VISUAL PHYSICS

SHORT NOTES

CHAPTER

Fluids

Fluids

Fluid is a substance that continually deform (flows) under an applied shear stress.

Pressure:

$$P = \frac{F \rightarrow \text{Applied force}}{A \rightarrow \text{Area of contact}}$$

(i) for solid, Area of contact $\neq$ Apparent Area. And hence pressure is not useful concept for solid analysis.

(ii) for liquid, gas, Area of contact = Apparent Area.

$\Rightarrow$ $\boxed{1 \text{ atm} = 1.013 \times 10^5 \text{ pa}}$

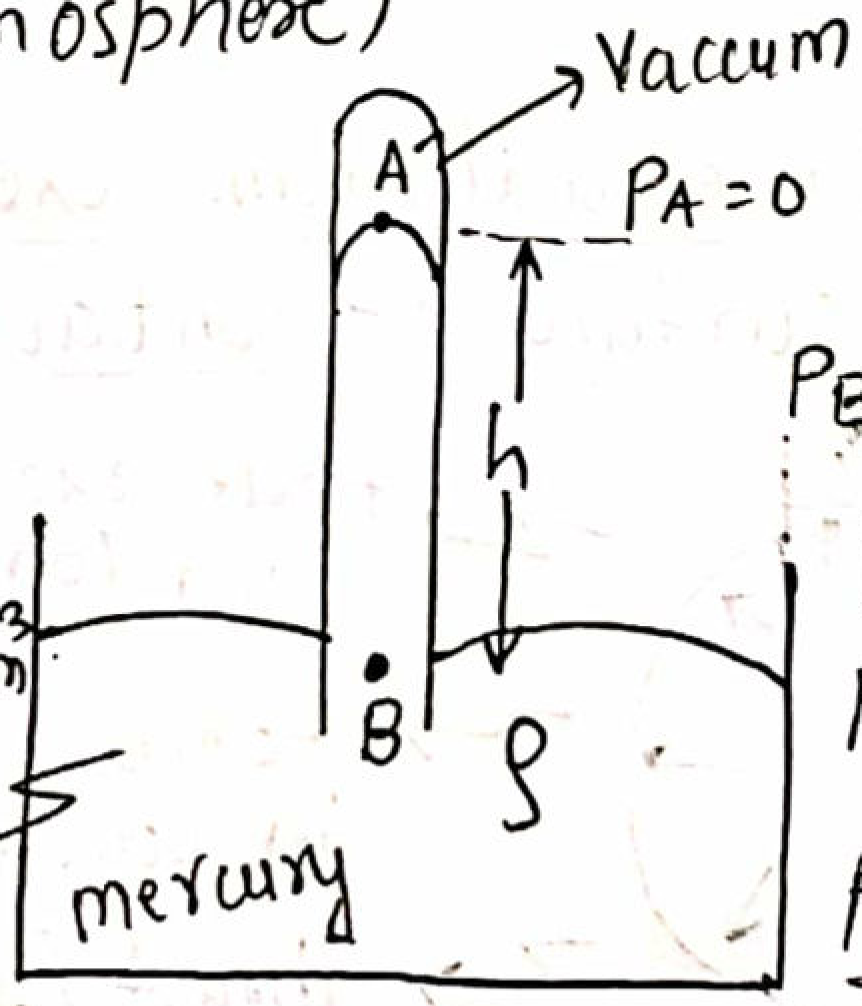
$\boxed{1 \text{ bar} = 10^5 \text{ pa}}$

Measuring pressure: (Atmosphere)

$\Rightarrow$ Mercury barometer

Invented by Torricelli.

$\rho = 13.6 \times 10^3 \text{ kg/m}^3$



$P_B = P_A + \rho g h$

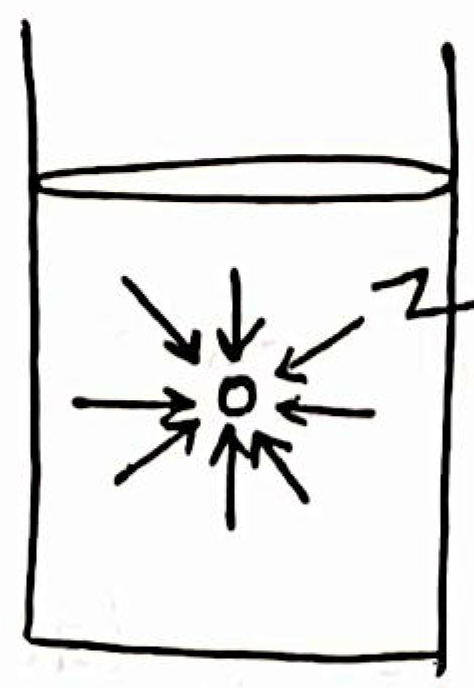
$h = 76 \text{ cm}$

$P_B = \rho \times 9.8 \times 0.76$

$\underline{P_B = 1.013 \times 10^5 \text{ Pa}}$

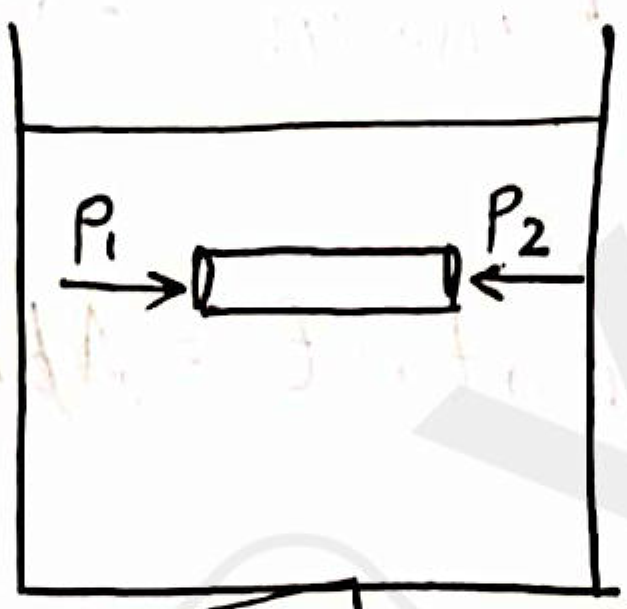
Properties of Pressure:

1. Pressure is isotropic:

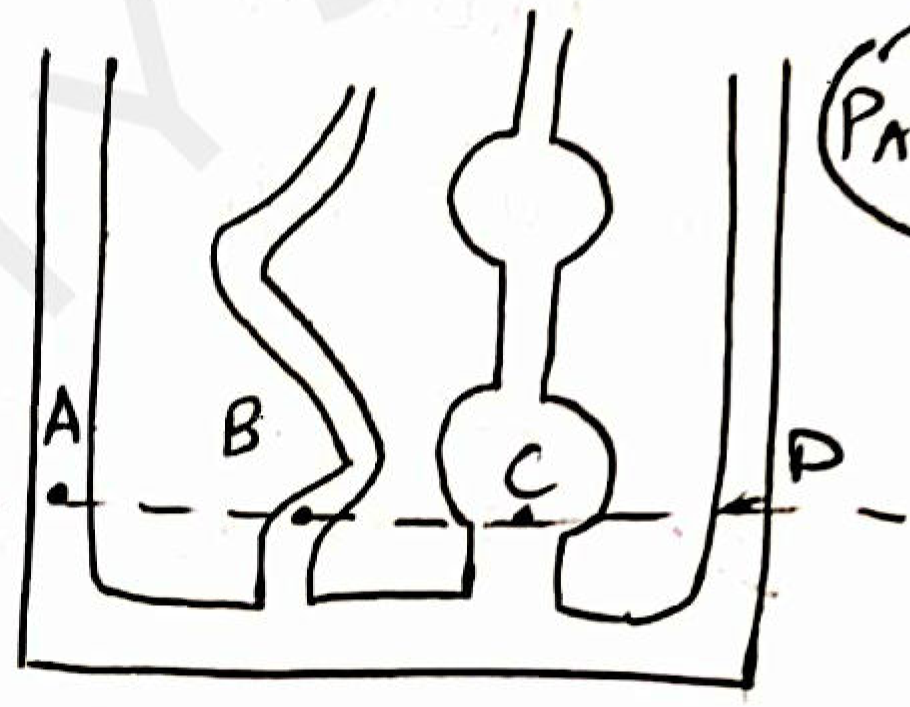


pressure exerted by fluid at a point is same from all direction.

2. Pressure is uniform on a horizontal plane (hydrostatic)



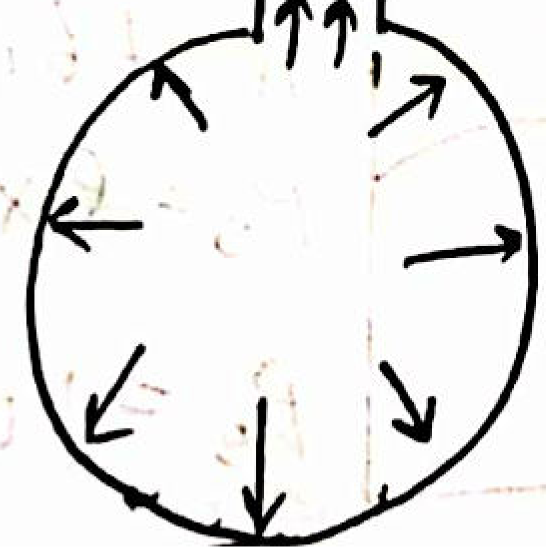
$P_1 = P_2$ → when fluid is at rest



$P_A = P_B = P_C = P_D$

Hydrostatics → Pressure is same on horizontal line. → when fluid is same ★

3. Fluids in equillibrium exert pressure at right angles to the surface of contact.

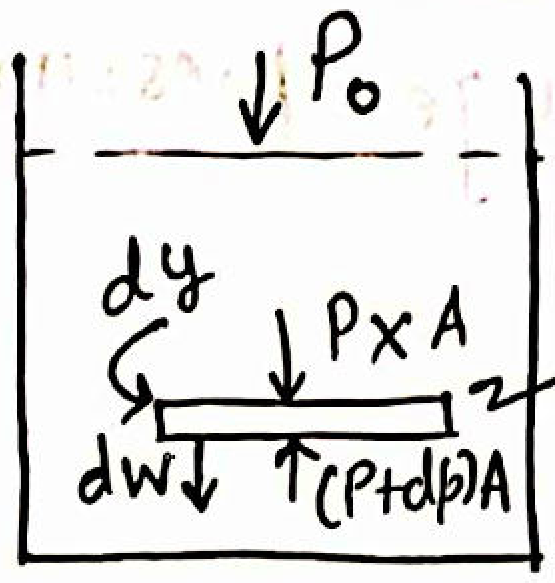


→ Force exerted perpendicular to surface in contact

→ force exerted by a liquid (→gas) on any surface in contact called thrust.

→ Thrust / Area = pressure

4. Pressure varies with depth and height:

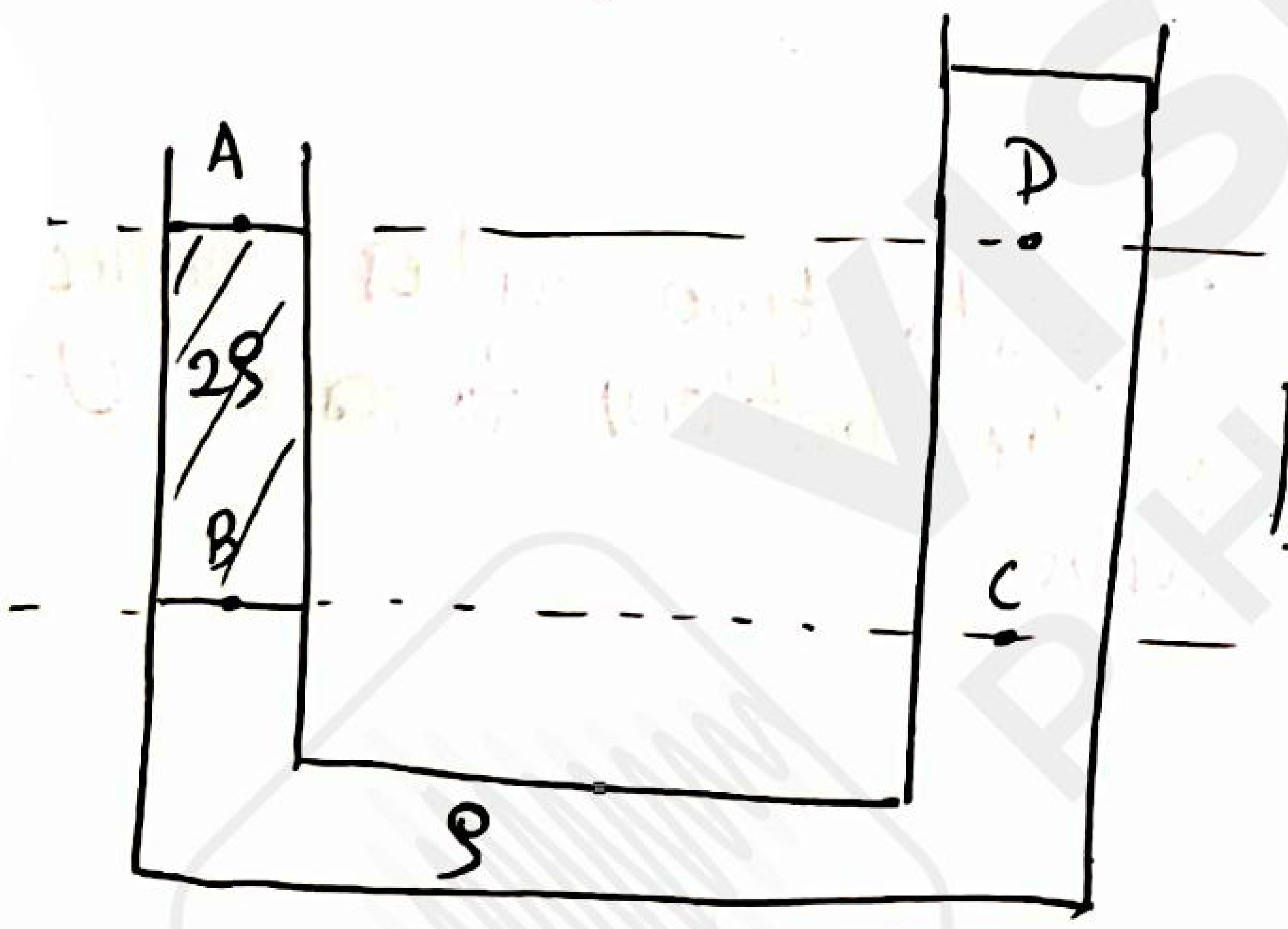


$$(P+dp)\times A - P\times A = dw$$

$dw \Rightarrow$ weight $= \rho\times A\times dy\times g$

$$\int_{P_0}^{P} dP = \int_0^h \rho g\,dy$$

$$\boxed{P = P_0 + \rho g h}$$



$\boxed{P_B = P_C}$ not $\boxed{P_A \neq P_D}$

Absolute pressure and Gauge pressure:

$$P_{absolute} = P_{gauge} + P_{atm}$$

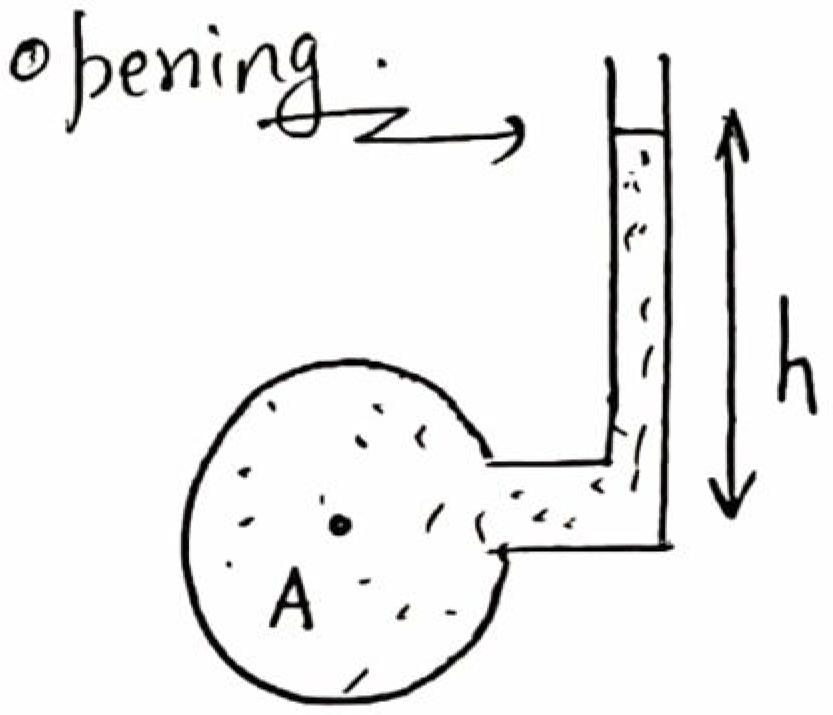
total pressure

reference from Pressure atmosphere

Manometers

Manometers → device used to measure **Gauge pressure**.

Simple monometer



$P_A = \rho g h$

- → Cannot measure pressure in a gas
- → Can measure only positive gauge pressure.
- → Cannot measure large pressure as tube will be long.

