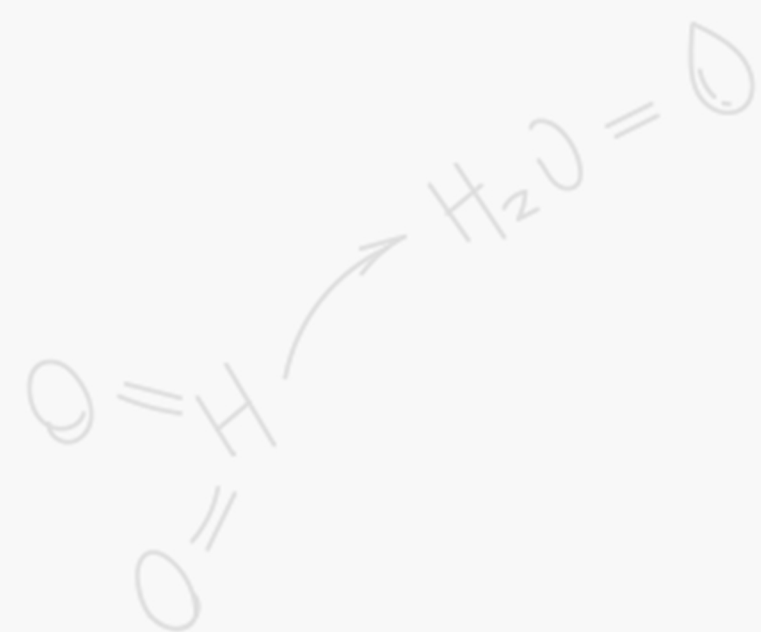




VISUAL
PHYSICS



V I S U A L P H Y S I C S

SHORT NOTES



C H A P T E R

Gauss's Law



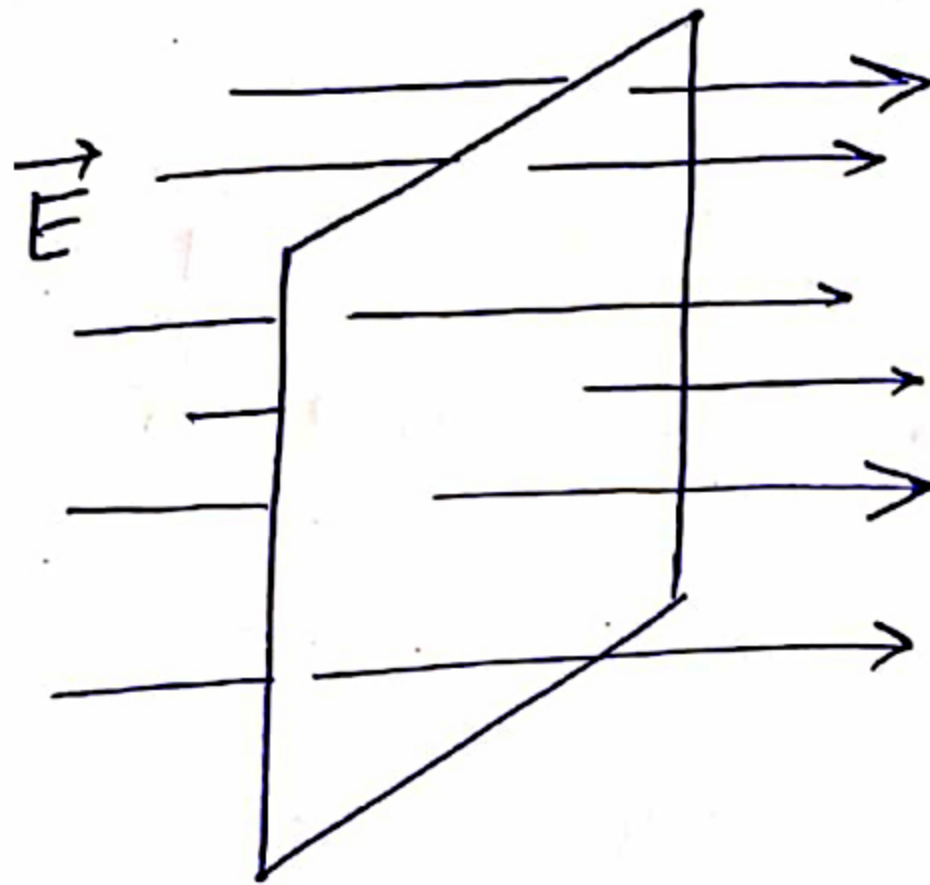
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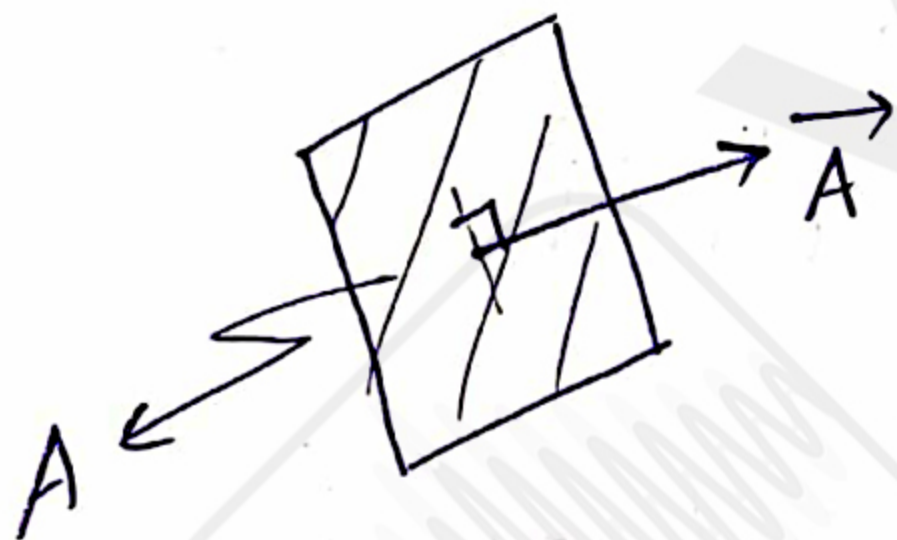
ELECTRIC FLUX & GAUSS LAW



$$\phi = \vec{E} \cdot \vec{A}$$

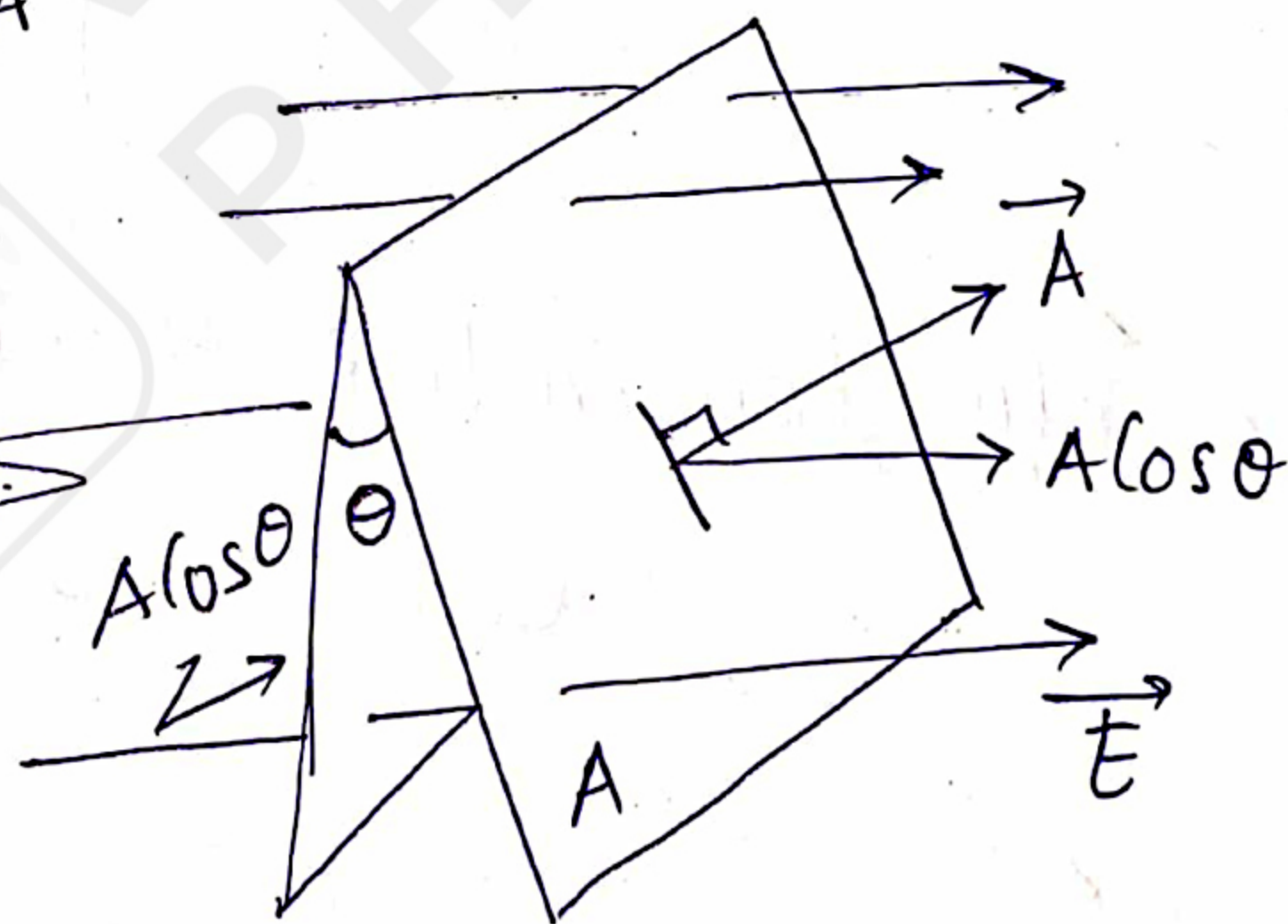
flux $\rightarrow$ no. of Electric field line passing through a given Area A.

