VISUAL PHYSICS

SHORT NOTES

CHAPTER

Electric Forces & Fields

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Electric Forces & fields

Electric charge: → SI unit Coulomb (C)

Electric charge: → fundamental attributes of particles of which matter is made of.

Electric charge: → It is physical property of certain fundamental particles (electron, proton)

matter → atom — made of → proton, electron & neutron

To distinguish the nature of interaction

charge → positive, negative

positive → proton

negative → electron

electron → orbits around nucleus

proton + and neutrons made → nucleus

neutral no charge

We represents charge on proton & electron by +e & -e

particle	mass	charge
electron	9.1×10^{-31} kg	-1.69×10^{-19} C
proton	1.67×10^{-27} kg	1.69×10^{-19} C
neutron	1.675×10^{-27} kg	0 C

so $|e| = 1.6 \times 10^{-19}$ C

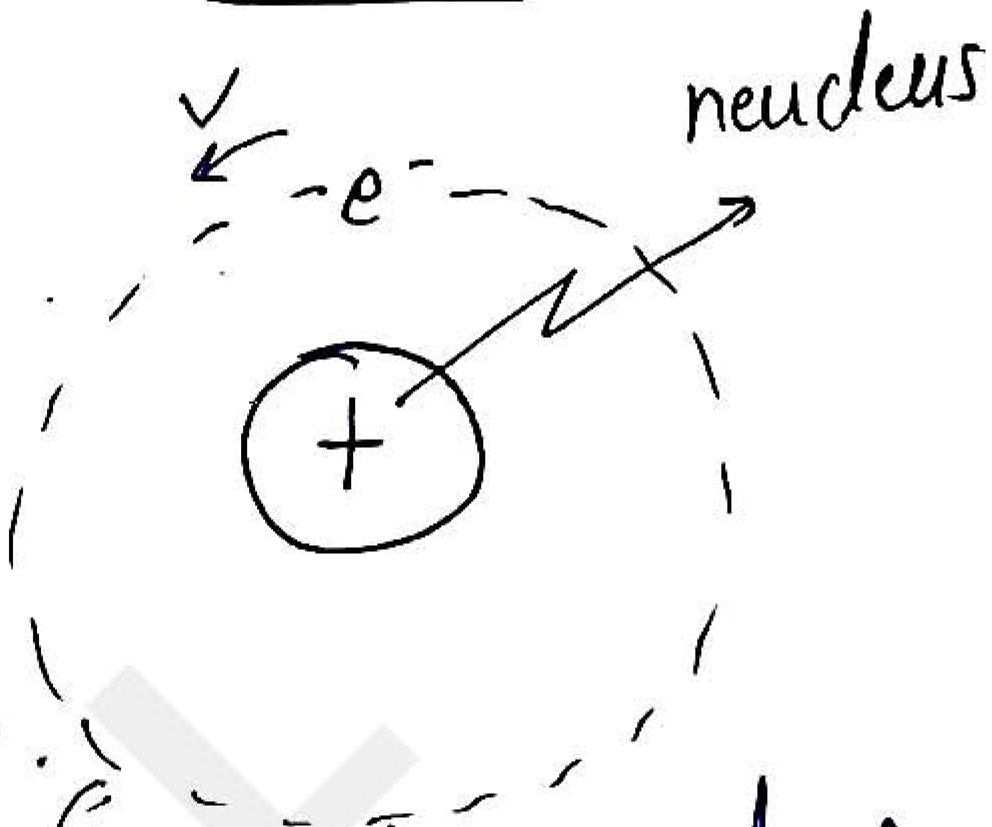
$|e|$ → represents electronic charge

⇒ like charges repels and unlike charges attracts.

⇒ ←(+) (+)→ → repels

←(−) (−)→ → repels

(−)→ ←(+) → attracts



↳ electron revolves around neucleus

ordinary matter contains equal number of protons and electrons

body → charged by → transfer of electrons

Electrons are in outer shells and it is easier to remove.

protons can't be removed from nucleus

" To charge a body negatively → electrons given to it "

" To charge a body positively → Some electrons are taken from it.

Work function → work to be done on a body in order to remove an electron from its surface

charging

friction

"when two bodies rubbed. one body transfer electron to other."

glass rod rubbed with silk

+

−

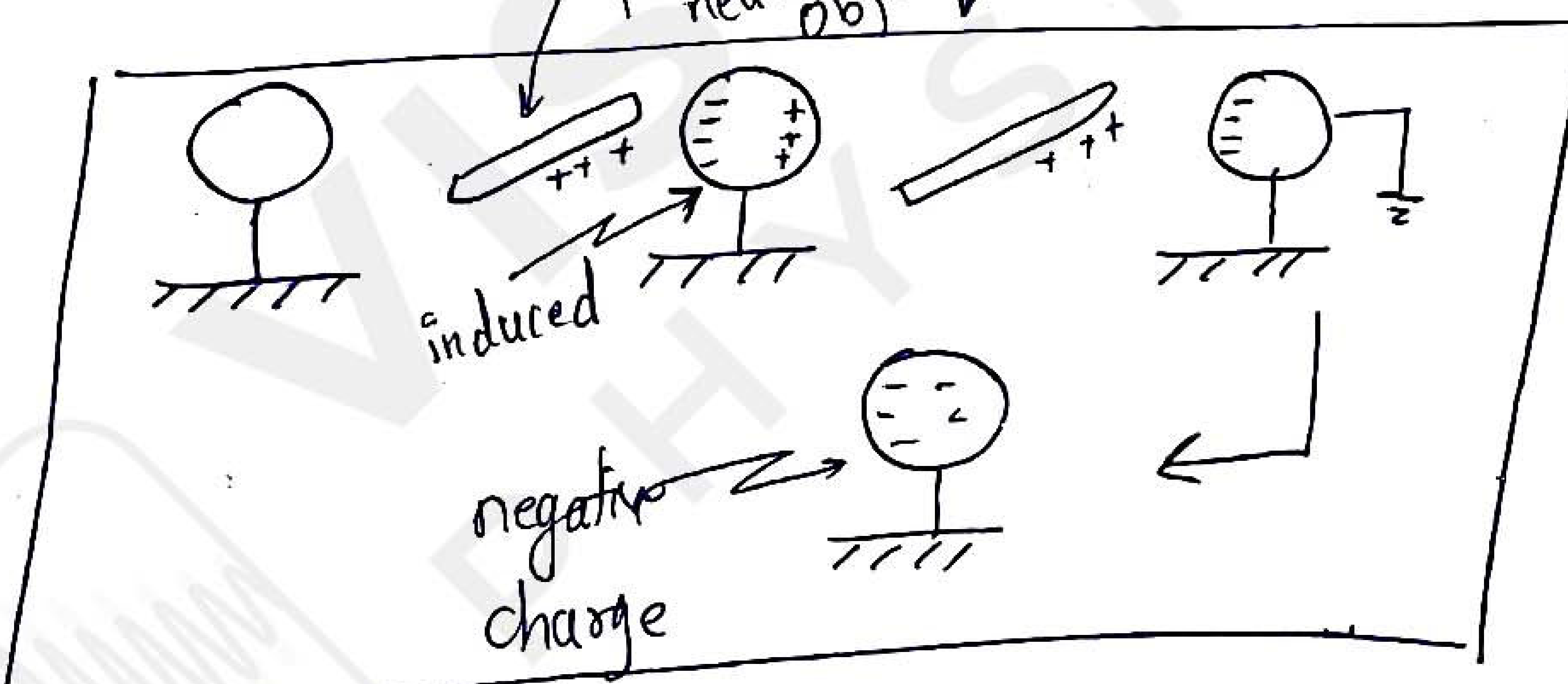
Conduction

transfer of charge from other body

Induction

charge body attracts opposite charge and repel similar charge. + Let bring positive charge

bringing positive to neutral object



When charge is stationary force between them is called "Electrostatic force"

Properties of Electric charge

Quantization of charge

$Q = \pm ne$
where $n = 0, 1, 2 \ldots$

Conservation of charge

total charge on isolate system is constant

Additivity

"net charge is algebric charge"

e.g 2C, −5C, 4C
net q = 2 − 5 + 4
= 1C

charge is Invarient

does not depend on speed

Coulomb's law:

$q_1 \longleftarrow r \longrightarrow q_2$

$$\boxed{|\vec{F}| = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}}$$

$|\vec{F}|$ → magnitude

$q_1 q_2$ → charge magnitude on charges

r^2 → seperation between them.

$\epsilon = \epsilon_0 \epsilon_r$

ϵ_r → (k = dielectric constant)

ϵ_r → relative permitivity of space

$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2C^{-2}$

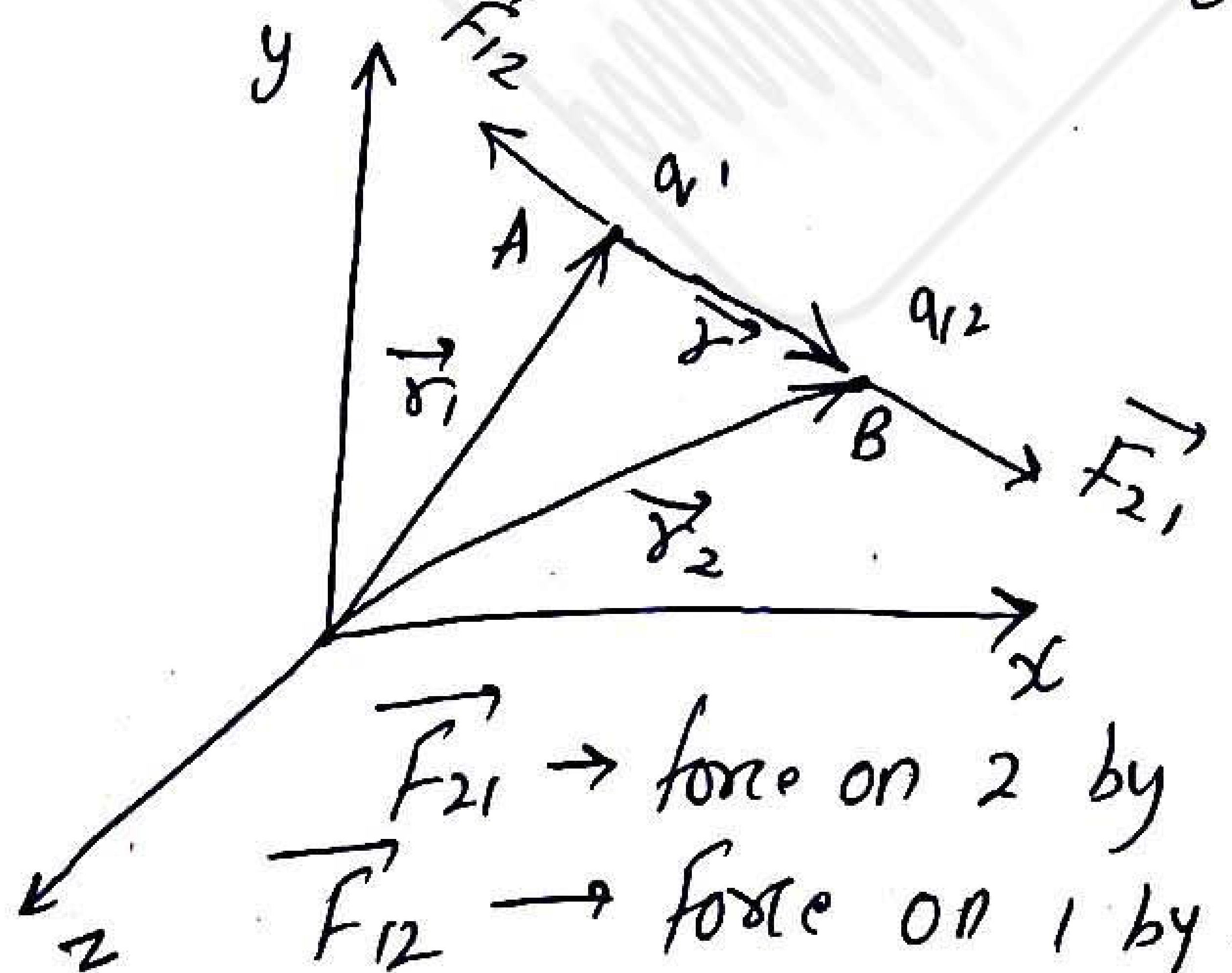
ϵ_0 → absolute permittivity of free space

$\epsilon_0 = 8.85\times10^{-12}\ C^2N^{-1}m^{-2}$

$\epsilon_r \to 1$ for air or vaccum

Coulomb's law:
- → applicable only for point charges
- → It is inversed square law
- → obey's Newton's third law
- → force acts along the line joining two particle
- → Electrostatic force is conservative force.

$|\vec{r}| = r$ = distance between q_1 & q_2



$\vec{F_{21}}$ → force on 2 by 1

$\vec{F_{12}}$ → force on 1 by 2

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$$

$$\vec{F_{21}} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}\frac{\vec{r}}{(r)}$$

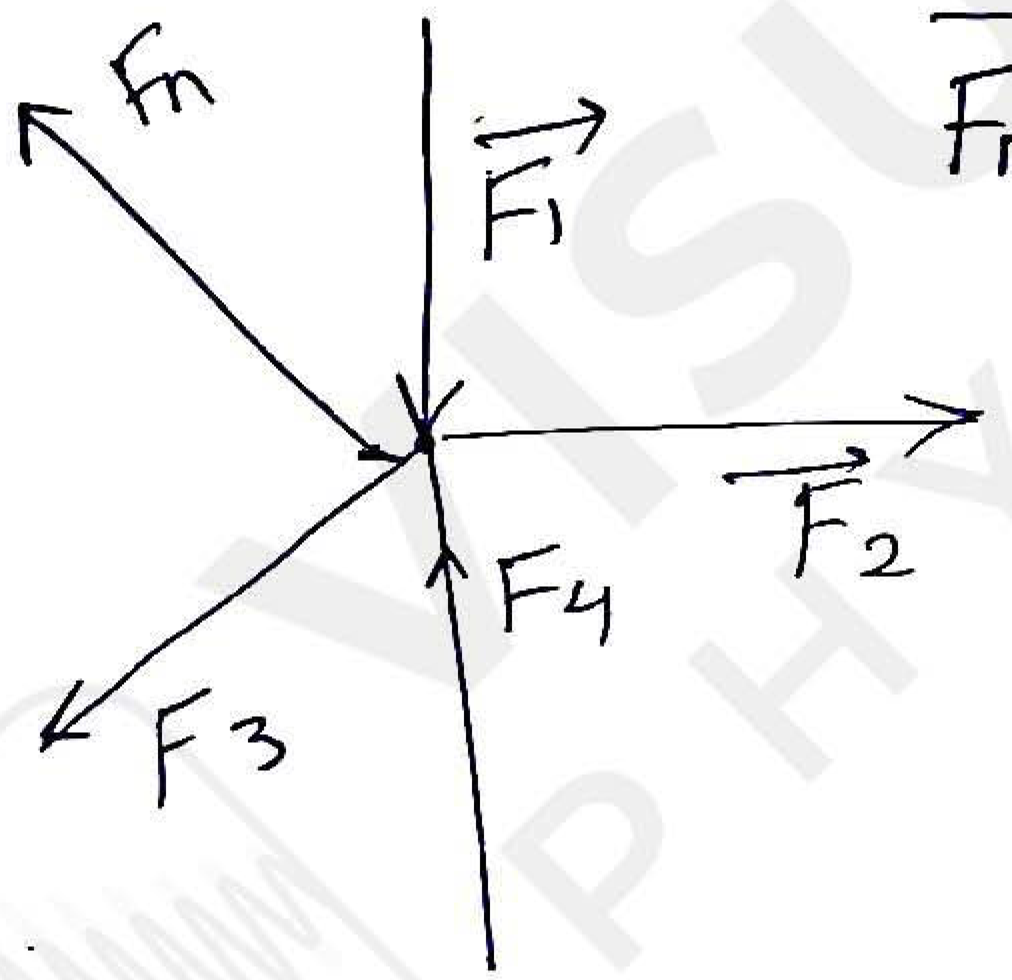
$$\boxed{\vec{F_{21}} = \frac{q_1 q_2(\vec{r_2} - \vec{r_1})}{4\pi\epsilon_0|\vec{r_2} - \vec{r_1}|^3}}$$

and as $|\vec{F_{12}}| = |\vec{F_{21}}|$ → Newton's third law

$\Rightarrow \vec{F_{12}} = -\vec{F_{21}}$

$$\boxed{\vec{F_{12}} = -\frac{q_1 q_2}{4\pi\epsilon} \frac{(\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}} = -\vec{F_{21}} \,($$

<u>superposition principle</u>:



$$\vec{F_{net}} = \vec{F_1} + \vec{F_2} + \ldots\ldots \vec{F_n}$$

\# To find the net force either we can use vector addition formula or can first resolve the vectors in given 2 or 3 directions (Depending on whether it is a 2D or 3D question) And then using algebric sum in each direction and $\vec{F_{net}} = \sqrt{F_x^2 + F_y^2 + F_z^2}$ to get final answer.

