



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Electric Potential



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ELECTRIC POTENTIAL

$$\Delta U = -W \Rightarrow U_b - U_a = -W \quad \begin{array}{l} \text{Work done} \\ \text{from going} \\ a \rightarrow b \end{array}$$

Potential energy at position 'b'

Potential energy at position 'a'

$$\text{As } W_{a \rightarrow b} = \int_{r_a}^{r_b} \vec{F} \cdot d\vec{l} = \int_{r_a}^{r_b} \left(\frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{r} \cdot d\vec{l} \right)$$

electrostatic force is conservative force

an also

$$-\Delta U = W_{a \rightarrow b} = \int_{r_a}^{r_b} \vec{F} \cdot d\vec{l} = \int_{r_a}^{r_b} q \vec{E} \cdot d\vec{l}$$

For any point 'r' from q_1, q_2 have

$$\text{potential} \Rightarrow U = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r}$$

potential energy at a given point in presence of many charges,

$$U = \frac{1}{4\pi\epsilon} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \dots \dots \dots \right]$$

Work is done

As potential is scalar quantity.

For a system, the Net potential of the system is:

$$U = \frac{1}{2} \left[\frac{1}{(4\pi\epsilon)} \sum_{i \neq j} \frac{q_i q_j}{r_{ij}} \right]$$

$\left[\frac{1}{2} \right]$ is used as each term in summation will appear twice

Now 'Potential energy' per unit charge at point a is known as potential

$$\Rightarrow V_a = \frac{U_a}{q_0} = \frac{1}{4\pi\epsilon} \times \frac{1}{r_a}$$

potential difference from point a $\rightarrow$ b is
= negative of work from a $\rightarrow$ b

$$\Rightarrow \Delta V = V_b - V_a = - \frac{W_{a \rightarrow b}}{q_0} = \frac{\Delta U_{a \rightarrow b}}{q_0}$$

$$[V] = \left[\frac{W}{q_0} \right]$$

$$= \left[\frac{ML^2T^{-2}}{AT} \right] = [ML^2T^{-3}A^{-1}]$$

Unit (JC^{-1})

So, net V at a point:

$$V = V_1 + V_2 + V_3 = \frac{1}{4\pi\epsilon} \sum \frac{q_i}{r_i}$$

for $\Delta V = 0 \Rightarrow -W = 0$

Now $\int q_0 \vec{E} \cdot d\vec{l} = W$

So for W to be zero

(i) $d\vec{l}$ and $\vec{E}$ must be perpendicular or

(ii) for $E=0$

"Now a surface with same potential is known as equipotential surface"

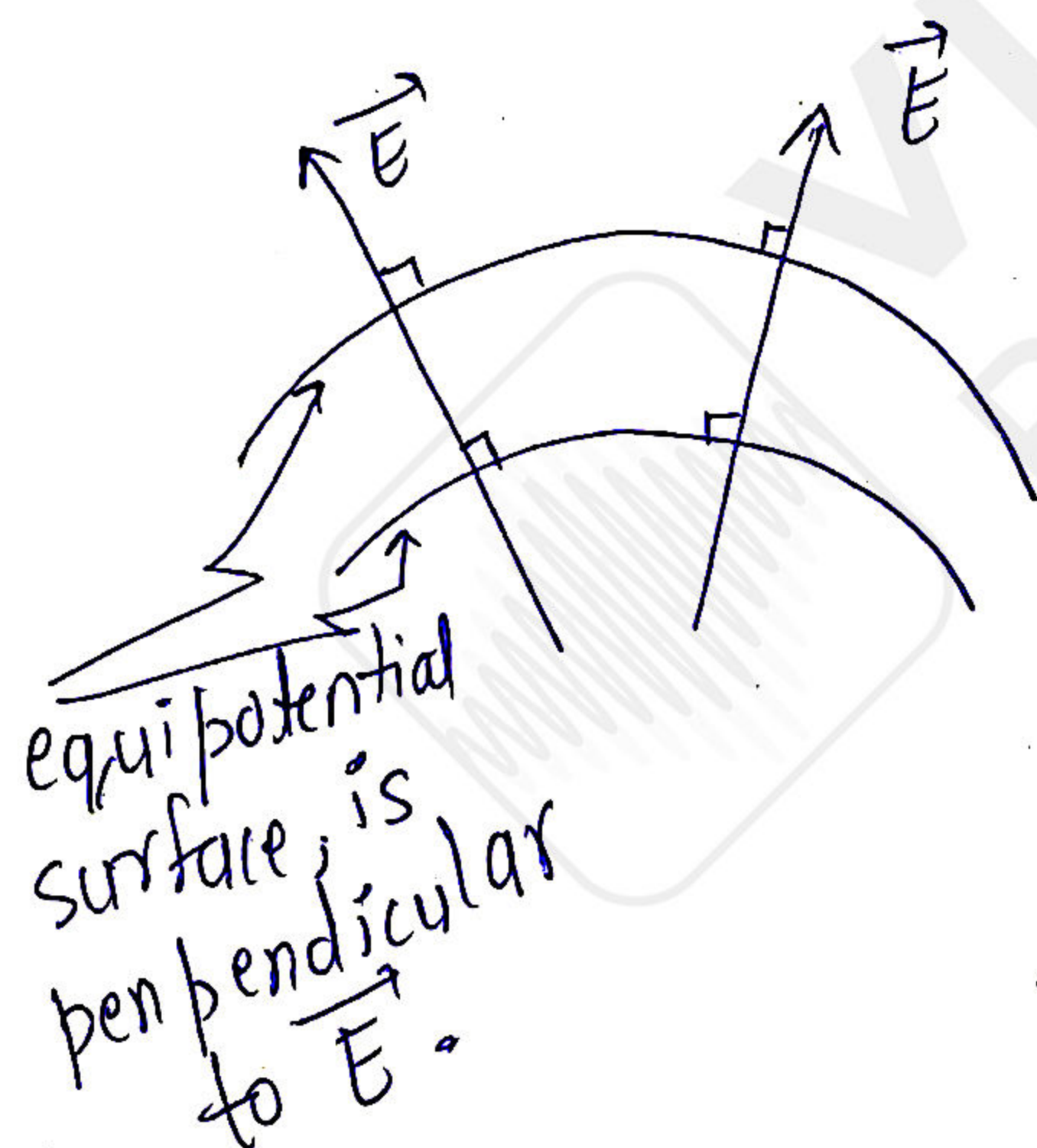
$\Rightarrow$ "So for a equipotential surface $\vec{E}$ is perpendicular on to it surface."