Area ($\vec{A}$) is perpendicular to the surface.



$$\phi = \vec{E} \cdot \vec{A}$$

Area perpendicular to $\vec{E}$ field.



$$\phi = EA \cos \theta$$

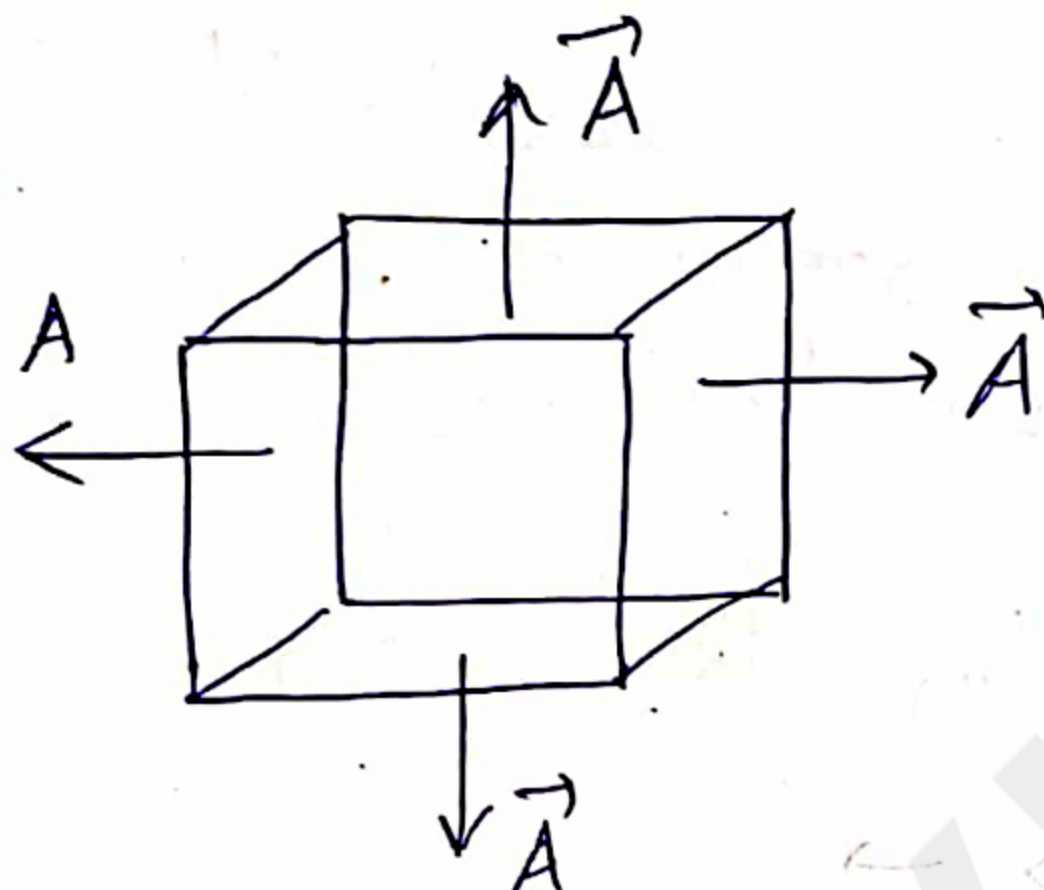
$$\boxed{\phi = \vec{E} \cdot \vec{A}}$$

Closed surface $\rightarrow$ enclosed a volume

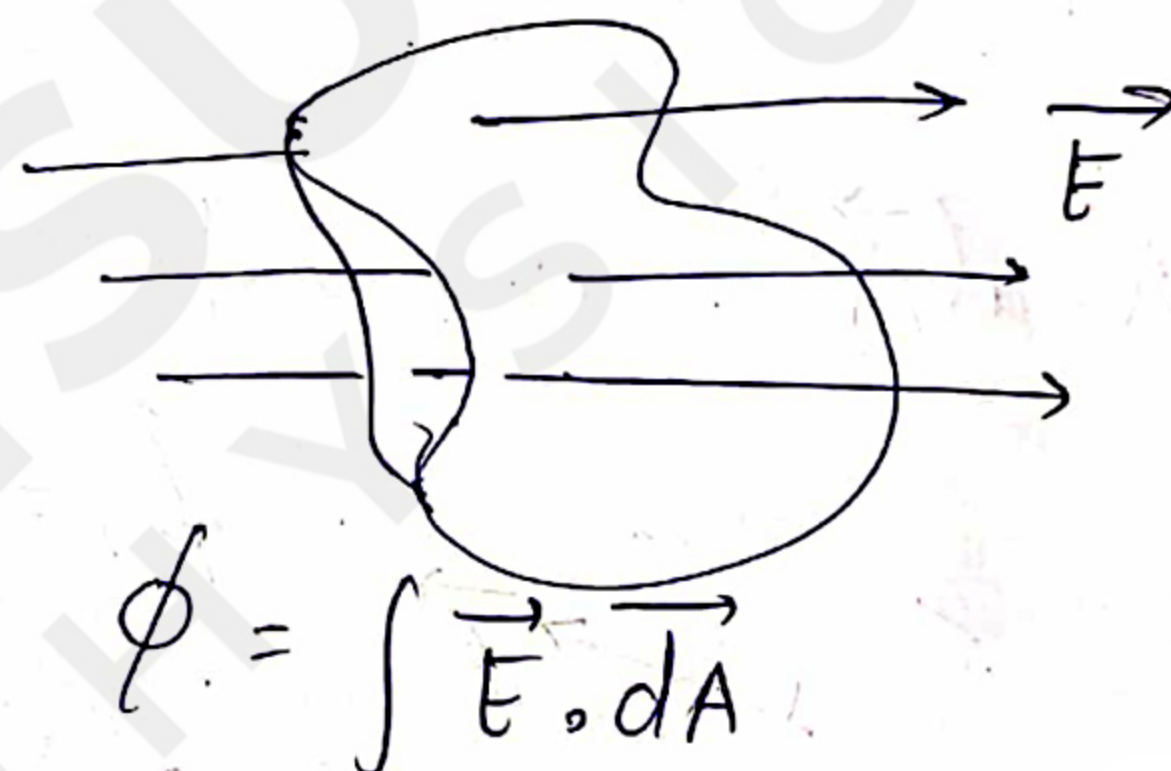
Open surface $\rightarrow$ just a surface and not
encloses any volume

⚡
direction of $\vec{A}$ not important

for closed surface $\rightarrow$ direction of $\vec{A}$
always outwards



$$\phi = \int \vec{E} \cdot d\vec{A}$$



$$\phi = \int \vec{E} \cdot d\vec{A}$$

⚡
sum of flux from each infinitesimal surfaces.

GAUSS'S LAW

Net flux through an enclosed surface
(closed surface) is equal to $\frac{1}{\epsilon}$ times

the net charge inside the closed surface.