Electric field

A space around a charge in which its influence can be felt by any other charged particle.

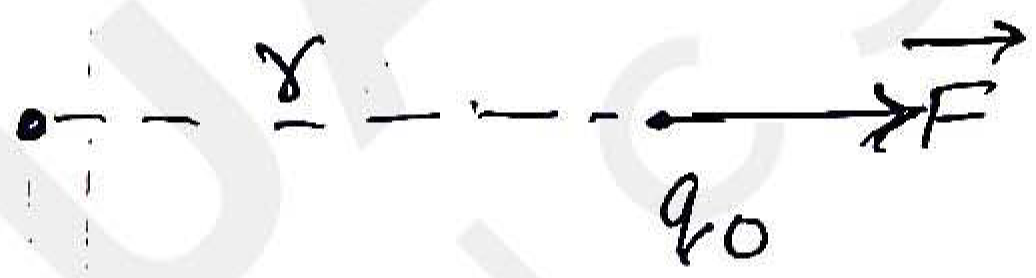
$$\boxed{E = \frac{1}{4\pi\epsilon}\frac{Q}{r^2}}$$

charge of the particle

distance at which we are finding intensity.

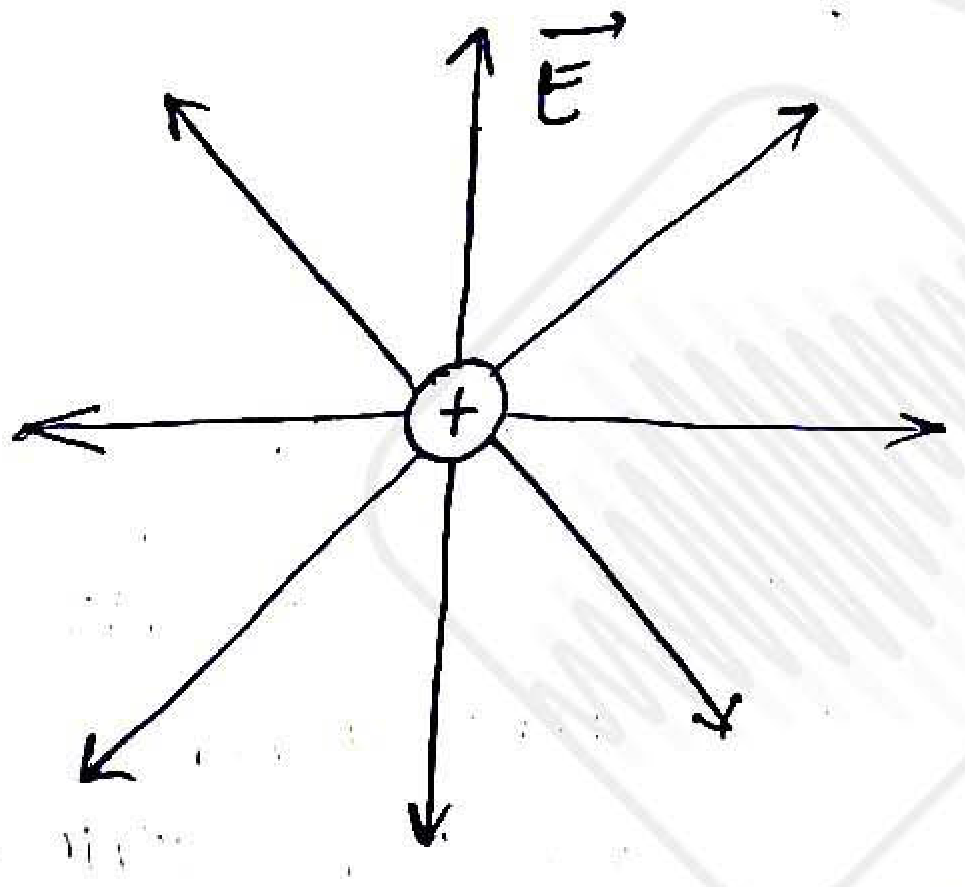
electric field intensity

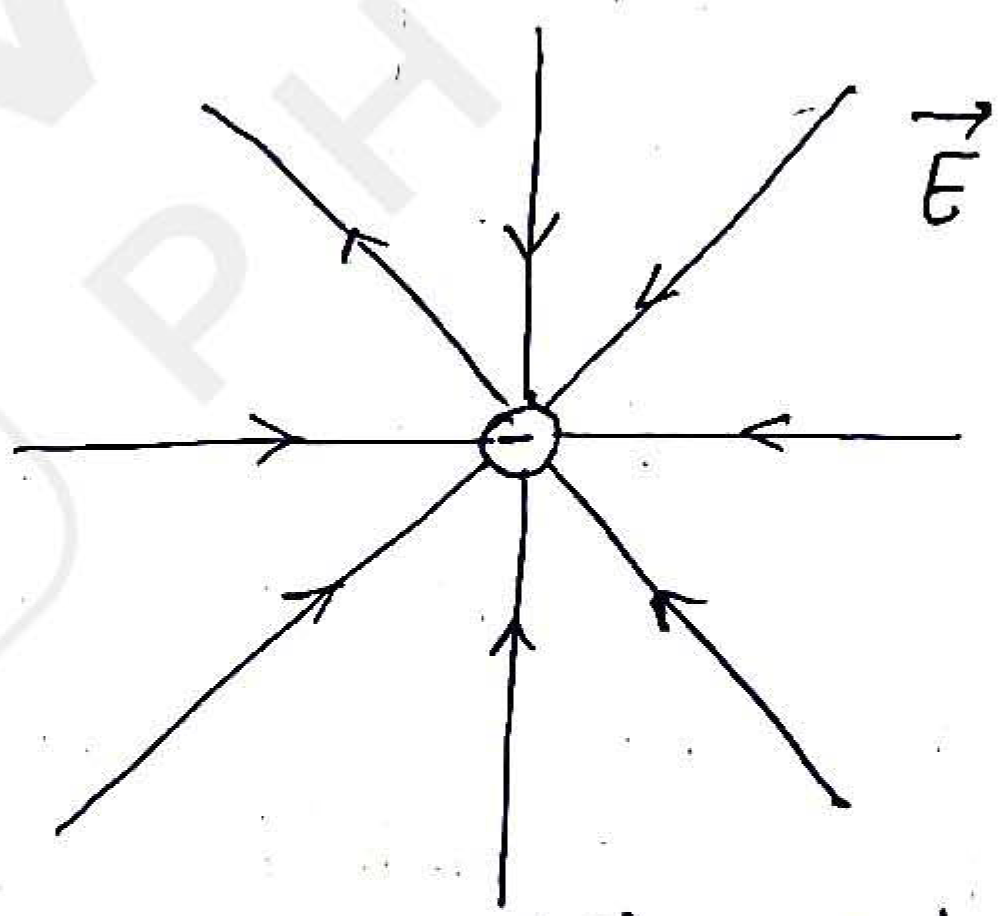
$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$



force on test charge q_0 due to , Q

unit positive charge

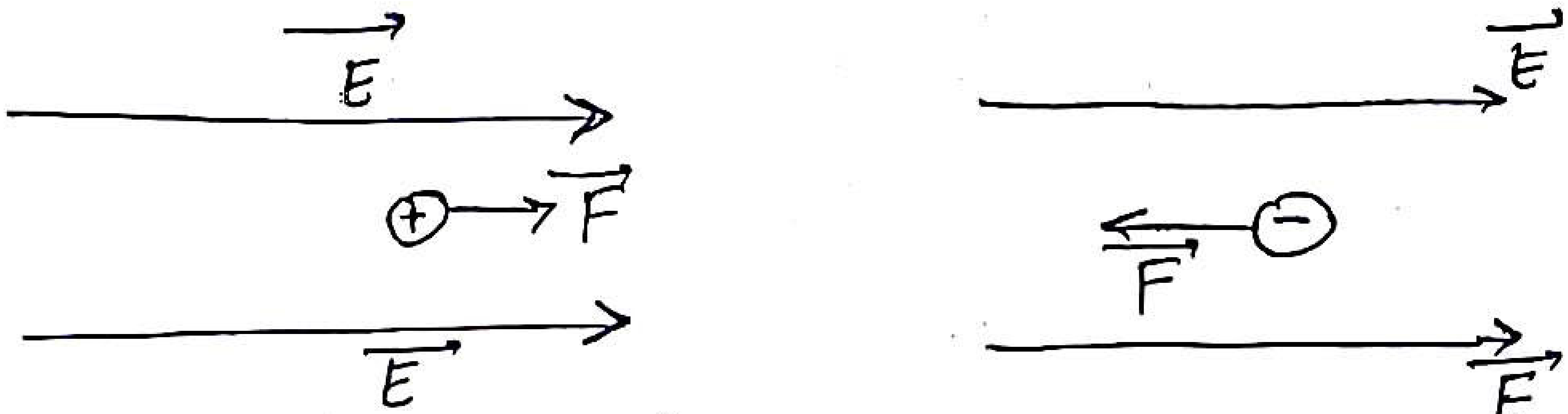




$\vec{E}$ point outwards for positive charge

$\vec{E}$ point inward for negative charge

$\boxed{\vec{E} \rightarrow \text{spherically symmetric}}$

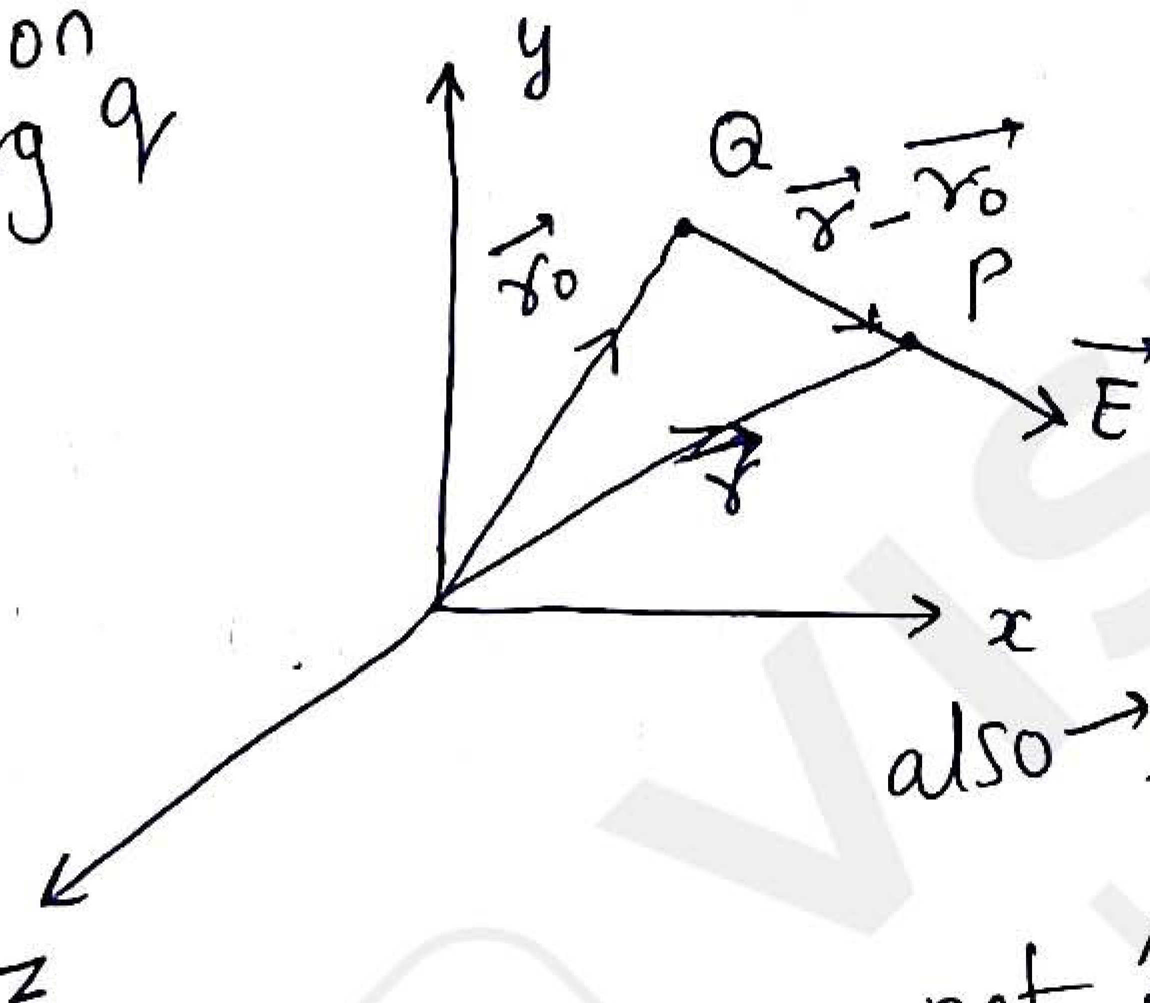


Force direction on charge when placed in External Electric field.

$$\boxed{\vec{F} = q\vec{E}}$$

force on charg q

External electric field



$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{(\vec{r}-\vec{r_0})}{|\vec{r}-\vec{r_0}|^3}}$$

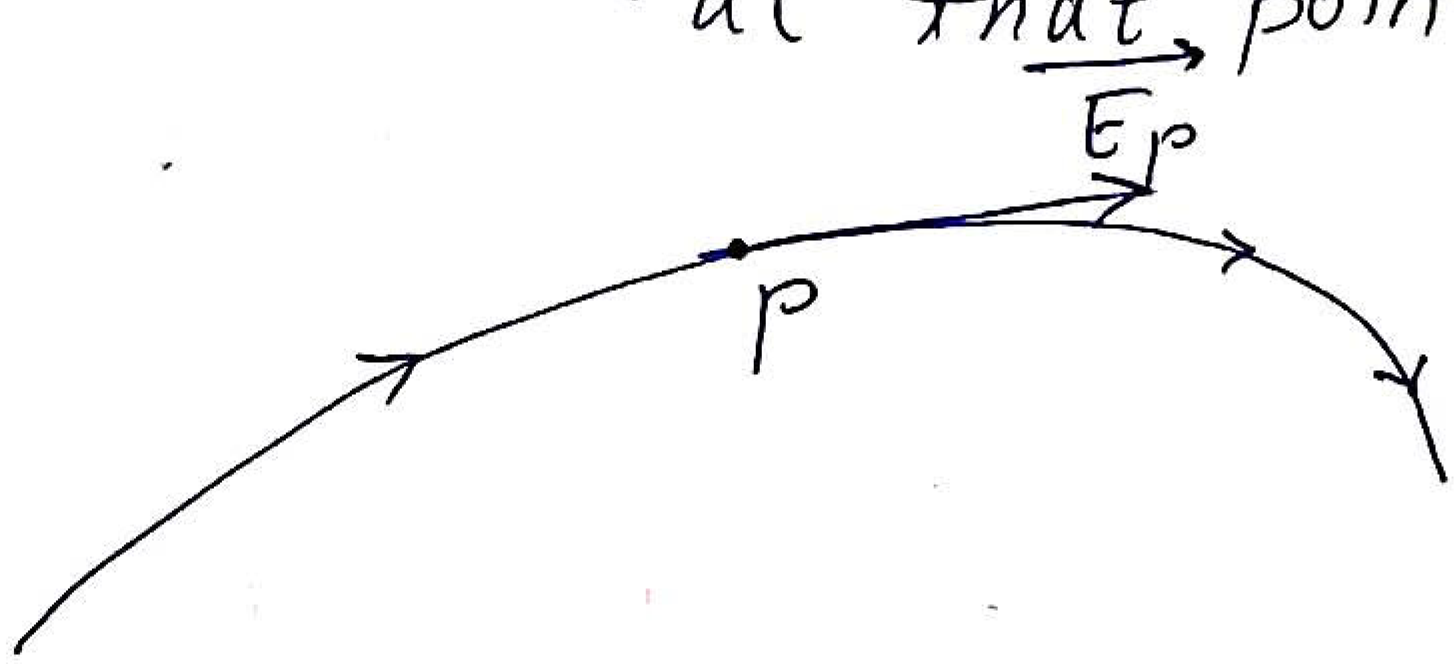
also → $\boxed{\vec{E}_{net} = \vec{E_1} + \vec{E_2} + \dots \vec{E_n}}$

net electric field at any point

Electric fields at given point

Lines of force:

→ drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point.