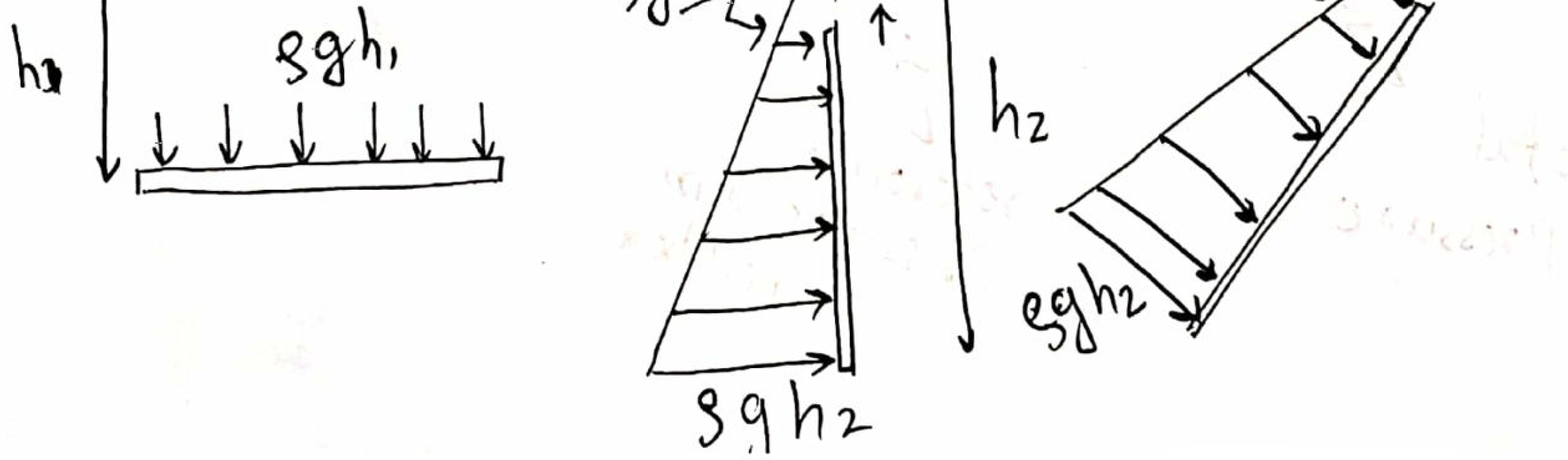
U-tube manometer

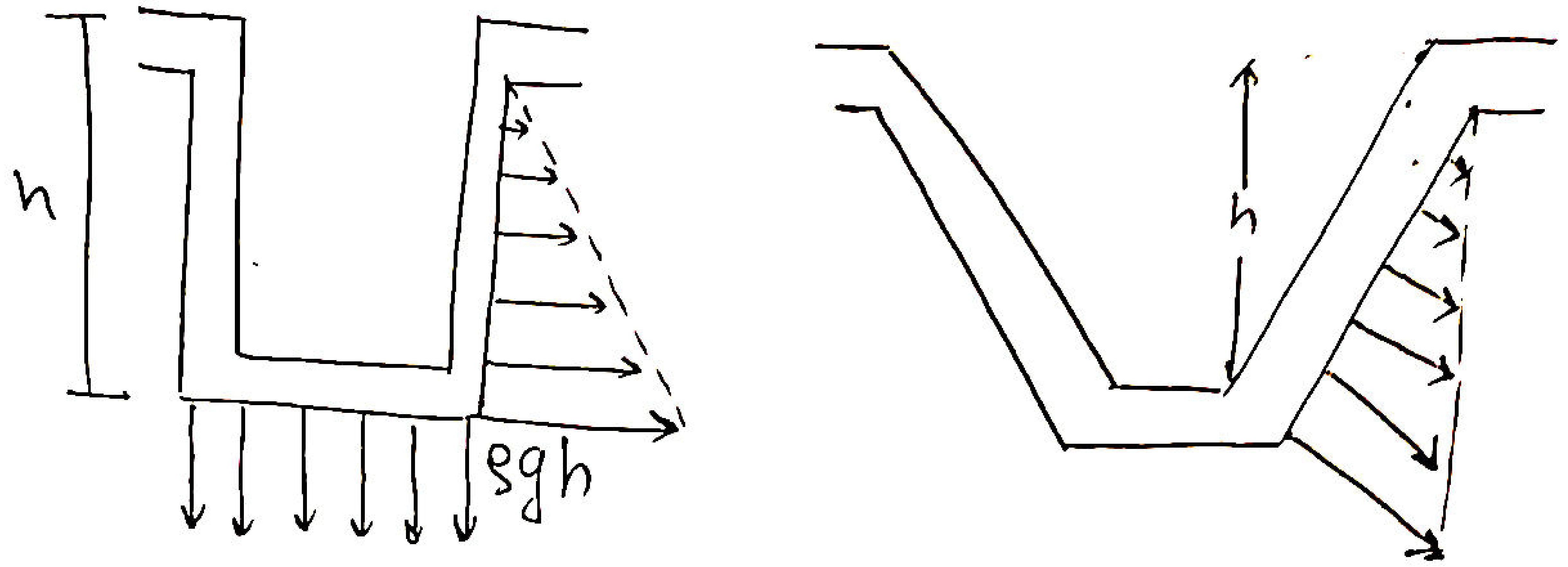
U-tube manometer → Consists two tubes connected at the bottom to form U-shape tube

↓

Can measure high pressure difference.

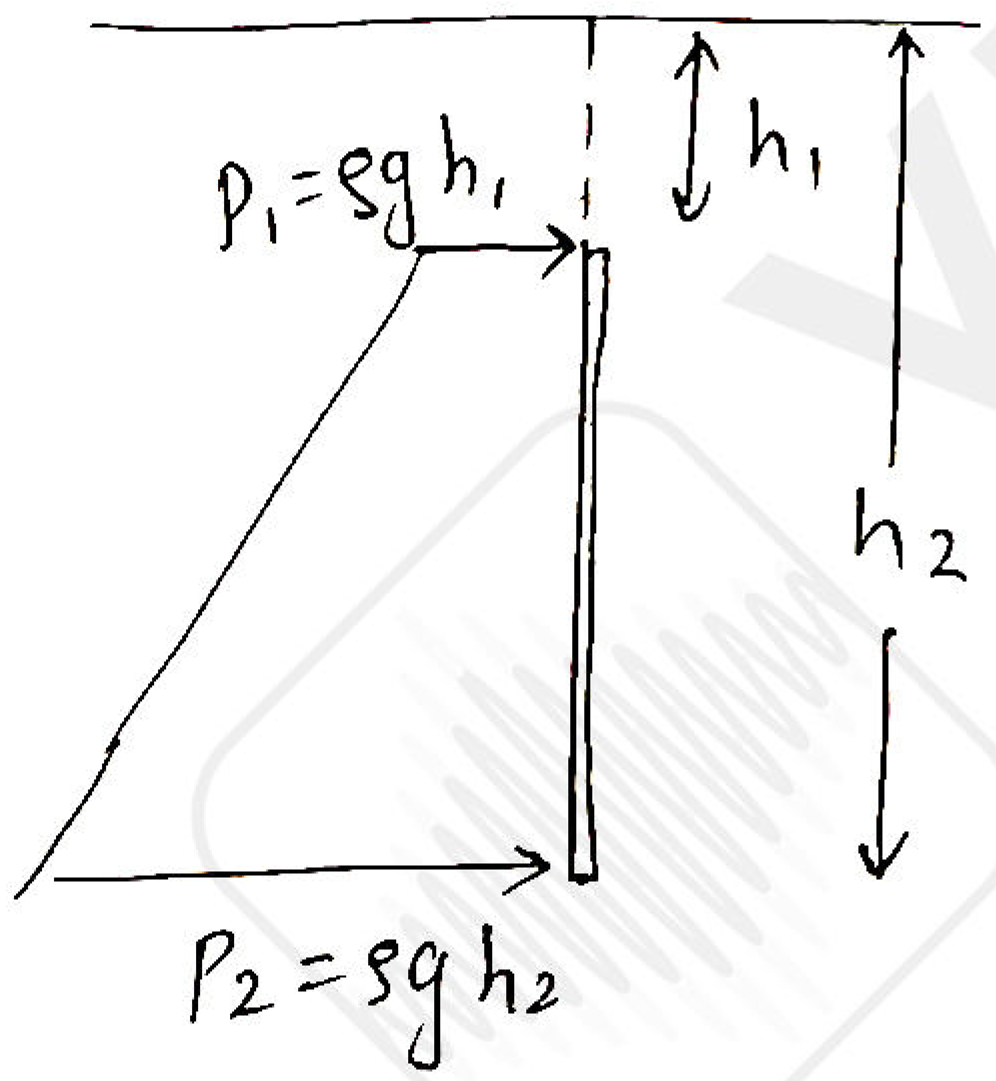
Pressure Diagrams and Force on Boundaries:





Force on Surface Immersed in a liquid and Centre of force

For any pressure diagram.



$$P_{average} = \frac{P_1 + P_2}{2} = P_{av}$$

force on the surface

$$= P_{av} \times Area$$

$$\boxed{F = \rho g \frac{(h_1 + h_2)}{2} \times A}$$

Thrust

$$\text{Thrust} = \int \rho g h b\, dh = \rho g \int b h\, dh$$

as $\bar{h}$ = centroid depth of the area $= \frac{\int (b\, dh) h}{A} = \bar{h}$

$$\Rightarrow \boxed{\text{Thrust} = (\rho g \bar{h}) A}$$

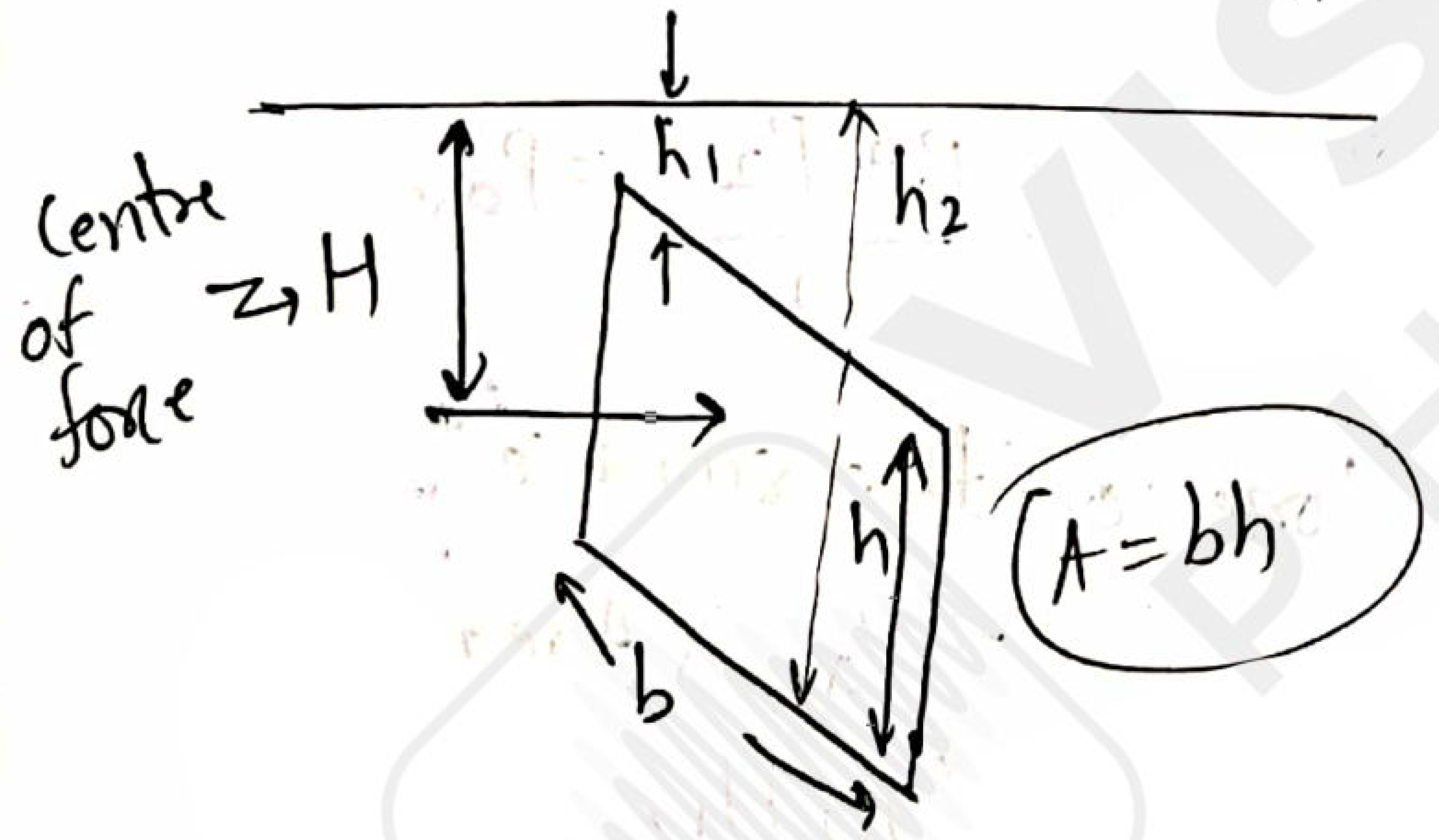
Centre of Force

↳ The point where the entire Force can be thought of acting.

↳ producing same rotational effect as the Individual thrust on each elemental strip.

$$M = \int_{h_1}^{h_2} (b\,dh)(h\rho g)h = \rho g \bar{h} A H$$ → Centre of force

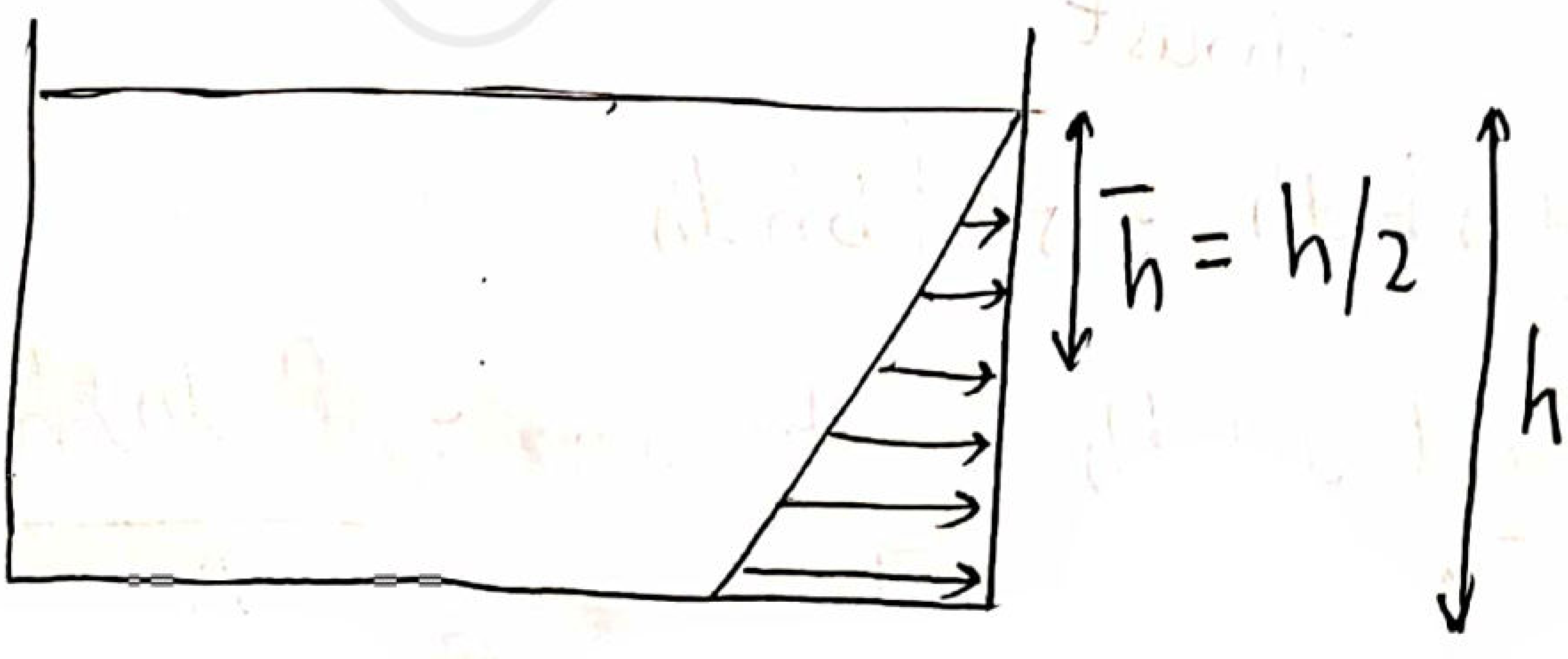
$$\Rightarrow H = \frac{1}{A\bar{h}} \int_{h_1}^{h_2} (bh^2)\,dh = \frac{1}{bh}\left(\frac{2}{h}\right) \times b\frac{h^3}{3}$$



$$\boxed{H = \frac{2}{3}h}$$

$A = bh$

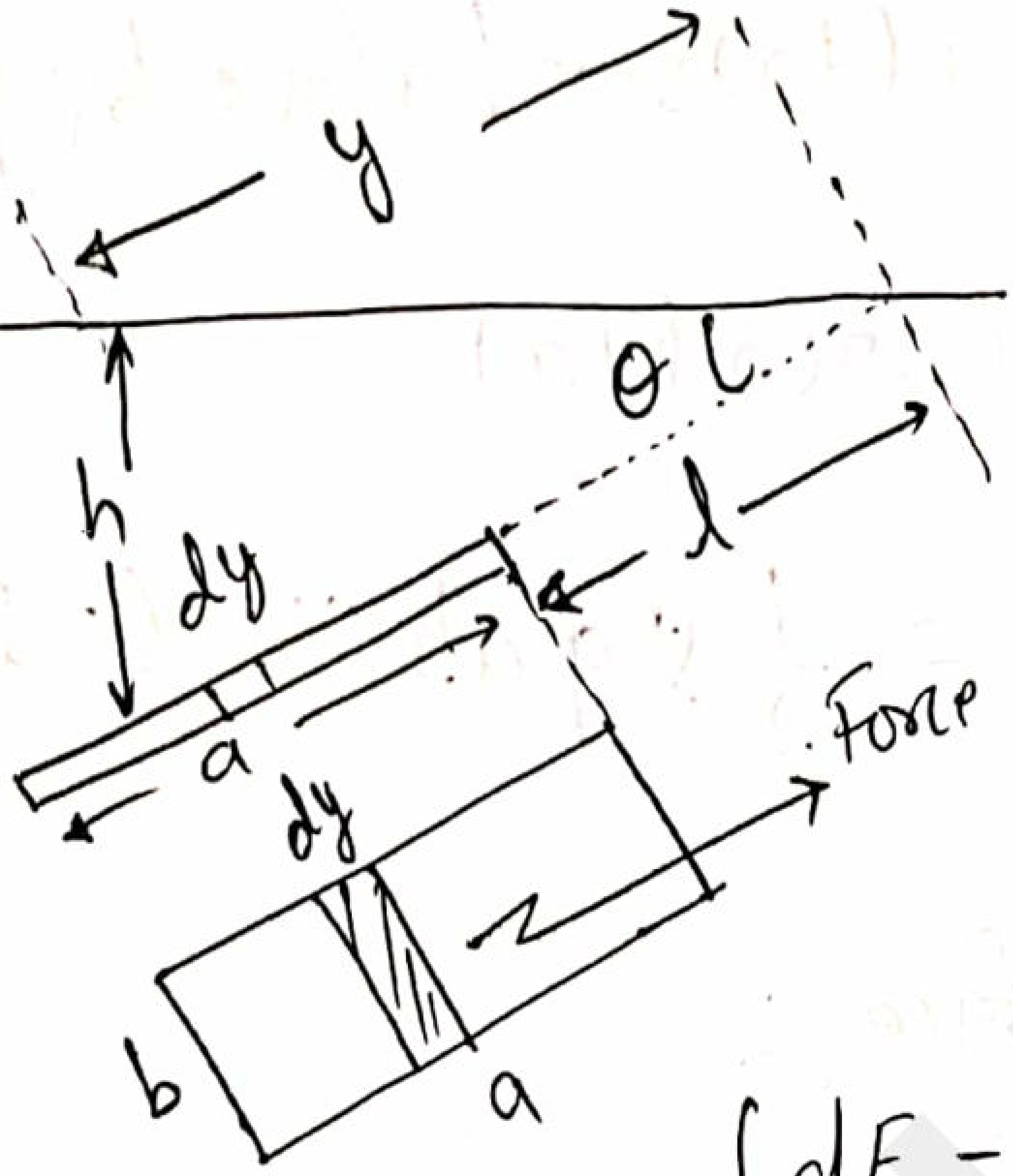
Force on vertical wall:



Force = Thrust $= (\rho g A)\bar{h}$

$$\boxed{\text{Force} = \frac{\rho g A h}{2}}$$

Hydrostatic Force on Inclined surfaces Immersed in Liquid



$$\boxed{h = y\sin\theta}$$

Force on element

$$dF = P(b\,dy)$$

$$dF = \rho g h\, b\,dy$$

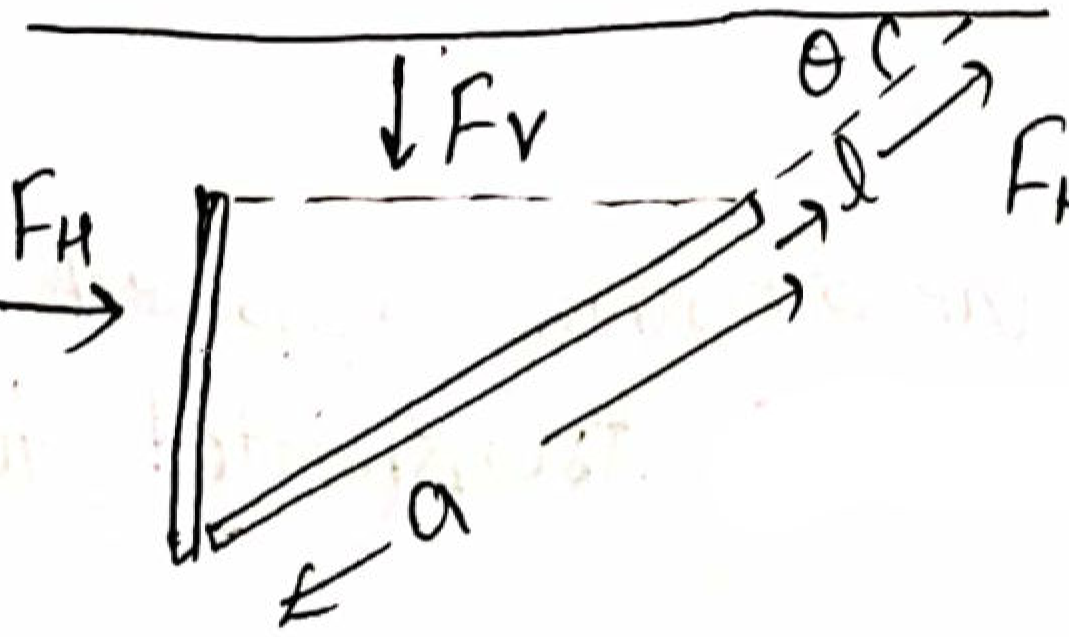
$$\int dF = F = \rho g b \sin\theta \int_{l}^{a+l} y\,dy$$

$$F = \rho g \frac{b\sin\theta}{2}\left[y^2\right]_{l}^{a+l}$$

$$F = \rho g b \frac{\sin\theta}{2}\left[(l+a)^2 - l^2\right]$$

$$\boxed{F = \frac{\rho g a b}{2}\left[a + 2l\right]}$$

Other method:



Projected area $= ab\sin\theta$

F_H = Force on area projected on vertical plane

$$F_H = \frac{1}{2}\left[\rho g l\sin\theta + \rho g (l+a)\sin\theta\right] \times (a\,b\sin\theta)$$

Projected area

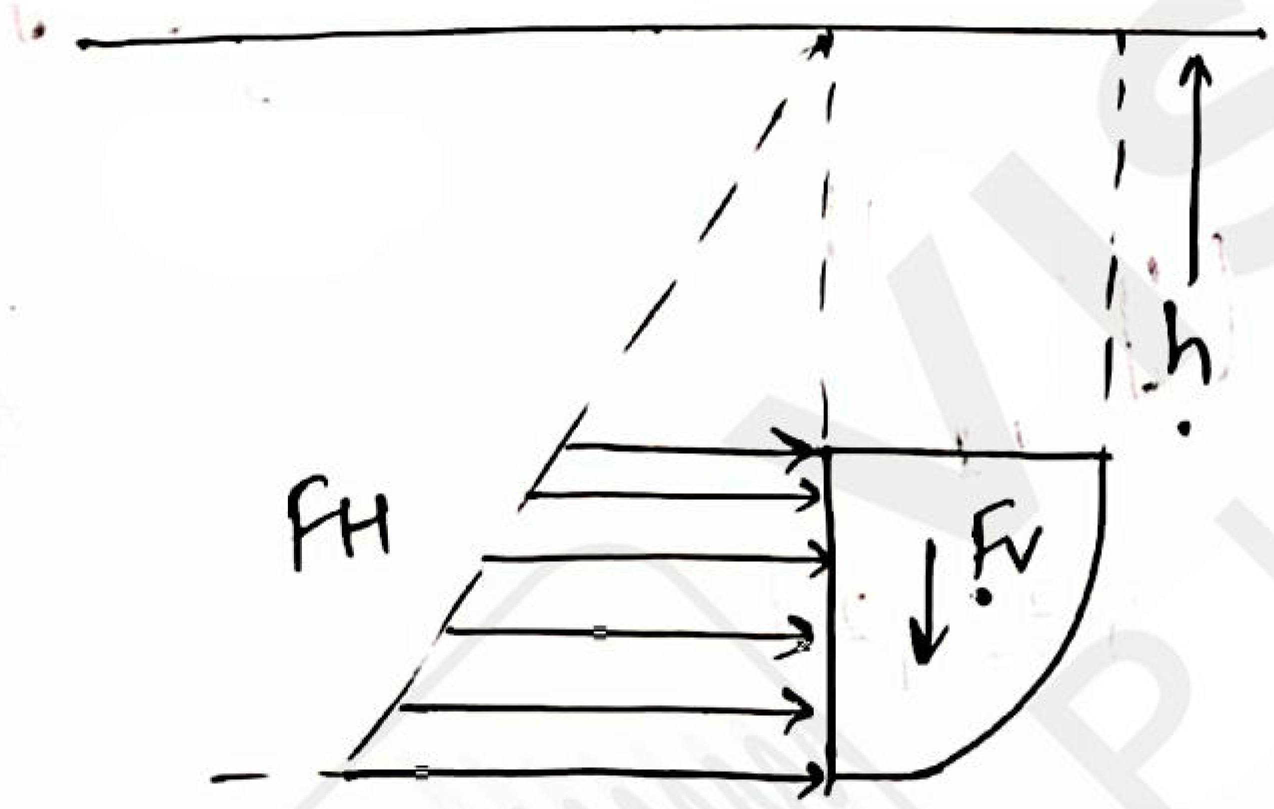
$F_V =$ Weight of water prism above Inclined surface

$$= \frac{1}{2}\left[l\sin\theta + (l+a)\sin\theta\right] a\cos\theta\, b\rho g$$

$$F_V = \frac{1}{2}\rho g\, ab\sin\theta\cos\theta(2l+a)$$

$$\boxed{F = \sqrt{F_H^2 + F_V^2} = \frac{1}{2}\rho g\, ab\sin\theta(2l+a)}$$

Force on Curved Surface :



$dF_H = dF\sin\theta = \rho g h\, dA\sin\theta$

or force on projected area.

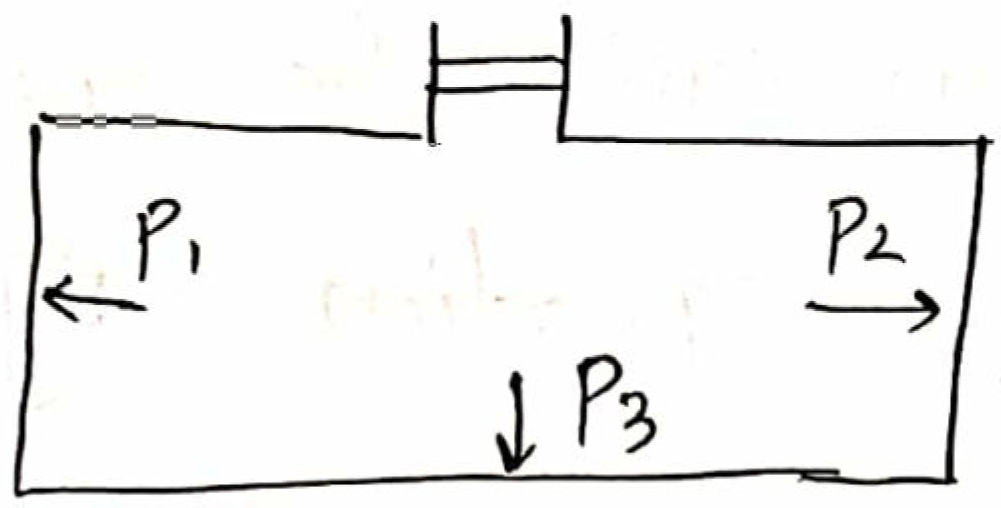
$F_H = \rho g \bar{h} A\sin\theta$

$F_V =$ Weight of water prism over the surface

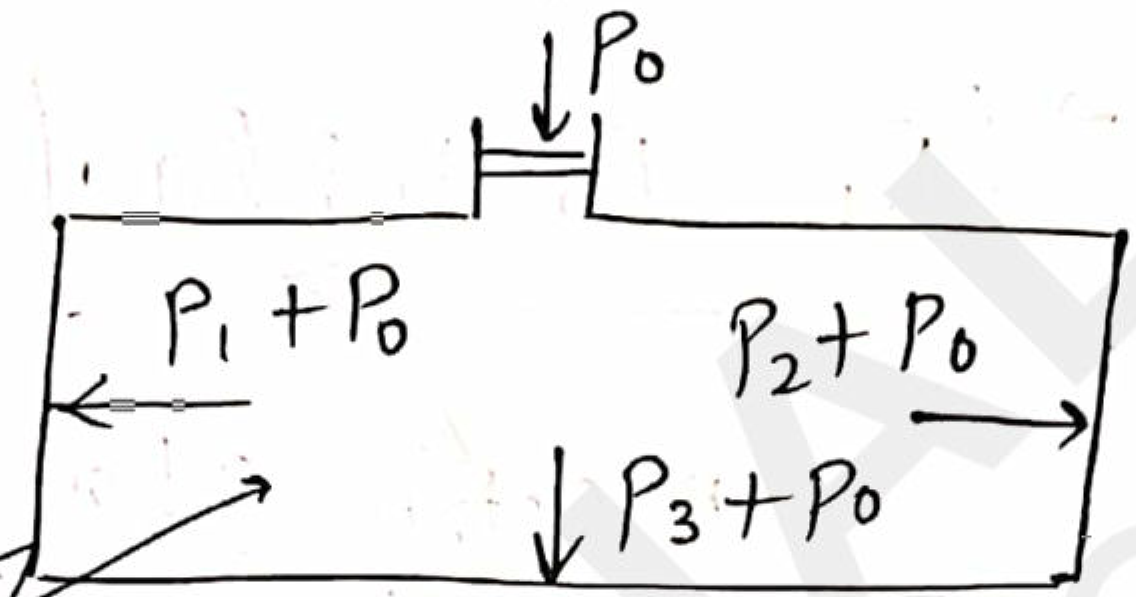
$F_V = \rho g \int h(dA)\cos\theta$

Pascal's law

"Any change in the pressure applied to an enclosed Incompressible Fluid is transmitted to every point of the fluid."

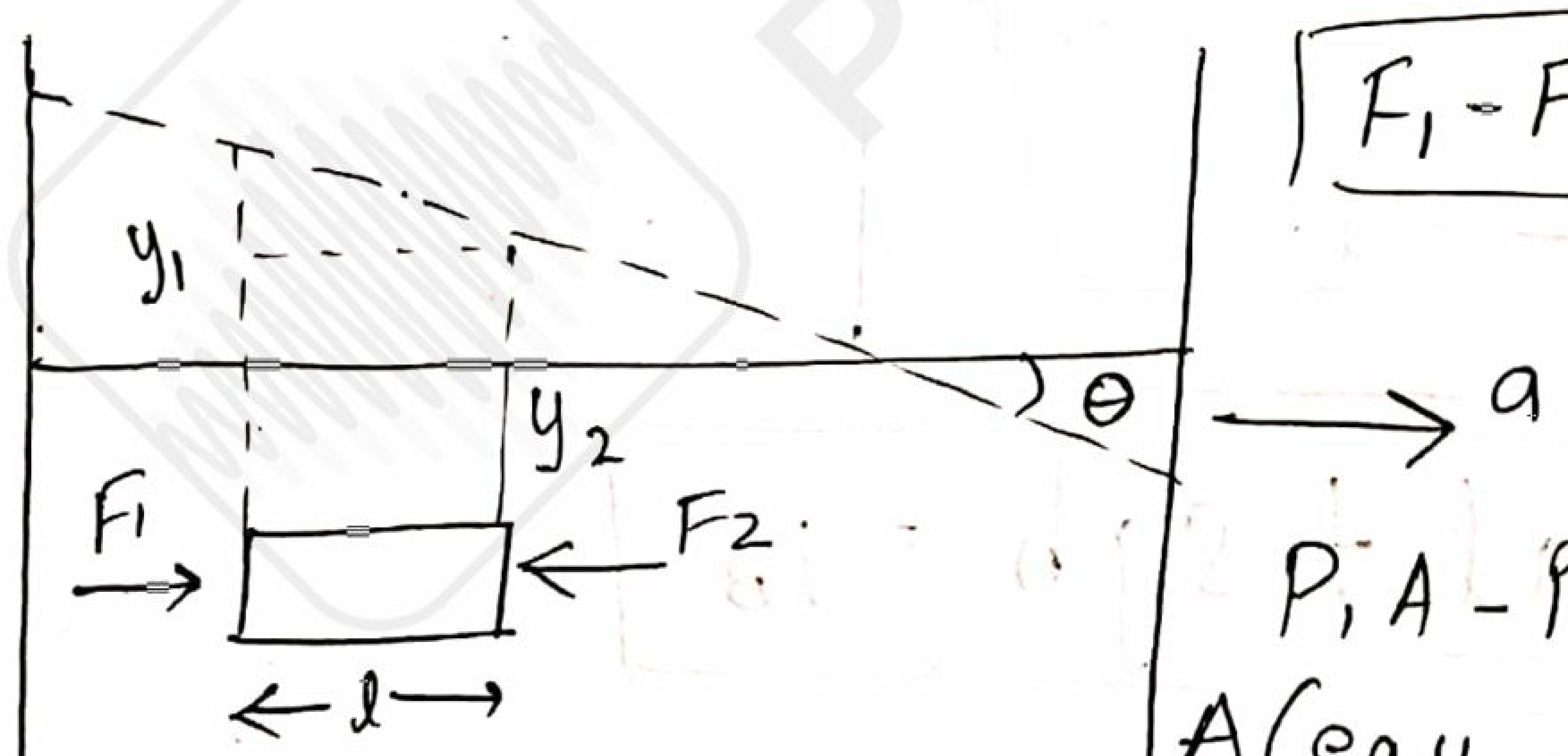


Now if we apply Pressure P_0



equal amount of increase of Pressure P_0 Throughout

Fluids in Uniformly Accelerating Motion



$$\boxed{F_1 - F_2 = ma}$$

$$P_1 A - P_2 A = ma$$

$$A(\rho g y_1 - \rho g y_2) = ma$$

as $m = \rho A l$

$$\text{as } \frac{y_1 - y_2}{l} = \frac{ma}{\rho g l} = \frac{\rho A l a}{\rho g l}$$

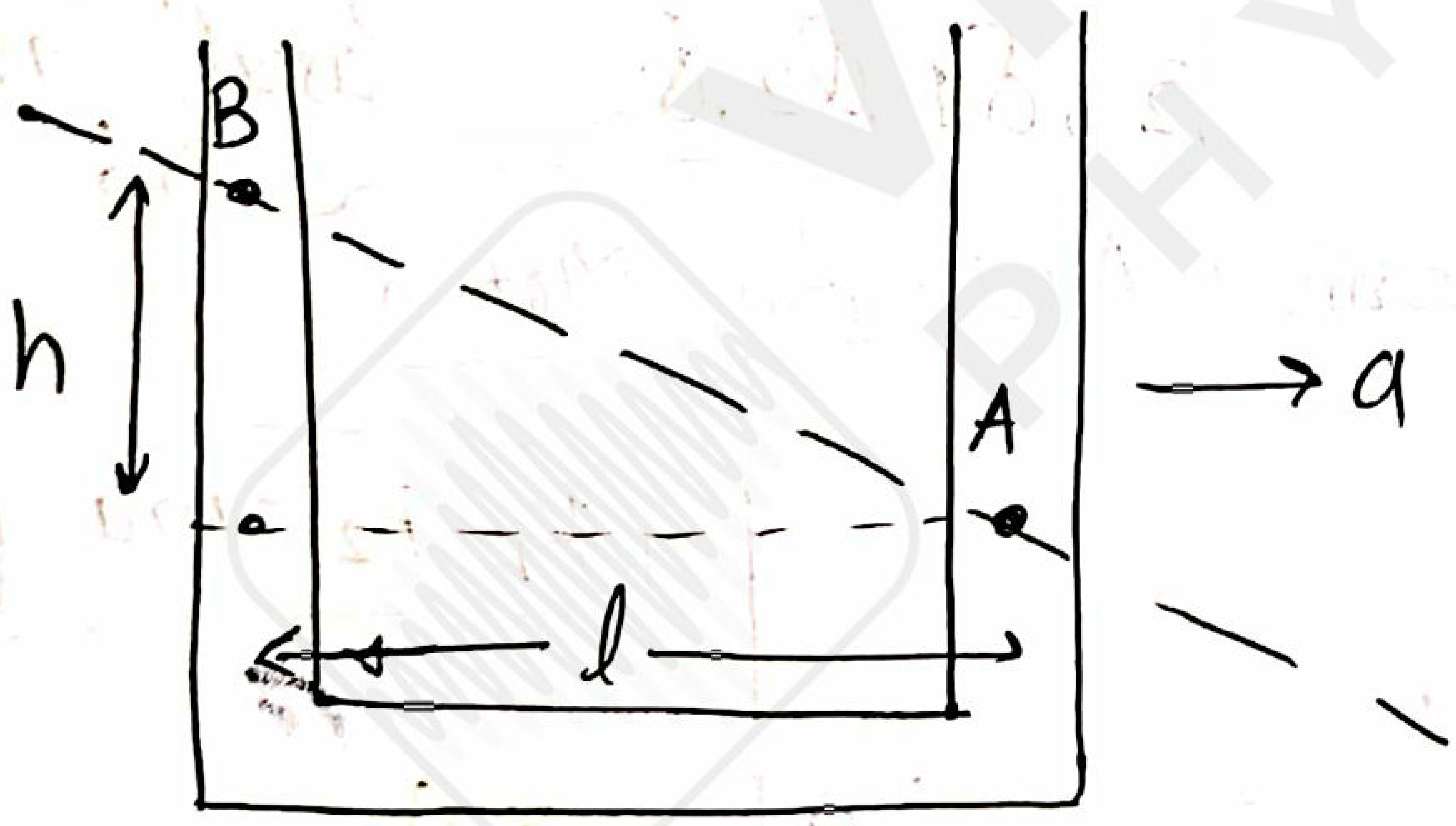
$$\Rightarrow \boxed{\frac{y_1 - y_2}{l} = \tan\theta = a/g}$$