$$\text{As, } V_f - V_i = - \int_i^f \frac{q_0 \vec{E} \cdot d\vec{s}}{q_0}$$

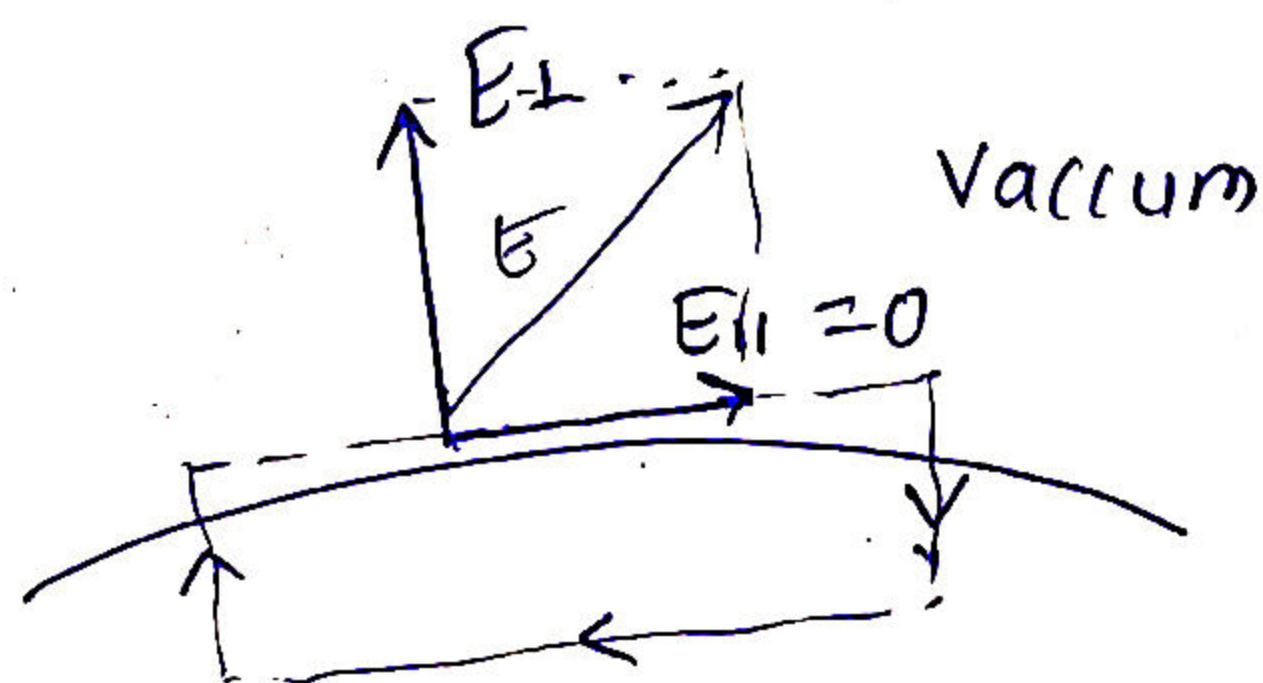
$$= - \int_i^f \vec{E} \cdot d\vec{s}$$

for $i \rightarrow \infty$, $V_i \rightarrow 0$ [at infinity for
our reference, $V = U = 0$
at $r \rightarrow \infty$]