→ Electric line of force start from poisitive charge and ends at negative charge.

→ The tangent drawn gives direction of force acting on positive charge.

→ Unit → N/C

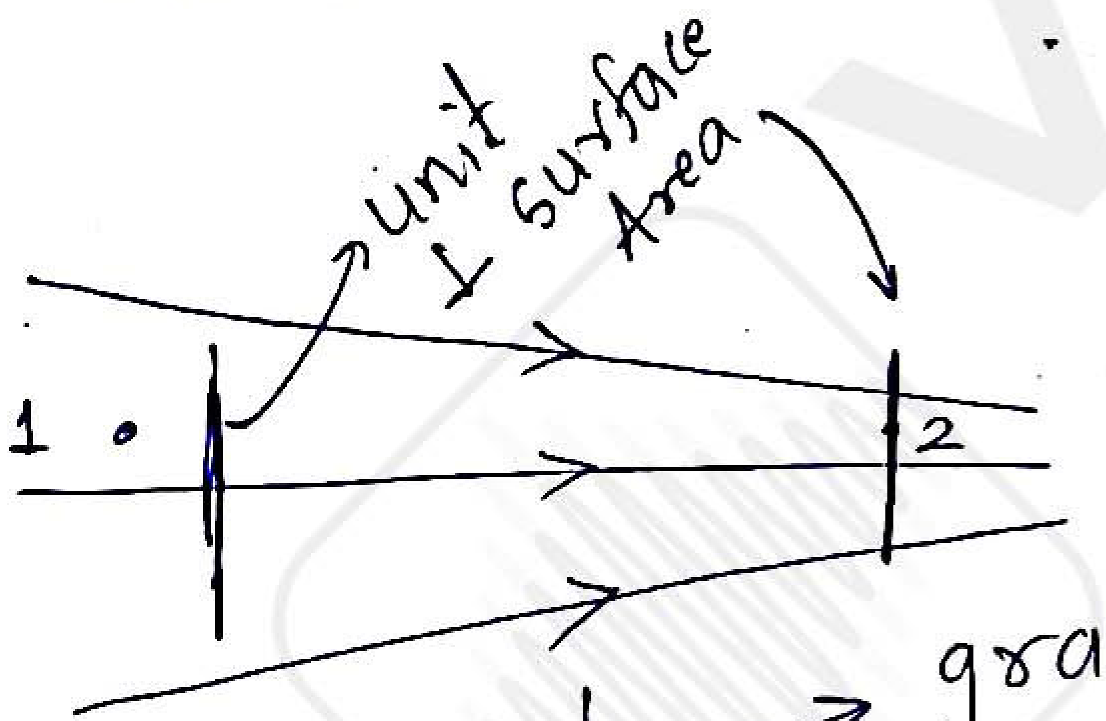
→ Can never be closed loop.

→ Electric field lines can never cross each other

→ $\vec{E}$ field inside a conductor is always zero, in static condition

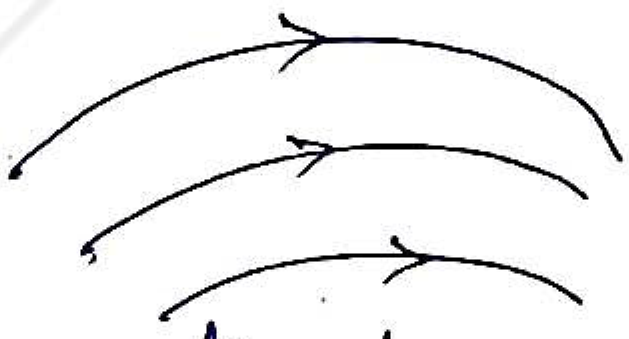
→ $\vec{E}$ field lines can never end and starts at same point.

★ <u>Electric field intensity</u> = number of $\vec{E}$ field lines passing through a unit perpendicular surface Area.

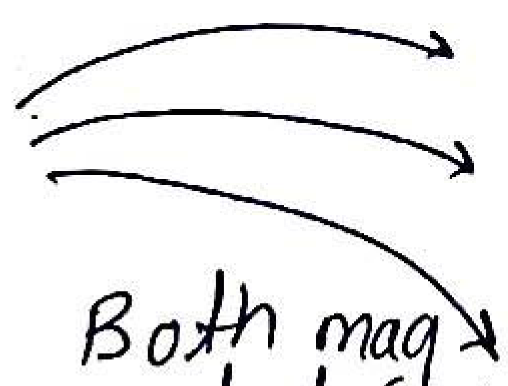


graphically we can say $\vec{E_1} > \vec{E_2}$

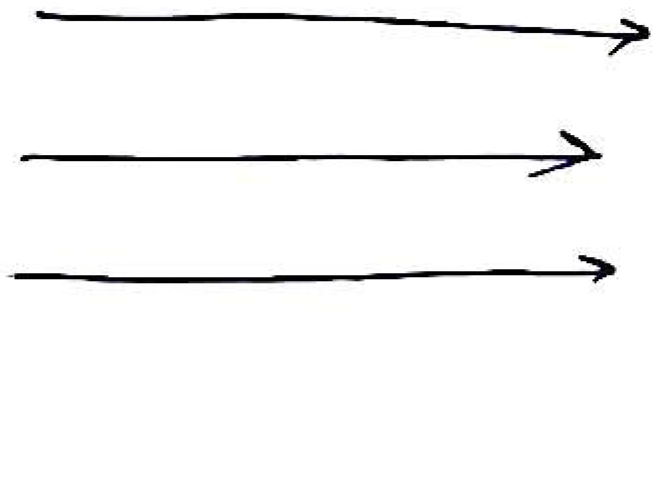
Magnitude of E not constant



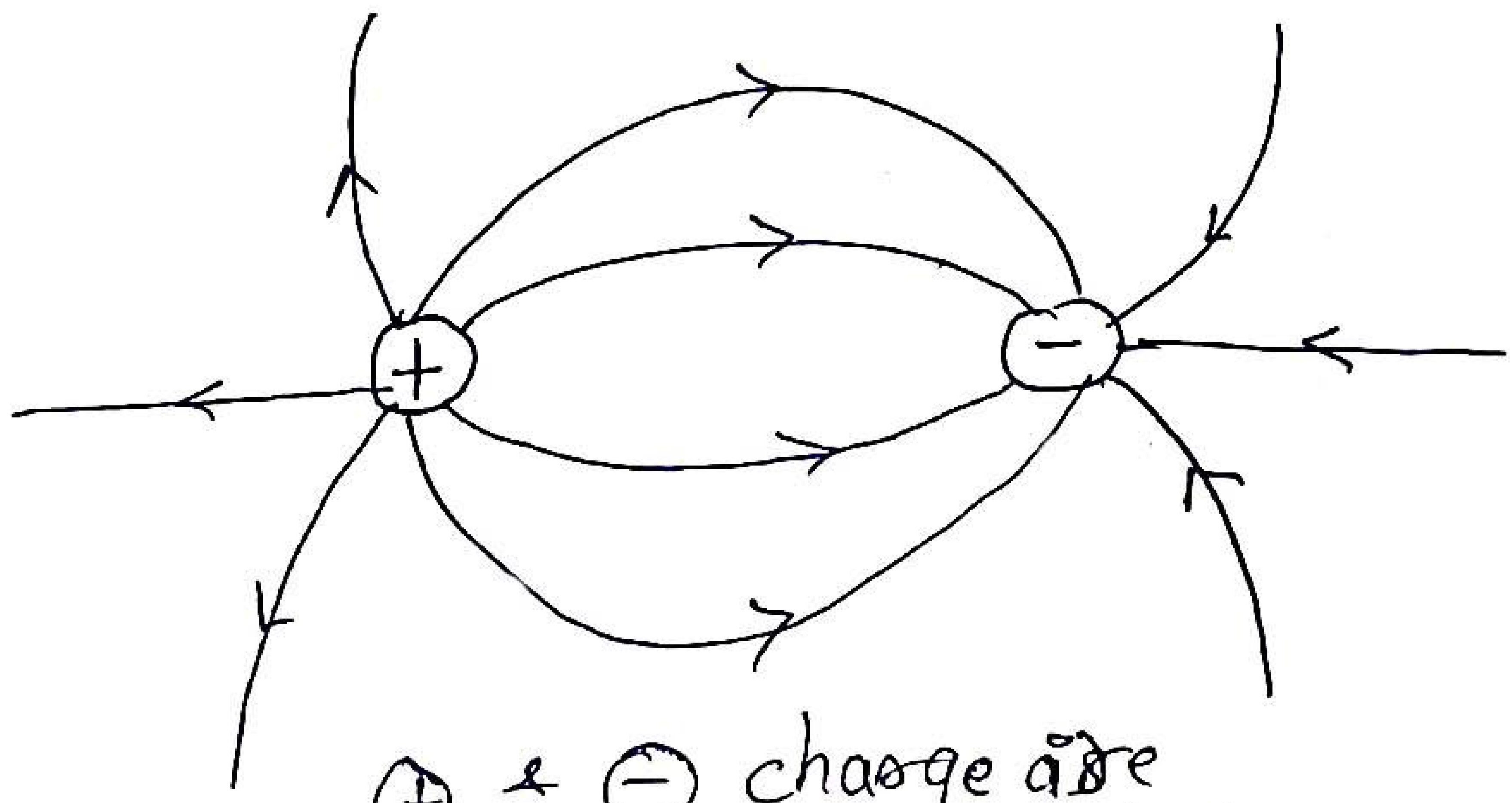
direction not constant



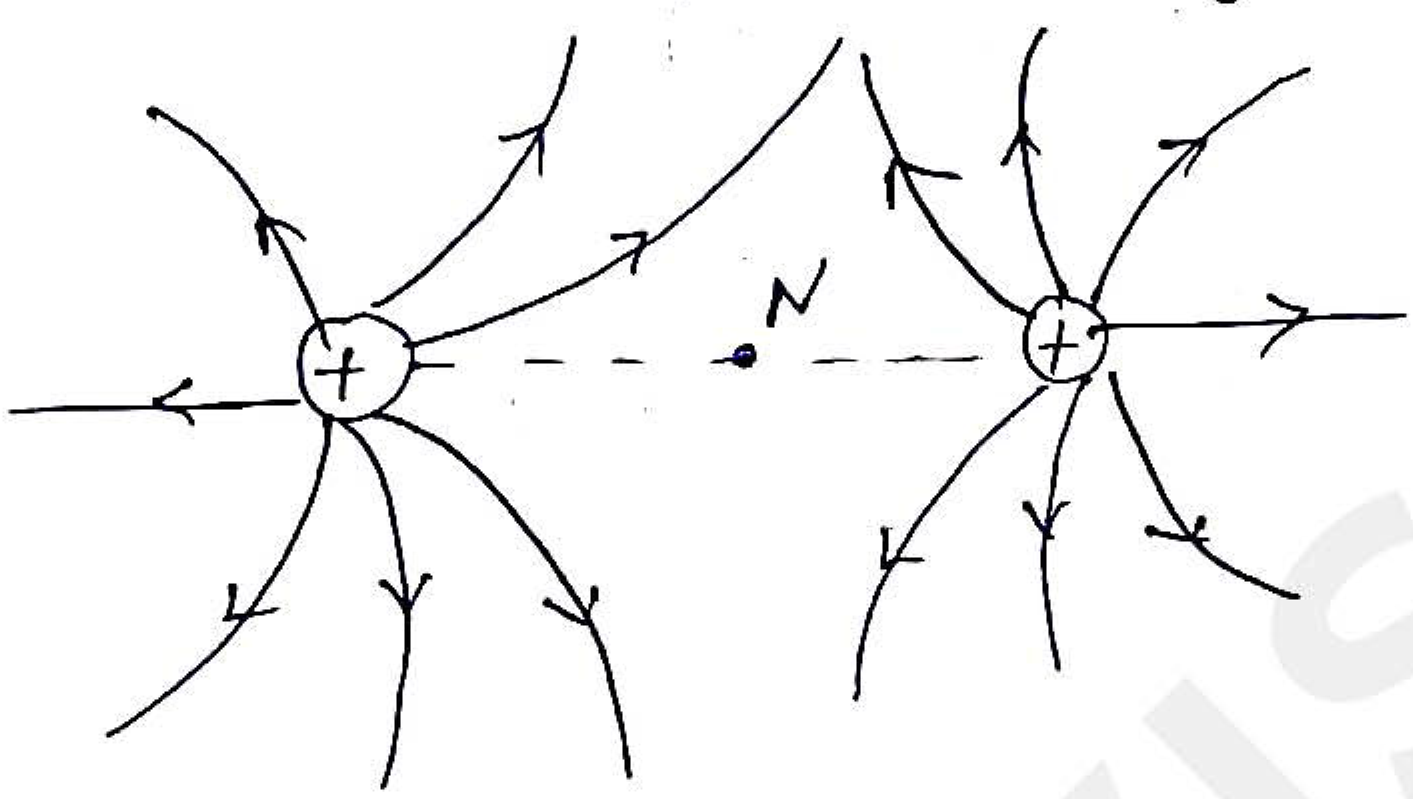
Both mag and direction not constant

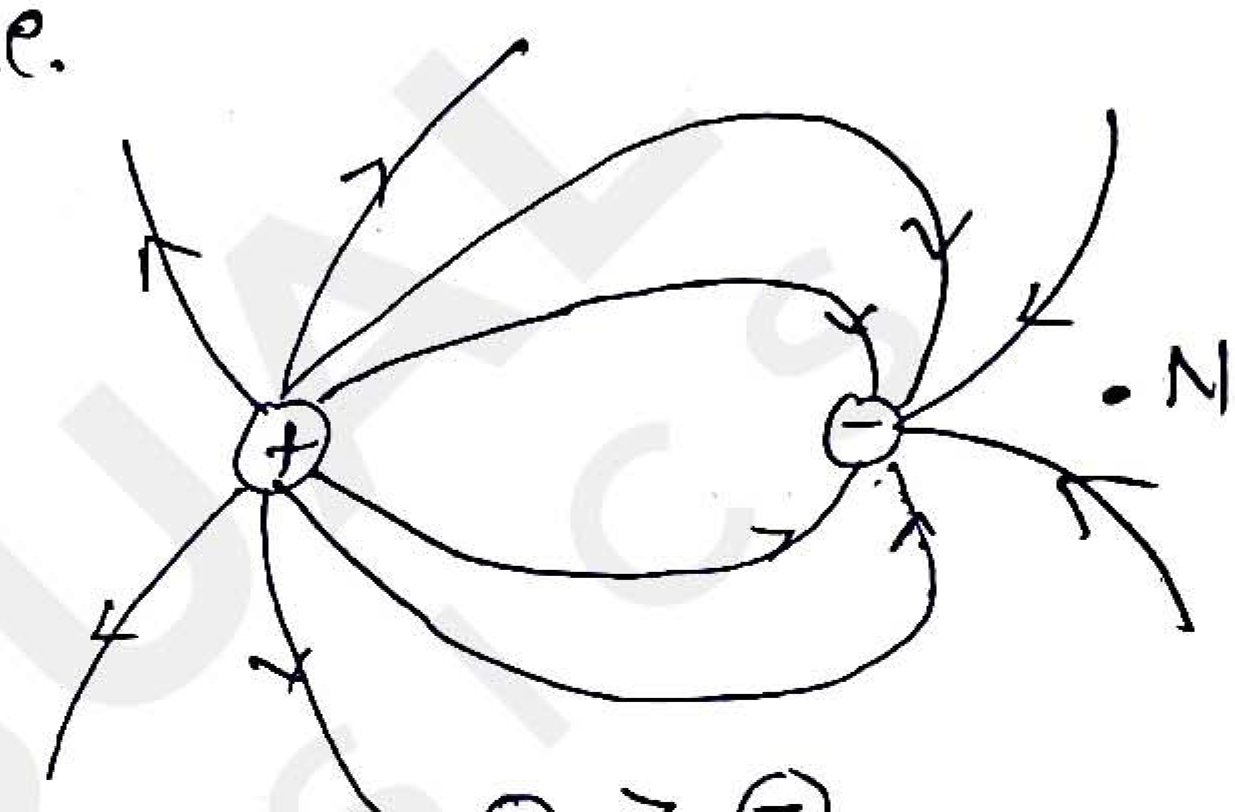


both mag and direction constant.