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

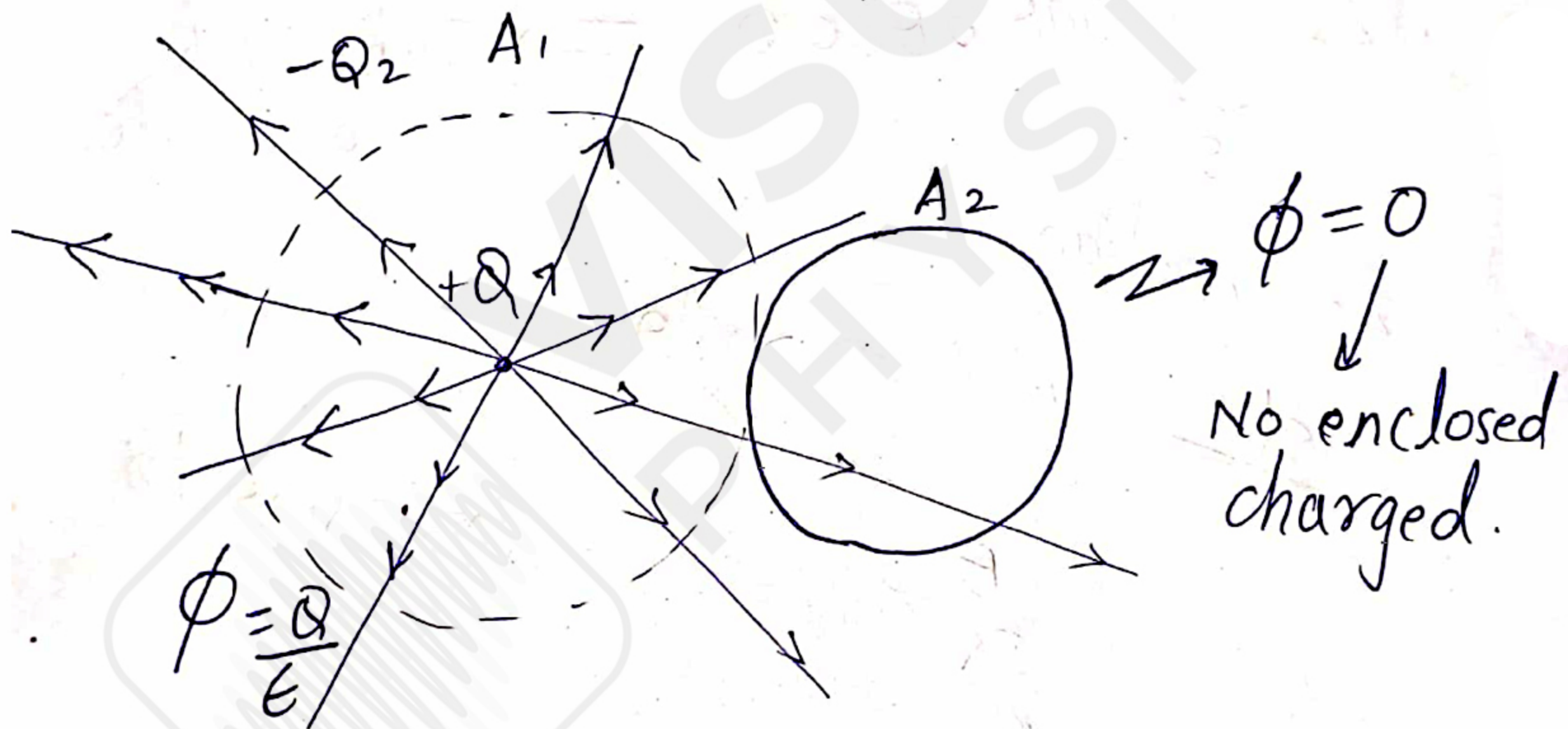
$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

closed integral.

charge that is only inside the closed surface

$\epsilon = \epsilon_0 \epsilon_r$

- positive flux → charge enclosed is positive
- negative flux → charge enclosed is negative
- Zero flux → No charge inside the surface.



Solid angle:

$$\Omega = \frac{A}{r^2}$$

solid angle

radius

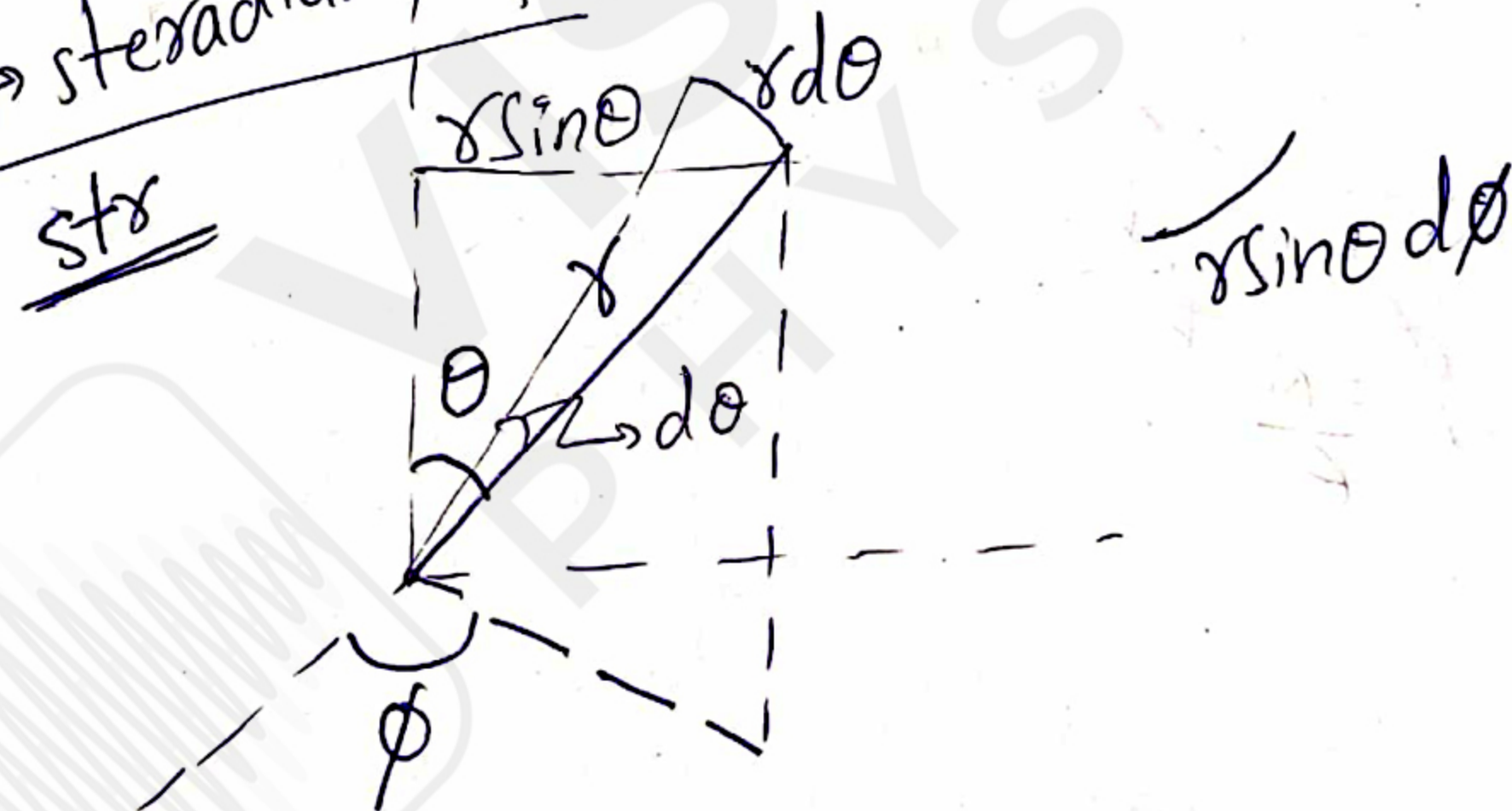
Projected Area



if we take small area element.

$$\Omega = \iint_S \sin\theta \, d\theta \, d\phi \rightarrow \text{Spherical coordinate}$$