★ now P_2 and P_1 can easily be found using static pressure equation $= \rho g h$

as $P_1 - P_2 = (\rho l)a$

$\Rightarrow \boxed{P_1 = P_2 + \rho l a}$

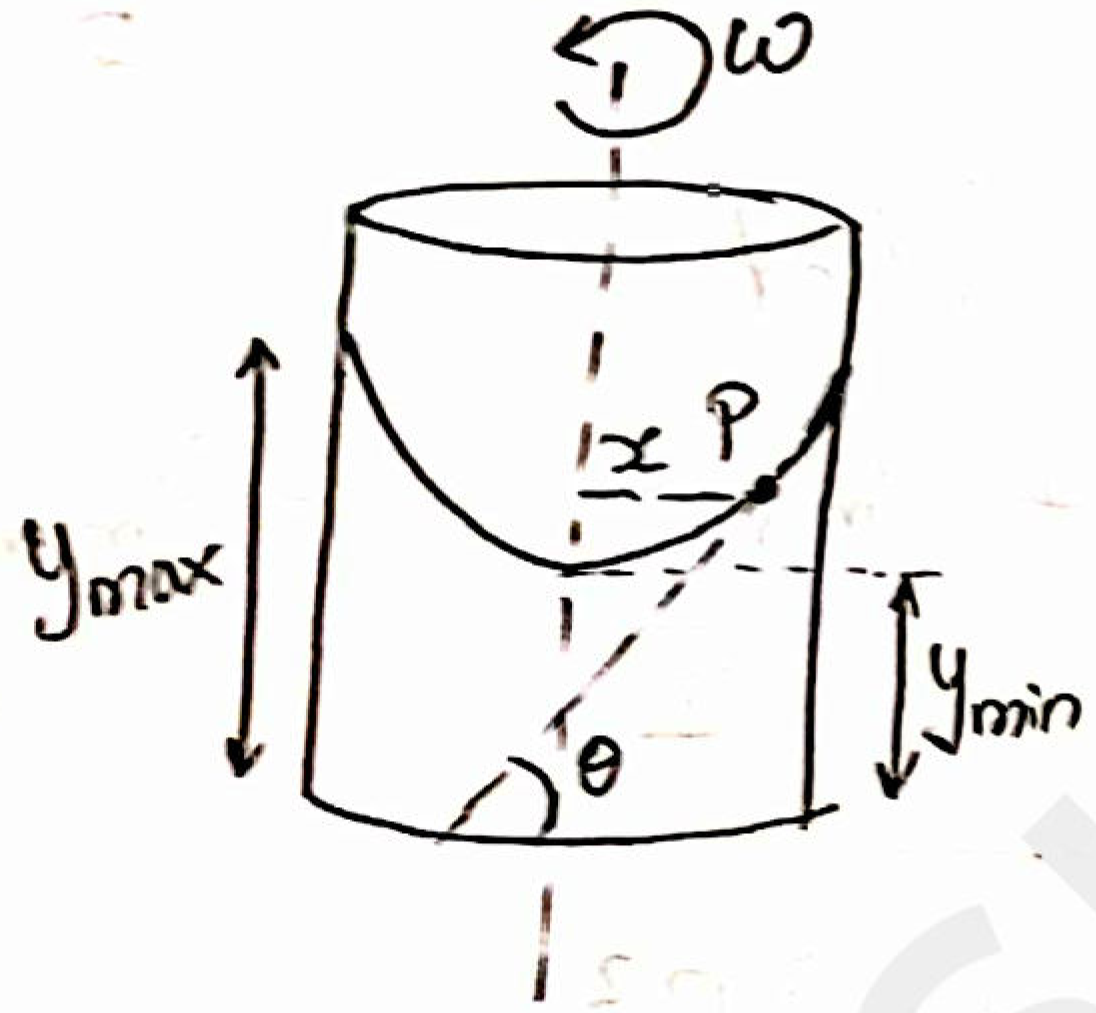
an extra $\rho l a$ term added while going horizontal in accelerating fluid.

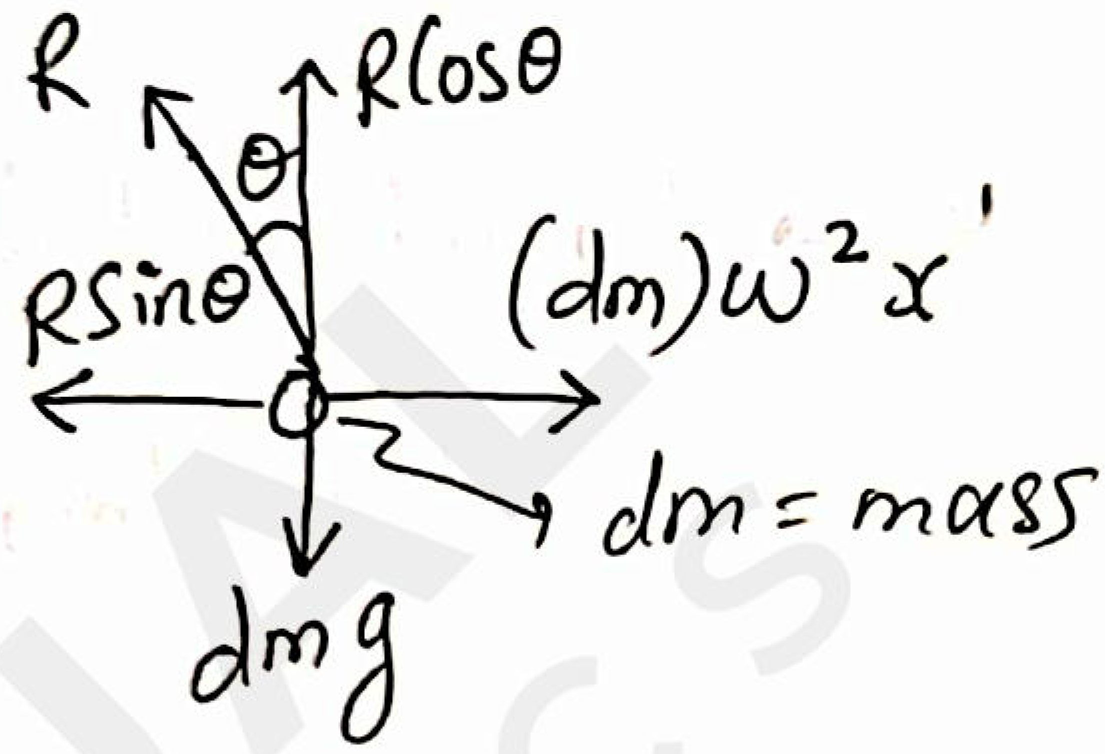


$$\boxed{\underline{P_A + \rho a l} + \rho g h = P_B}$$

Liquid in a Vessel Rotating with Constant angular velocity:

from vessel frame:





R → force exereted on the element due to neighbouring elements of fluid

So, $R\cos\theta = dm\, g$

$R\sin\theta = (dm)\omega^2 x.$

$$\boxed{\tan\theta = \frac{\omega^2 x}{g}} = \frac{dy}{dx}$$

$$\Rightarrow \int dy = \int \frac{\omega^2 x}{g} dx$$

$$y = \frac{\omega^2 x^2}{2g} + C$$

at $x = R$, $y = y_{max}$

at $x = 0$, $y = y_{min} = C$

$$\Rightarrow \boxed{y_{max} = \frac{R^2\omega^2}{2g} + y_{min}}$$

Volume of paraboloid of revolution is half that of circumscribing cylinder

$$\text{volume of paraboloid} = \frac{(y_{max} - y_{min})}{2} \times A$$

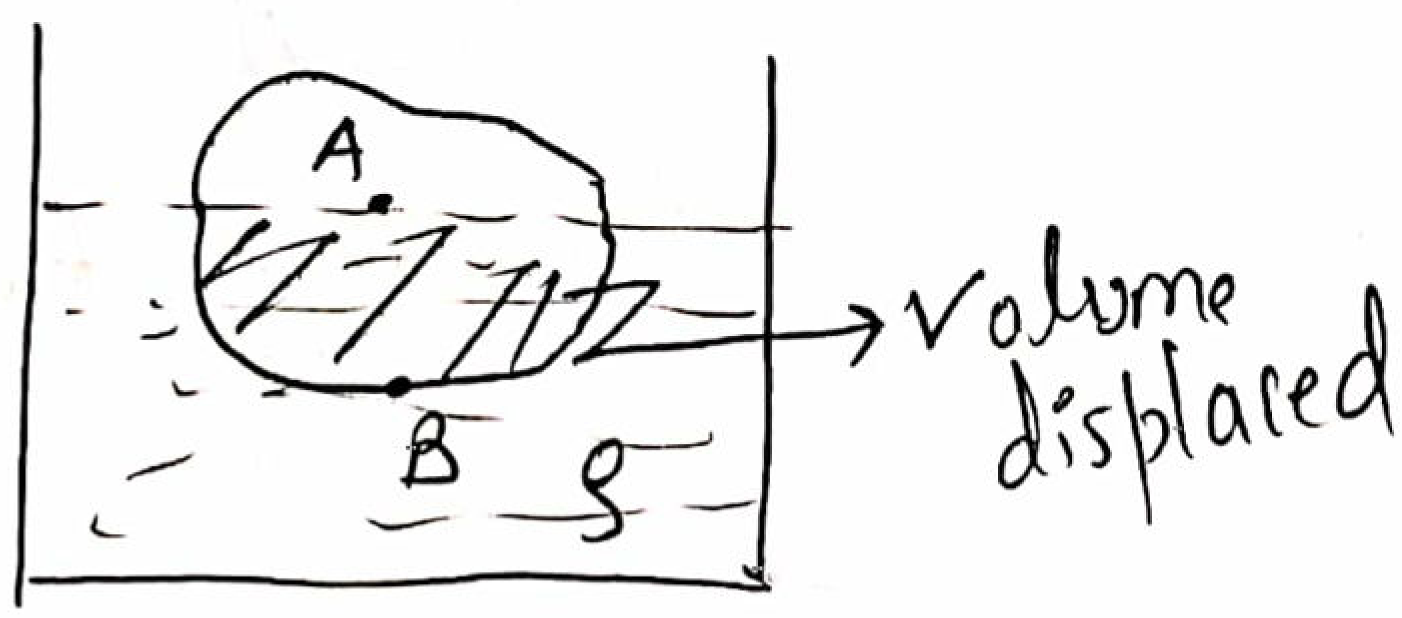
$y_0 \rightarrow$ liquid initially filled

$$y_0 = y_{min} + \frac{y_{max} - y_{min}}{2} = \frac{y_{max} + y_{min}}{2}$$

$$\Rightarrow \boxed{y_{max} = y_0 + \frac{\omega^2 R^2}{4g}}$$

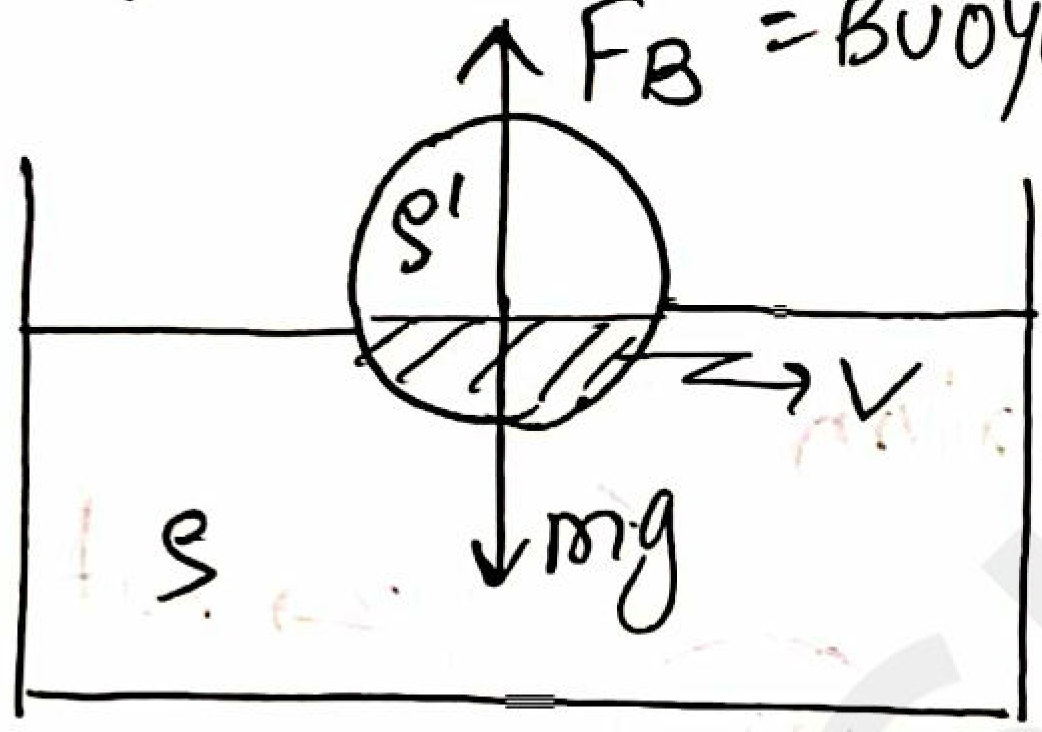
Archimede's Principle

magnitude of upward force = weight of liquid displace = Buoyant Force = $\rho V g$ = weight displaced



Buoyant force → upward force
= Weight of liquid displaced or gas

<u>Floating</u> <u>Body</u>



as body is floating

$mg = F_B$

$mg = \rho V g$

V' = net volume of body

$\Rightarrow \rho' V' g = \rho V g$

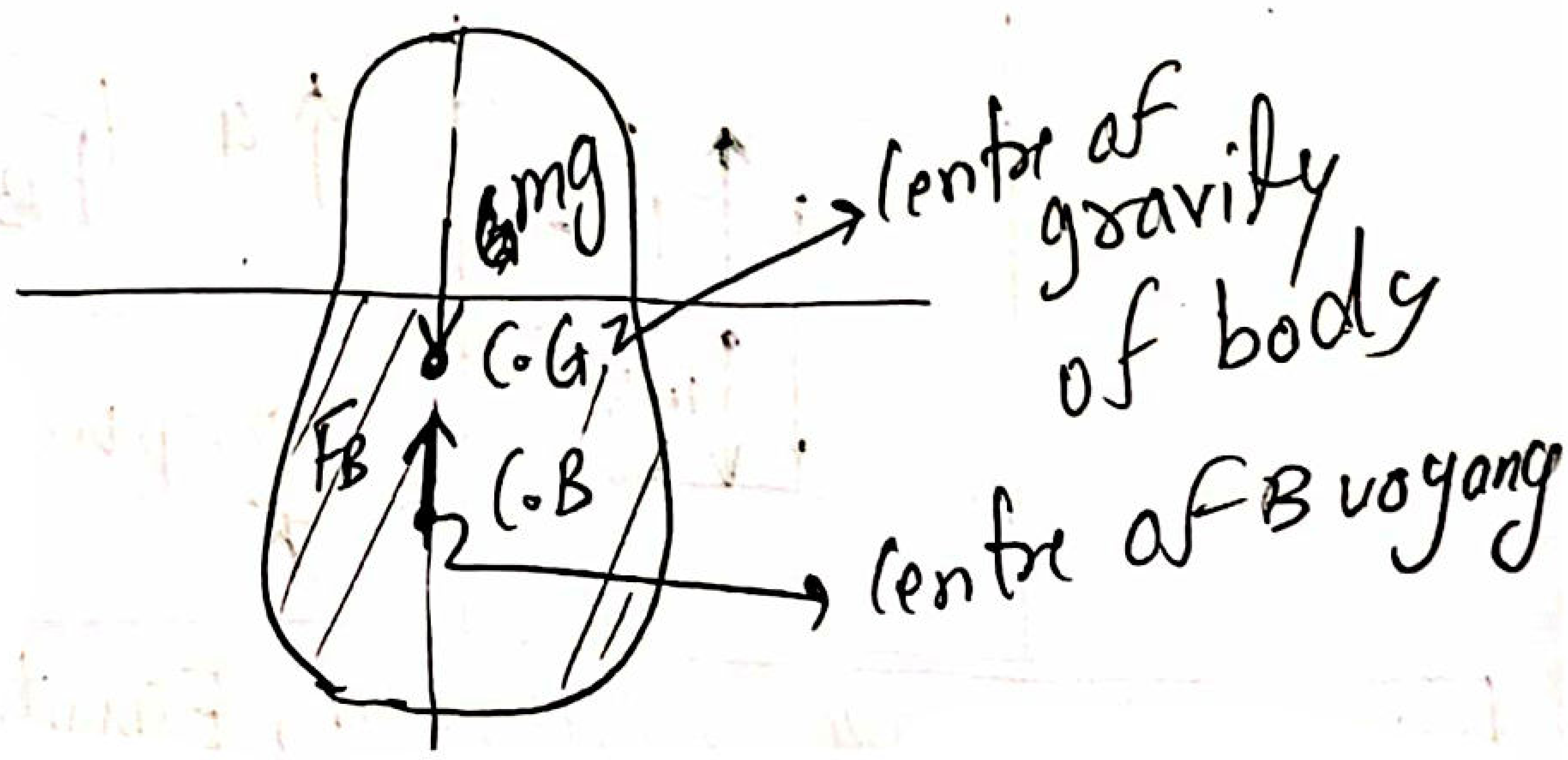
V = volume displaced

$$\boxed{V = \frac{\rho' V'}{\rho}}$$

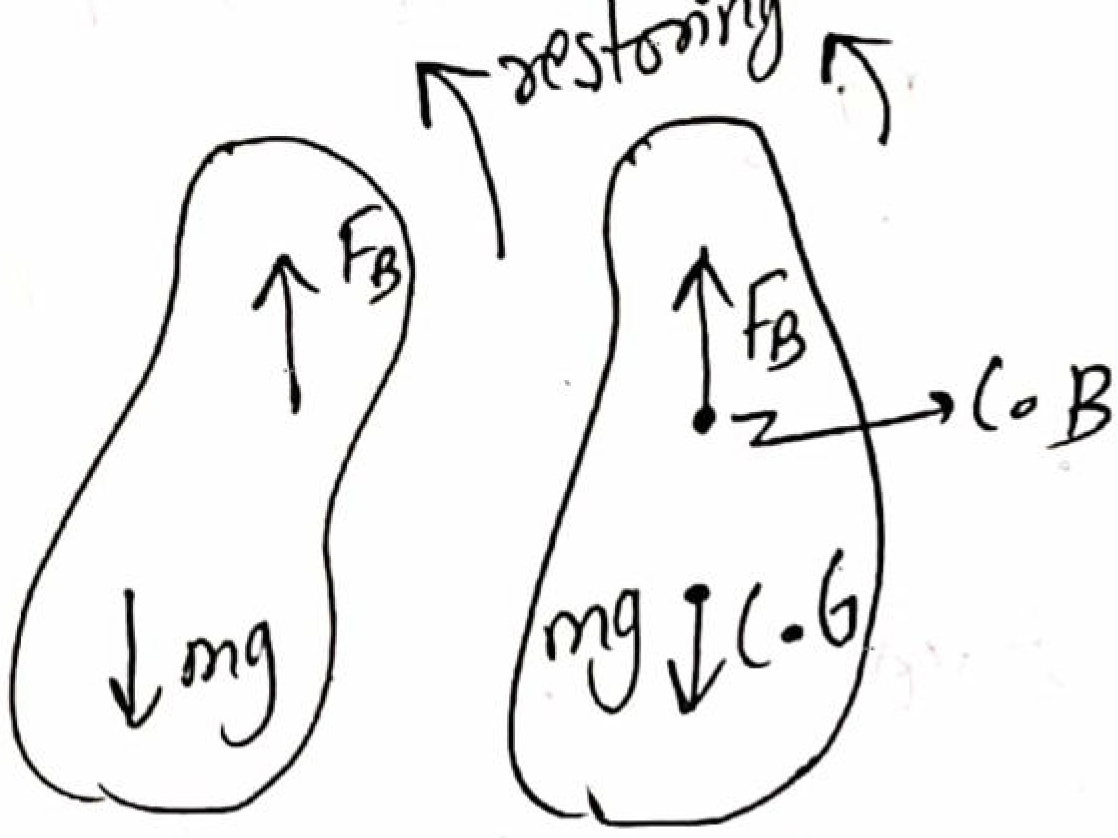
<u>Centre of Buoyancy</u>:

↳ Centre of gravity of displaced fluid

Point where all Buoyancy force acts.



for stable equillibrium :



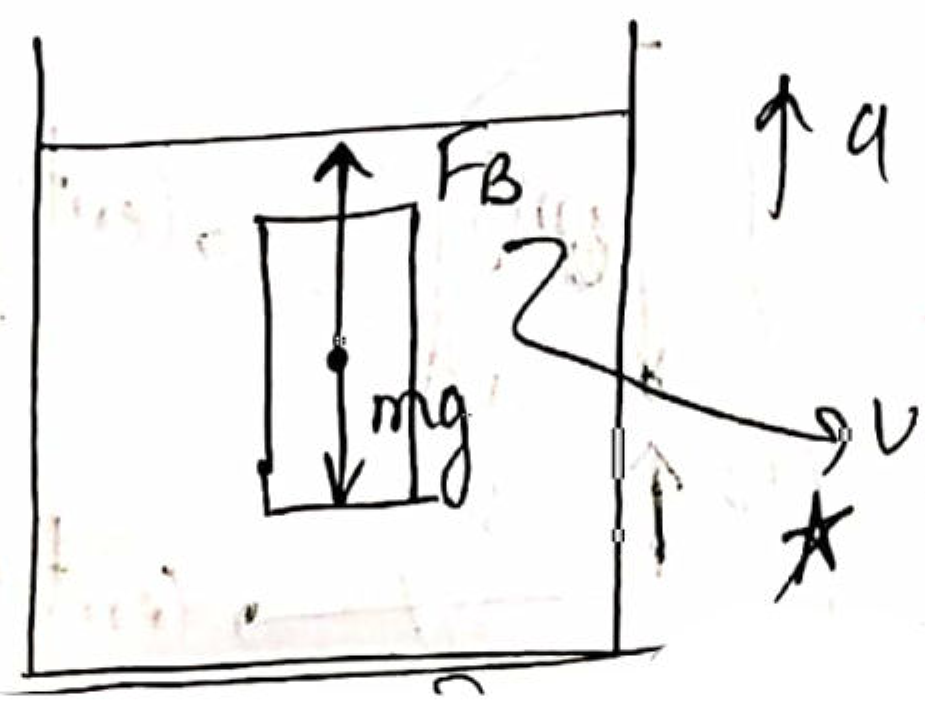
C.B should lie above C.G

hence, always restoring torque

For unstable equillibrium

C.B lies below C.G

Buoyant force in accelerating Fluid



$F_B = mg + ma$

upward force = buoyancy force

Hydrogynamics :

$\hookrightarrow$ Fluids in motion (bulk motion)

Type of flow:

(i) steady and unsteady Flow:

steady $\rightarrow$ not change in velocity, pressure, density with time.

$$\frac{dv}{dt} = 0 , \frac{dP}{dt} = 0 \;\&\; \frac{d\rho}{dt} = 0$$

unsteady $\rightarrow$ velocity at a point in the flow varies with time.

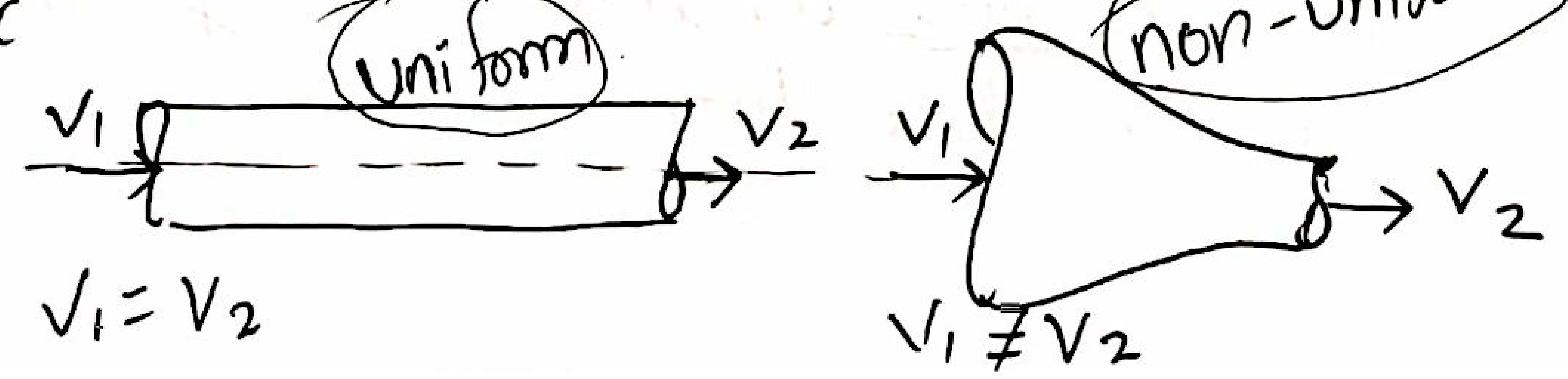
i.e. $\frac{dv}{dt} \neq 0$

(ii) Uniform & non-uniform flow:

* Flow is uniform at any instant of time if the velocity does not vary along the direction of flow

$$\frac{dv}{dx} = 0$$

* if $\frac{dv}{dx} \neq 0 \Rightarrow$ non-uniform flow



stream line flow:

if every point of a steadily flowing liquid follows exactly the same path followed by particles preceding it.
↳ Flow is streamlined.

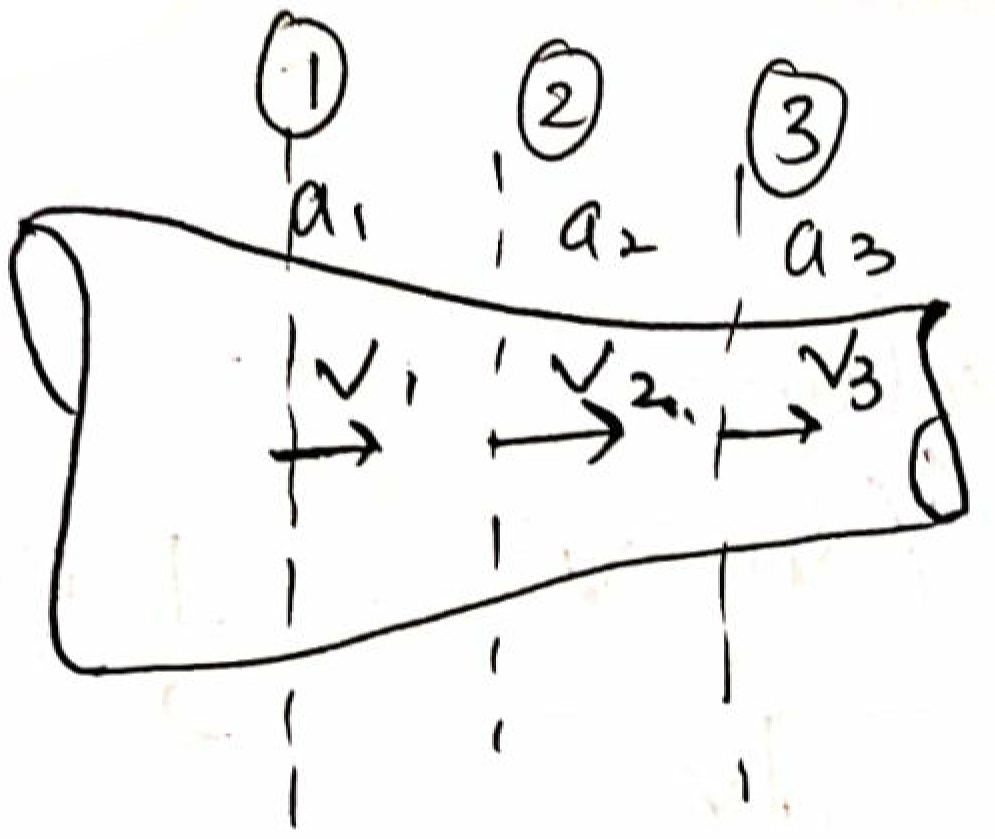
Turbulent Flow

If velocity exceeds a certain value, Flow become complicated. Random, irregular, local. Currents develop throughout fluid.

Principle of Continuity: The continuity equation:

mass Conservation

In steady state flow:
** ↳ amount of liquid entering a section is the same as that leaving another section in a given time Interval.



$a_1, a_2, a_3 \rightarrow$ Area of cross-section

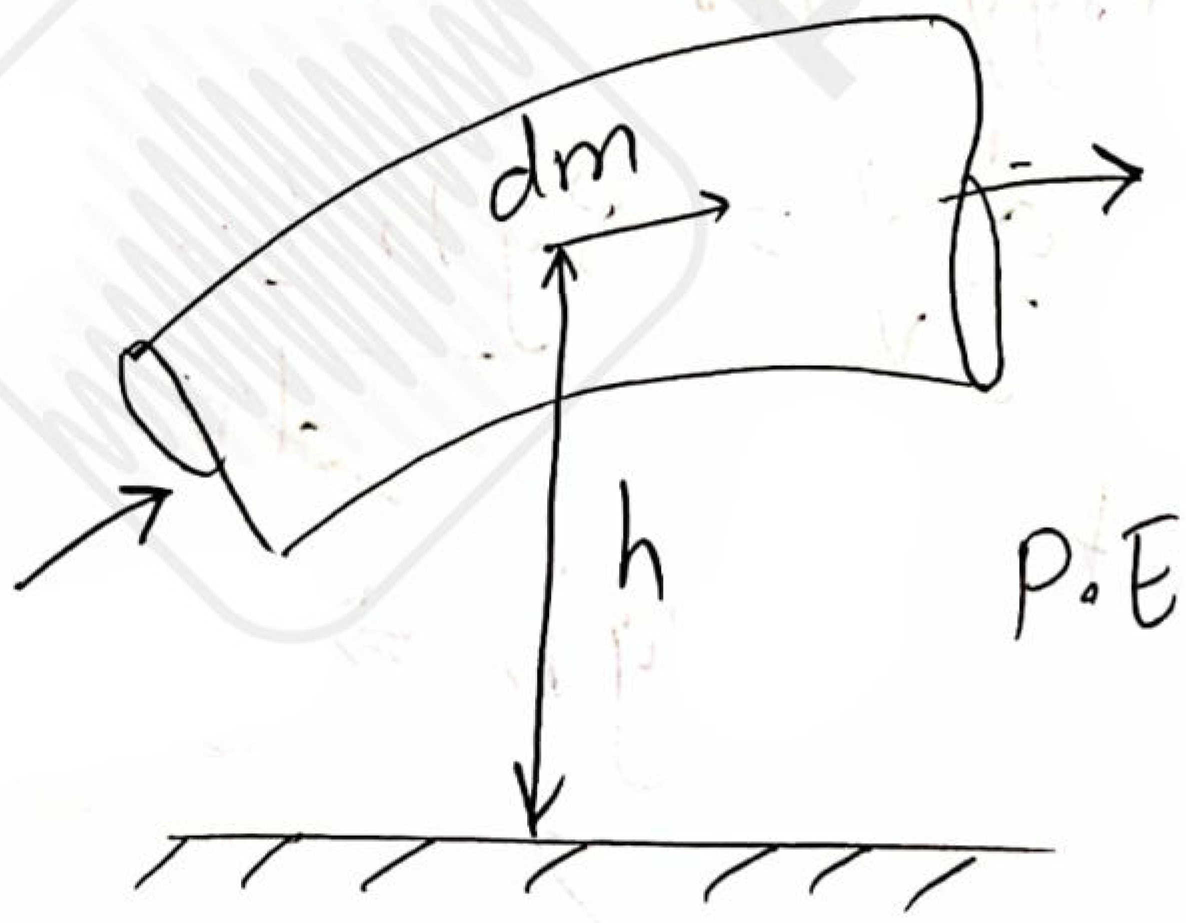
$m_1 = m_2 = m_2 \Rightarrow \rho_1 v_1 a_1 \Delta t = \rho_2 v_2 a_2 \Delta t = \rho_3 v_3 a_3 \Delta t$

$$\Rightarrow \boxed{v_1 a_1 = v_2 a_2 = v_3 a_3}$$

Continuity equation

Energy Associated with moving Liquid :

Potential Energy :



$P.E = dmgh$

P.E per unit volume

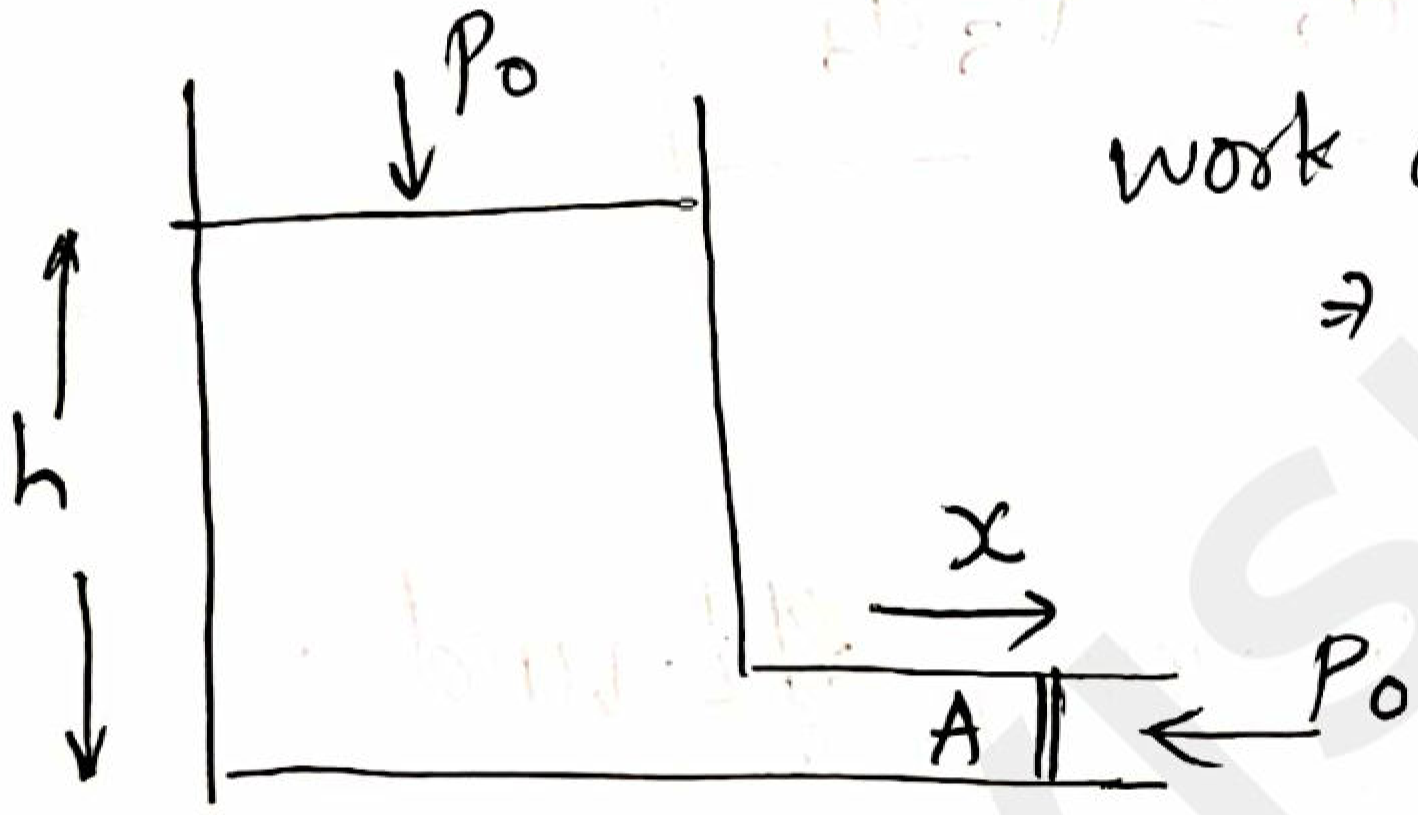
$$\Rightarrow \boxed{\frac{P.E}{dV} = \frac{dm}{dV} gh = \rho gh}$$