⊕ & ⊝ charge are equal in magnitude.





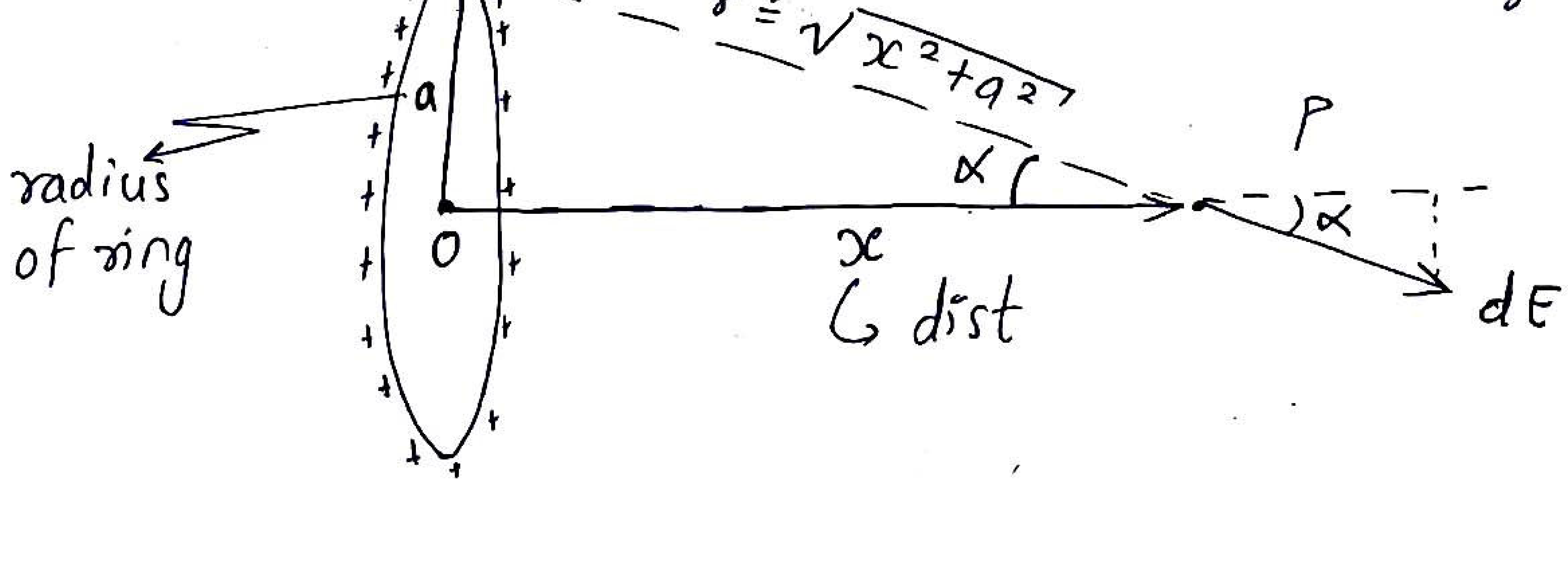
⊕ > ⊝

N → Neutral point in space, where net Electric field is zero

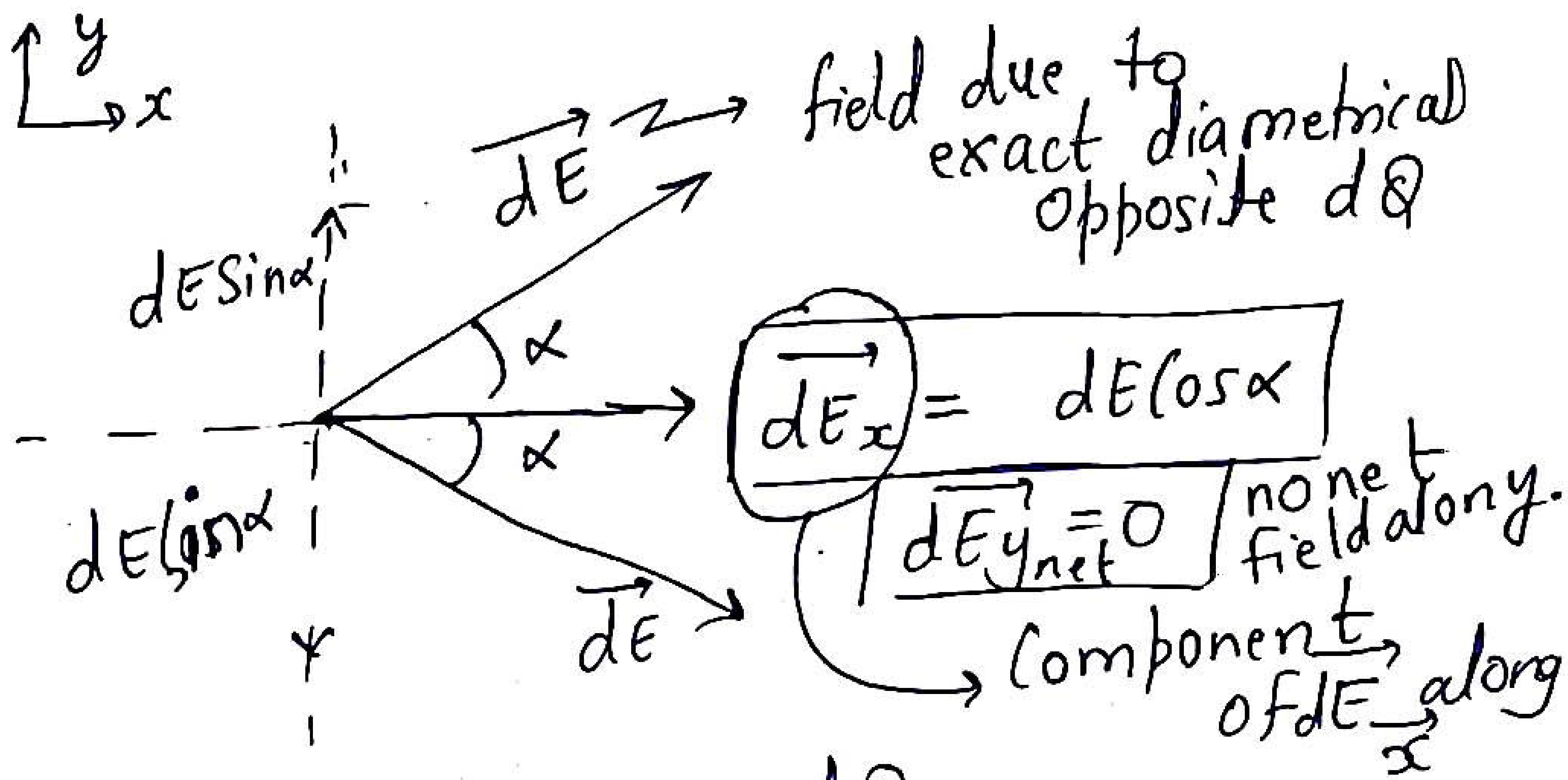
Video: → Electric field, Q8, Q9, Q7 → Theory section
Q7, Q9 → solved

Field of ring charge: (on the axis of ring)

$\cos\alpha = \dfrac{x}{r}$



so ring, the $\vec{E}$ is along the axis only



$$dE = \frac{1}{4\pi\epsilon}\frac{dQ}{(r^2)} = \frac{1}{4\pi\epsilon}\frac{dQ}{(x^2+a^2)}$$

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\epsilon}\frac{x\,dQ}{(x^2+a^2)^{3/2}}$$

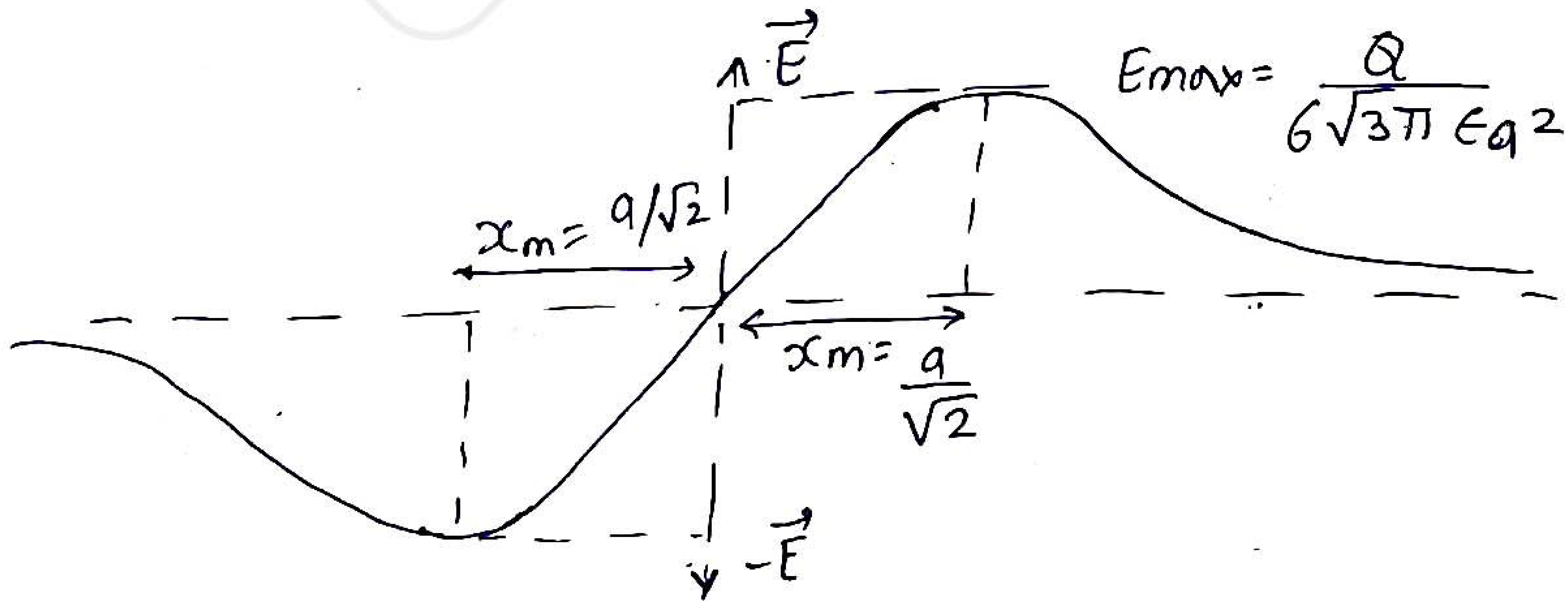
$$E_{net} = \int dE_x = \int \frac{1}{4\pi\epsilon}\frac{x\,dQ}{(x^2+a^2)^{3/2}}$$

$$\boxed{E_{net} = E_x\hat{i} = \frac{1}{4\pi\epsilon}\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i}}$$

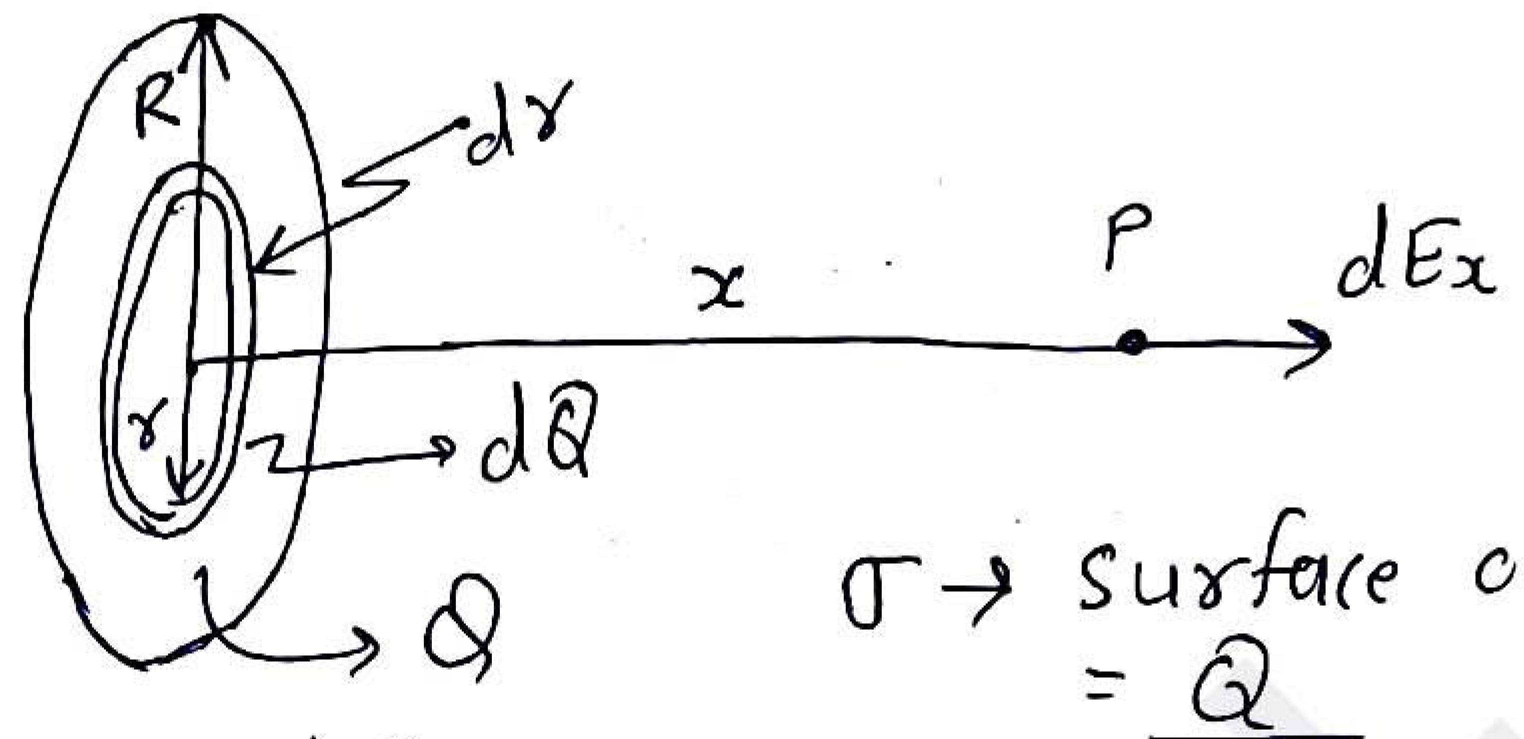
$x \to 0, \quad E_{net} \to 0$

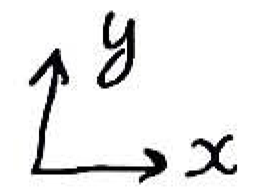
$$E_{max} = \frac{d}{dx}E_{net} = 0$$

$$\Rightarrow \boxed{x_m = \pm\frac{a}{\sqrt{2}}}$$



Field of uniformly charged disk : (along axis)