$$\Rightarrow \boxed{V_f = V = - \int_{\infty}^f \vec{E} \cdot d\vec{s}}$$

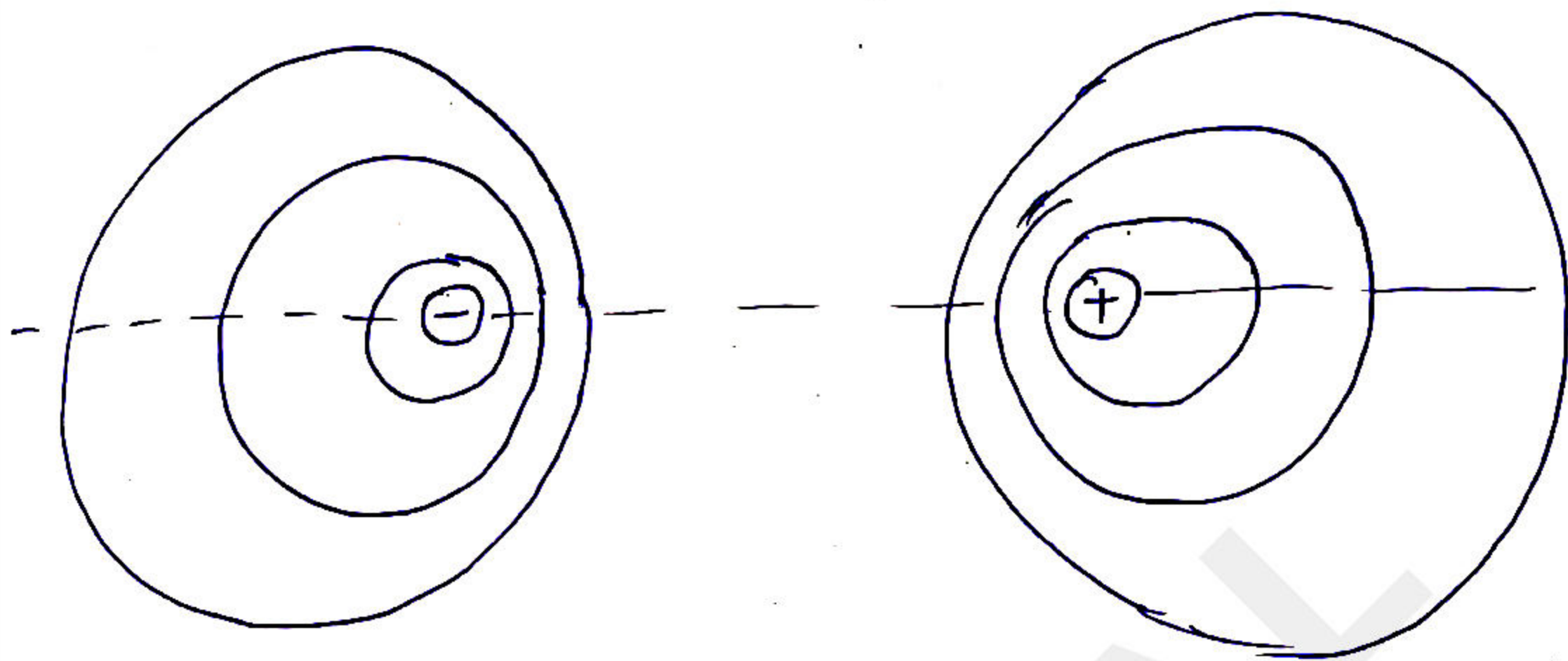


just above the conductor
 $\vec{E}$ must be perpendicular to conductor

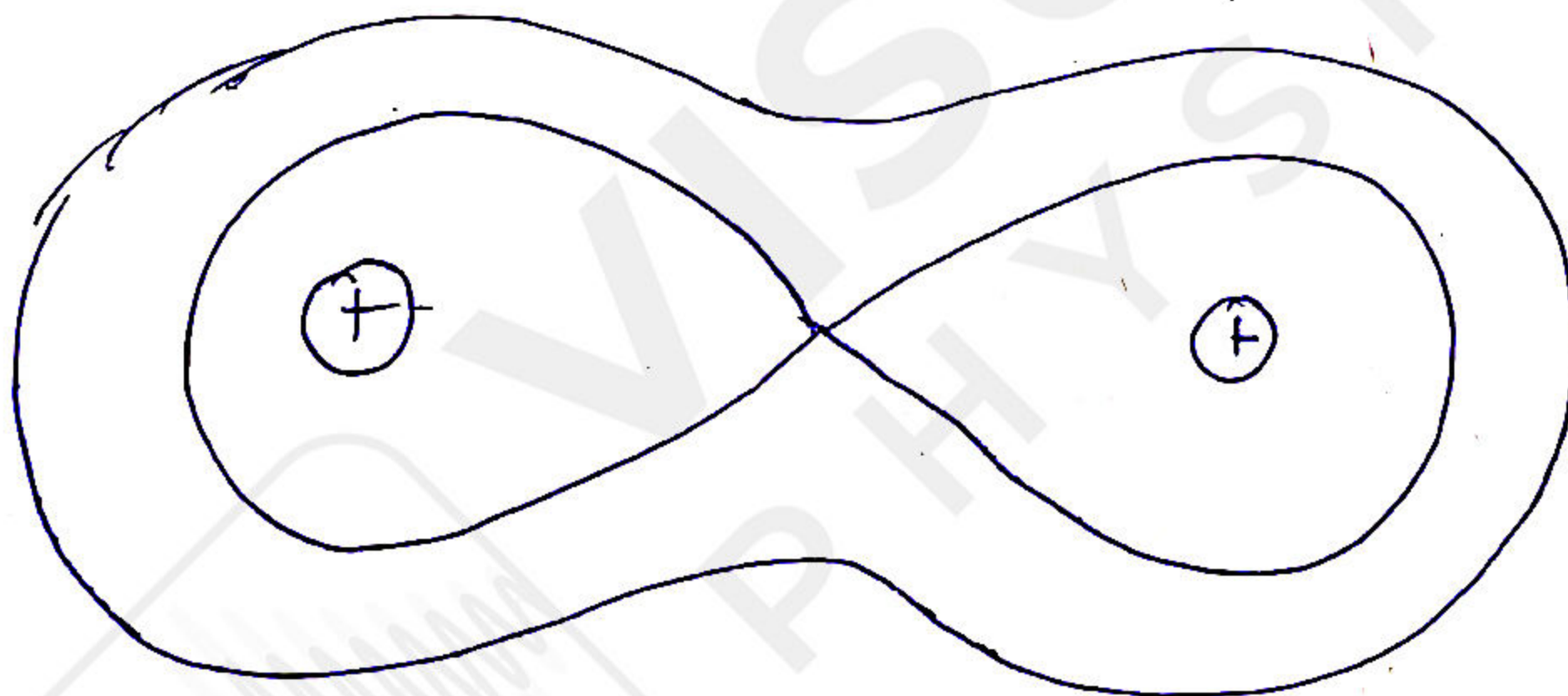


$E = 0$ Conductor

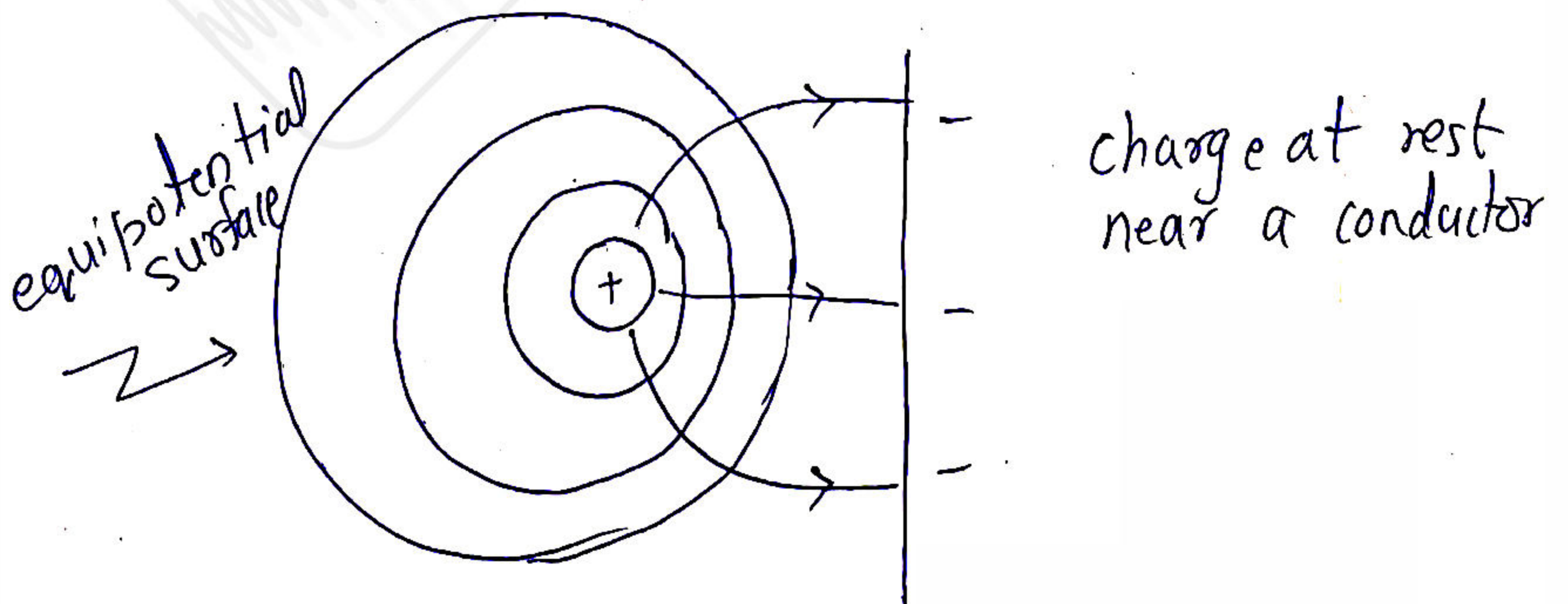
As if $E_{\parallel} \neq 0$, electron must be moving and hence non zero work by E . but E is conservative so $E_{\parallel} = 0$



Equipotential surface of dipole



Two equal positive charge

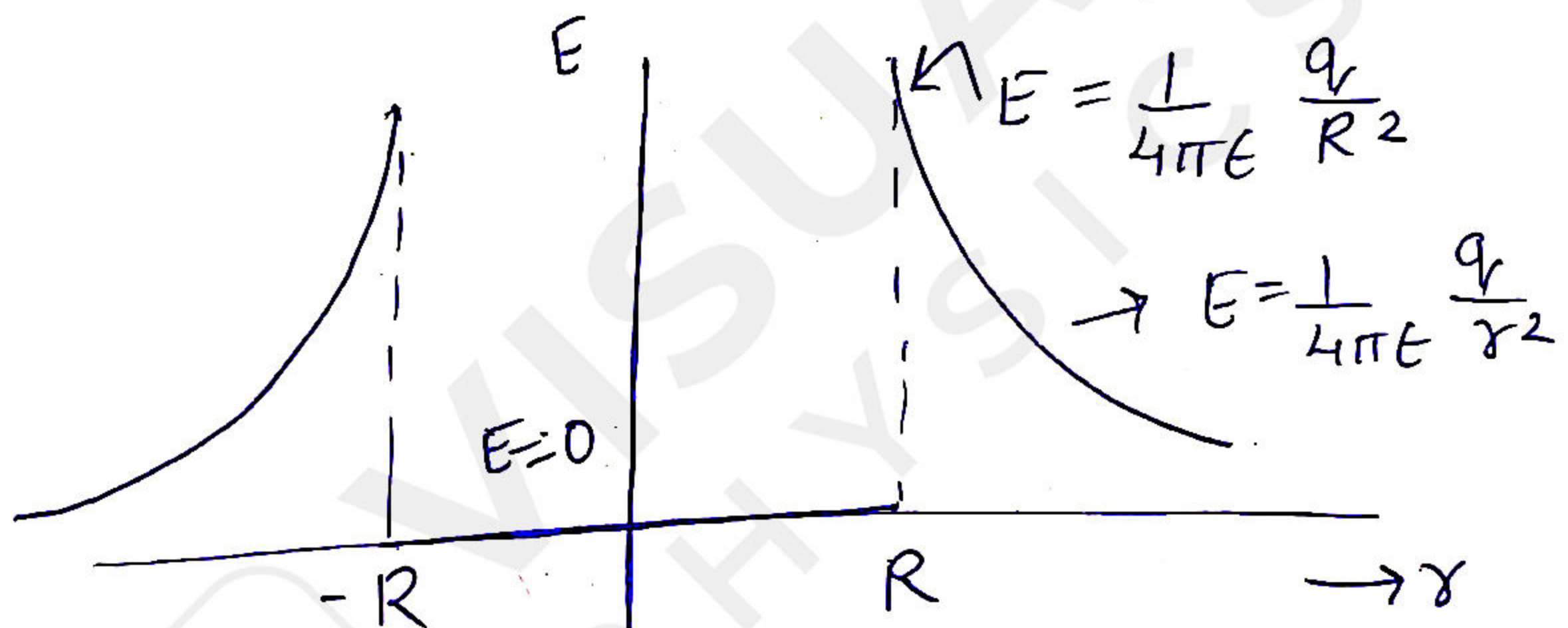


Potential due to a conducting shell.

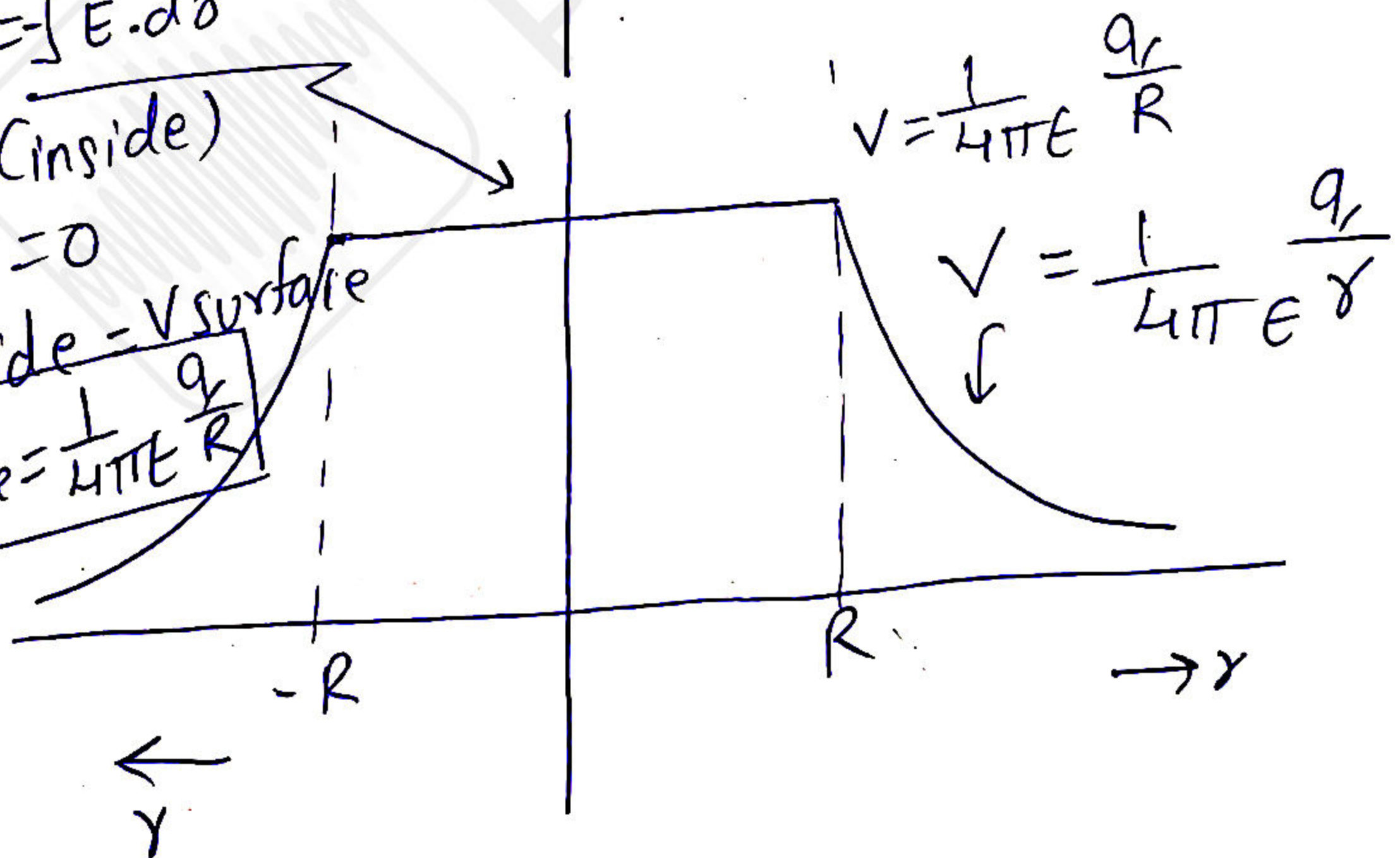
$$\int_{\infty}^{\gamma} dV = \int_{\infty}^{\gamma} -\vec{E} \cdot d\vec{r} \quad [V_{\infty} = 0]$$