unit $\rightarrow$ steradian,
str

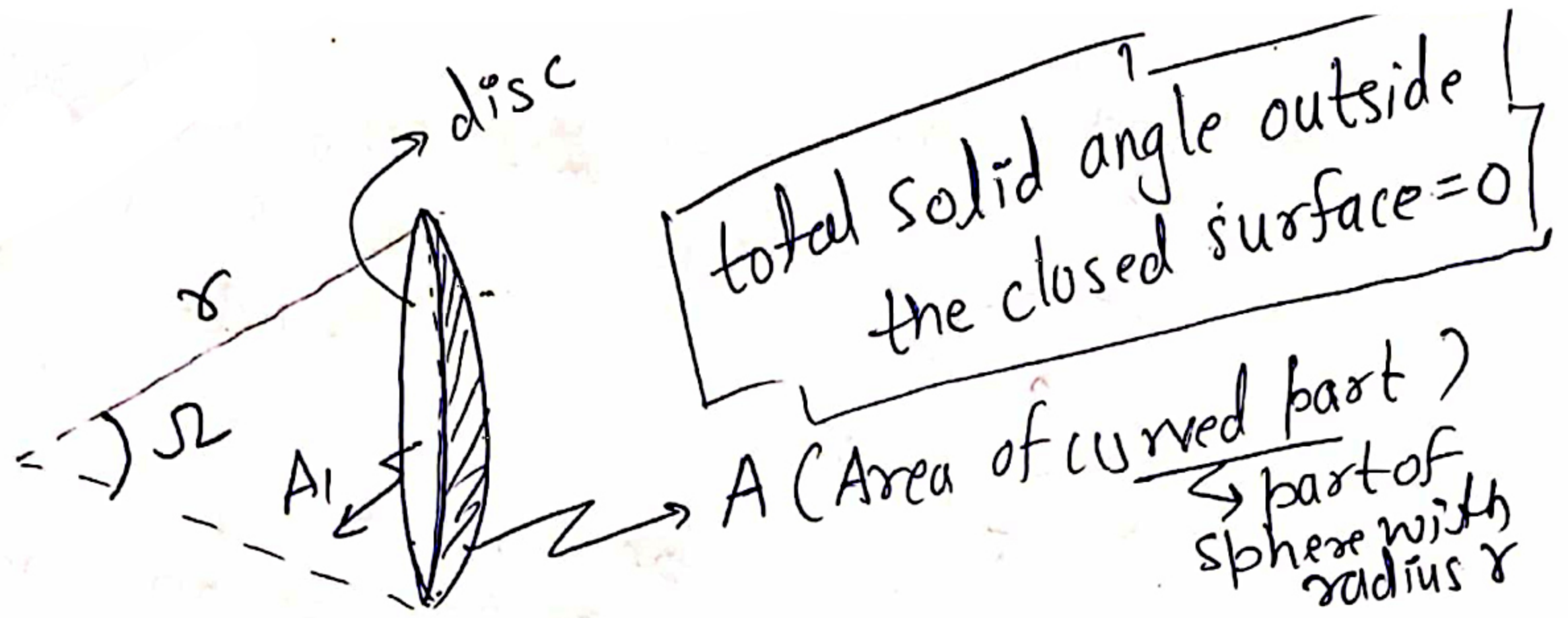


$$\text{so, } dA = r \sin\theta \, r \, d\theta \, d\phi$$

$$\text{so, } \Omega = \iint_S \frac{dA}{r^2}$$

total solid
angle = 4π

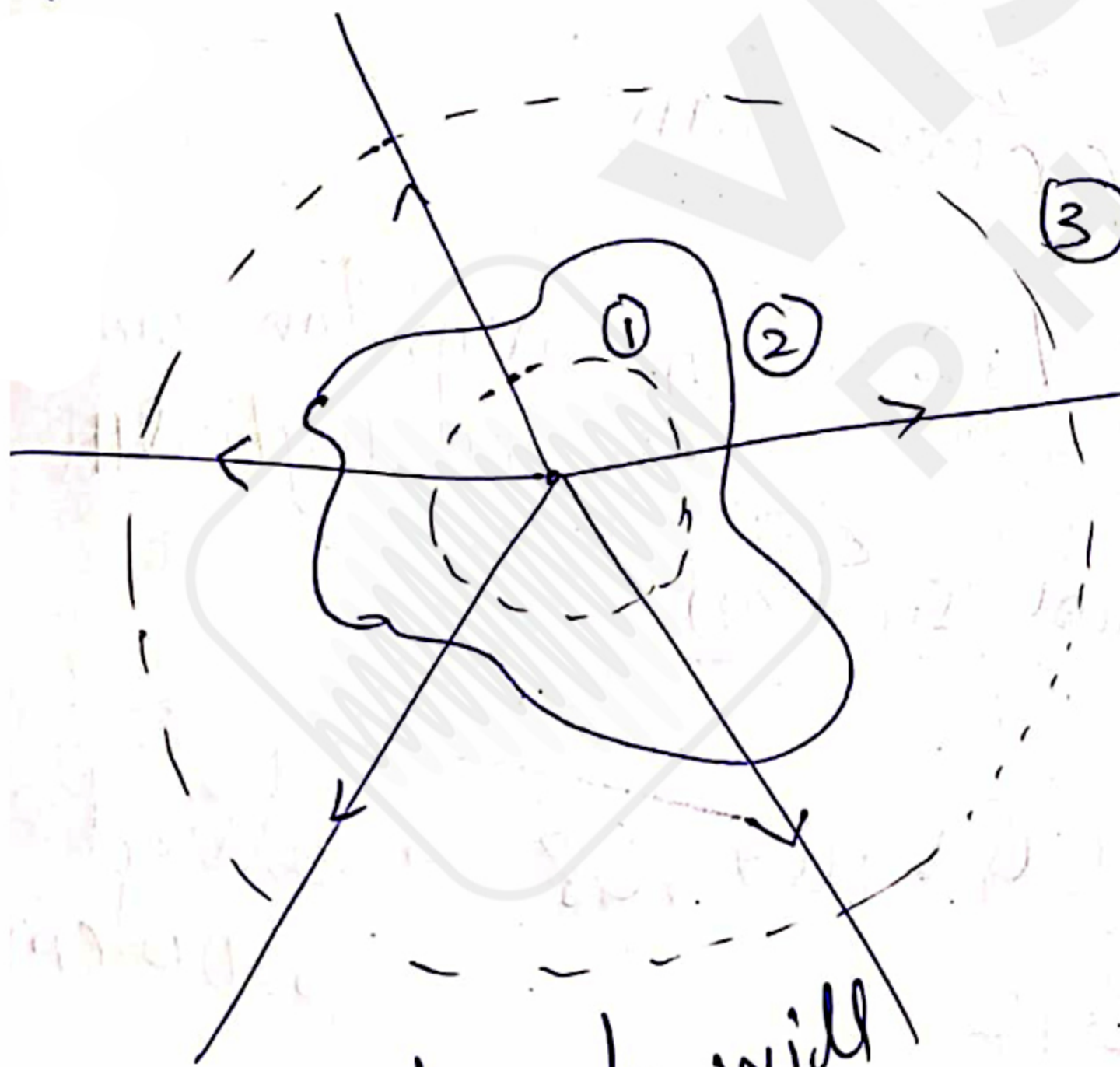
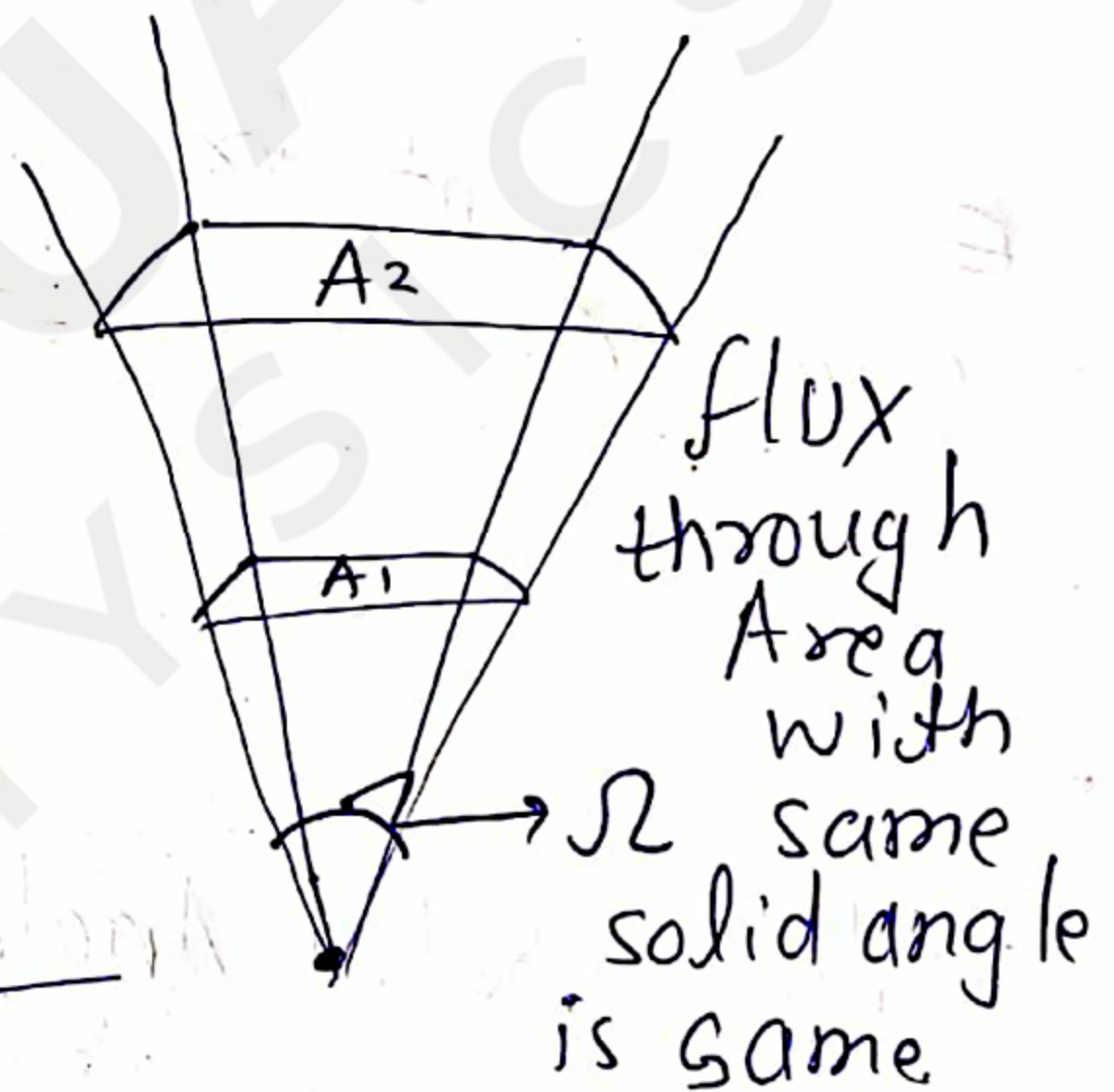
$$\Omega = \iint_S \sin\theta \, d\theta \, d\phi$$