$\sigma \rightarrow$ surface charge density $= \dfrac{Q}{4\pi R^2}$

$$dQ = \sigma(2\pi r\,dr)$$

$$dE_x = \frac{1}{4\pi\epsilon}\frac{(dQ)\,x}{(x^2+r^2)^{3/2}} = \frac{1}{4\pi\epsilon}\frac{(2\pi r\sigma)\,x\,dr}{(x^2+r^2)^{3/2}}$$

$$E_x = \int dE_x = \int_0^R \frac{1}{4\pi\epsilon}\frac{(2\pi\sigma r\,dr)\,x}{(x^2+r^2)^{3/2}}$$

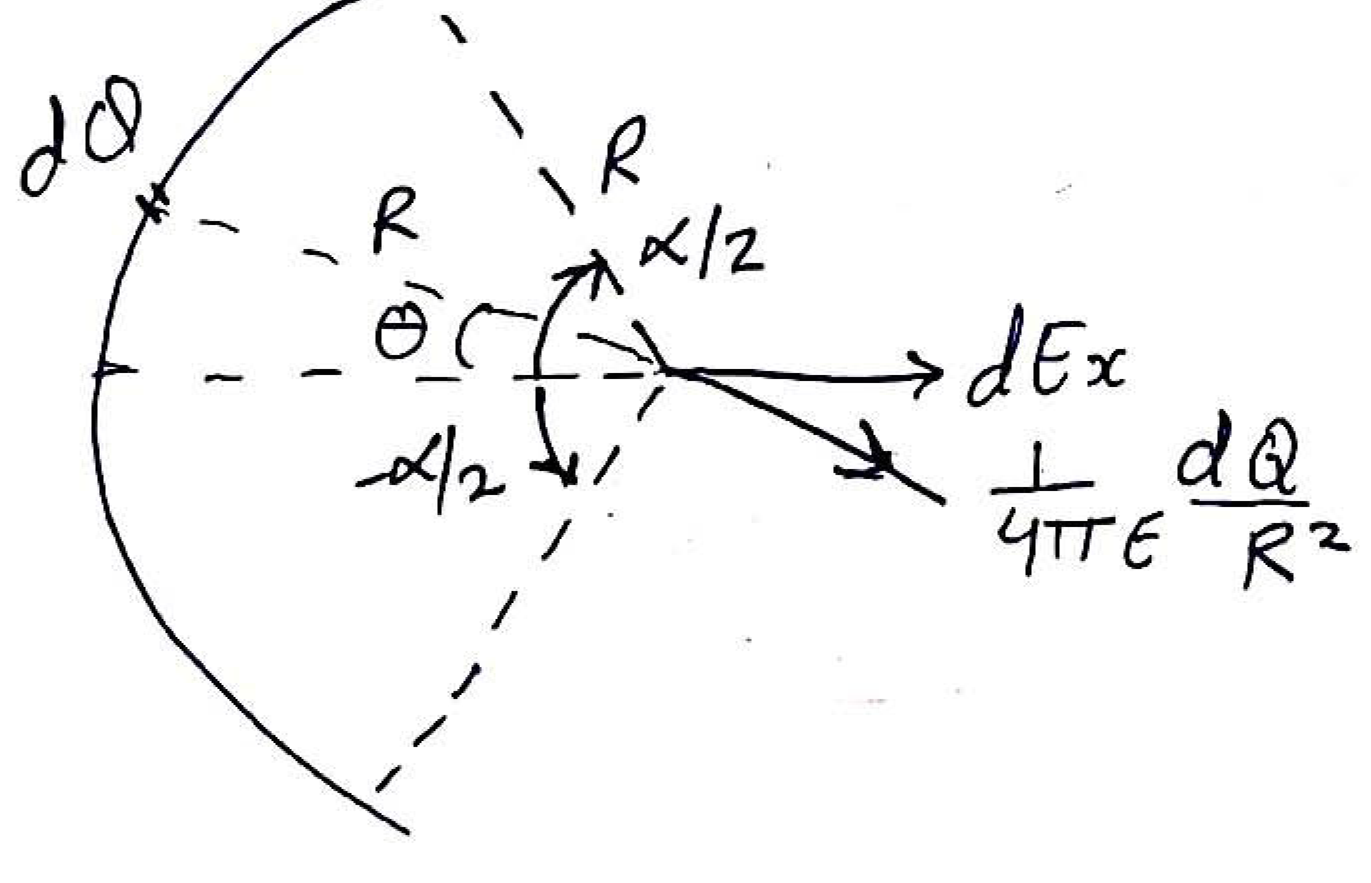
$$\boxed{E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

Net field along x-axis

$\cancel{x \ll R}$ $\quad R \gg x \longrightarrow \boxed{E_x \rightarrow \dfrac{\sigma}{2\epsilon}}$

$x \gg R \quad \boxed{E_x \longrightarrow 0}$

Electric field due to arc:



$\lambda \rightarrow$ linear charge density

$$\lambda = \frac{Q}{R\alpha}$$

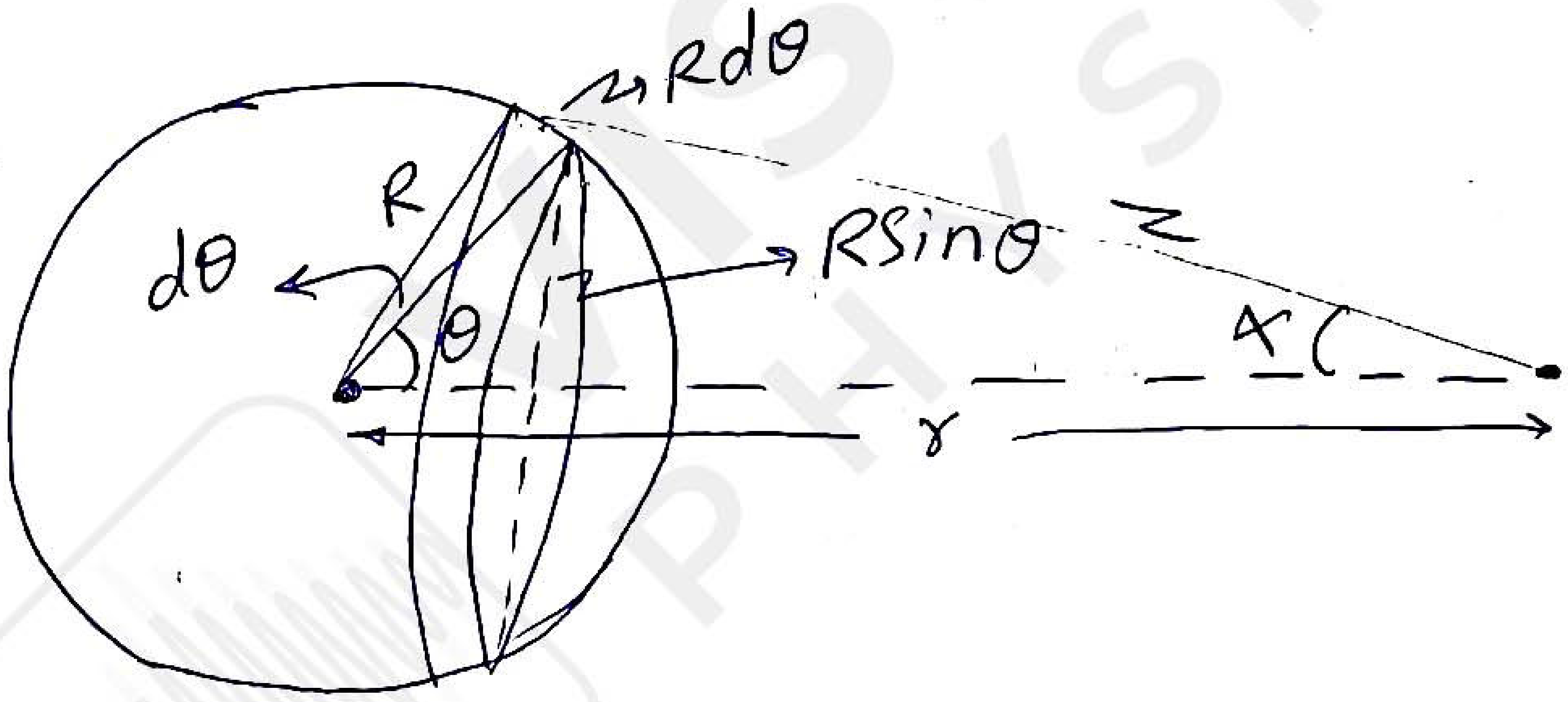
$$\boxed{dE_x = \frac{\cos\theta\,\lambda R\,d\theta}{4\pi\epsilon\,R^2}}$$

$$E_{net} = \int_{-\alpha/2}^{\alpha/2} \frac{1}{4\pi\epsilon} \frac{\lambda R d\theta}{R^2} \cos\theta$$

$$E_{net} = \frac{1}{4\pi\epsilon} \frac{\lambda}{R} \int_{-\alpha/2}^{\alpha/2} \cos\theta \, d\theta$$

$$\boxed{E_{net} = \frac{k\lambda}{R} 2\sin\left(\alpha/2\right)}$$

$$\boxed{\vec{E_{net}} = \frac{k\lambda}{R} 2\sin\left(\alpha/2\right)\hat{i}}$$



$$dQ = (2\pi R\sin\theta)(R d\theta)\frac{Q}{4\pi R^2} = \frac{Q}{2}\sin\theta \, d\theta$$

$$dE = \frac{1}{4\pi\epsilon} \frac{dQ}{z^2} \cos\alpha$$

$$z^2 = R^2 + r^2 - 2Rr\cos\theta$$

$$z\,dz = Rr\sin\theta \, d\theta$$

$$\cos\alpha = \frac{z^2 + r^2 - R^2}{2zr}$$

$$dE = \frac{\frac{1}{4\pi\epsilon}\frac{Q}{2}\frac{z\,dz}{R\gamma}}{z^2}\qquad \frac{z^2+\gamma^2-R^2}{2z\gamma}$$

$$\boxed{k \to \frac{1}{4\pi\epsilon}}$$

$$dE = \frac{kQ}{4R\gamma^2}\left[1 - \frac{(R^2-\gamma^2)}{z^2}\right]dz$$

$$E = \int dE = \int_{\gamma-R}^{\gamma+R} \frac{kQ}{4R\gamma^2}\left[1 - \frac{(R^2-\gamma^2)}{z^2}\right]dz$$

$$= \frac{kQ}{4R\gamma^2}\left[z - (R^2-\gamma^2)\left(\frac{-1}{z}\right)\right]_{\gamma-R}^{\gamma+R}$$

$$E = \frac{kQ}{4R\gamma^2}\left[z + \frac{R^2-\gamma^2}{z}\right]_{\gamma-R}^{\gamma+R}$$

$$E = \frac{kQ}{4R\gamma^2}\left[\gamma+R + \frac{(R+\gamma)(R-\gamma)}{\gamma+R} - \left(\gamma-R + \frac{(R+\gamma)(R-\gamma)}{\gamma-R}\right)\right]$$

$$E = \frac{kQ}{4R\gamma^2}\left[2R - (-2R)\right]$$

$$\boxed{E_{shell} = \frac{kQ}{\gamma^2}}$$

when $\gamma > R$

$$\boxed{E_{shell} = \frac{1}{4\pi\epsilon}\frac{Q}{\gamma^2}}$$

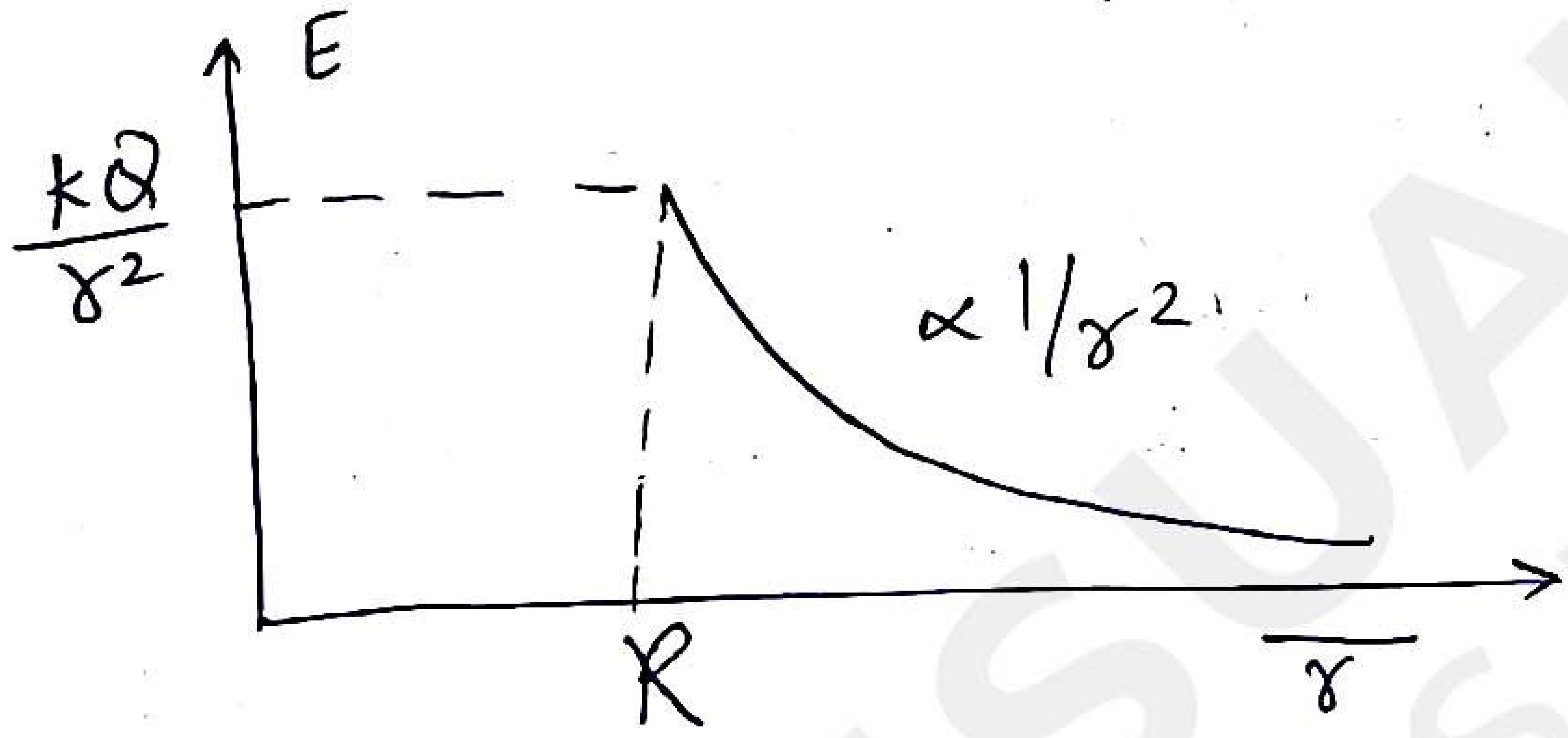
~~sh~~ same as if a Q is placed at centre.