$$V = - \int_{\infty}^{\gamma} \frac{1}{4\pi\epsilon} \frac{q}{r^2} dr$$

$$V = \frac{1}{4\pi\epsilon} \frac{q}{\gamma}$$



as $dV = -\vec{E} \cdot d\vec{r}$
 $E = 0$ (inside)
 so, $dV = 0$
 so $V_{\text{inside}} = V_{\text{surface}}$
 $\Rightarrow V_{\text{inside}} = \frac{1}{4\pi\epsilon} \frac{q}{R}$



A Non-conducting solid sphere
outside

$$E_{\text{outside}} = \frac{1}{4\pi\epsilon} \frac{q}{r^2}$$

$q \rightarrow$ total charge

$$\int_{\infty}^r dV = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$$

$$V = - \int_{\infty}^r \frac{1}{4\pi\epsilon} \frac{q}{r^2} dr$$

$$\boxed{V = \frac{1}{4\pi\epsilon} \frac{q}{r}}$$

At surface, $V = \frac{1}{4\pi\epsilon} \frac{q}{R}$
 $\hookrightarrow$ Radius

Inside

$$E_{\text{inside}} = \frac{1}{4\pi\epsilon} \frac{q}{R^3} r$$

$$\int_{V_s}^V dV = - \int_R^r \frac{1}{4\pi\epsilon} \frac{q}{R^3} r dr$$

$$V - V_s = - \frac{1}{4\pi\epsilon} \frac{q}{R^3} \left[\frac{r^2}{2} \right]_R^r$$

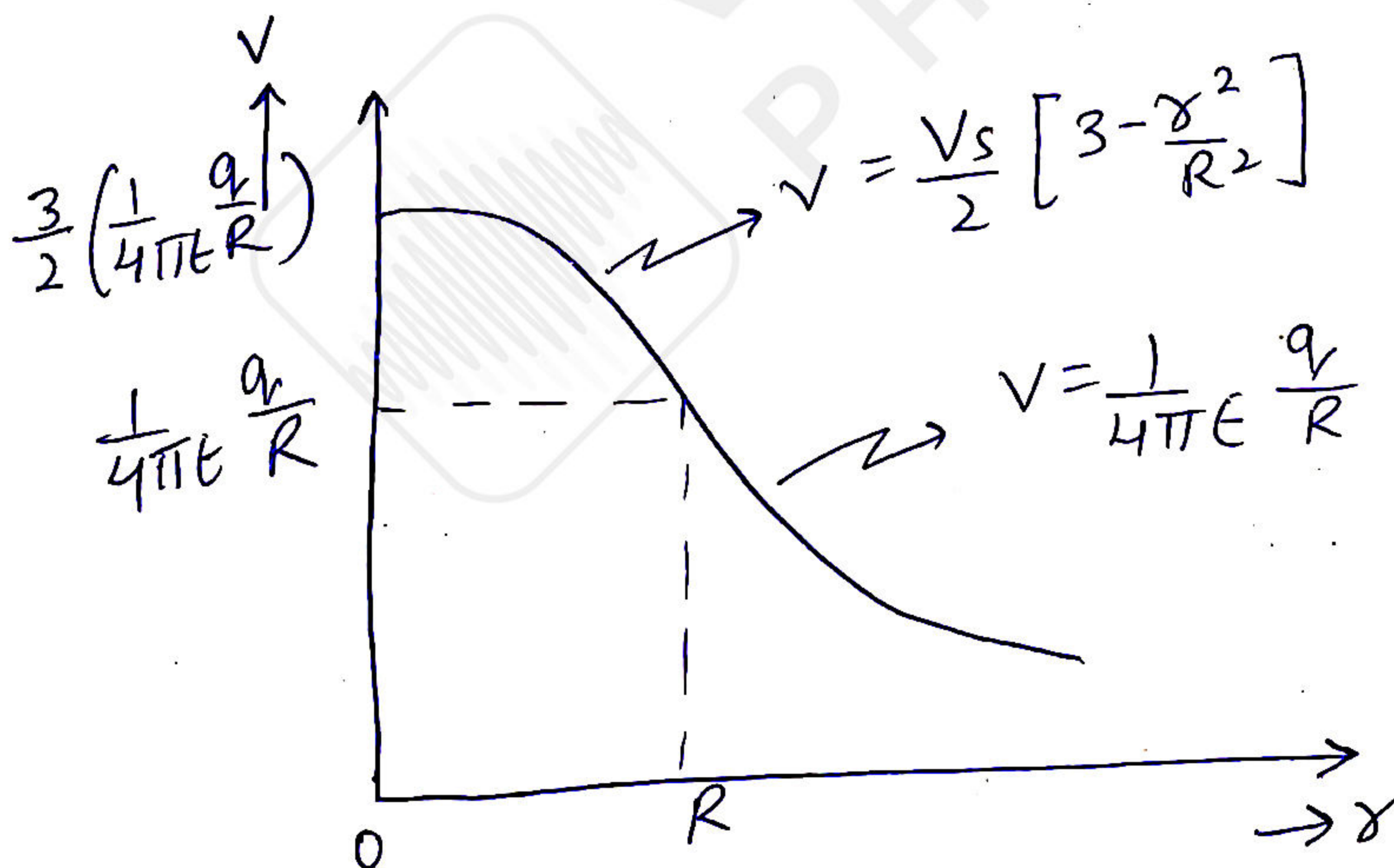
$$\text{as, } V_s = \frac{1}{4\pi\epsilon} \frac{q}{R}$$

$$\text{So, } V = -\frac{1}{4\pi\epsilon} \frac{q}{R^3} \left[\frac{r^2}{2} - \frac{R^2}{2} \right] + V_s$$