Kinetic Energy:

energy per unit volume:

$$= \frac{1}{2}\frac{(dm)v^2}{dV} = \frac{1}{2}\rho v$$

Pressure Energy:



work done by displacement 'x'

$$\Rightarrow dW = (P_0 + \rho g h - P_0)A \times dx$$

$$\text{Force} = (P_0 + \rho g h - P_0)A$$

$$\Rightarrow dW = \rho g h A dx$$

so energy per unit volume

$$\Rightarrow \frac{dW}{dV} = \rho g h \frac{\cancel{(Adx)}}{\cancel{dV}}$$

Pressure energy per unit volume

$$= \rho g h \Rightarrow \text{gauge pressure}$$

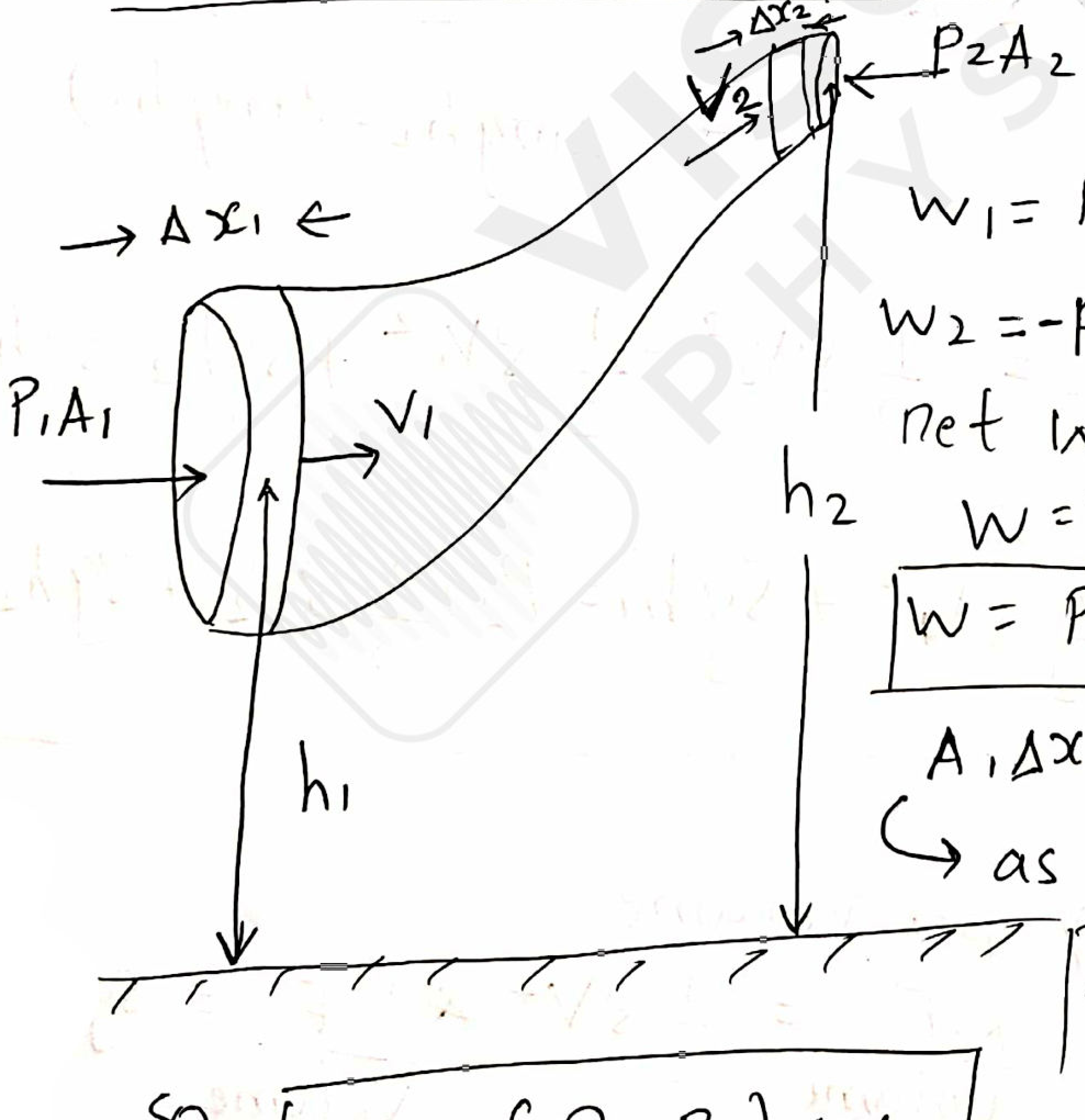
★ hence pressure has energy

Bernoulli's Equation :

↳ sum of total of these three kinds of energy remains constant along a streamline.

⇒ $$\boxed{\frac{1}{2}\rho v^2 + \rho g y + p = \text{Constant}}$$

Proof of Bernoulli's Equation :



$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$

$W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2$

net work

$W = W_1 + W_2$

$$\boxed{W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2}$$

$A_1 \Delta x_1 = A_2 \Delta x_2 = \Delta V$

↳ as $A_1 V_1 = A_2 V_2$

$$\boxed{A_1 \frac{\Delta x_1}{\Delta t_1} = A_2 \frac{\Delta x_2}{\Delta t_2}}$$

so, $$\boxed{W = (P_1 - P_2)\Delta V}$$

change in kinetic-Energy

$\Rightarrow \quad \Delta k = \frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2$

& $\Delta P.E = \Delta m g h_2 - \Delta m g h_1$

so energy Conservation:

$\Rightarrow \quad \Delta W = \Delta k + \Delta U$

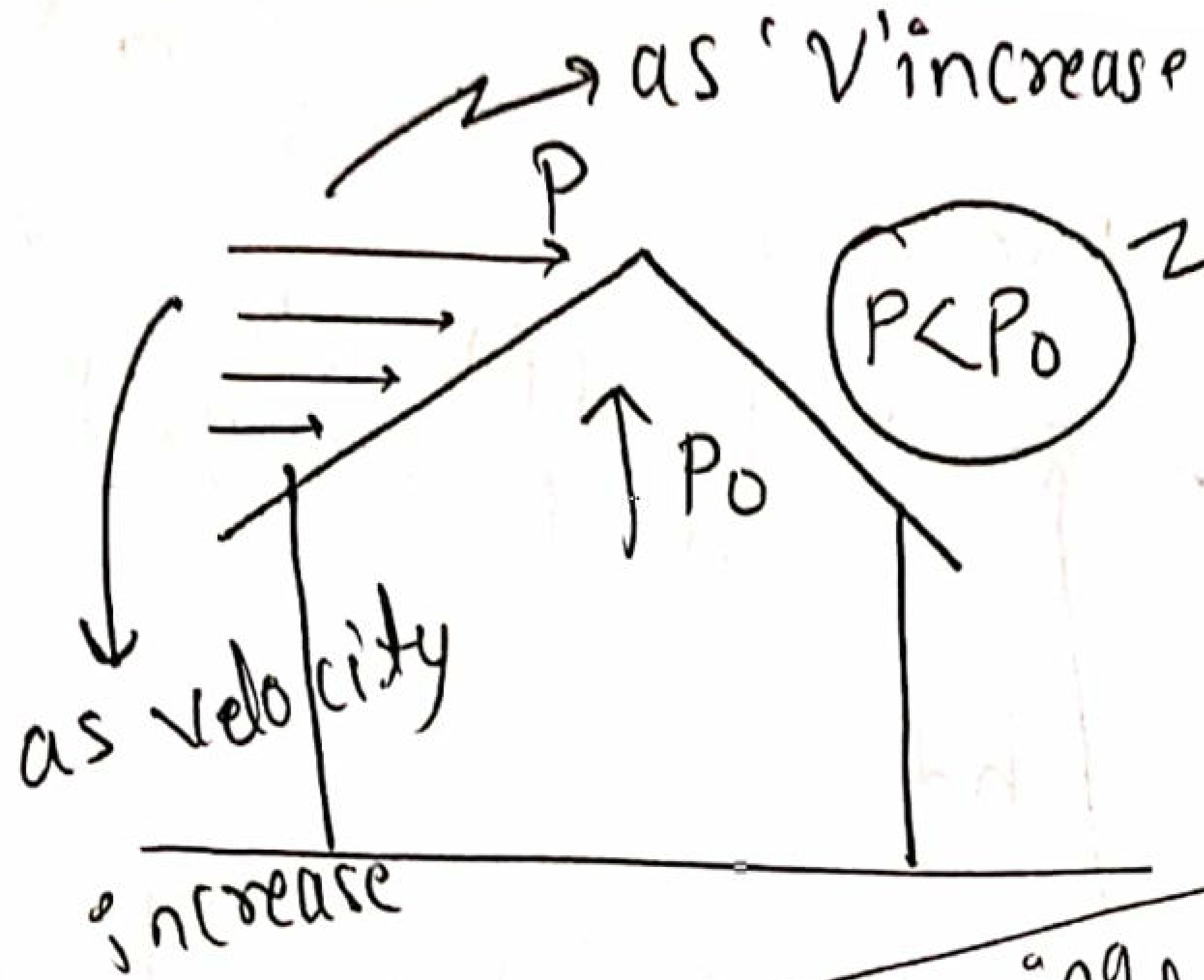
$\Rightarrow \quad (P_1 - P_2)\Delta V = \left(\frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2\right) + (\Delta m g h_2 - \Delta m g h_1)$

$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 + \rho g h_2 - \rho g h_1$

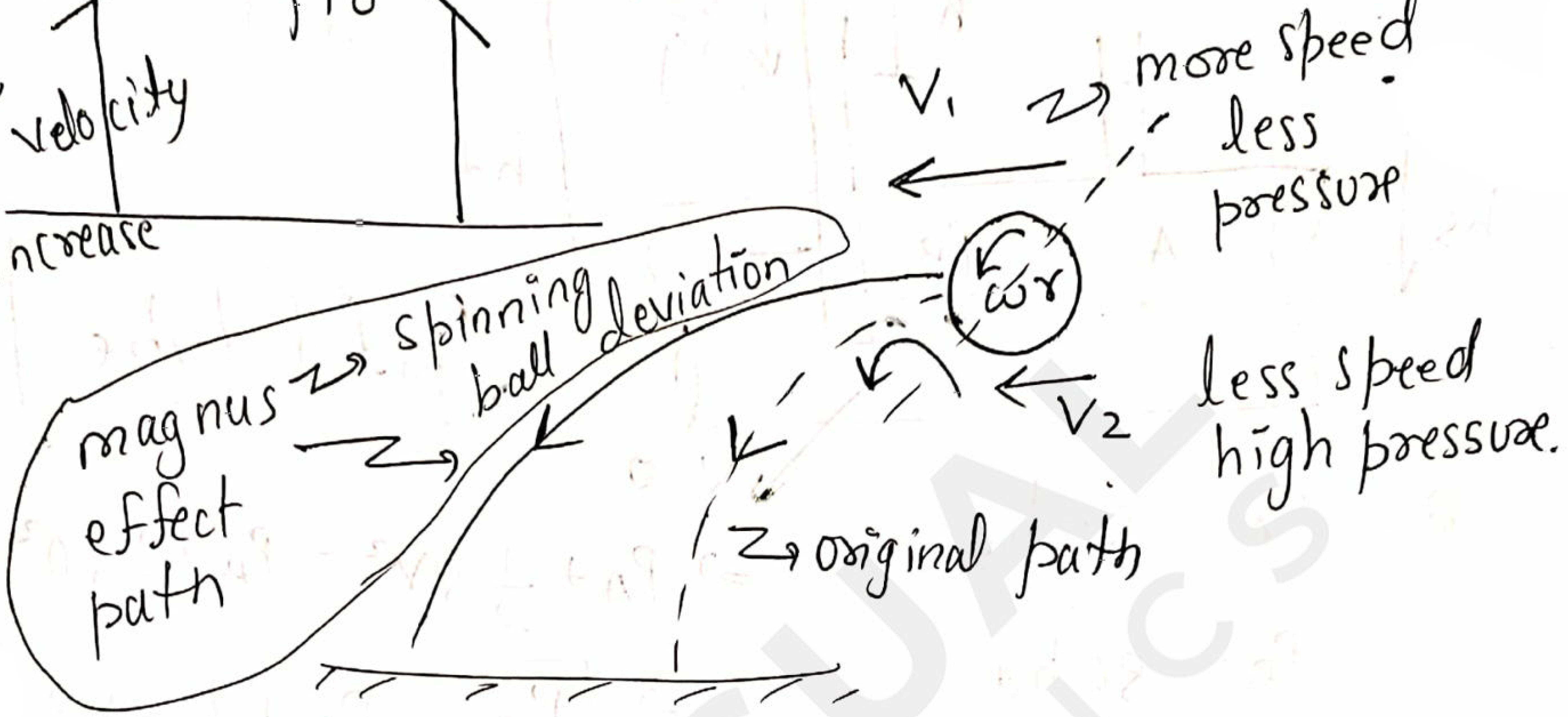
$\Rightarrow \quad \boxed{P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2}$

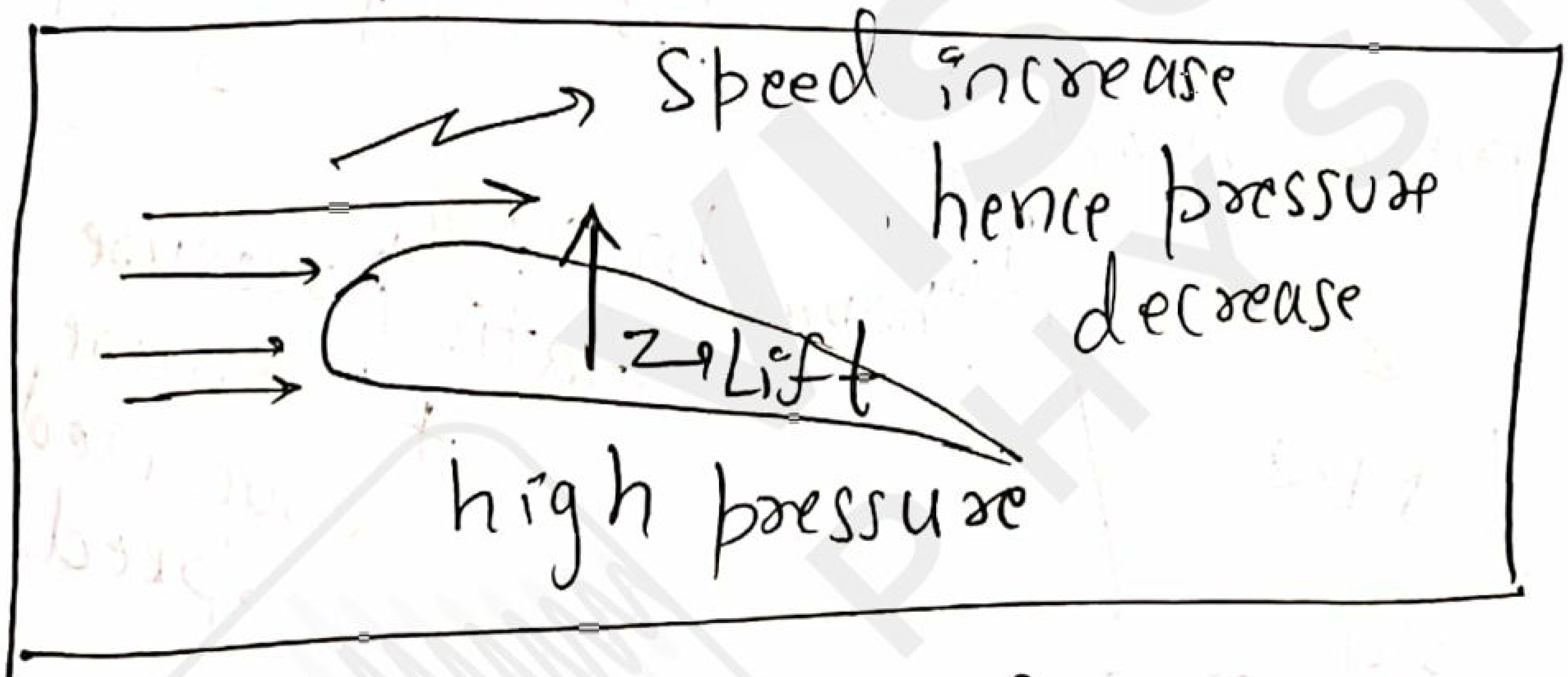
As energy per volume

$\frac{K.E}{Volume} = \frac{1}{2}\rho V^2$ & $\frac{P.E}{Volume} = \rho g h$

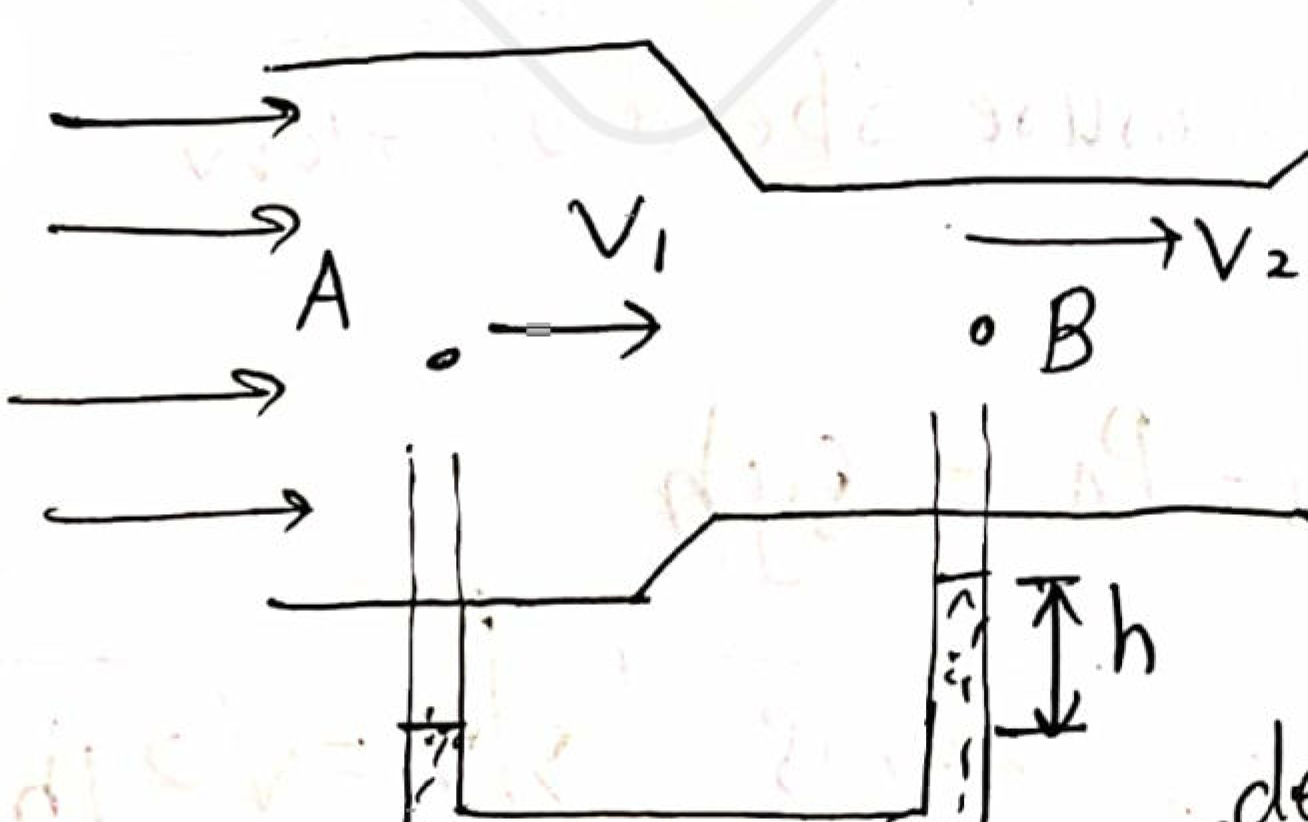


→ as velocity increase hence pressure decrease.