When point is inside

$(R-r) \rightarrow (R+r)$

on solving $E \rightarrow 0$

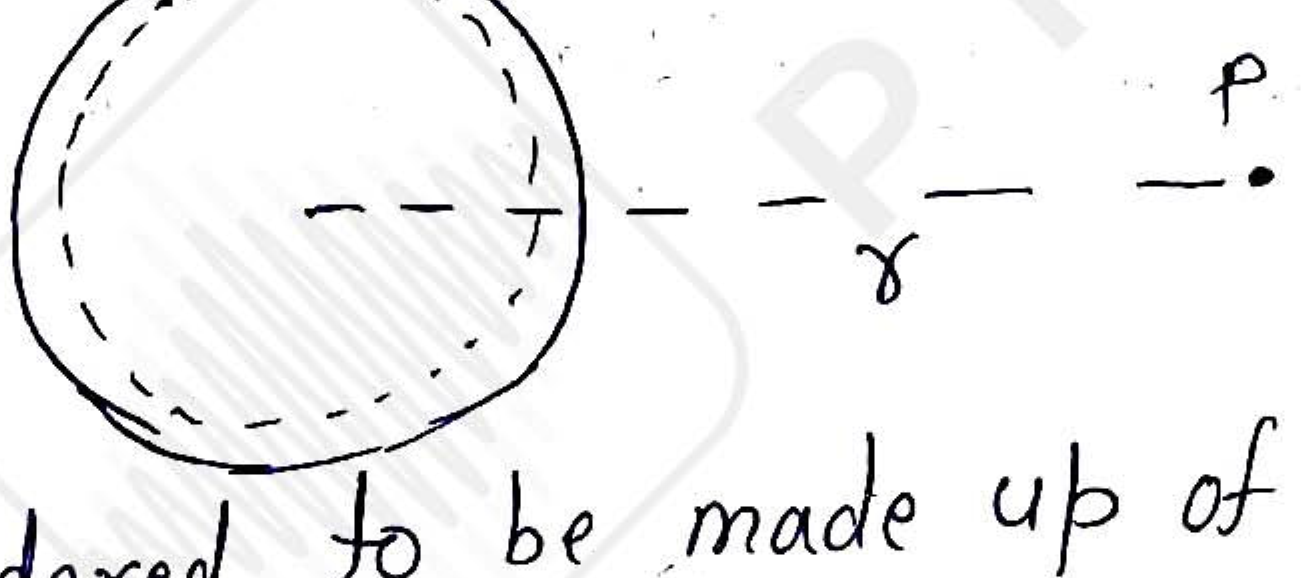
$\Rightarrow$ field inside a shell $= 0$



Field due to sphere: $\rightarrow$ solid

$\rho \rightarrow$ volume charge density

$Q \rightarrow$ Total charge.



Considered to be made up of infinite shells, and as shell of dQ charge

$$dE = \frac{k\,dQ}{r^2} = \frac{1}{4\pi\epsilon}\frac{dQ}{r^2}$$

so $E_{net} = \int dE = \frac{1}{4\pi\epsilon}\int\frac{dQ}{r^2}$

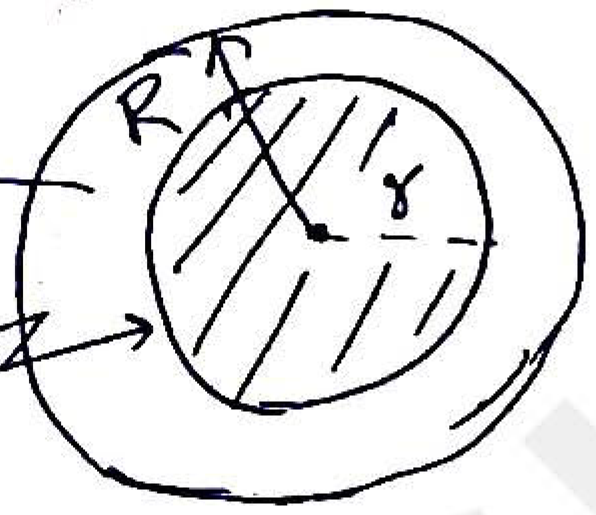
$$\boxed{E_{net} = \frac{Q}{4\pi\epsilon r^2}}$$

now $E = \frac{kQ}{r^2} \rightarrow r \geq R$

$E \neq \frac{kQ}{r^2} \rightarrow r < R$

So when point is inside, $Q' = \rho V'$

neglected



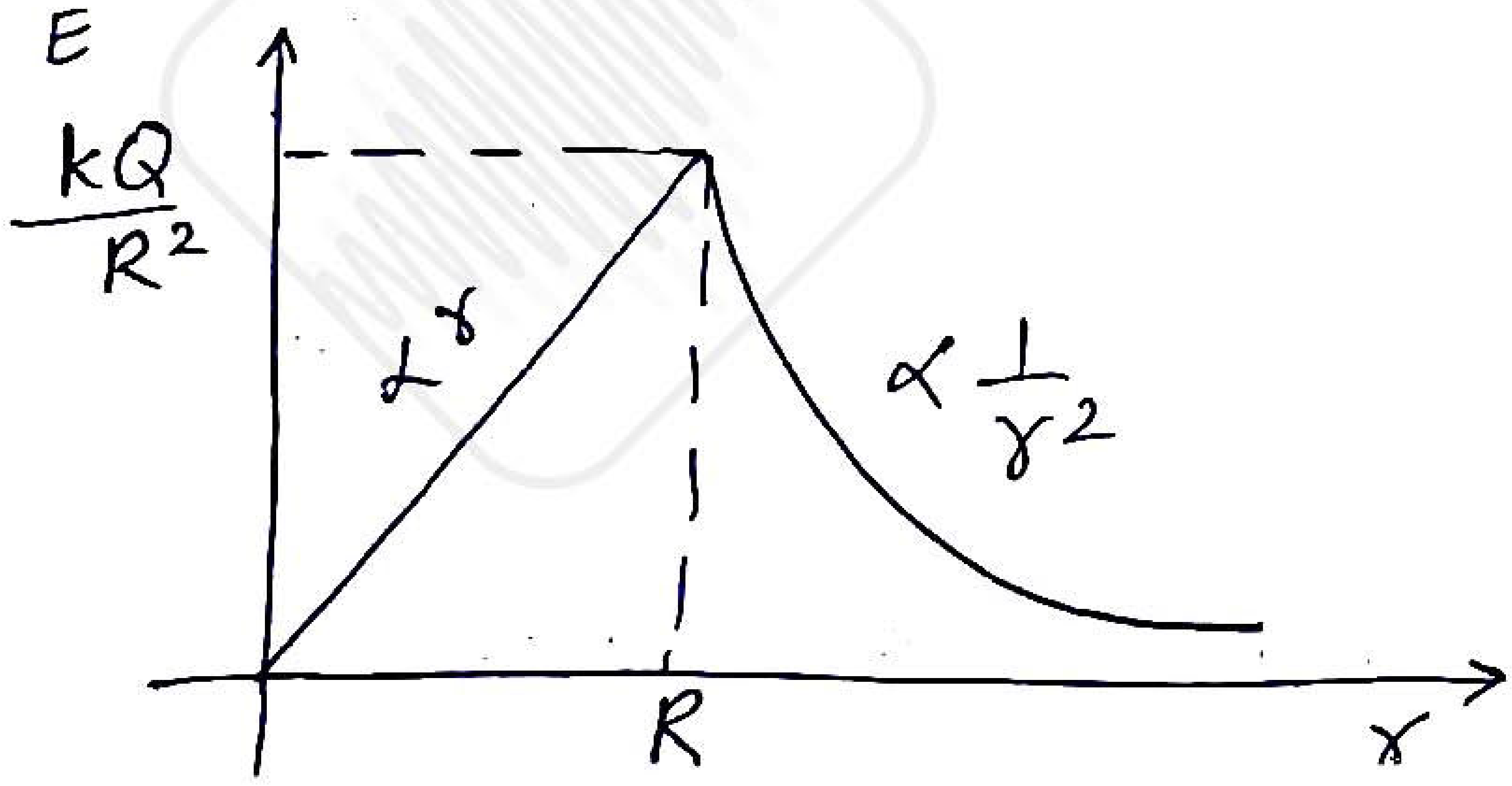
Consider only charge on sphere of radius r.

$$Q' = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

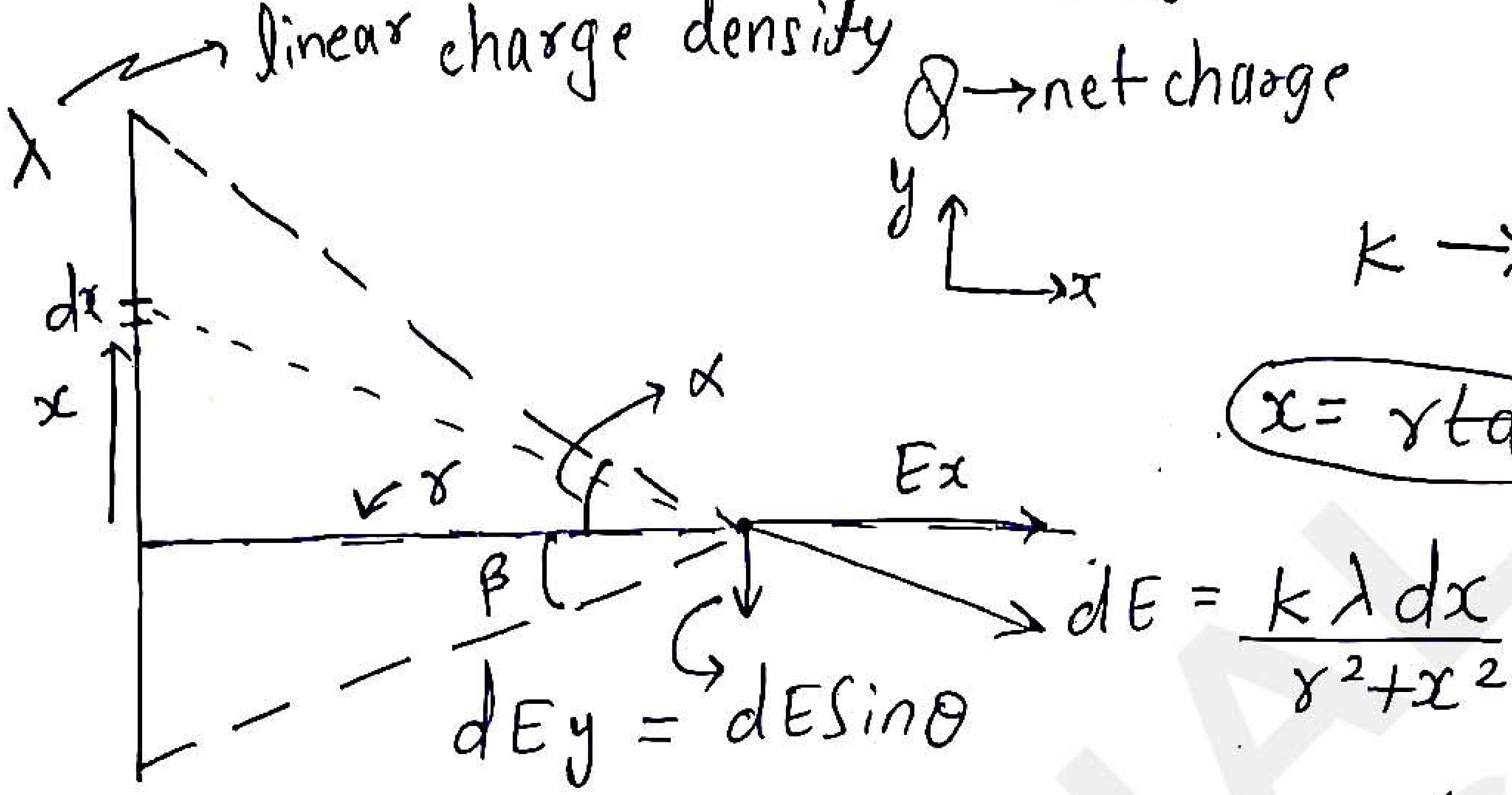
$$Q' = \frac{Q r^3}{R^3}$$

So,
$$\boxed{E_r = \frac{1}{4\pi\epsilon}\left(\frac{Q r^3}{R^3}\right) \times \frac{1}{r^2}}$$

$$\boxed{E_r = \frac{kQ}{R^3} r \text{ or } \frac{1}{4\pi\epsilon}\frac{Q r}{R^3}}$$



Field due to a line of charge



$Q \rightarrow$ net charge

$k \rightarrow \frac{1}{4\pi\epsilon}$

$x = r\tan\theta$

$dE = \frac{k\lambda\, dx}{r^2+x^2}$

$dE_y = dE\sin\theta$

$dx = r\sec^2\theta\, d\theta$

$$E_y = \int \frac{\lambda k}{r^2+x^2}\sin\theta\, dx = \frac{k\lambda}{r}\int_{-\beta}^{\alpha} \frac{\sec^2\theta \sin\theta\, d\theta}{1+\tan^2\theta}$$

$x = r\tan\theta$

$$\boxed{E_y = \frac{k\lambda}{r}\left[\cos\beta - \cos\alpha\right]}$$

$$E_x = \int dE_x = \int dE\cos\theta = \frac{k\lambda}{r}\int_{-\beta}^{\alpha} \frac{\sec^2\theta\cos\theta\, d\theta}{1+\tan^2\theta}$$

$$\boxed{E_x = \frac{k\lambda}{r}\left[\sin\alpha + \sin\beta\right]}$$

if line charge is infinitly long

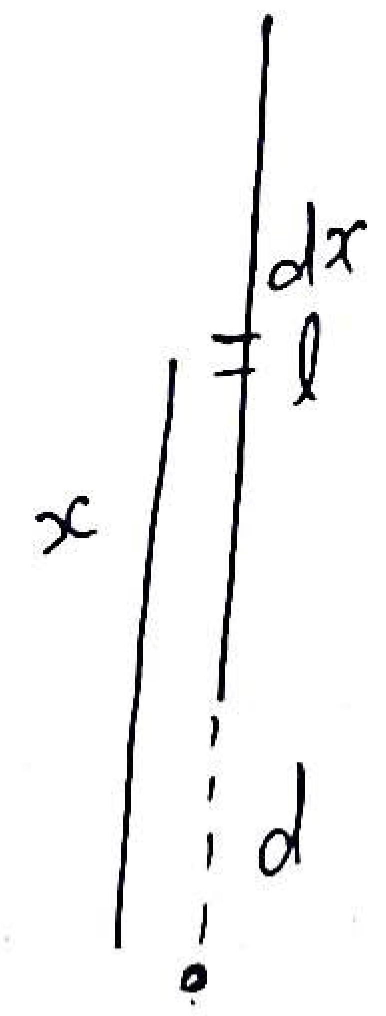
$\alpha \rightarrow 90, -\beta \rightarrow -90$