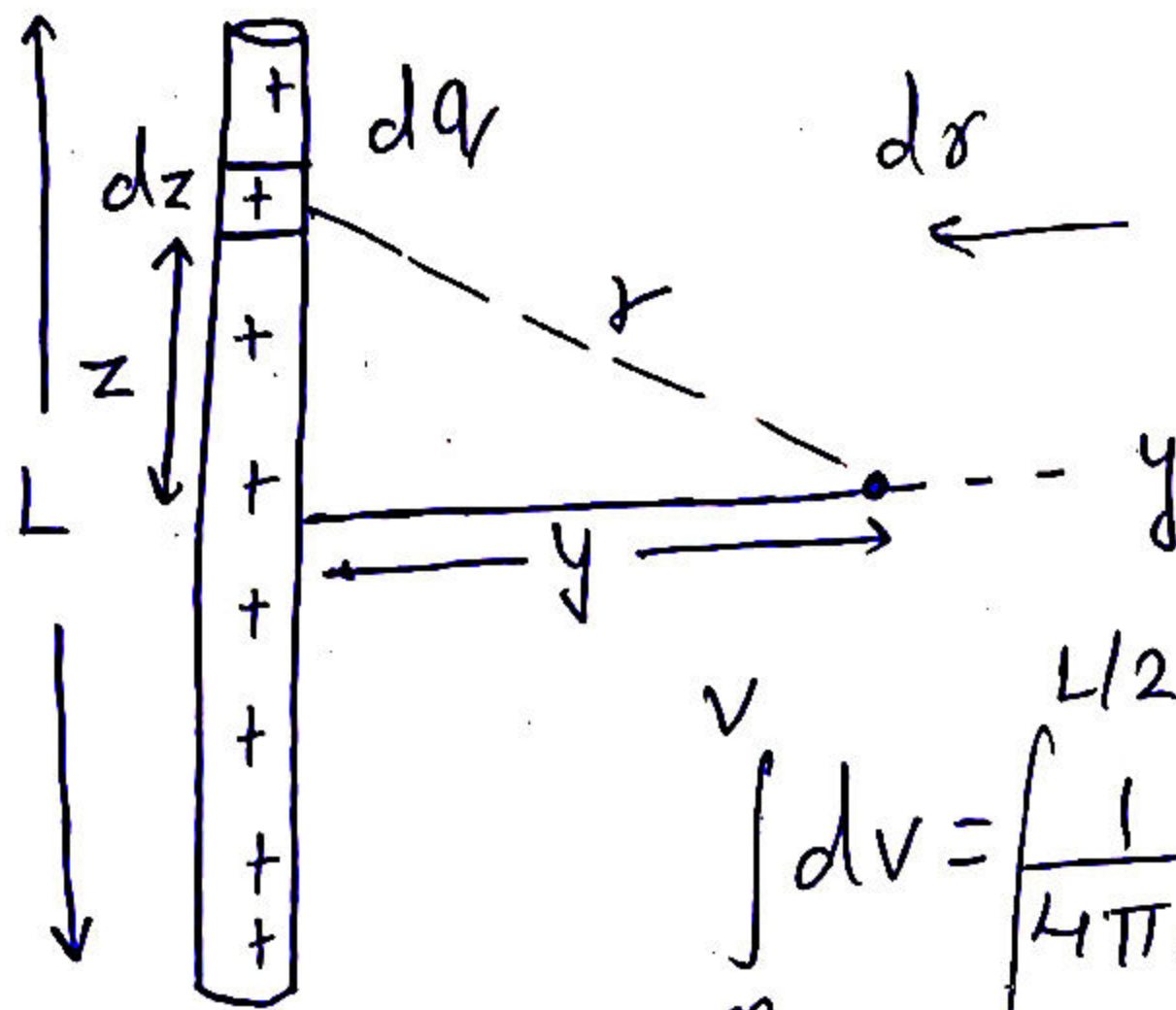
$$V = \frac{1}{4\pi\epsilon} \frac{q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right]$$

$$V = \frac{1}{4\pi\epsilon} \frac{q}{R} \left[3 - \frac{r^2}{R^2} \right]$$

$$\text{or } V = \frac{V_s}{2} \left[3 - \frac{r^2}{R^2} \right]$$



A uniform line of charge:



$$dV = \frac{1}{4\pi\epsilon} \frac{dq}{r} = \frac{1}{4\pi\epsilon} \frac{dq}{\sqrt{y^2 + z^2}}$$

now $dq = \lambda dz$

linear charge density

$$\int_{-\infty}^{\infty} dV = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon} \frac{\lambda dz}{\sqrt{z^2 + y^2}}$$

for linear
length L

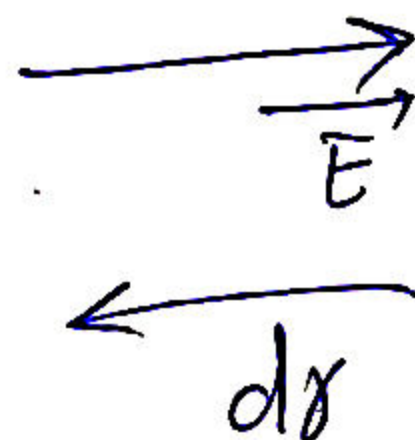
$$V = \frac{\lambda}{4\pi\epsilon} \ln \left[\frac{L/2 + \sqrt{L^2/4 + y^2}}{-L/2 + \sqrt{L^2/4 + y^2}} \right]$$

for infinite rod

$$V_b - V_a = \int_a^b -\vec{E} \cdot d\vec{r} = \int_a^b \frac{1}{4\pi\epsilon} \frac{2\lambda}{r} dr$$

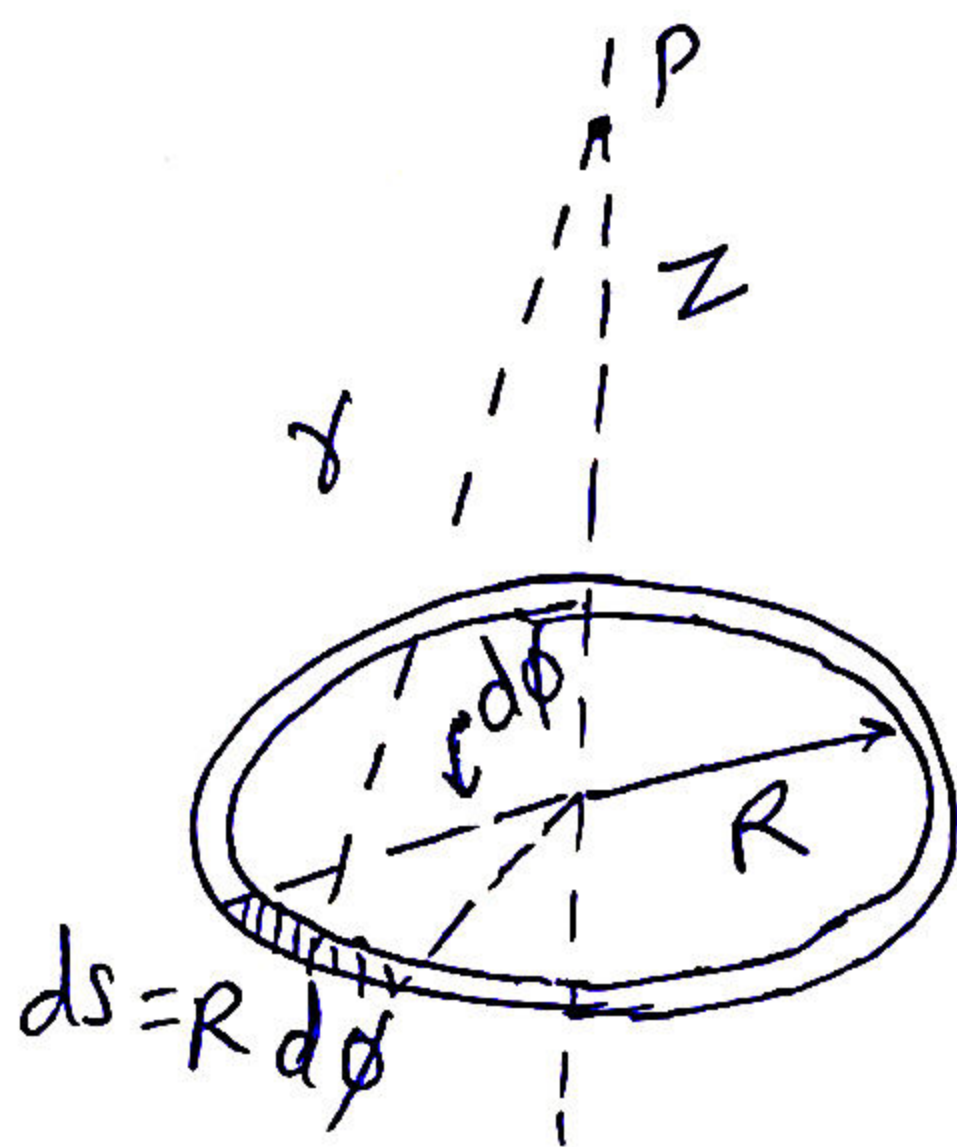
$$V_b - V_a = \frac{\lambda}{2\pi\epsilon} \ln(b/a)$$