Venturi-meter: Used for measuring rate of flow



$$P_A + \frac{\rho V_1^2}{2} = P_B + \frac{\rho V_2^2}{2}$$

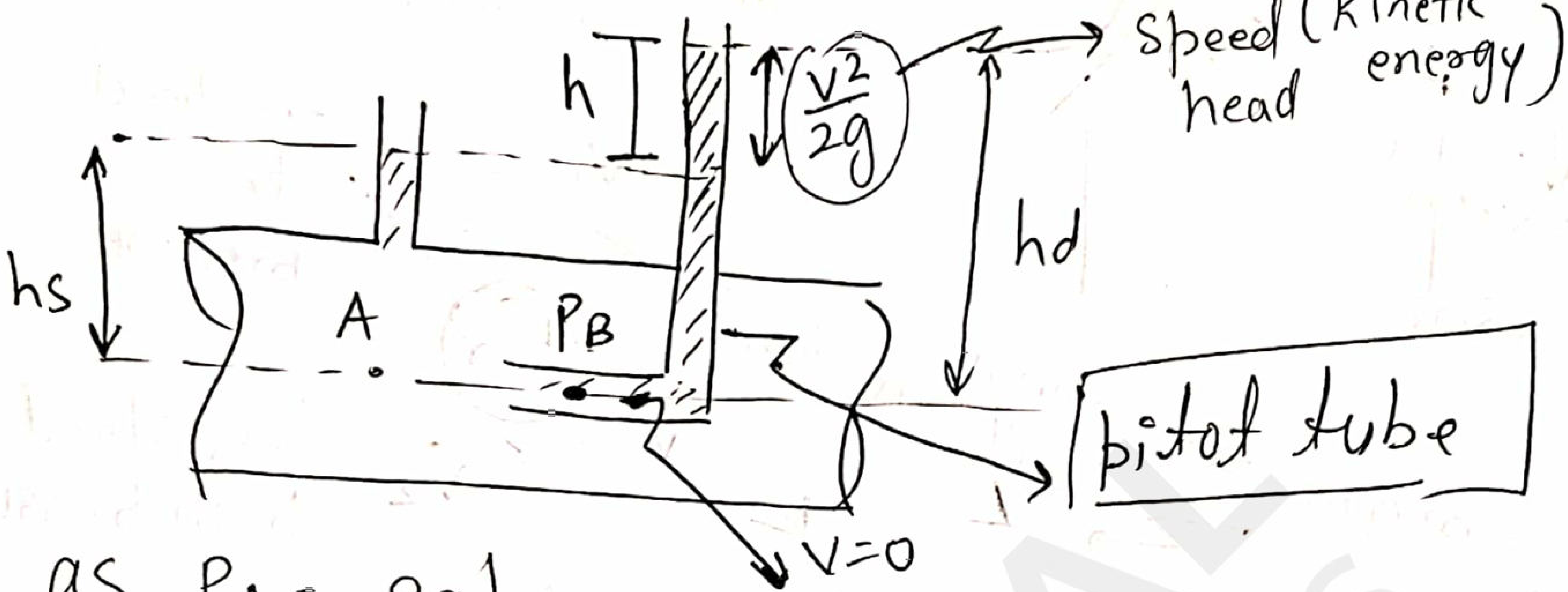
$$\Rightarrow \boxed{V_2^2 - V_1^2 = 2gh}$$

as $\boxed{P_A - P_B = \rho g h}$

$$Q = AV$$

$$\boxed{Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

Static pressure and dynamic Pressure:



as $P_A = \rho g h_s$

$P_B = \rho g h_d$

$\Rightarrow P_A + \frac{1}{2}\rho v^2 = P_B + \frac{1}{2}\rho 0^2$

$\Rightarrow \boxed{P_B = P_A + \frac{1}{2}\rho v^2}$

now in terms of head

dynamic pressure = static pressure + pressure associated with speed.

$$\boxed{h_d = h_s + \frac{\frac{1}{2}\rho v^2}{\rho g}}$$

$\boxed{\text{dynamic head = static head + head associated with speed}}$

★ pitot tube used to measure speed of flow

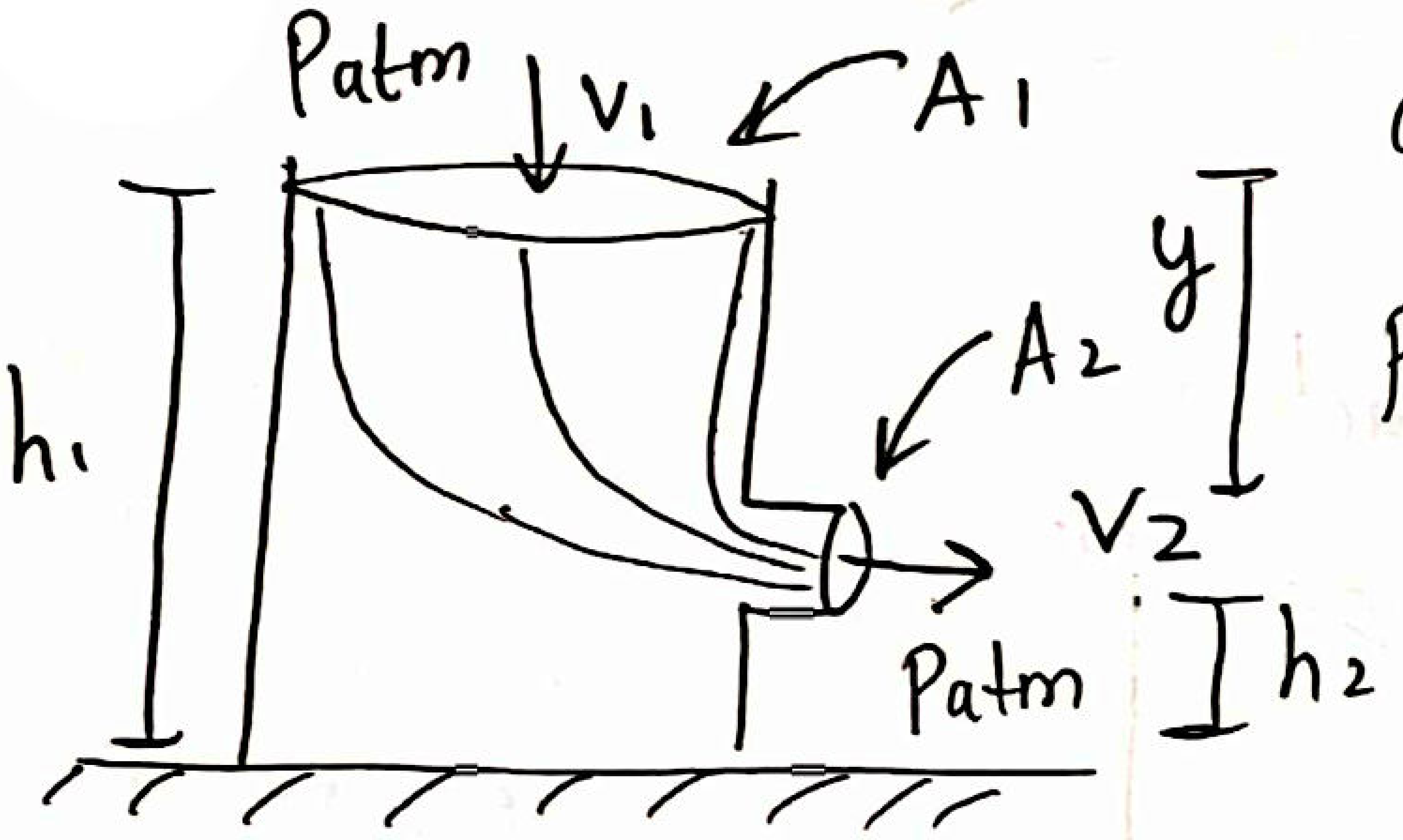
$v_B = 0$

$\Rightarrow \frac{1}{2}\rho v^2 = P_B - P_A = \rho g h$

$\Rightarrow v = \sqrt{\frac{2gh\rho}{\rho}} \Rightarrow \boxed{v = \sqrt{2gh}}$

Velocity of EFFLUX

↪ liquid comes out of a hole made at a depth y.



as $A_1 v_1 = A_2 v_2$

$$P_{atm} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_{atm} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow h_1 - h_2 = y$$

$$\Rightarrow v_2^2 = v_1^2 + 2gy$$

$$\Rightarrow \boxed{v_2 = \sqrt{\frac{2gy}{1-\left(\frac{A_2}{A_1}\right)^2}}}$$

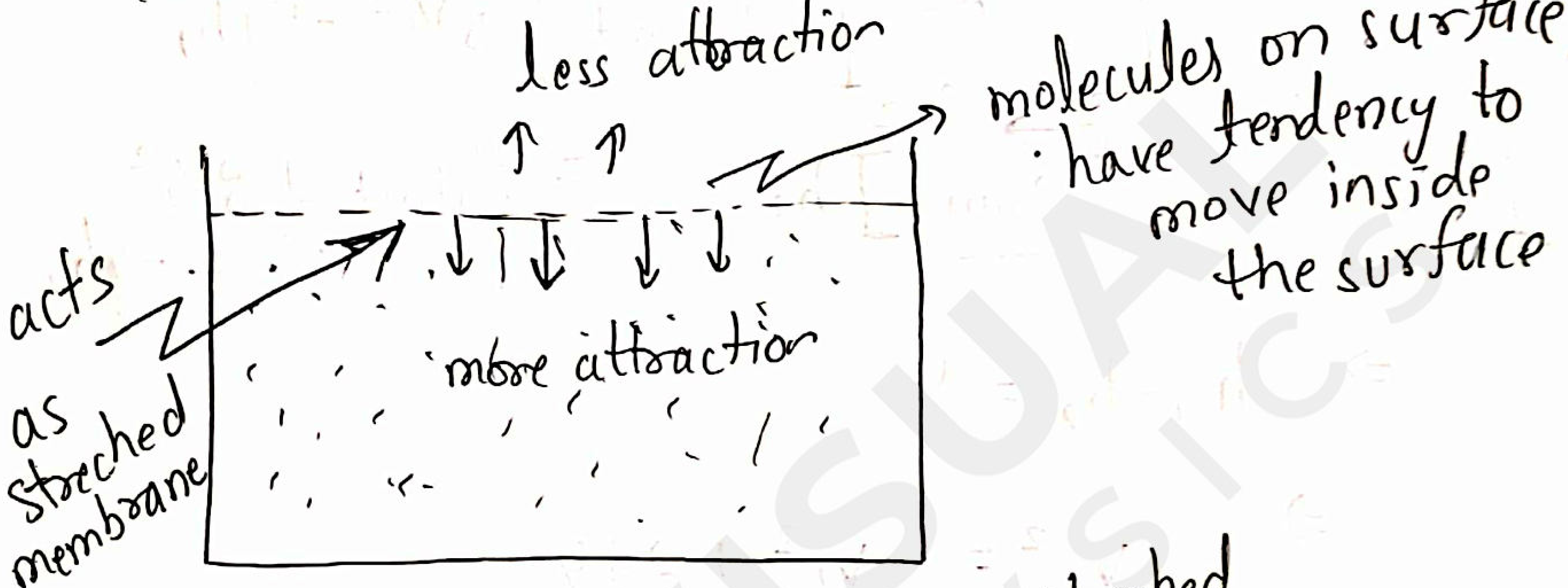
using $v_1 = (A_2/A_1) v_2$

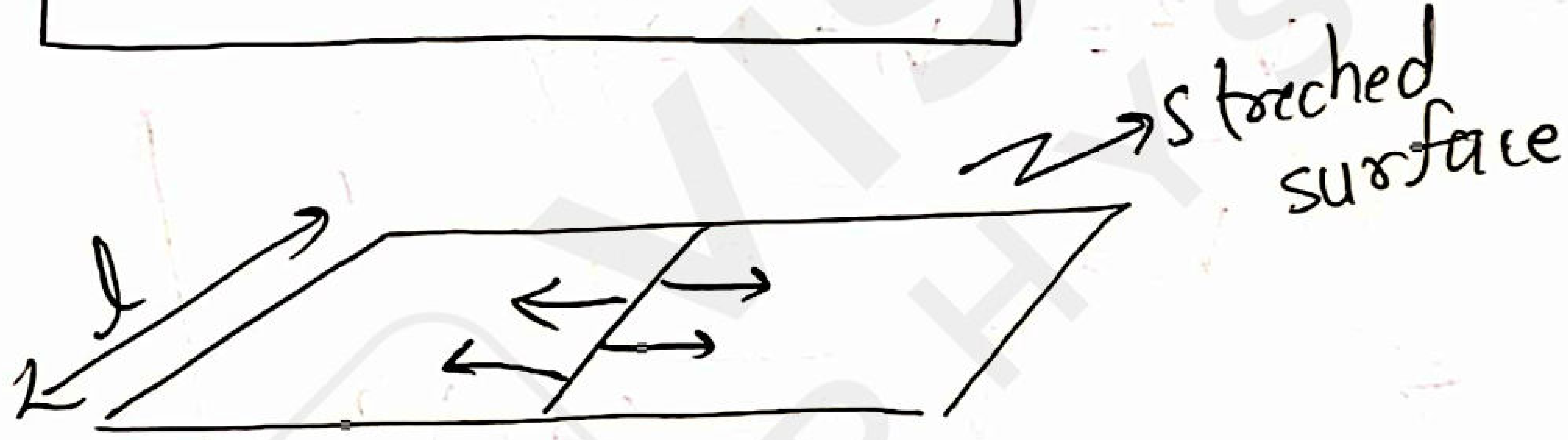
so, time to fall on ground = t

$$\Rightarrow t = \sqrt{\frac{2h_2}{g}}$$

$$\boxed{\text{Range} = v_2 t = \sqrt{\frac{2gy}{1-\left(\frac{A_2}{A_1}\right)^2}}\sqrt{\frac{2h_2}{g}}}$$

Surface tension:

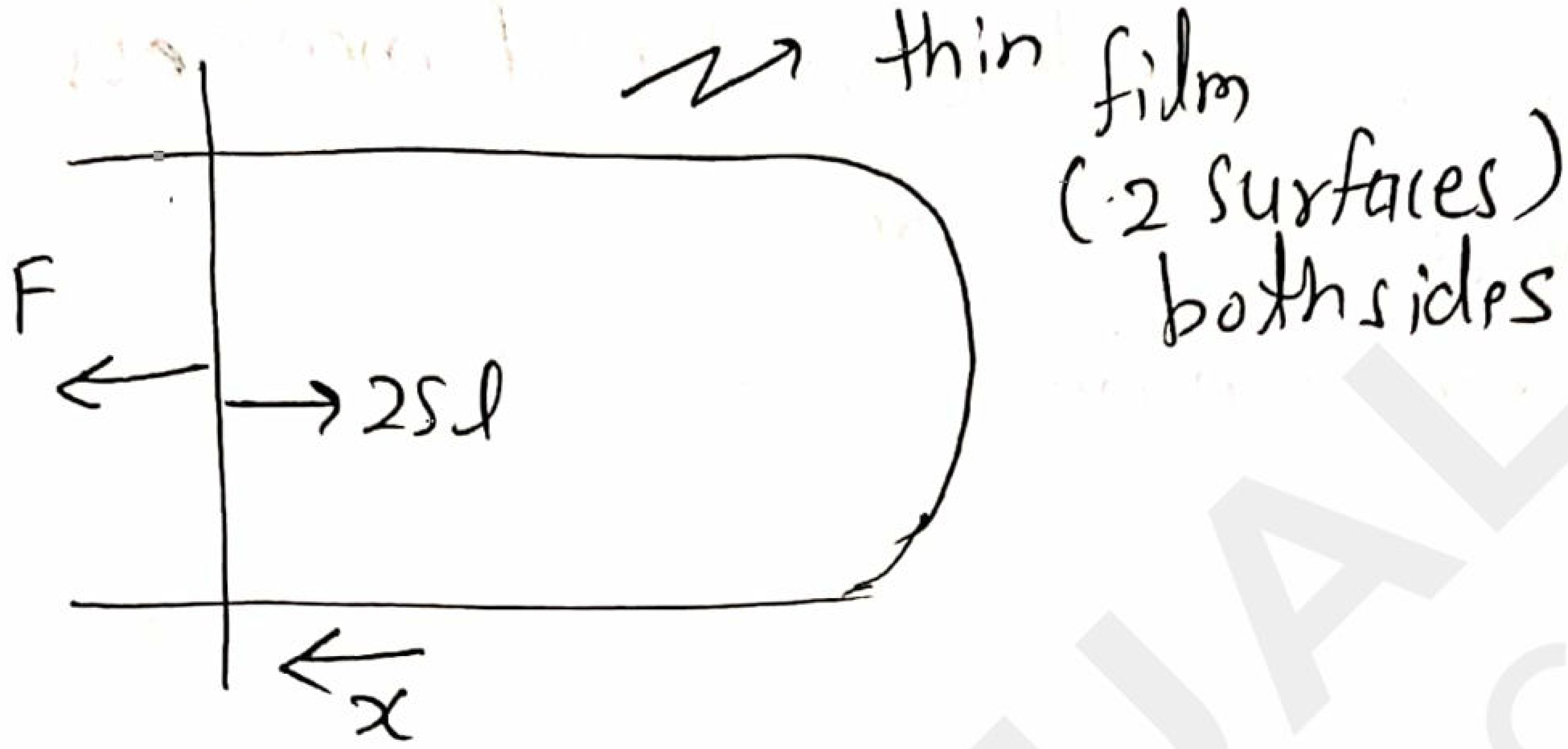




now $\frac{\text{force}}{\text{length}}$ = surface tension = S

★ hence because of this → surface tends to have smaller area

Surface Energy: = $\frac{\text{potential energy}}{\text{Area of surface}}$ = Surface tension



