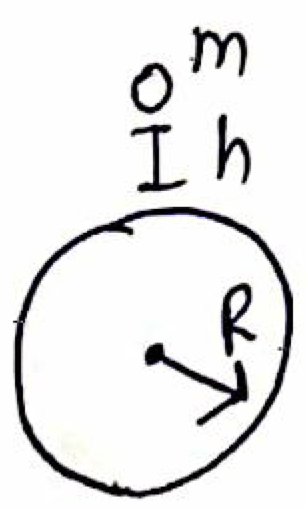
$R \rightarrow$ Radius of earth

$M \rightarrow$ Mass of earth

Variation of Acceleration due to Gravity:

I. Effect of Altitude:

$$g' = \frac{F}{m} = \frac{GMm}{(R+h)^2 m}$$



$$g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{GM}{(R+h)^2}\frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

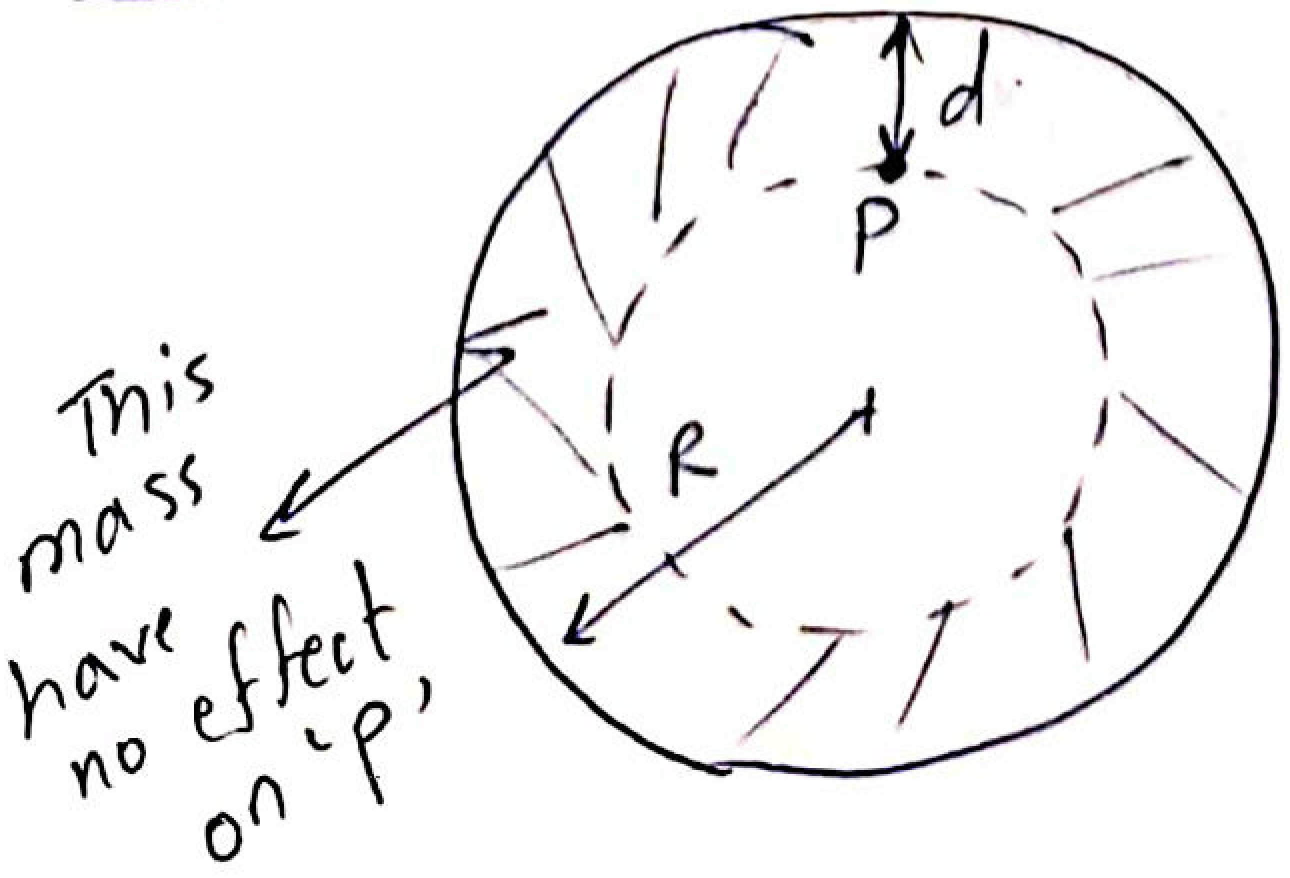
$\boxed{\text{if and only if } h << R}$

$$\left(1+\frac{h}{R}\right)^{-2} \rightarrow 1 - \frac{2h}{R}$$

so, $$\frac{g'}{g} = \left(1 - \frac{2h}{R}\right) \quad [\text{when } h << R]$$

$$\Rightarrow \boxed{g' = g\left(1 - \frac{2h}{R}\right)} \Rightarrow \text{only valid when } \boxed{h << R}$$