$\Rightarrow \beta = 90$

$$\boxed{E_y \rightarrow 0}, \quad \boxed{E_x = \frac{2k\lambda}{r}}$$

When, $\alpha \to 90, \beta \to 0$

$$\boxed{E_y = \frac{k\lambda}{r}, \quad E_x = \frac{k\lambda}{r}}$$



$$dE = k\lambda \frac{dx}{x^2}$$

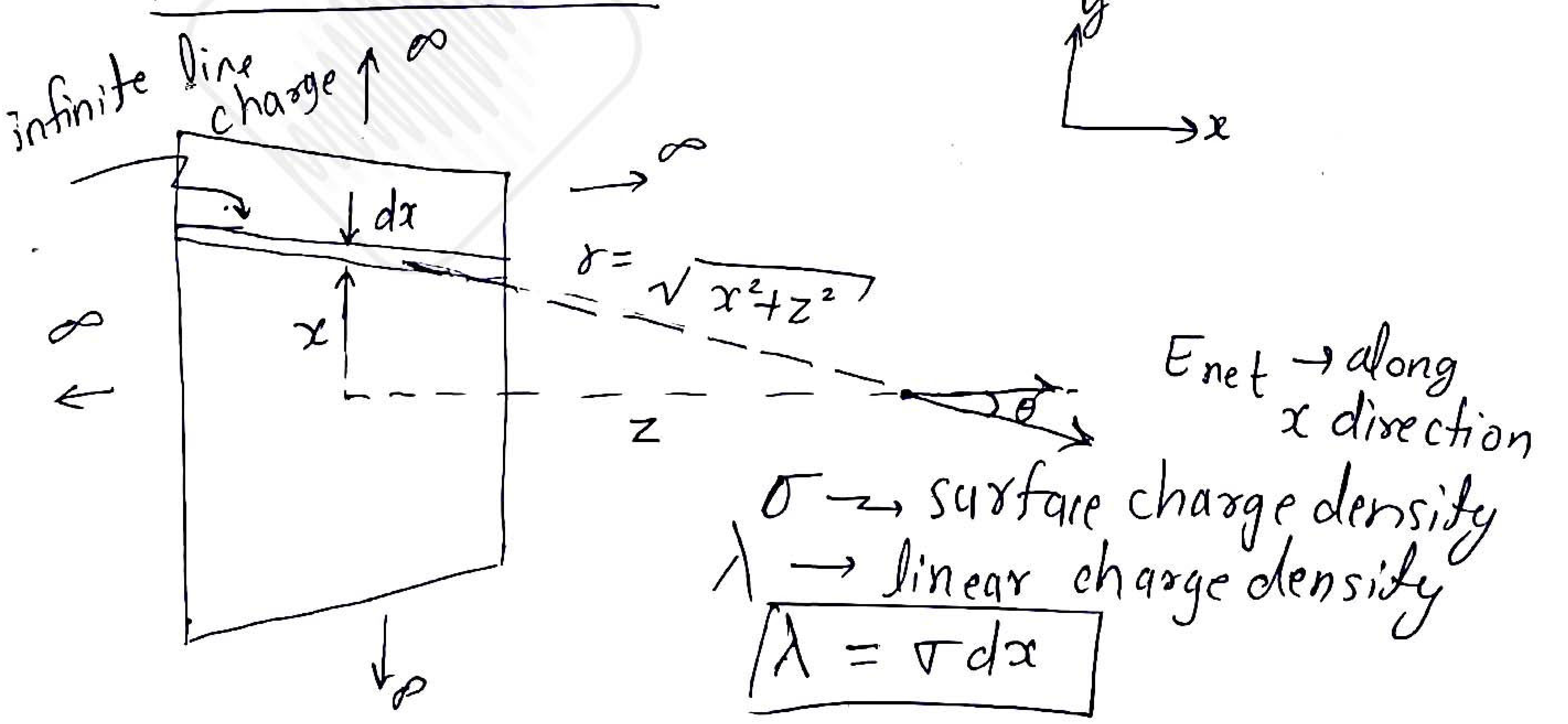
$$E_y = \int_d^{d+l} \frac{k\lambda}{x^2}\, dx$$

$$\boxed{E_y = k\lambda \frac{l}{d(d+l)}}$$

When $l >> d$ $\rightarrow$ $E_y = k\lambda \frac{\cancel{l}}{d(\cancel{l})}$

$$\boxed{E_y = \frac{k\lambda}{d}}$$

Field due to sheet:



$\sigma \to$ surface charge density

$\lambda \to$ linear charge density

$$\boxed{\lambda = \sigma\, dx}$$

"E due to infinite sheet is constant does not depends on distance from sheet and position."

So, $dE = \frac{2k\lambda}{r} = \frac{2k\sigma dx}{\sqrt{x^2+z^2}}$

$dE_x = dE\cos\theta = \frac{2k\sigma dx}{\sqrt{x^2+z^2}} \frac{z}{\sqrt{x^2+z^2}}$

$E_{net} = \int dE_x = 2k\sigma z \int_{-\infty}^{\infty} \frac{dx}{x^2+z^2}$

$E_{net} = 2k\sigma z \left[\frac{1}{z}\tan^{-1}\left(\frac{x}{z}\right)\right]_{-\infty}^{\infty}$

$E_{net} = 2k\sigma \left[\tan^{-1}\left(\frac{\infty}{z}\right) - \tan^{-1}\left(\frac{-\infty}{z}\right)\right]$

$= 2k\sigma\left(\pi/2 + \pi/2\right)$

$= \frac{2}{4\pi\epsilon}\sigma\pi$

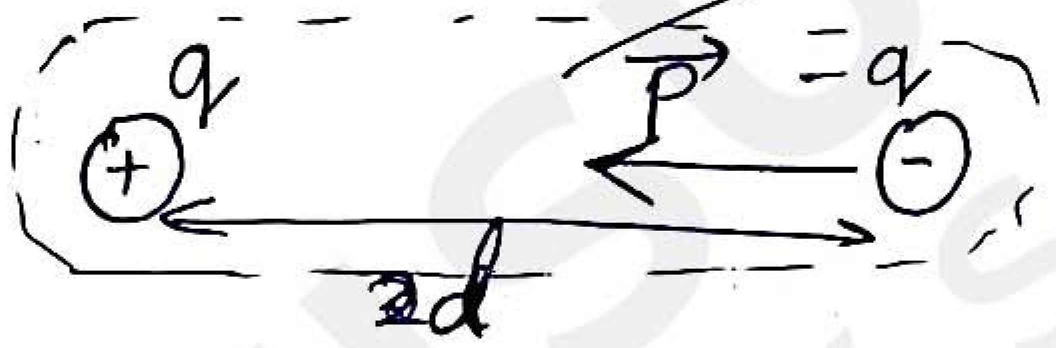
$\boxed{E_{net} = \frac{\sigma}{2\epsilon}}$

Electric Dipole :

Two equal and opposite point charge placed at small distance

electric dipole.

net charge
$= 0$
$= q, -q$

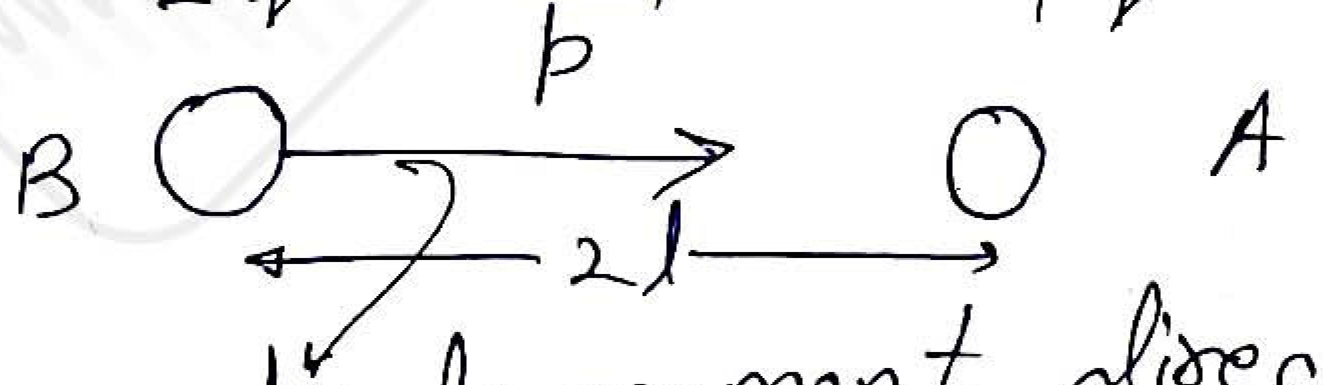


$\vec{P} \rightarrow$ dipole moment $\Rightarrow$ $|\vec{P}| = \cancel{2} qd$

charge × distance

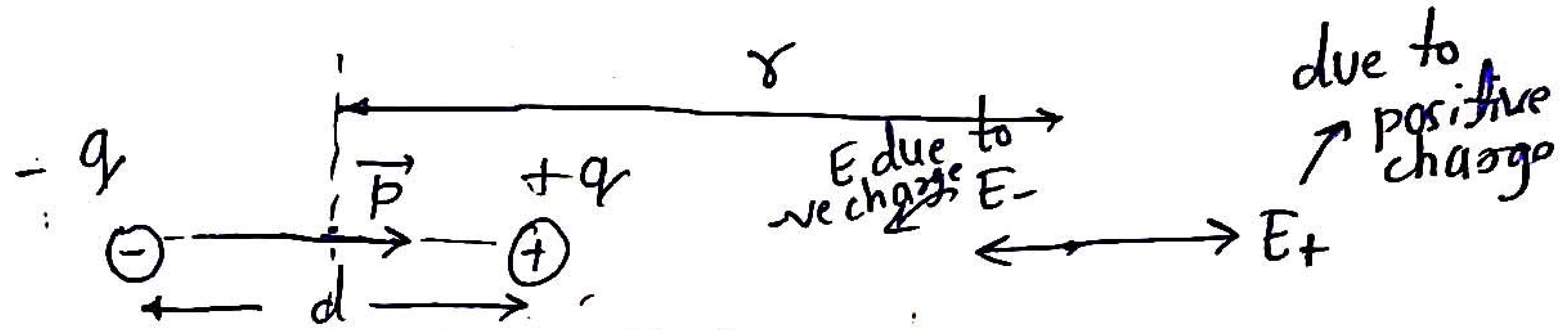
points from −ve to +ve

$\Rightarrow$ ~~direction of $\vec{p}$ is~~ opposite ~~to direction~~



dipole moment direction

Electric field at axis of dipole:



$E_{net} = E_+ - E_-$

$$\boxed{K = \frac{1}{4\pi\epsilon}}$$

$$\Rightarrow E_{net} = \frac{kq}{(r-d/2)^2} - \frac{kq}{(r+d/2)^2}$$

$$E_{net} = kq\left[\frac{4r(d/2)}{[r^2-(d/2)^2]^2}\right]$$

$r >> d/2$

$$E_{net} = kq\left[\frac{4r(d/2)}{r^4}\right] \to E_{net} =$$

$r^4 \longrightarrow (r^2 - \cancel{0})^2$ as $d/2 << r$

$$\Rightarrow E_{net} = \frac{2kqd}{r^3}$$

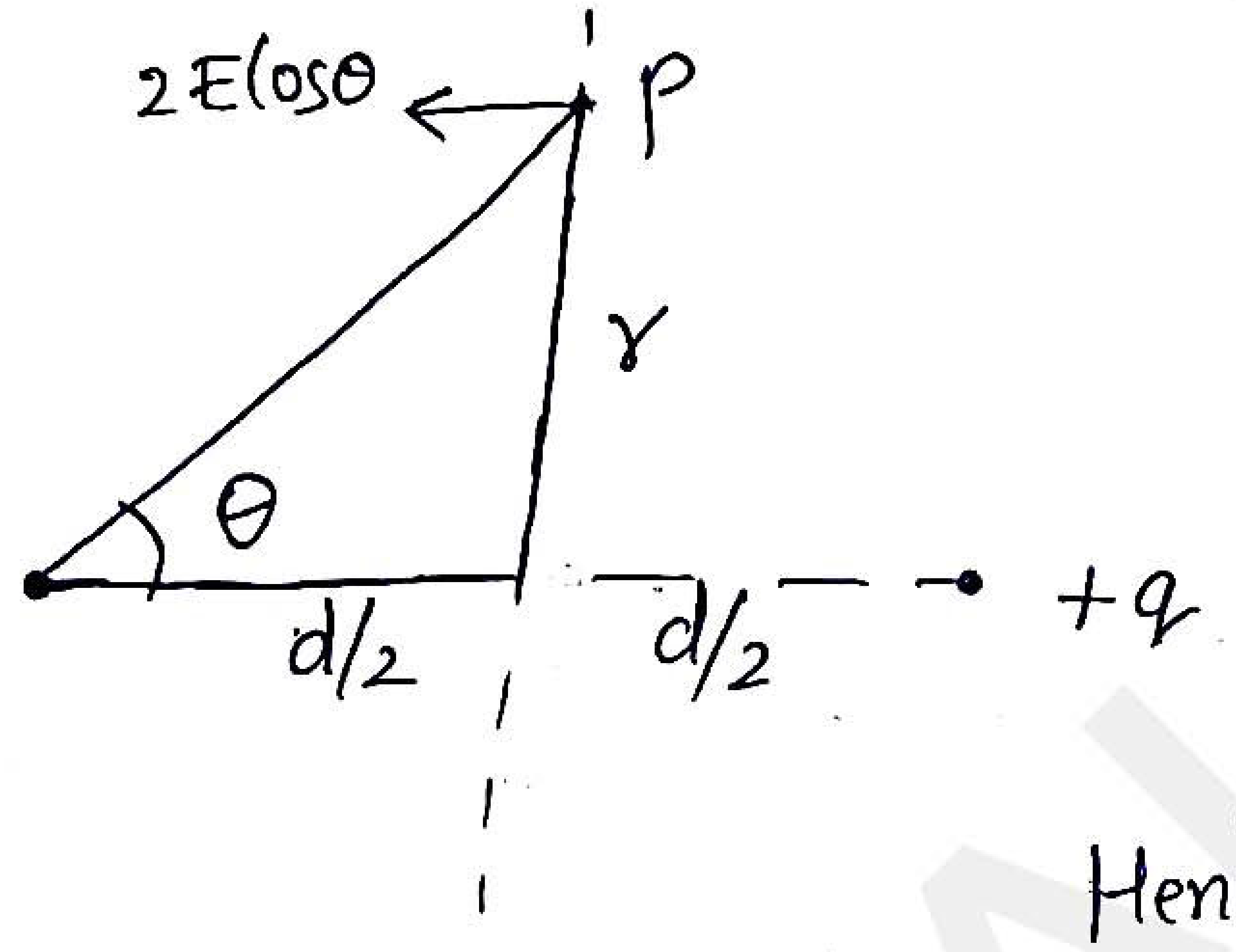
$qd \to$ dipole moment

$$E_{net} = \frac{1}{4\pi\epsilon}\frac{2p}{r^3}$$

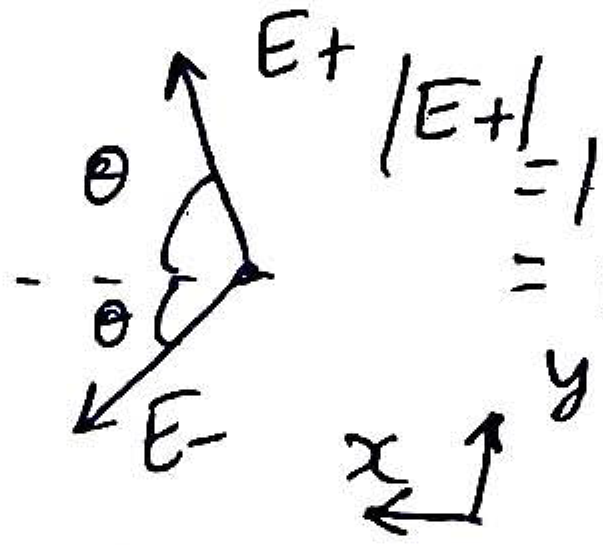
as $\vec{p}$ points in $\vec{E}$ directio

$$\Rightarrow \boxed{\vec{E}_{net} = \frac{1}{4\pi\epsilon}\frac{2\vec{p}}{r^3}}$$