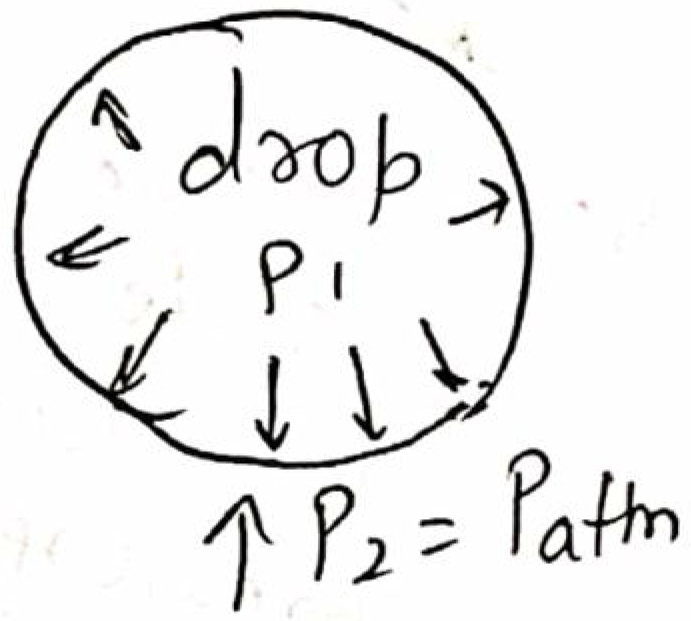
$$\Delta W = (2Sl)x = \Delta U$$

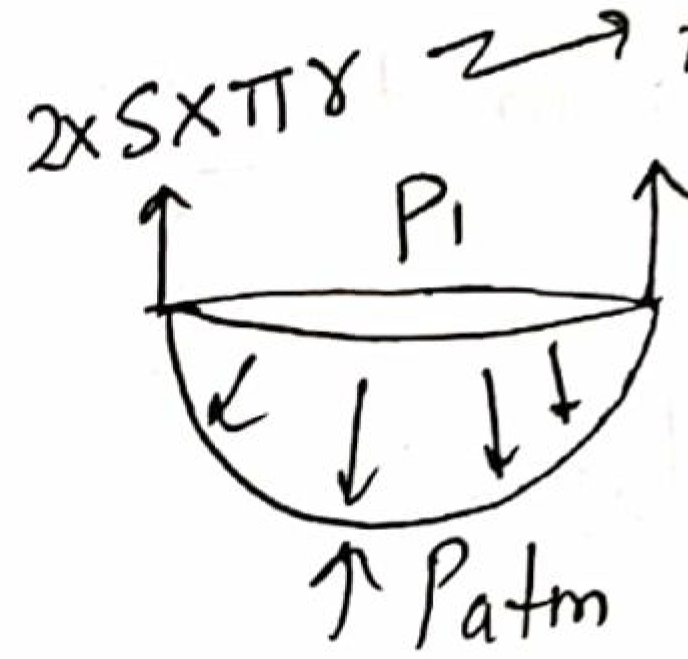
$$\Delta A = 2lx$$

so $\boxed{\text{surface energy} = \frac{\Delta U}{\Delta A} = S = \text{Surface tension}}$

★ for given volume → sphere have lowest surface area
Hence liquid drops are in spherical shape

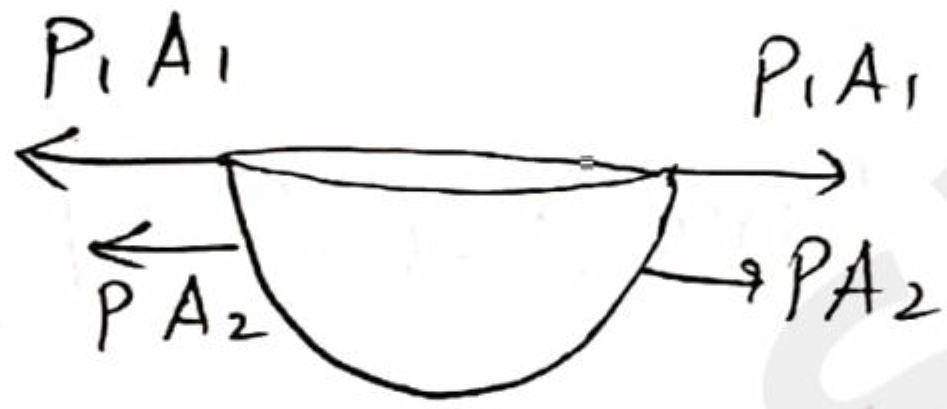
Pressure Inside Bubble and drop



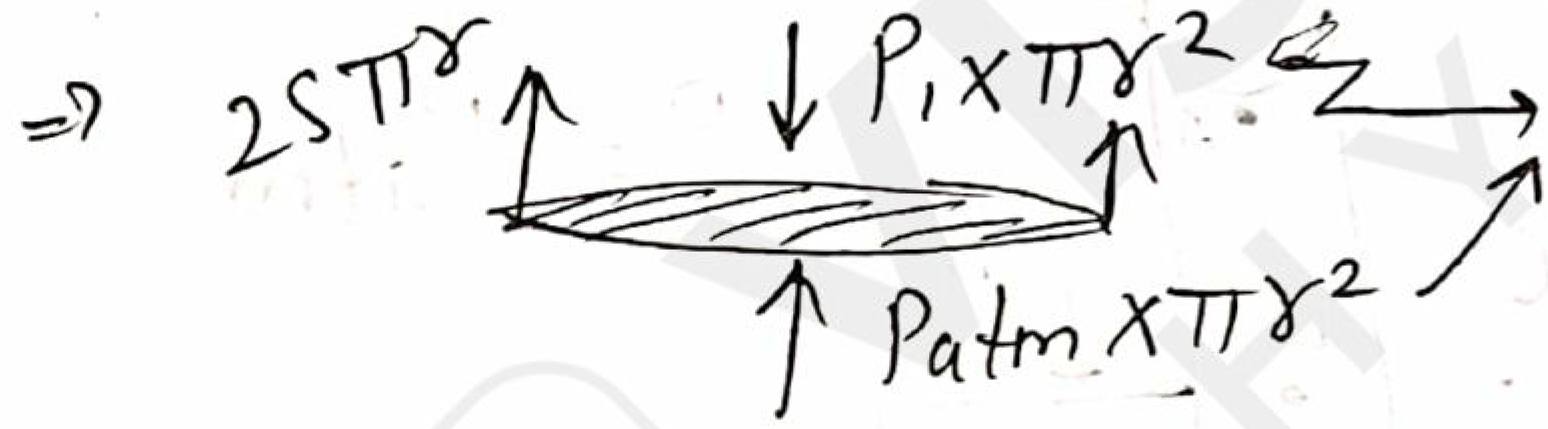


→ force due to surface tension

now net force is force on projected area



⇒ as force in horizontal direction will cancel out



→ area of circle

$$\Rightarrow \quad 2S\pi r + P_{atm} \times \pi r^2 = P_1 \times \pi r^2$$

$$\Rightarrow \quad \boxed{P_1 = P_{atm} + \frac{2S}{r}}$$

Not for bubble there are two surface, inside & outside

Hence $\boxed{P_1 = P_{atm} + 2\left(\frac{2S}{r}\right)} \Rightarrow \left(\frac{4S}{r}\right)$

hence for bubble additional pressure

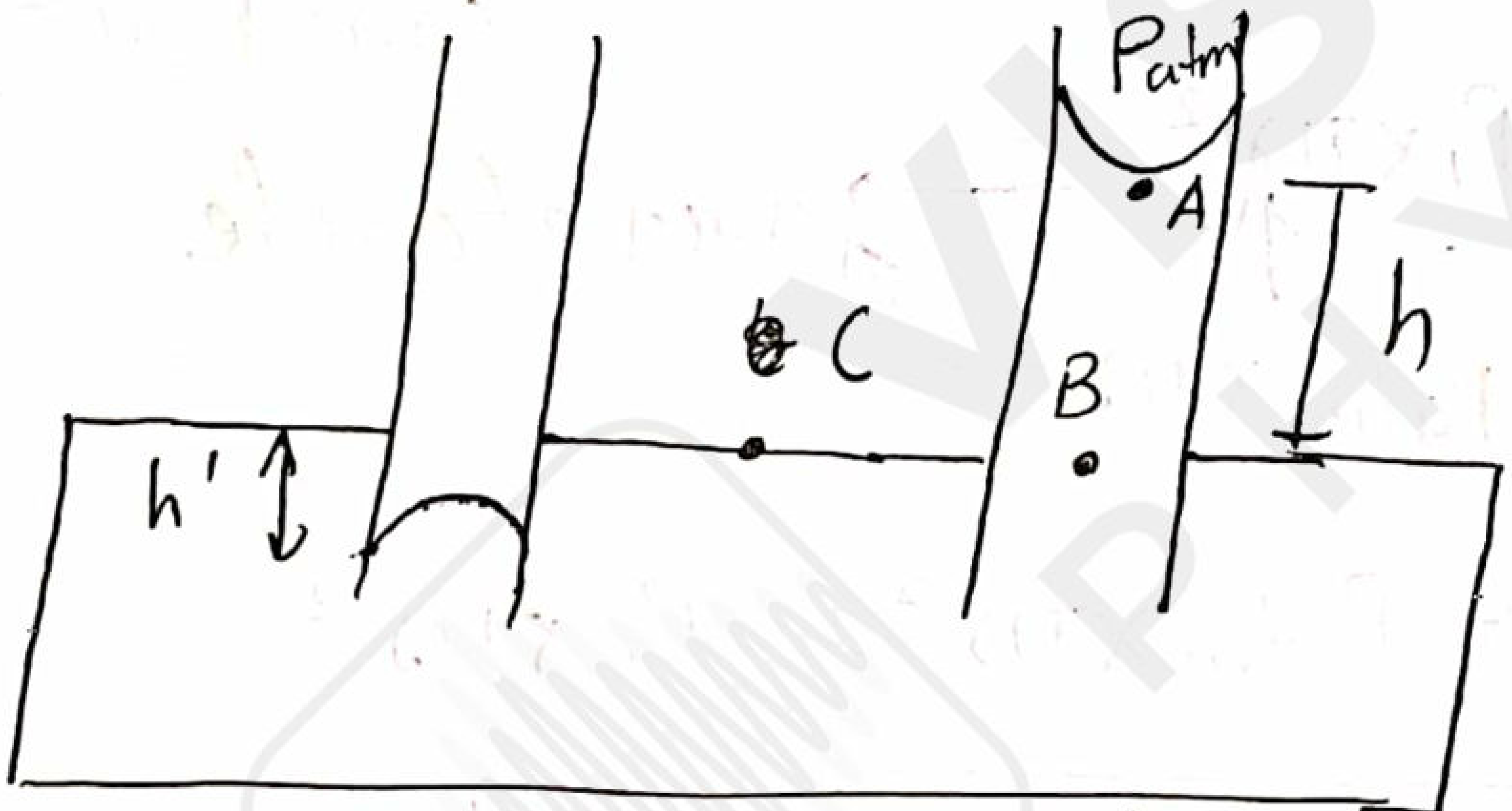
$$\boxed{P_1 = P_{atm} + \frac{4S}{r}}$$

★ So concave side have additional pressure!

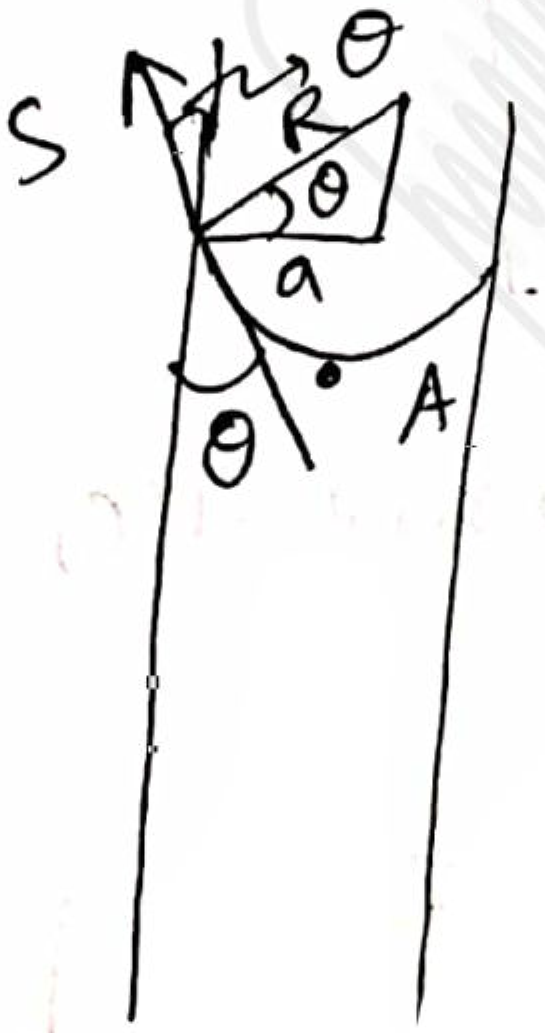
& for drop:

$$\boxed{P_1 = P_{atm} + \frac{2S}{r}}$$

Capillary tube (capillary rise / fall)



$P = P_{atm}$



a → radius of tube
R → radius of surface

now $\boxed{P_A = P_{atm} - \frac{2S}{R}}$

as concave side pressure have additional pressure $\frac{2S}{R}$

hence convex side pressure

now $P_B = P_C = P_{atm}$

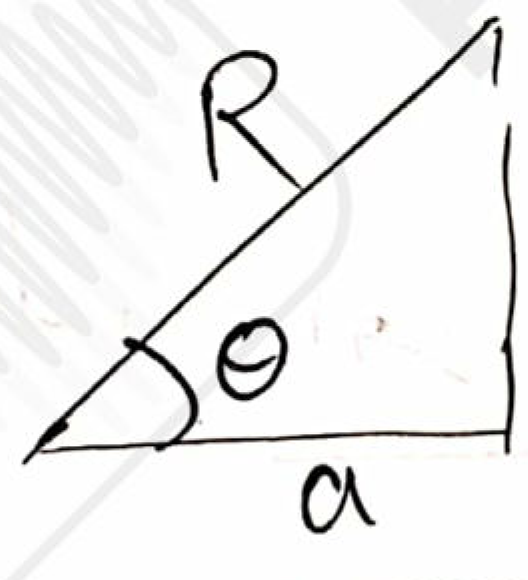
now $P_B = P_A + \rho g h$

$$P_{atm} = P_{atm} - \frac{2S}{R} + \rho g h$$

$$\Rightarrow \boxed{h = \frac{2S}{\rho g R}}$$

☆ same for capillarly fall

now



$a = R\cos\theta$

$R = \frac{a}{\cos\theta}$

$$\Rightarrow \boxed{h = \frac{2S\cos\theta}{a\rho g}}$$

Viscocity:

Ideal fluid → no friction

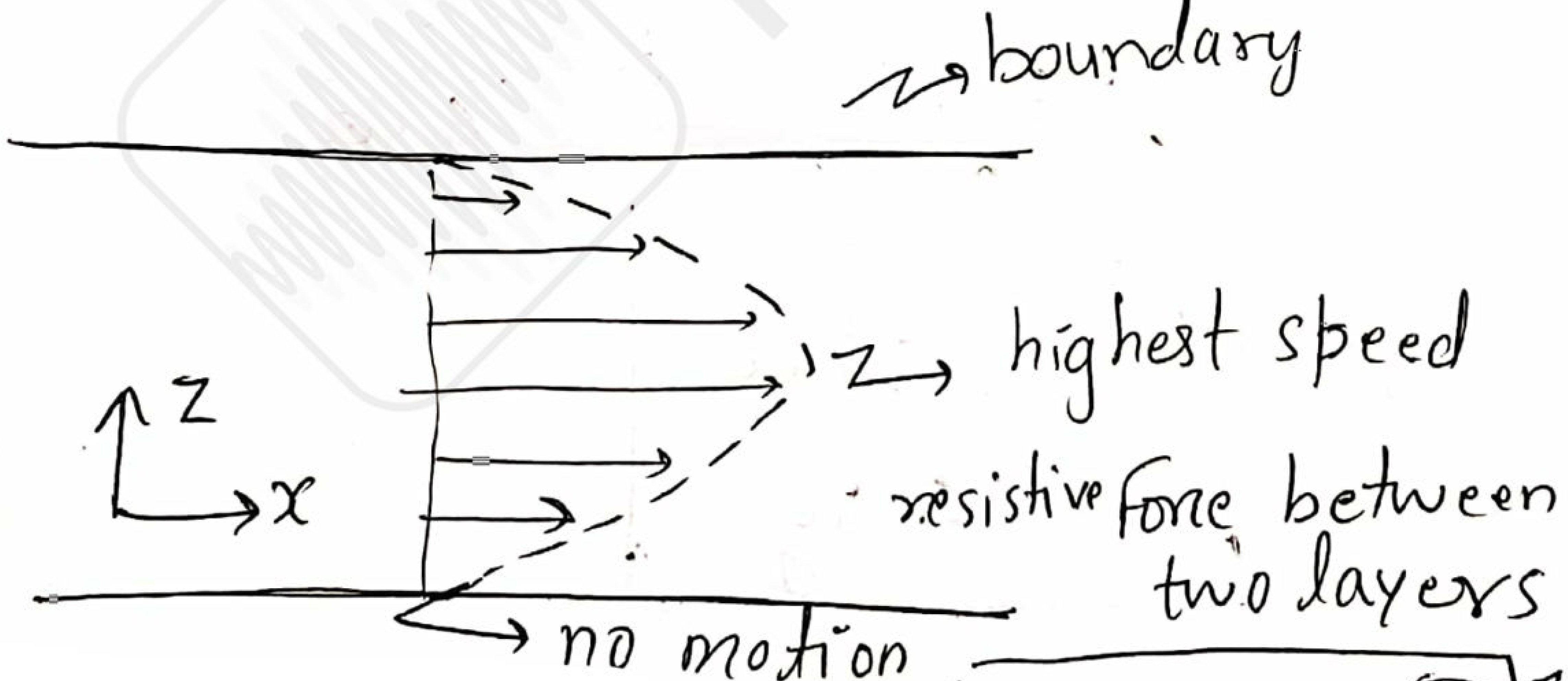
(and the cases derived were for ideal fluid)

Real fluid → have Internal friction.

It means layers of fluids applies drag on each other.

* Now this internal resistance in fluids layers is known as viscocity.

so velocity profile:



$$\boxed{F = -\eta A \frac{dv}{dz}}$$

velocity difference

coefficient of viscocity