Effect of Depth



$M \rightarrow$ net mass

$$g = \frac{GM}{R^2}$$

$M = \frac{4}{3}\pi R^3 \rho \longrightarrow$ density of earth (assumed uniformed)

M' = Mass of unshaded part

$$M' = \frac{4}{3}\pi (R-d)^3 \rho$$

$$M' = \frac{4}{3}\pi (R-d)^3 \frac{M}{\frac{4}{3}\pi R^3}$$

$$\boxed{M' = \frac{M(R-d)^3}{R^3}}$$

$$g'' = \frac{GM'}{(R-d)^2} = \frac{GM(R-d)^3}{(R-d)^2 R^3}$$

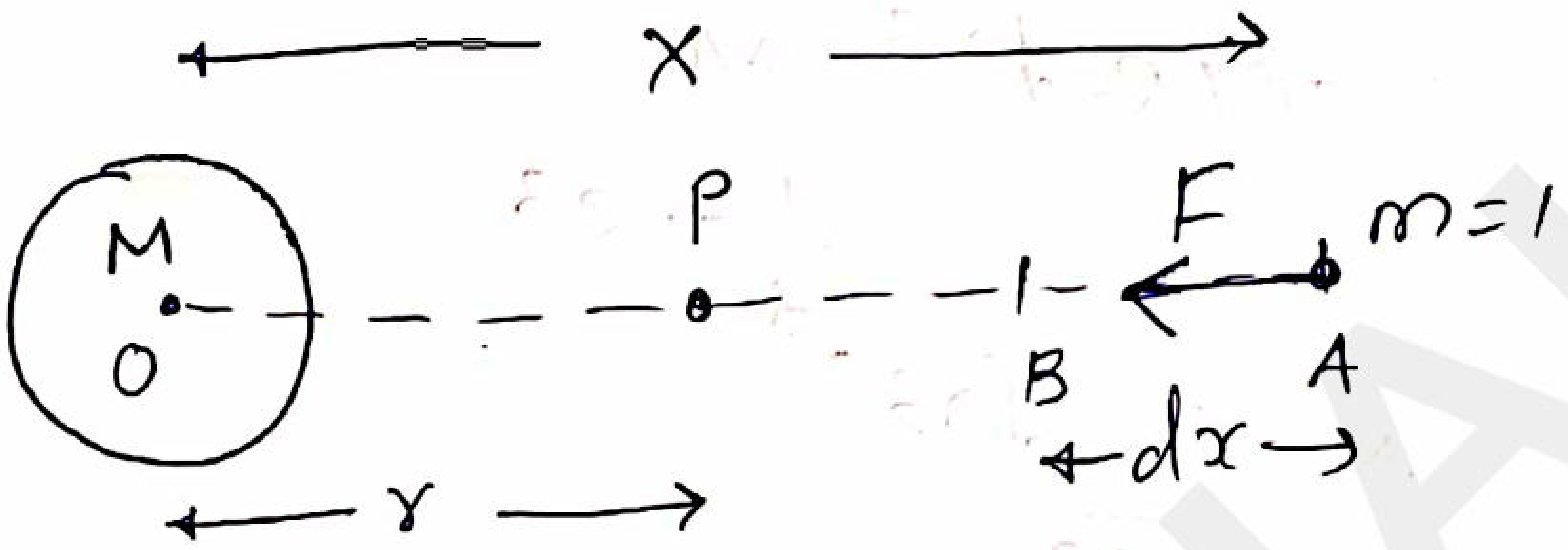
$$= \left(\frac{GM}{R^2}\right)\frac{(R-d)}{R}$$

$$\boxed{g'' = g\left(1 - \frac{d}{R}\right)}$$

Valid for any d.