as



so, $\cos\theta = -1 = \cos 180^\circ$

A ring of charge:



$$dq = \lambda ds = \lambda R d\phi$$

$$dV = \frac{1}{4\pi\epsilon} \frac{dq}{r}$$

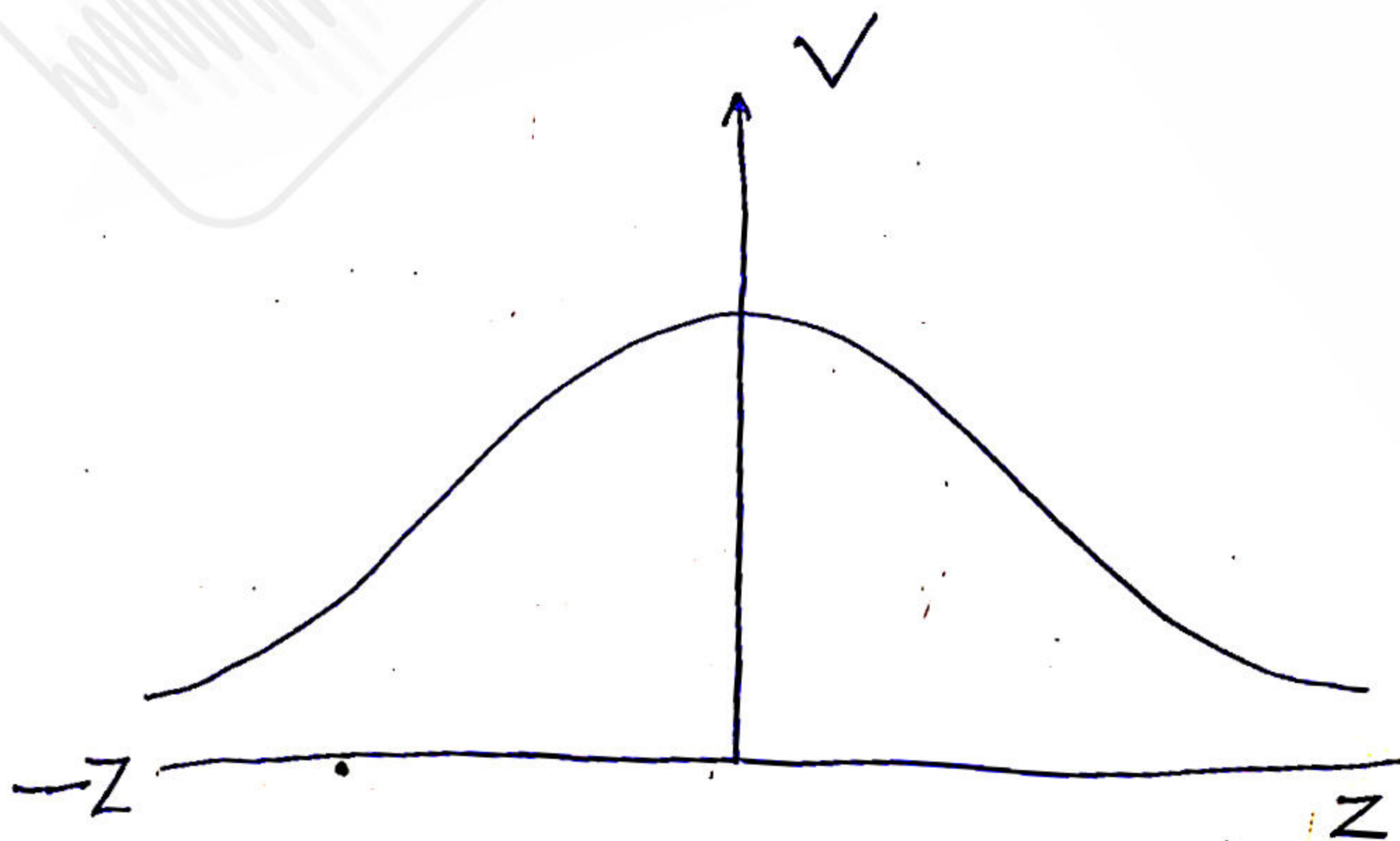
$$= \frac{1}{4\pi\epsilon} \frac{\lambda R d\phi}{\sqrt{R^2 + z^2}}$$

$$V = \frac{1}{4\pi\epsilon} \frac{\lambda R}{\sqrt{R^2 + z^2}} \int_0^{2\pi} d\phi$$

$$V = \frac{1}{4\pi\epsilon} \frac{2\pi\lambda R}{\sqrt{R^2 + z^2}}$$

as $q = 2\pi R \lambda$, and $r = \sqrt{R^2 + z^2}$

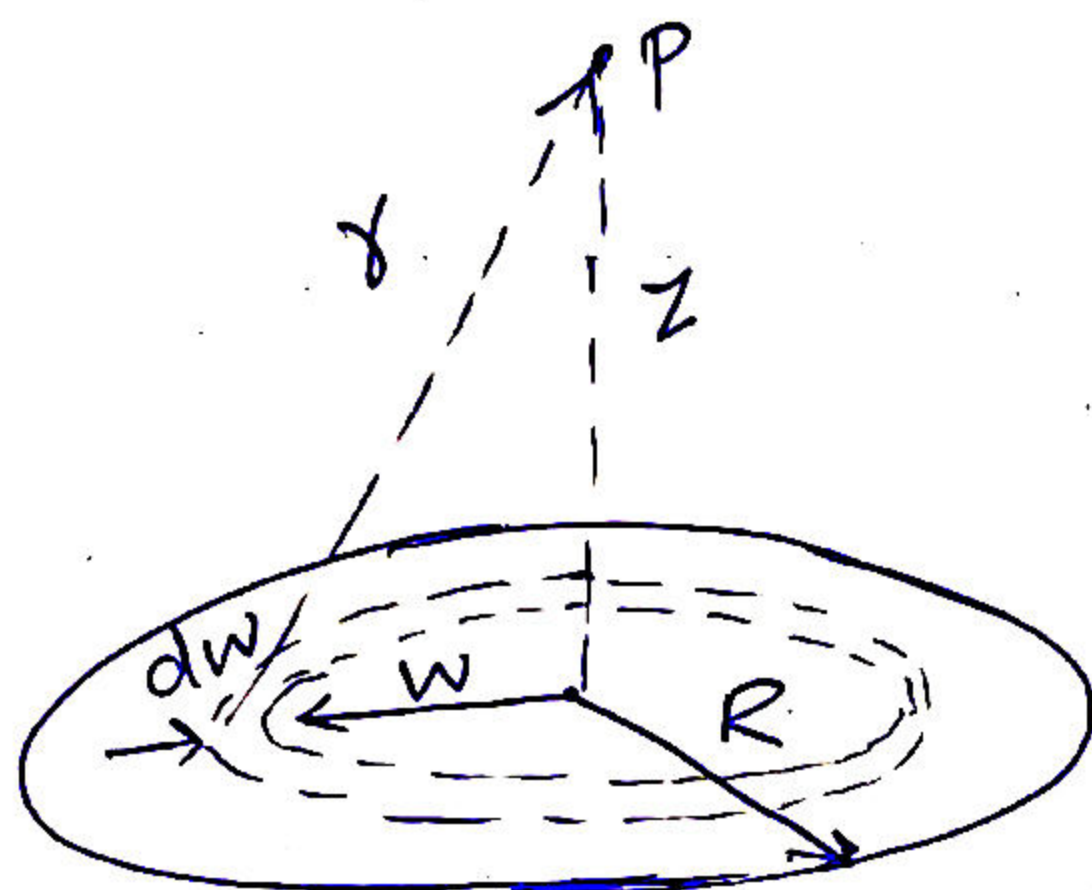
$$V = \frac{1}{4\pi\epsilon} \frac{q}{r}$$