$$\Omega = \frac{A}{r^2} \quad \checkmark$$

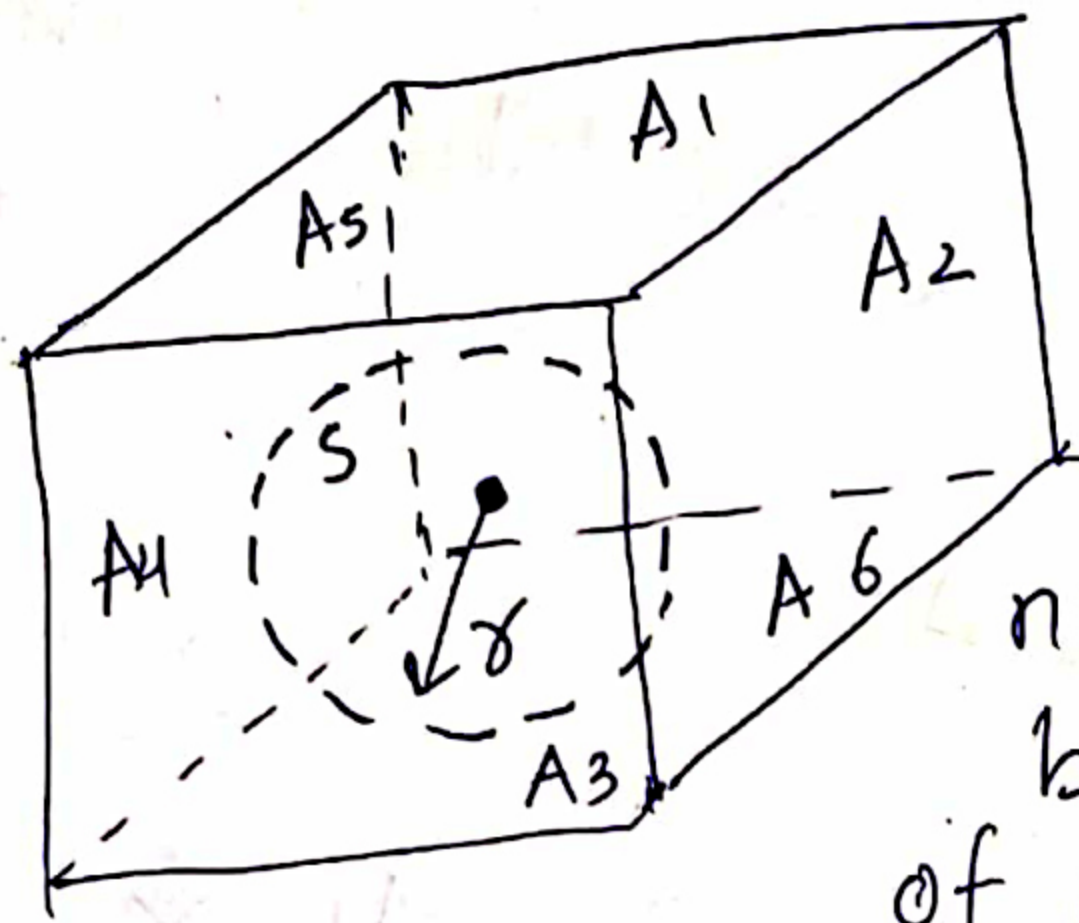
$$\Omega = \frac{A_1}{r^2} \times$$

total Ω Any closed surface = 4π ster



flux through surface (2) = flux through its projection on surface (1)

As solid angle will be same for all surfaces which is equal to surface (3) flux



ϕ = flux through
 $A_1 + A_2 + A_3 + A_4$
 $+ A_5 + A_6$

now same flux will
 be through projection
 of area of cube on sphere
 S

so, $\phi = \int \vec{E} \cdot d\vec{s} = E \int ds$

E is constant for
 given r

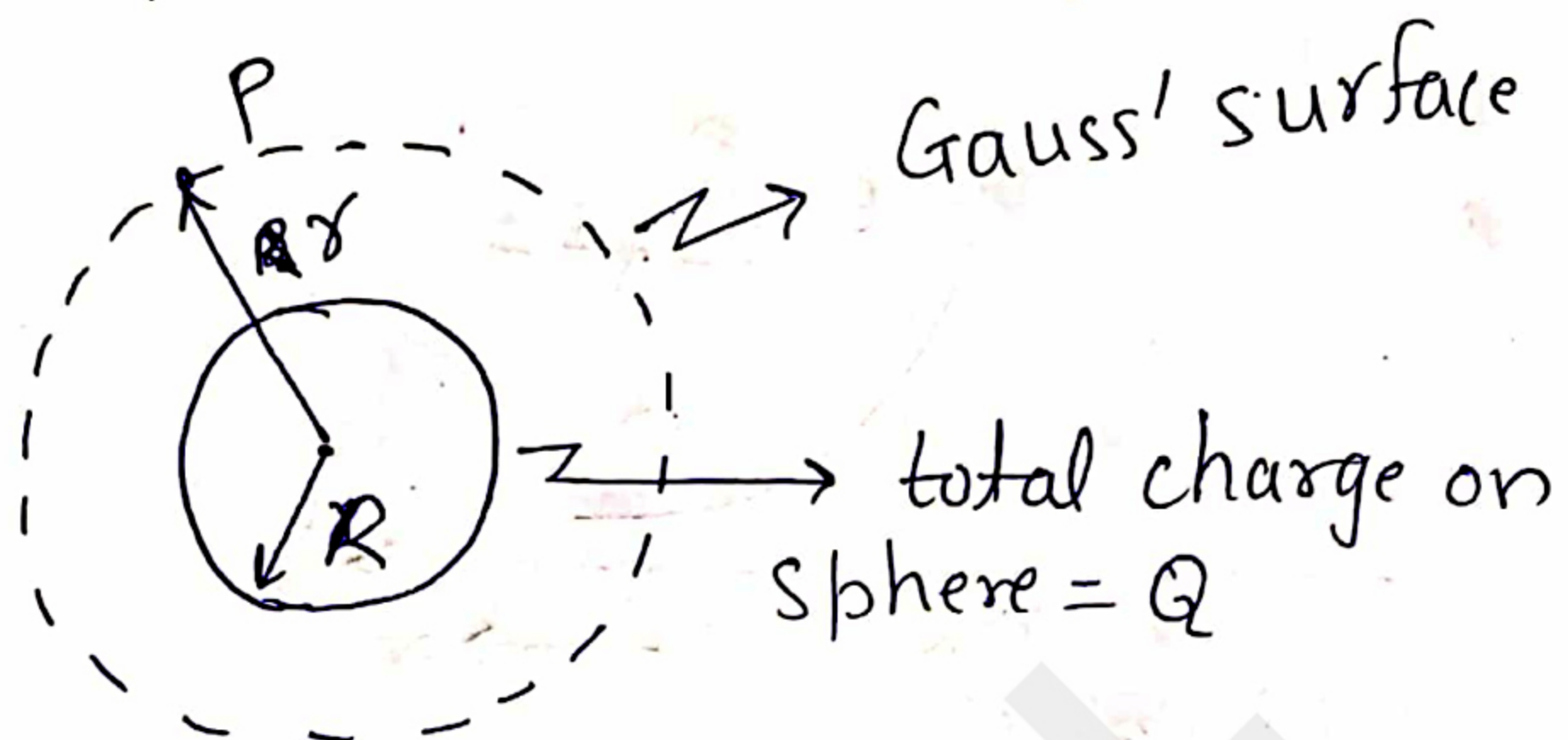
$$= E \times 4\pi r^2$$

$$= \frac{Q}{4\pi r^2 \epsilon} \times 4\pi r^2$$

$$\Rightarrow \boxed{\phi = \frac{Q}{\epsilon}}$$

gauss's law

Electric Field outside the sphere (shell) & (solid)



Electric field at point 'P' is:

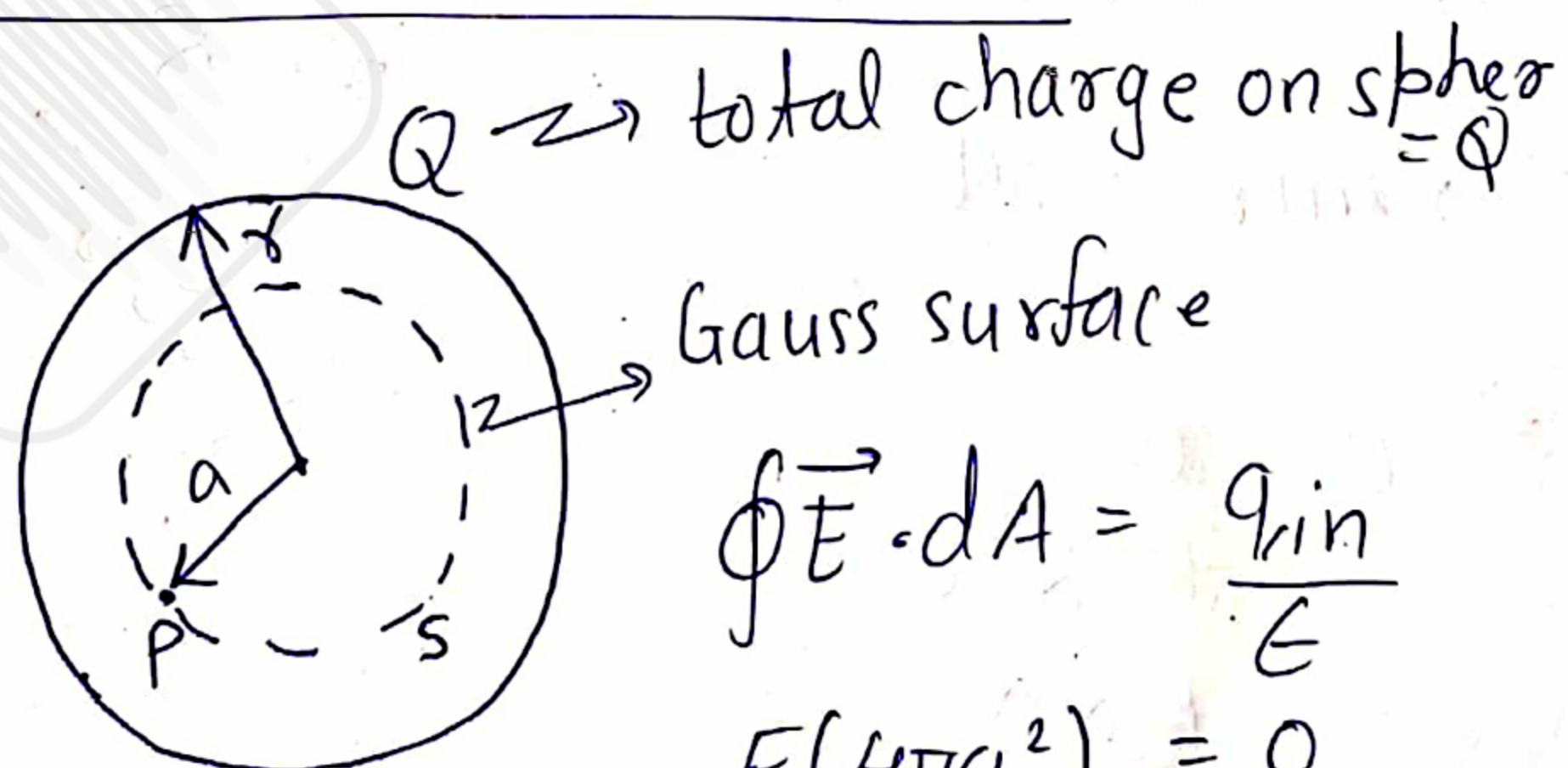
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon} \quad [\text{Gauss' law}]$$