GRAVITATIONAL POTENTIAL:

Work done in bringing a unit mass from infinite to gravitational field of body.



$$F = \frac{GM(1)}{x^2} = \frac{GM}{x^2}$$

$$\int_{\infty}^{r} dW = F\,dx = \int_{\infty}^{r} \frac{GM}{x^2}\,dx$$

$$W = \int_{\infty}^{r} \frac{GM}{x^2}\,dx = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = -\frac{GM}{r}$$

$$\boxed{V_P = \text{potential at point 'P'} = -\frac{GM}{r}}$$

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Gravitational potential at a point due to earth :

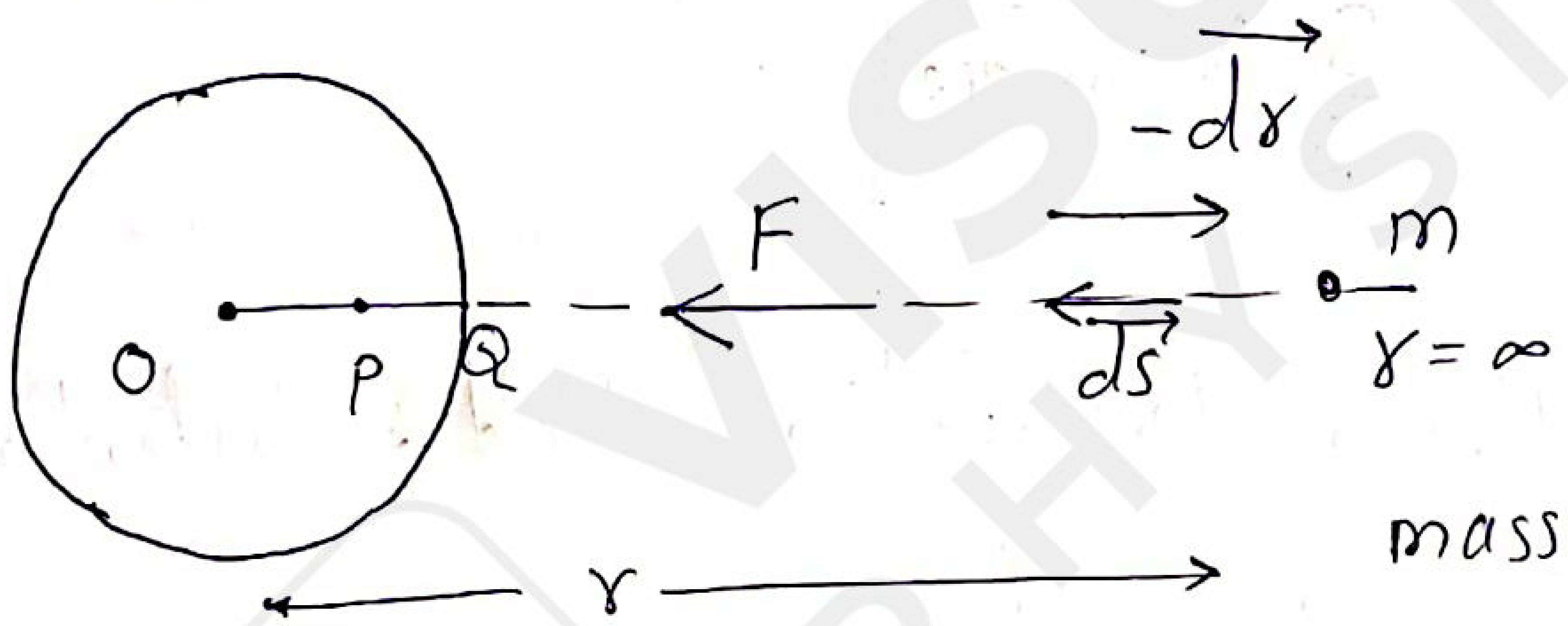
Outside the earth :

$r > R$

$$\boxed{V = \frac{-GM}{r}}$$

→ as earth have mass M, similar in derivation as we just did

Inside the earth :



mass of 'r' radius

$\rho \times \frac{4}{3}\pi r^3$

$$\boxed{M' = \frac{Mr^3}{R^3}}$$

$$V_P = \frac{[W_{\infty \to P}]}{m}$$

$$V_P = +\left[\frac{W_{\infty \to Q} + W_{Q \to P}}{m}\right]$$

$$= +\frac{1}{m}\left[\int_{\infty}^{R} \frac{GMm}{r^2}\, dr + \int_{R}^{r} \frac{GMm}{R^3} r(dr)\right]$$

$$\boxed{V_P = -GM\left[\frac{3R^2 - r^2}{2R^3}\right]}$$

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→ $V = -\frac{GM}{r}$, $r > R$

→ $V = -\frac{GM}{R}$, $r = R$

→ $V = -GM\left[\frac{3R^2 - r^2}{2R^3}\right]$ $r < R$

$r = 0$

→ $V = \frac{-3GM}{2R}$

$$\boxed{V_{centre} = \frac{3}{2} V_{surface}}$$

Relation between Gravitational Field & potential:

$$\boxed{E = -\frac{dV}{dr}}$$

Gravitational potential energy = Work done in bringing 'm' from $\infty \rightarrow P$.