A CHARGED DISK:

$$dA = 2\pi w dw$$

$$dV = \frac{1}{4\pi\epsilon} \frac{dq}{\sqrt{w^2 + z^2}}$$



now

$$dq = 2\pi\sigma w dw$$

$$dV = \frac{1}{4\pi\epsilon} \frac{2\pi\sigma w dw}{\sqrt{w^2 + z^2}}$$

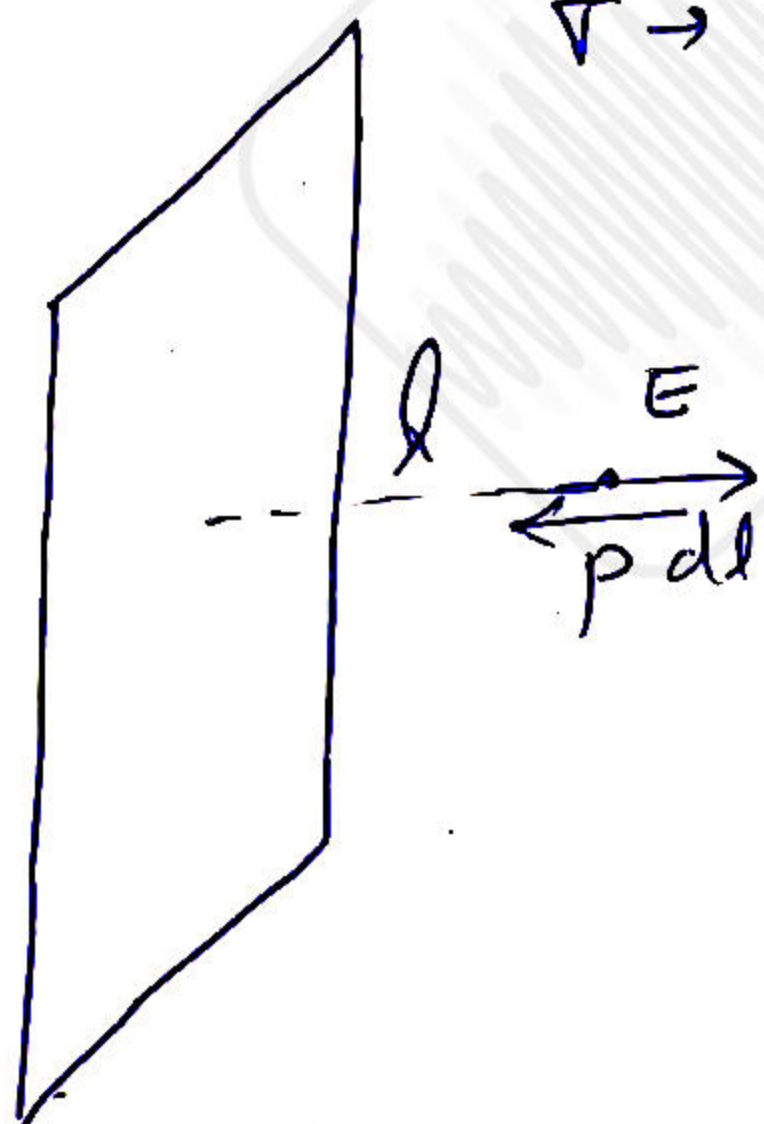
$$V = \frac{\sigma}{2\epsilon} \int_0^R \frac{w dw}{\sqrt{w^2 + z^2}}$$

$$V = \frac{\sigma}{2\epsilon} (\sqrt{R^2 + z^2} - |z|)$$

So, $R \rightarrow \infty$, $V \rightarrow \infty$ so can't define absolute potential

Infinite charge plane.

$\sigma \rightarrow$ surface charge density



$$E = \frac{\sigma}{2\epsilon}$$

$$\text{so, } \int_{\infty}^l dV = - \int_{\infty}^l \vec{E} \cdot d\vec{l}$$

$$V - 0 = \int_{\infty}^l \frac{\sigma}{2\epsilon} dl = \frac{\sigma}{2\epsilon} [l - \infty]$$

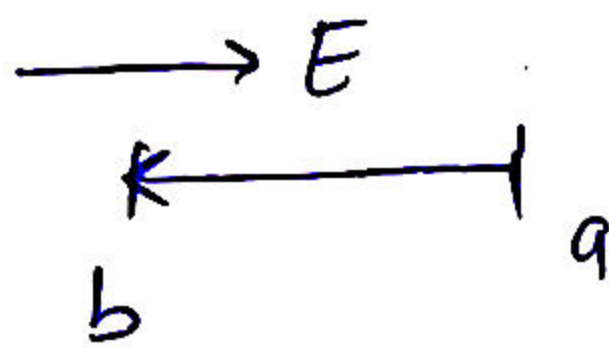
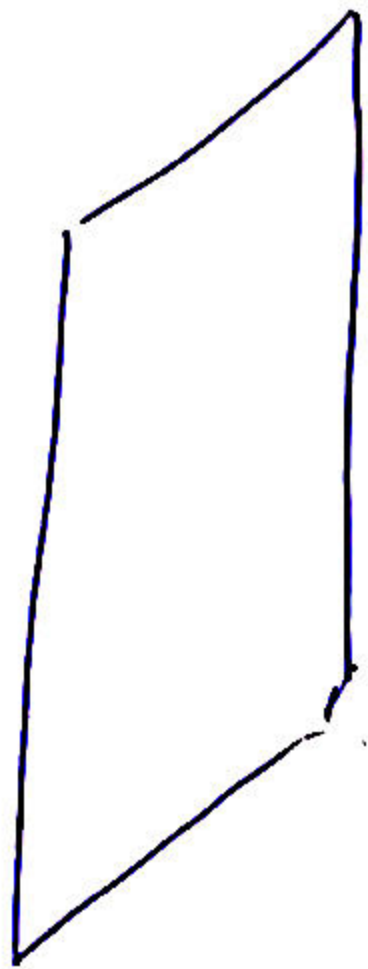
V absolute not define

but we can define potential difference between two points.

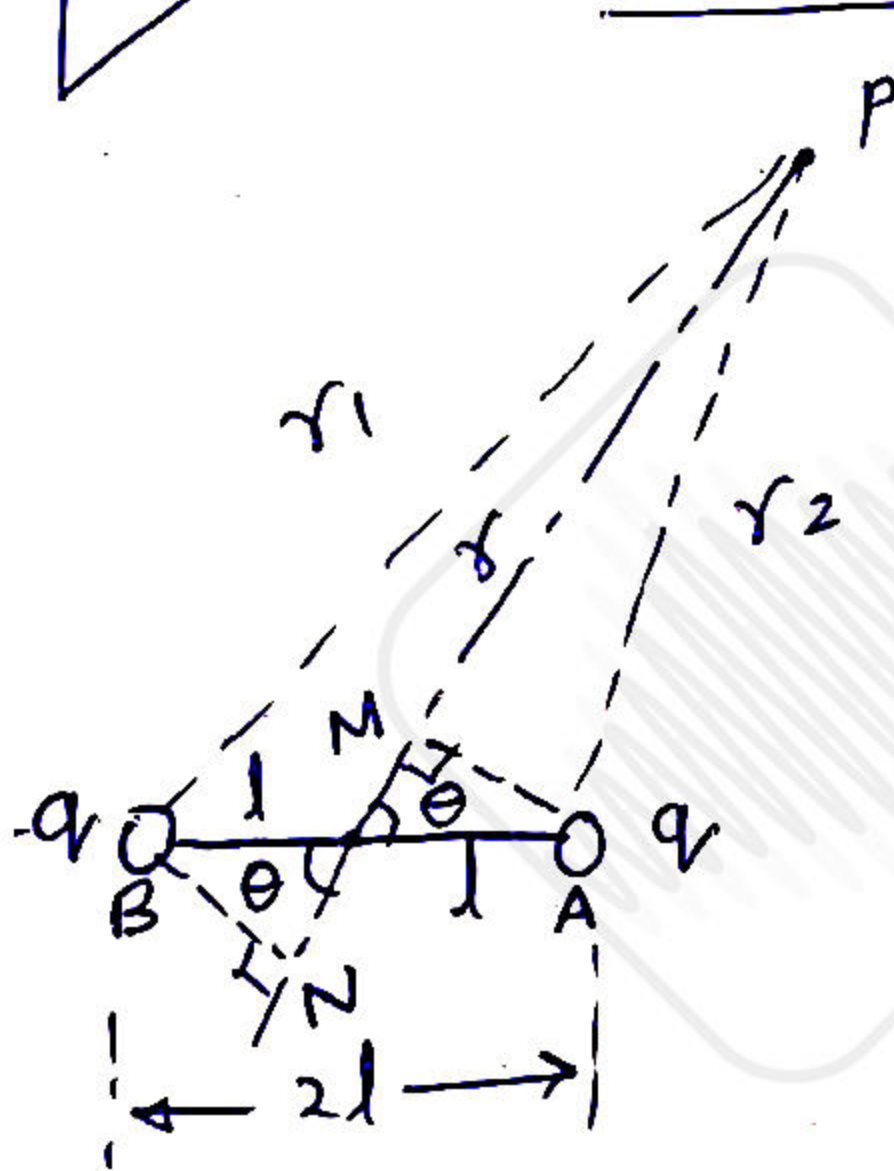
$$\int_a^b dV = - \int_a^b \vec{E} \cdot d\vec{l}$$