$$E \times 4\pi r^2 = \frac{Q}{\epsilon} \quad (A = 4\pi r^2)$$

Same for both
Shell & solid sphere

$$\Rightarrow \boxed{E = \frac{Q}{4\pi r^2 \epsilon}} \quad \boxed{E = \frac{Q}{4\pi r^2 \epsilon}}$$

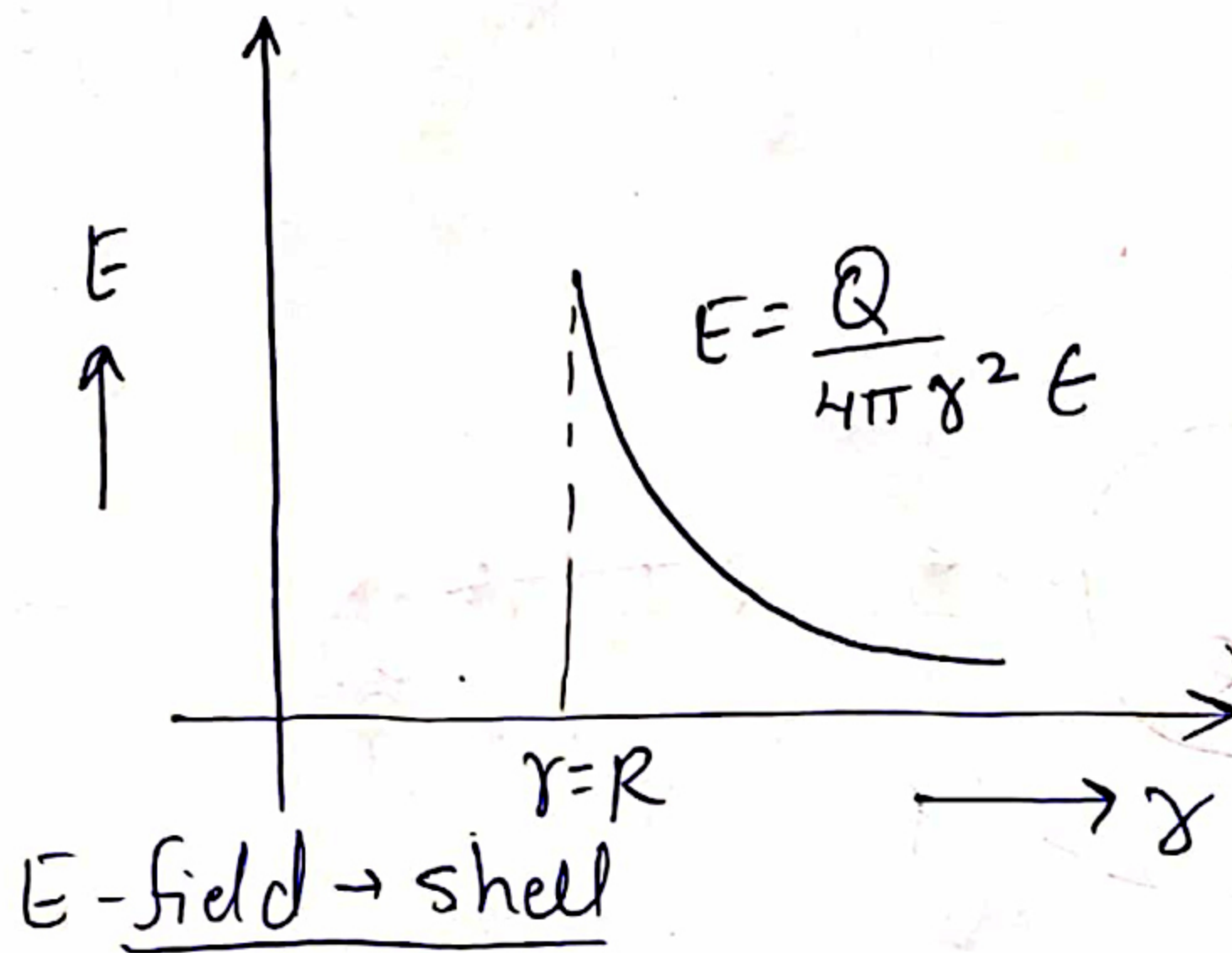
Electric field inside the shell:



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$E(4\pi a^2) = \frac{0}{\epsilon}$$

$$\Rightarrow \boxed{E = 0}$$



Electric field inside a solid uniformly charged sphere:

$\rho \rightarrow$ charge density $= \frac{Q}{\frac{4\pi R^3}{3}}$

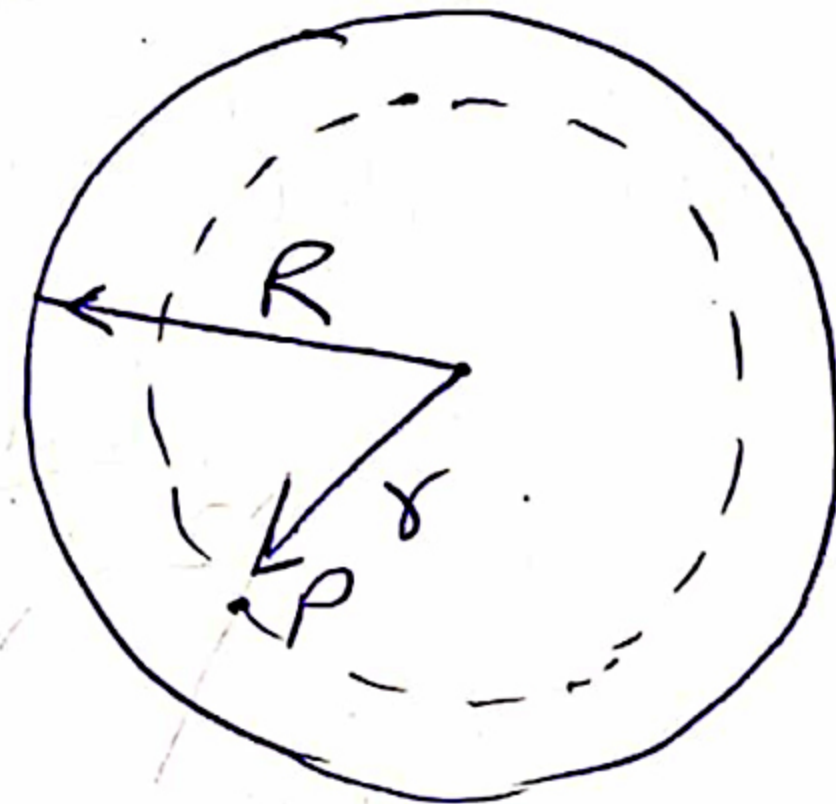
$q_{in} \rightarrow \frac{Q}{\frac{4\pi R^3}{3}} \times \frac{4\pi r^3}{3} = \frac{Q r^3}{R^3}$