⇒ $\boxed{U_P = mV_P}$ or $\boxed{U_P = -\frac{GMm}{r}}$

→ negative sign shows potential energy is due to attractive gravitational force. system is bounded.

→ for earth, $r_1 = R$, $r_2 = R+h$

$$\underbrace{\Delta U}_{\text{change in potential energy}} = \underbrace{-\frac{GMm}{(R+h)}}_{\text{final}} - \underbrace{\left(-\frac{GMm}{R}\right)}_{\text{Initial}}$$

$$\Delta U = \frac{GMm}{R}\left[1 - \frac{R}{(R+h)}\right]$$

$$= \frac{GMm}{R}\left[1 - \frac{1}{(1+h/R)}\right]$$

if $h << R$

$$\Rightarrow \Delta U = \frac{GMm}{R}\left[1 - \left(1 - \frac{h}{R}\right)\right]$$

$$\Delta U = \left(\frac{GMm}{R^2}\right)h$$

$$\boxed{\Delta U = m\left(\frac{GM}{R^2}\right)h = mgh}$$

if there are $m_1, m_2 \dots m_n$ masses,
so U of m_0 in this system will be:

$$U = -\left[\frac{Gm_1m_0}{r_{01}} + \frac{Gm_2m_0}{r_{02}} + \dots\right]$$

distance b/w m_1 & m_0

distance b/w m_2 & m_0

→ if there are three particles, m_1, m_2, m_3
U of system will be:

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \frac{Gm_1m_3}{r_{13}}\right]$$

★ for isolated system, gravitational potential energy for all possible pairs of its constituent particle.

→ This is superposition principle

ESCAPE VELOCITY:

↳ minimum speed required to project, so that it never return to the surface is called Escape velocity.

means, $r_f \longrightarrow \infty$ [no potential or say Gravitation effect]

$r_i \rightarrow R$
↳ Radius of earth

⇒ All from mechanical energy Conservation

$$E_i = E_f$$

$$-\frac{GMm}{R} + \frac{1}{2}mV^2 = -\frac{GMm}{\infty} + 0$$

$$\Rightarrow \boxed{V = \sqrt{\frac{2GM}{R}}}$$

on putting values

$$\boxed{V = 11.2 \text{ km/s}}$$

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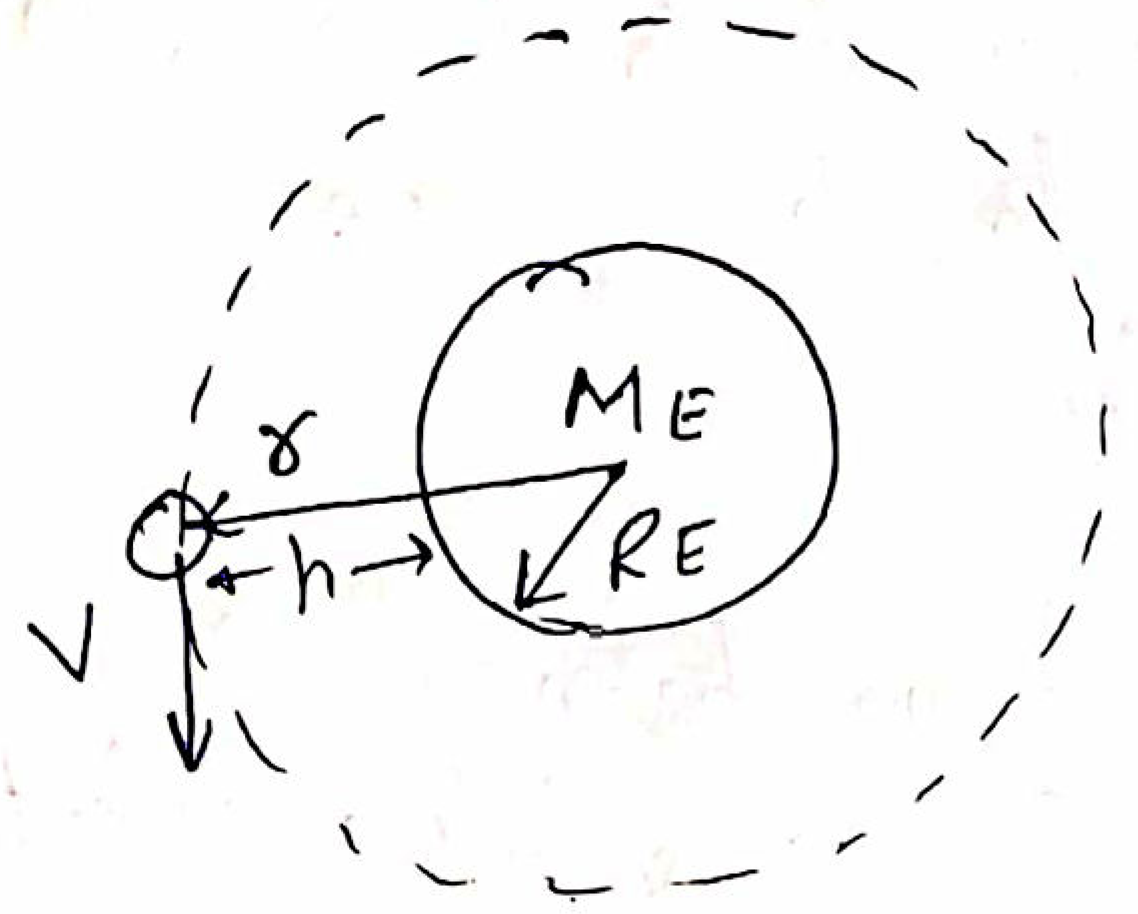
ORBITAL SPEED of satellite :