$$V_b - V_a = \int_a^b E dl \cos 180^\circ = - \frac{\sigma}{2\epsilon} \int_a^b dl$$

$$\Rightarrow \boxed{V_b - V_a = \frac{\sigma}{2\epsilon} (b-a)}$$



Potential due to an electric dipole:



$$OP = r, \angle AOP = \theta$$

$$BP = r_1, AP = r_2$$

$$r_1 \approx PN = r + l \cos \theta$$

$$r_2 = AP = MP = r - l \cos \theta$$

$$V = \frac{-q_r}{4\pi\epsilon r_1} + \frac{q_r}{4\pi\epsilon r_2} = \frac{q_r}{4\pi\epsilon} \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

$$V = \frac{q_r}{4\pi\epsilon} \left[\frac{r + l \cos \theta - r + l \cos \theta}{r^2 - l^2 \cos^2 \theta} \right]$$

$$V = \frac{1}{4\pi\epsilon} \frac{q_r (2l \cos \theta)}{(r^2 - l^2 \cos^2 \theta)}$$

$$\therefore p = q_r (2l)$$

$$\boxed{V = \frac{1}{4\pi\epsilon} \frac{p \cos \theta}{(r^2 - l^2 \cos^2 \theta)}}$$

→ if $l \ll r$

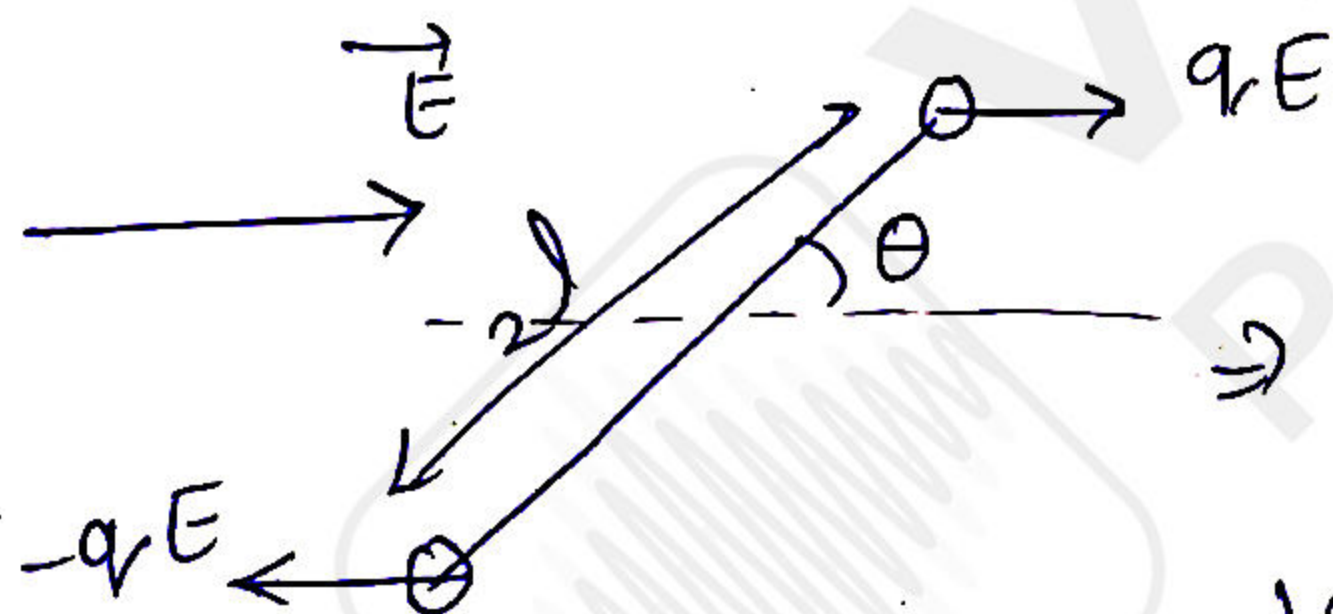
$$V = \frac{1}{4\pi\epsilon} \frac{P \cos\theta}{r^2}$$

→ $\theta = 0^\circ$
$$V = \frac{1}{4\pi\epsilon} \frac{P}{(r^2 - l^2)}$$

→ $\theta = 90^\circ$, $\boxed{V = 0}$

Work done in rotating Electric dipole in a uniform Electric Field.

as, $dw = \text{Torque} \times d\theta$



$$\begin{aligned} \text{Torque} &= qE(2l \sin\theta) \\ &= PE \sin\theta \end{aligned}$$

$$\Rightarrow dw = PE \sin\theta d\theta$$

$$W = \int_0^\theta PE \sin\theta d\theta =$$

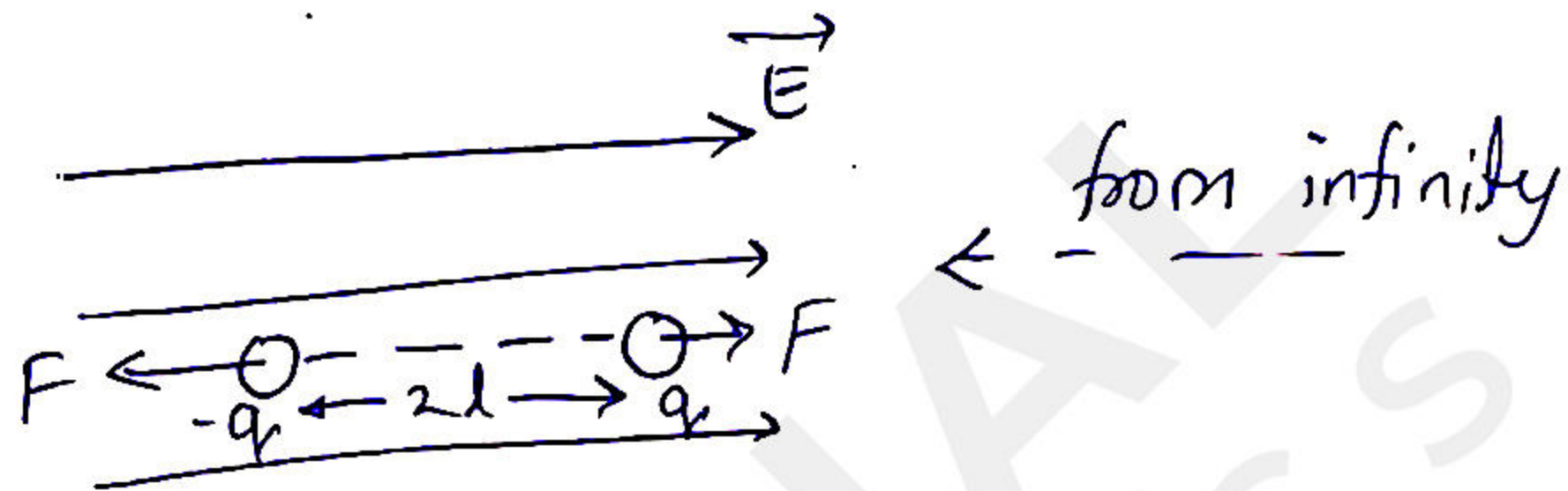
$$= PE (-\cos\theta)_0^\theta$$

$$\Rightarrow W = PE [-\cos\theta + \cos 0]$$

$$\boxed{W = PE(1 - \cos\theta)}$$

Potential Energy of an electric dipole in a uniform Electric field:

→ Work done in bringing an electric dipole from infinity to some point inside the field is equal to the potential energy of an electric dipole.



Work in bringing dipole = Force on charge $(-q)$ $\times$ extra distance moved

$$\Rightarrow \boxed{U = -PE}$$

(orientation is $\theta = 0$)

Now in rotating from $\theta = 0 \rightarrow \theta$

$$\text{Work} = PE(1 - \cos\theta)$$

so net potential at θ

$$U_0 = U + W = -PE + PE(1 - \cos\theta)$$

so work
in $\theta_i \rightarrow \theta_f$

$$W = U_f - U_i$$

$$W = -PE(\cos\theta_i - \cos\theta_f)$$

$$U_0 = -PE\cos\theta$$

$$\boxed{U_0 = -\vec{P} \cdot \vec{E}}$$