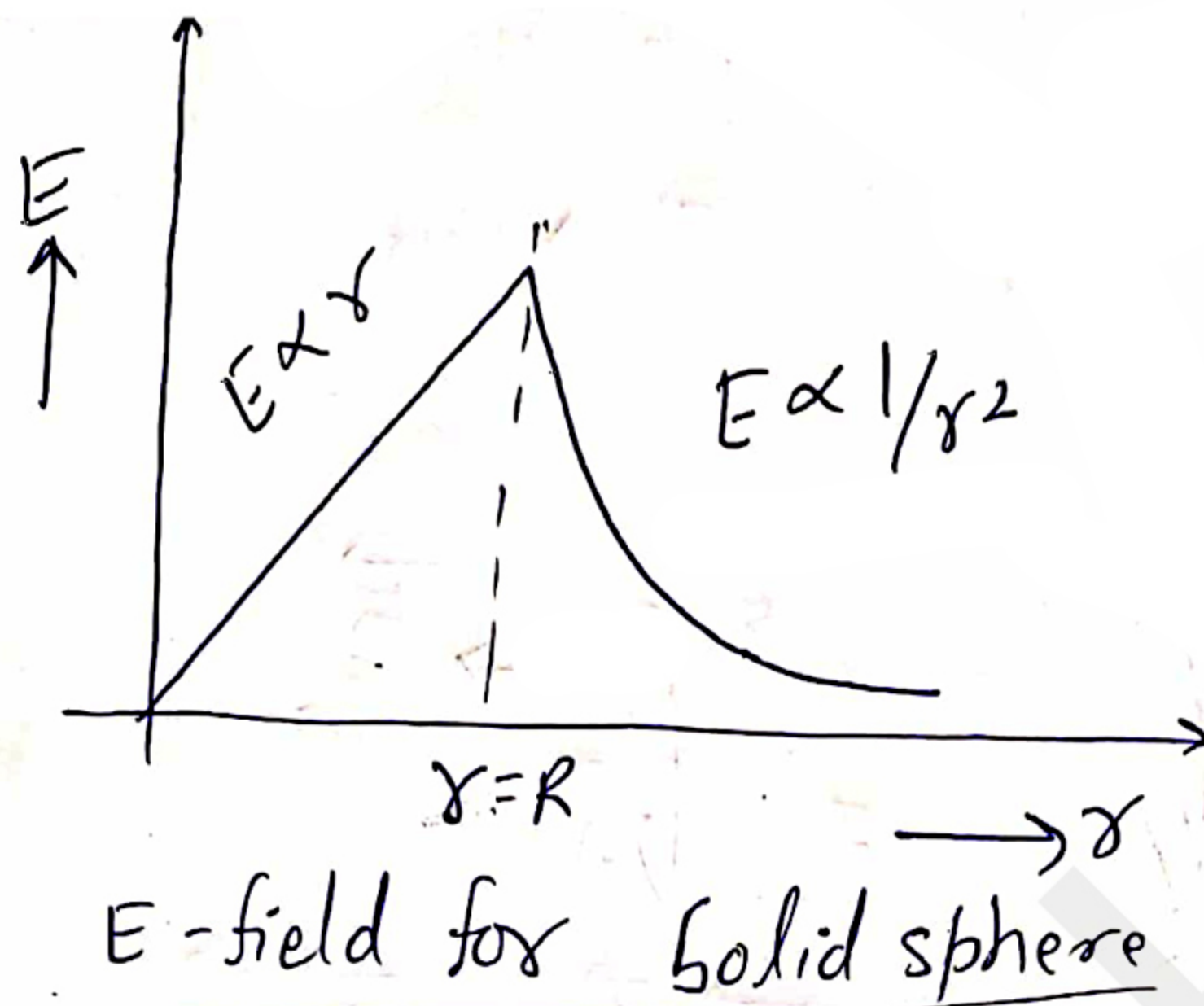
Electric field at P:

$$E \times 4\pi r^2 = \frac{q_{in}}{\epsilon}$$

$$\Rightarrow E = \frac{Q}{4\pi R^3} r$$

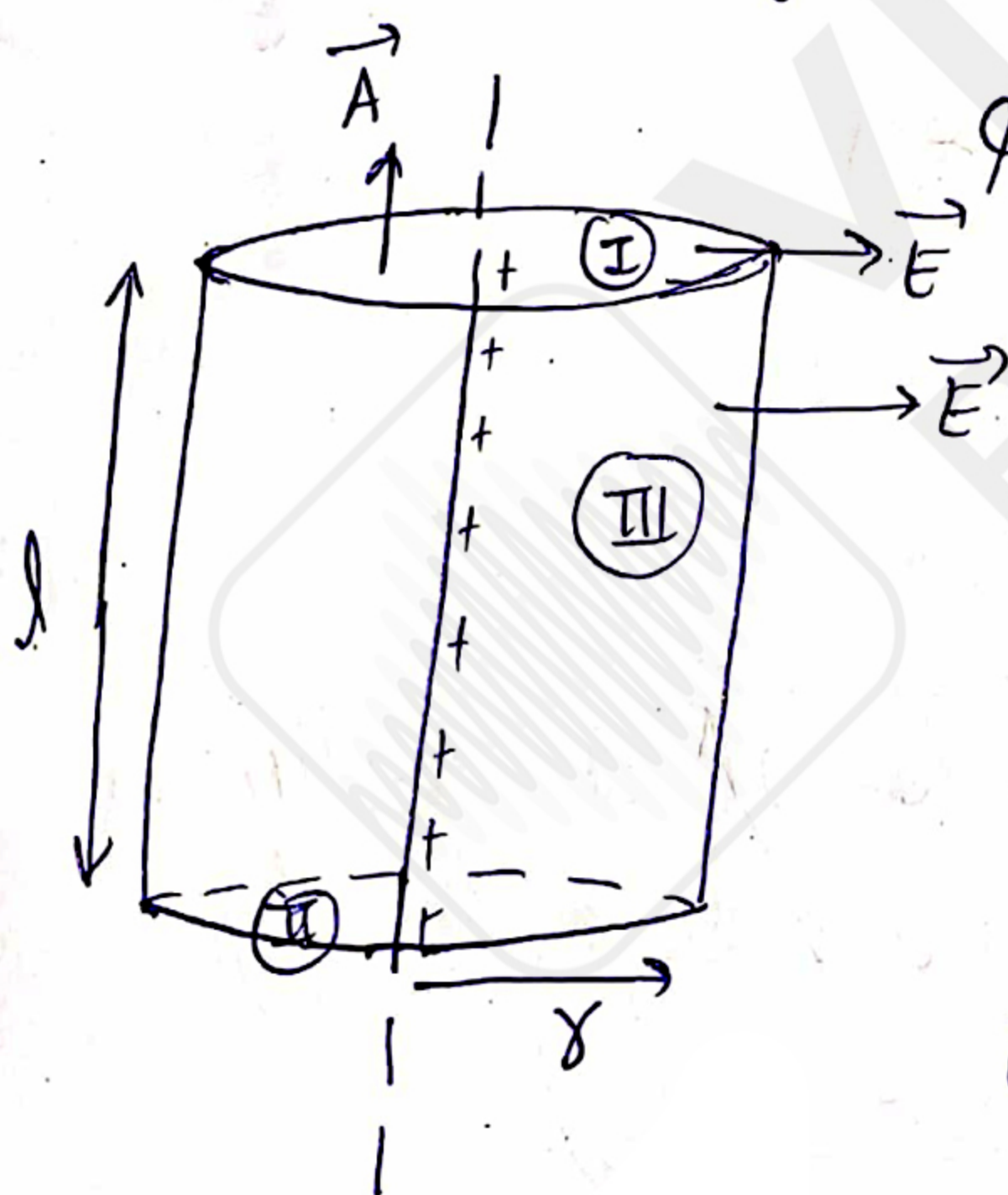
$$\Rightarrow E \propto r$$





FIELD OF A LINE CHARGE:

$\lambda \rightarrow$ line charge density $\Rightarrow$ (assumed positive)



$$\begin{aligned}\phi &= \vec{E} \cdot \vec{A}_I + \vec{E} \cdot \vec{A}_{II} + \vec{E} \cdot \vec{A}_{III} \\ &= EA_I \cos 90^\circ + EA_{II} \cos 0^\circ + EA \cos 0^\circ\end{aligned}$$

$$\phi = \frac{q_{in}}{\epsilon} = \frac{\lambda l}{\epsilon}$$

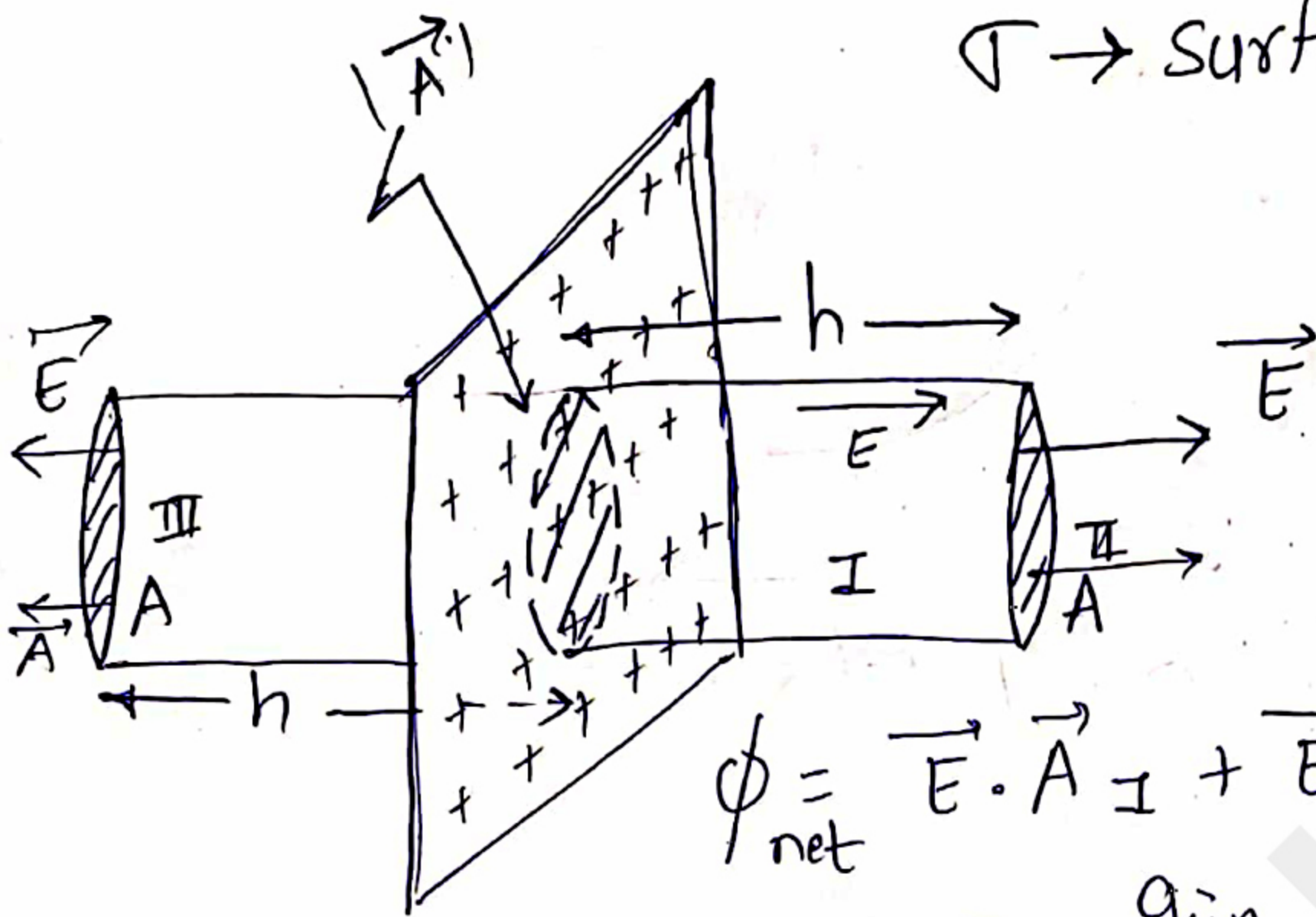
$$EA \cos 0 = \frac{\lambda l}{\epsilon}$$