Electric field at equitorial line by dipole



$$\boxed{K = \frac{1}{4\pi\epsilon}}$$



$|E_+| = |E_-| = E$

So, $\boxed{E_y = E\sin\theta - E\sin\theta = 0}$

Hence field only in x direction

now

$$E_{net} = 2E\cos\theta$$

$$= 2\left[\frac{kq}{[r^2 + (d/2)^2]}\right]\left[\frac{d/2}{\sqrt{r^2 + (d/2)^2}}\right]$$

$$E_{net} = \frac{2kq}{[r^2 + (d/2)^2]} \frac{d/2}{\sqrt{r^2 + (d/2)^2}}$$

$$\boxed{E_{net} = \frac{k\underbrace{qd}_{p}}{[r^2 + (d/2)^2]^{3/2}}}$$

when $d << r$

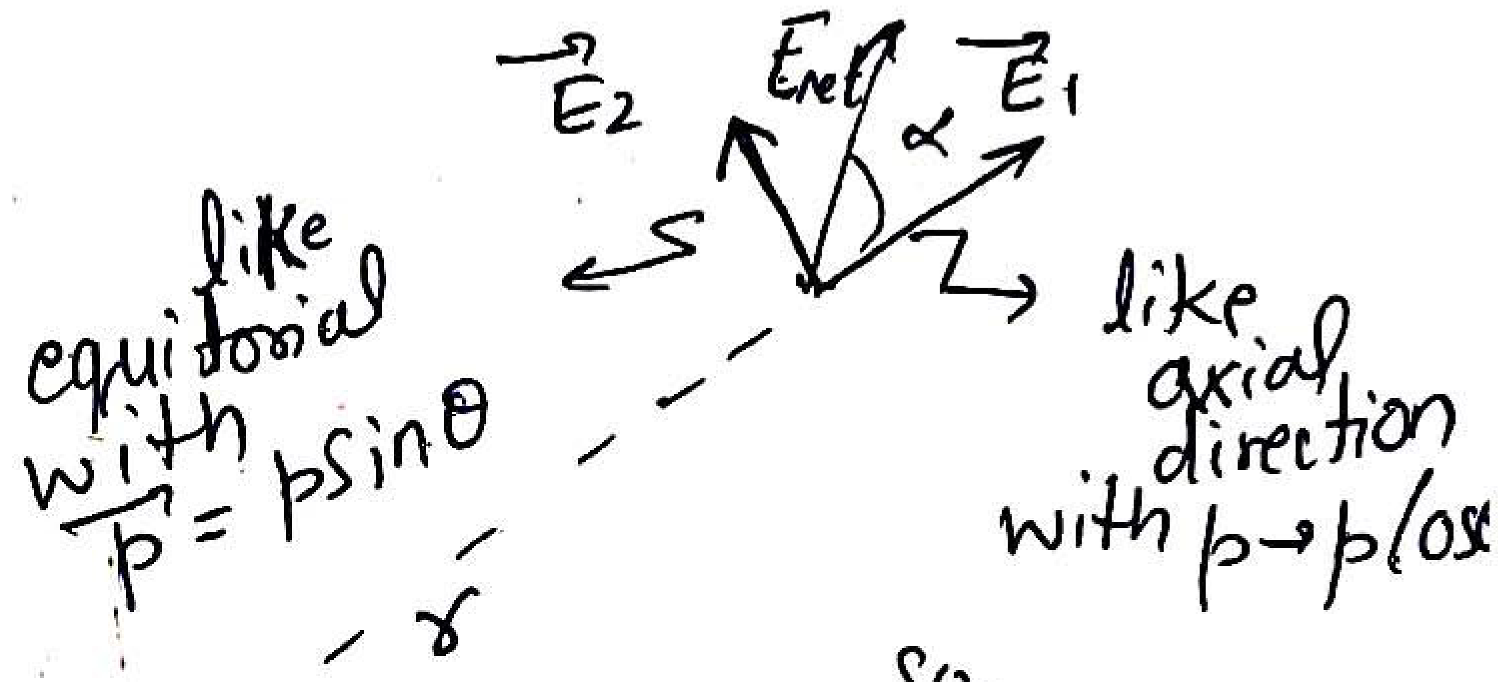
$$\Rightarrow \quad E_{net} = \frac{1}{4\pi\epsilon} \frac{p}{(r^2)^{3/2}} = \frac{p}{4\pi\epsilon r^3}$$

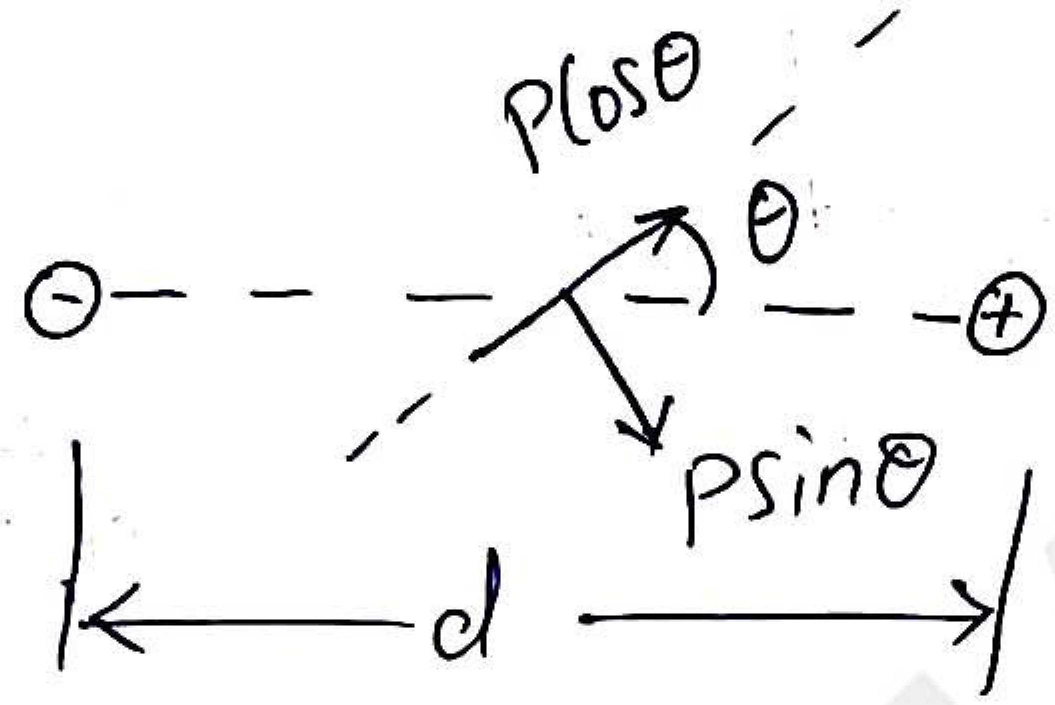
now as E points opposite to dipole moment

So, $$\boxed{\vec{E}_{net} = \frac{-\vec{p}}{4\pi\epsilon r^3}}$$

General Expression

$r >> d$





So,

$$\vec{E_1} = \frac{1}{4\pi\epsilon}\frac{2(P\cos\theta)}{r^3}$$

$$\vec{E_2} = -\frac{1}{4\pi\epsilon}\frac{P\sin\theta}{r^3}$$

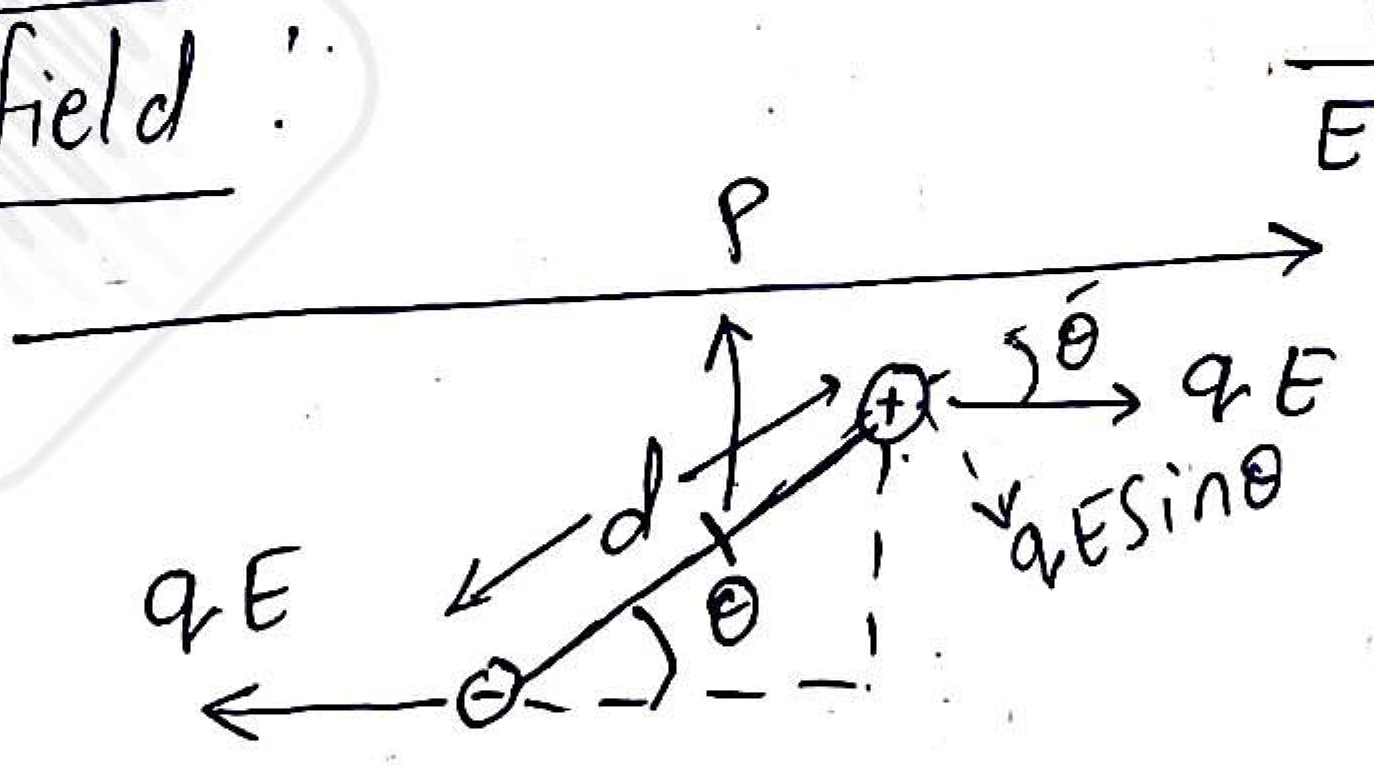
$$|\vec{E_{net}}| = \sqrt{(\vec{E_1})^2 + (\vec{E_2})^2}$$

$$|\vec{E_{net}}| = \sqrt{\left(\frac{2(P\cos\theta)}{4\pi\epsilon r^3}\right)^2 + \left(\frac{P\sin\theta}{4\pi\epsilon r^3}\right)^2}$$

$$\boxed{|\vec{E_{net}}| = \frac{P}{4\pi\epsilon r^3}\sqrt{1+3\cos^2\theta}}$$

Dipole in External field:

Unifor field:



$\boxed{F_{net} = 0}$

$$\text{net torque about } P = (qE\sin\theta)(d/2) + (qE\sin\theta)(d/2)$$

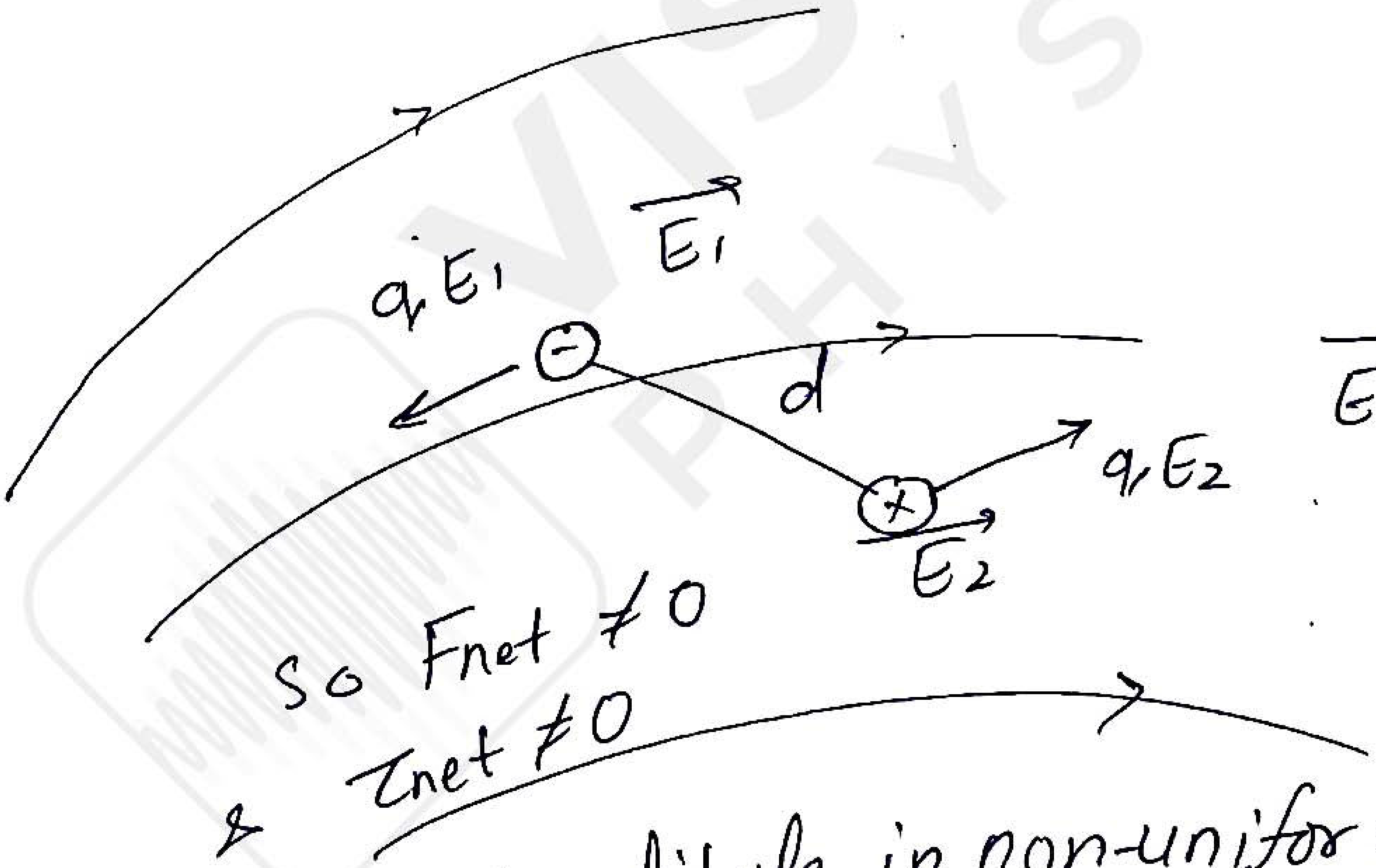
$$\boxed{\tau = qEd\sin\theta}$$

or $\boxed{\tau = pE\sin\theta}$ $\quad p = qd$

or $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

if $\theta \to 0$, $\tau \to 0$ $\longrightarrow$ stable Equilibrium
↳ as Torque is restoring

if $\theta \to 180$ $\tau \to 0$ unstable equilibrium
↳ as Torque is not restoring



$\vec{E} \neq$ Constant

so $F_{net} \neq 0$
& $\tau_{net} \neq 0$

so dipole in non-uniform field
$\longrightarrow$ $F_{net} \neq 0$
& also $\tau_{net} \neq 0$