as Centripetal force is provided by Gravitational force

$$\Rightarrow \frac{mv^2}{r} = \frac{GM_E m}{r^2}$$

$$V = \sqrt{\frac{GM_E}{r}}$$

$$g = \frac{GM_E}{R_E^2}$$

$$\Rightarrow V = \sqrt{\frac{GM_E}{R_E^2}\frac{R_E^2}{r}} = R_E\sqrt{\frac{g}{r}}$$

$$\boxed{V = R_E\sqrt{\frac{g}{r+R_E}}}$$

→ Independent of mass of satellite

→ Depend on mass of planet & Radius

Time period of satellite :

$$T = \frac{2\pi r}{v} = \frac{\text{distance}}{\text{orbital velocity}}$$

$$\boxed{T = \frac{2\pi r}{R}\sqrt{\frac{r}{g}} = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}} \Rightarrow \boxed{T^2 \propto r^3}$$

$$M = \frac{4}{3}\pi R^3 \rho \quad , \quad g = \frac{GM}{R^2} \quad , = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

$$g = \frac{4\pi \rho R G}{3}$$

$$T = \frac{2\pi}{R}\sqrt{\frac{3(R+h)^3}{4\pi \rho R G}}$$

$$\boxed{T = \sqrt{\frac{3\pi (R+h)^3}{G \rho R^3}}}$$

for $h << R$

$$h + R \rightarrow R$$

$$\boxed{T = \sqrt{\frac{3\pi}{G\rho}} \quad \text{or} \quad 2\pi\sqrt{\frac{R}{g}}}$$

Altitude or Height of satellite

$$T^2 = \frac{4\pi^2 (R+h)^3}{R^2 g} = \frac{4\pi^2 r^3}{R^2 g}$$

$$\Rightarrow \quad (R+h)^3 = \frac{T^2 R^2 g}{4\pi^2}$$

$$R+h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3}$$

$$\boxed{h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R}$$

Angular momentum, L

$$L = mvr = mr\sqrt{\frac{GM}{r}}$$

$$\boxed{L = (m^2 G M r)^{1/2}}$$

dependent on $\underline{M, m, r}$

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Energy of a satellite:

↳ Kinetic Energy + potential Energy

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{GM}{r} \right)$$

& $$U = -\frac{GMm}{r}$$

so, $E = K + U$

$$\boxed{E = -\frac{GMm}{2r}}$$

★ → That means system is bounded as total energy is negative.

GEOSTATIONARY OR GEOSYNCHRONOUS:

↳ satellite that appears to be fixed at a definite heights.

→ This satellite revolves around the earth with same angular speed in the same direction as done by earth.

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$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

$R = 6.4\times10^6\,m$, $g = 9.8\,m/s^2$, $T = 24\,h$

$T = 24\times60\times60\,s$

on solving we get

$$\boxed{h = 3.6\times10^7\,m}$$

$\rightarrow$ for this $\boxed{v = 3.1\,km/s}$

→ Sense of rotation is same as that of earth.

→ It should revolve in an orbit concentric and coplanar with equitorial plane.

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