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon}$$

$$E = \frac{\lambda}{2\pi r \epsilon}$$

Electric field Thin sheet [uniformly charged]

$\sigma \rightarrow$ surface charge density



$$\phi_{\text{net}} = \vec{E} \cdot \vec{A}_I + \vec{E} \cdot \vec{A}_{II} + \vec{E} \cdot \vec{A}_{III}$$

$$= \frac{q_{\text{in}}}{\epsilon}$$

$$\Rightarrow EA \cos 90^\circ + EA \cos 0 + EA \cos 0 = \frac{\sigma A}{\epsilon}$$

$$2EA = \frac{\sigma A}{\epsilon}$$

$$E = \frac{\sigma}{2\epsilon}$$

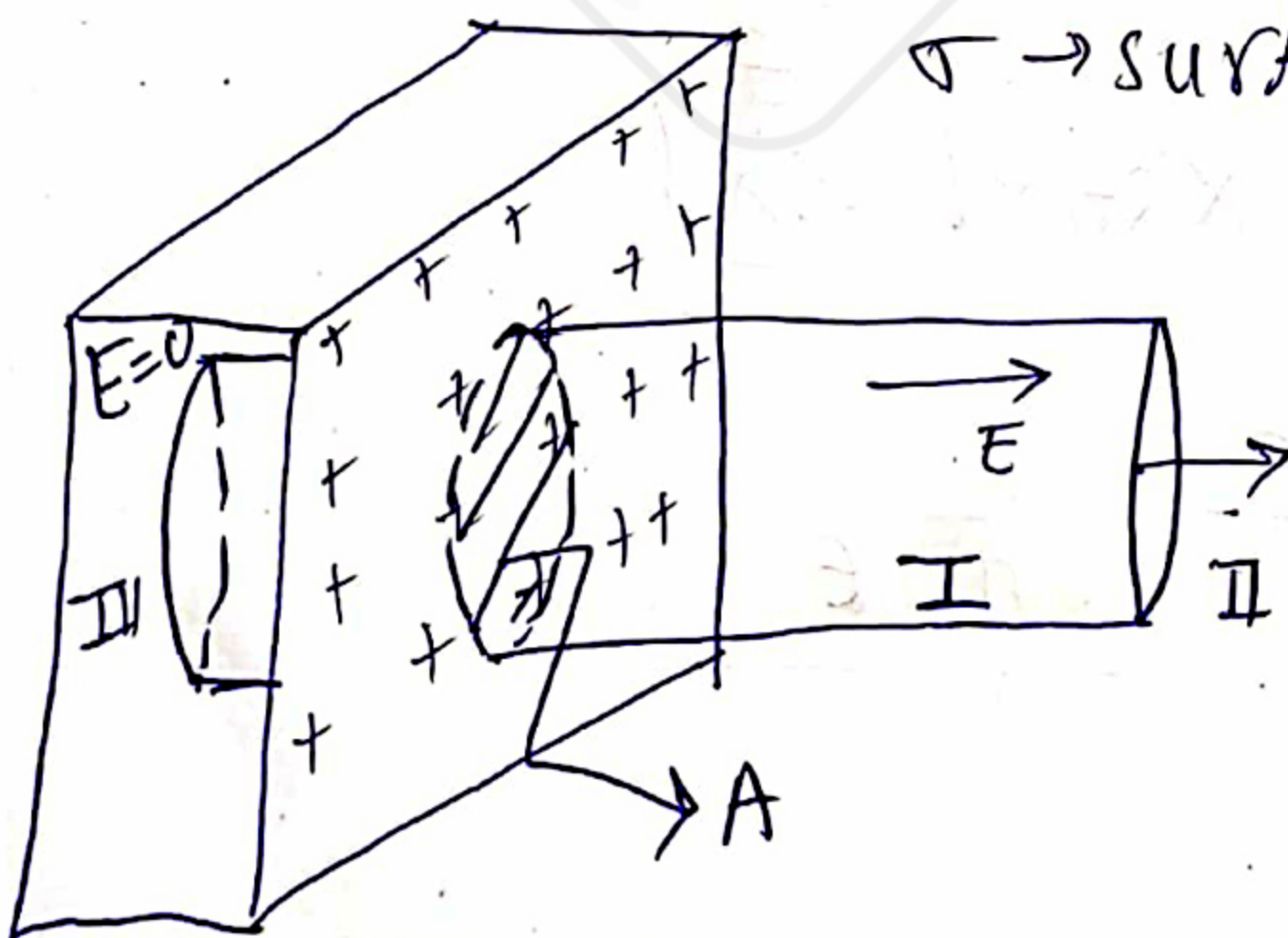
FIELD AT The surface of a conductor:

$\sigma \rightarrow$ surface charge density

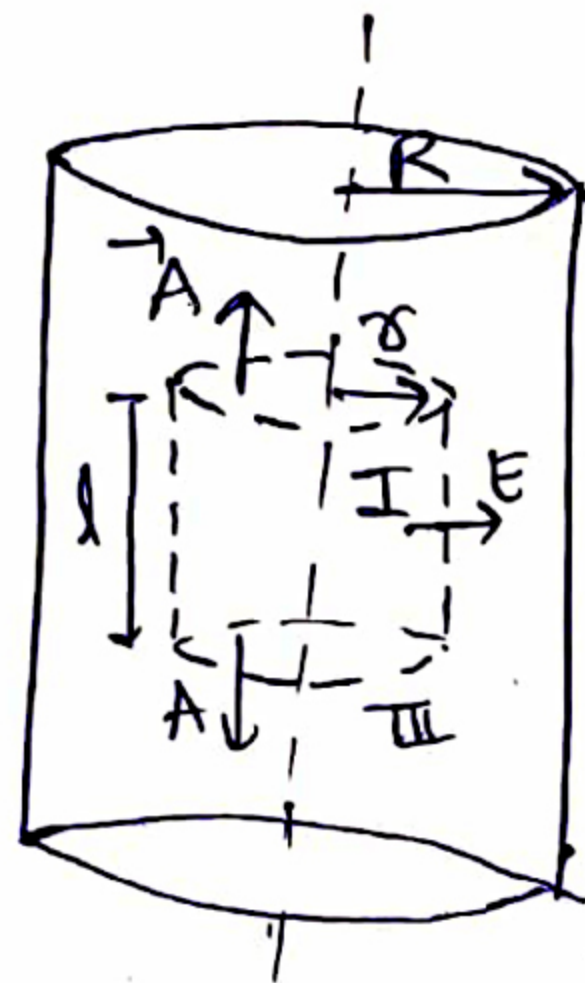
$$\phi = \frac{q_{\text{in}}}{\epsilon}$$

$$EA = \frac{\sigma A}{\epsilon}$$

$$E = \frac{\sigma A}{A\epsilon} = \frac{\sigma}{\epsilon}$$



Electric field due to a long uniformly charged cylinder:



$\rho \rightarrow$ volume charge density
 $R \rightarrow$ radius

$$q_{in} = \rho \times \pi r^2 l$$

$$\phi = q_{in} / \epsilon$$

$$E(2\pi r l) + E A \cos 90^\circ = \frac{\rho \pi r^2 l}{\epsilon}$$

$$\Rightarrow \boxed{E = \frac{\rho r}{2\epsilon}} \quad | \quad \boxed{E \propto r}$$

and outside the cylinder:

$$q_{in} = \rho \times \pi R^2 l$$

$$\Rightarrow E(2\pi r l) = \frac{\rho \pi R^2 l}{\epsilon}$$

$$E = \frac{\rho R^2}{2\epsilon r} \Rightarrow \boxed{E \propto \frac{1}{r}}$$

