



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Elasticity



+91 999021287



www.visualphysics.in



GET IT ON

Google Play

ELASTICITY

Elasticity → property by virtue of which it regains its original configuration, when external deforming force is removed.

Plasticity → By virtue of which it does not regain its original configuration when external force is removed.

★ IF a force produces a change in normal positioning of the molecules, hence changing configuration of body in length, volume etc. That force is called as deforming Force

Perfectly elastic body → Body that regain original configuration immediately & completely after removal of deforming force. e.g. Quartz & phosphor bronze (nearly perfectly elastic)

Perfectly plastic body:
→ A Body which does not regain its original configuration at all after deforming force is removed. e.g. putty, mud (nearly perfectly plastic)

★ Nothing is perfectly elastic or perfectly plastic

↳ only degree of elasticity or plasticity differs from body to body.

Stress:

Deformation $\xrightarrow{\text{produce}}$ Internal restoring forces

Stress $\leftarrow \frac{\text{(Internal restoring Forces)}}{\text{(Unit Area)}}$

$$\Rightarrow \boxed{\text{Stress} = \frac{\text{Restoring Force}}{\text{Area}}}$$

★ if no permanent deformation or absence of plastic behaviour

↳ $\boxed{\text{Restoring Force} = \text{External deforming Force}}$

Hence at that case,

$$\boxed{\text{Stress} = \frac{\text{External deforming Force}}{\text{Area}}}$$

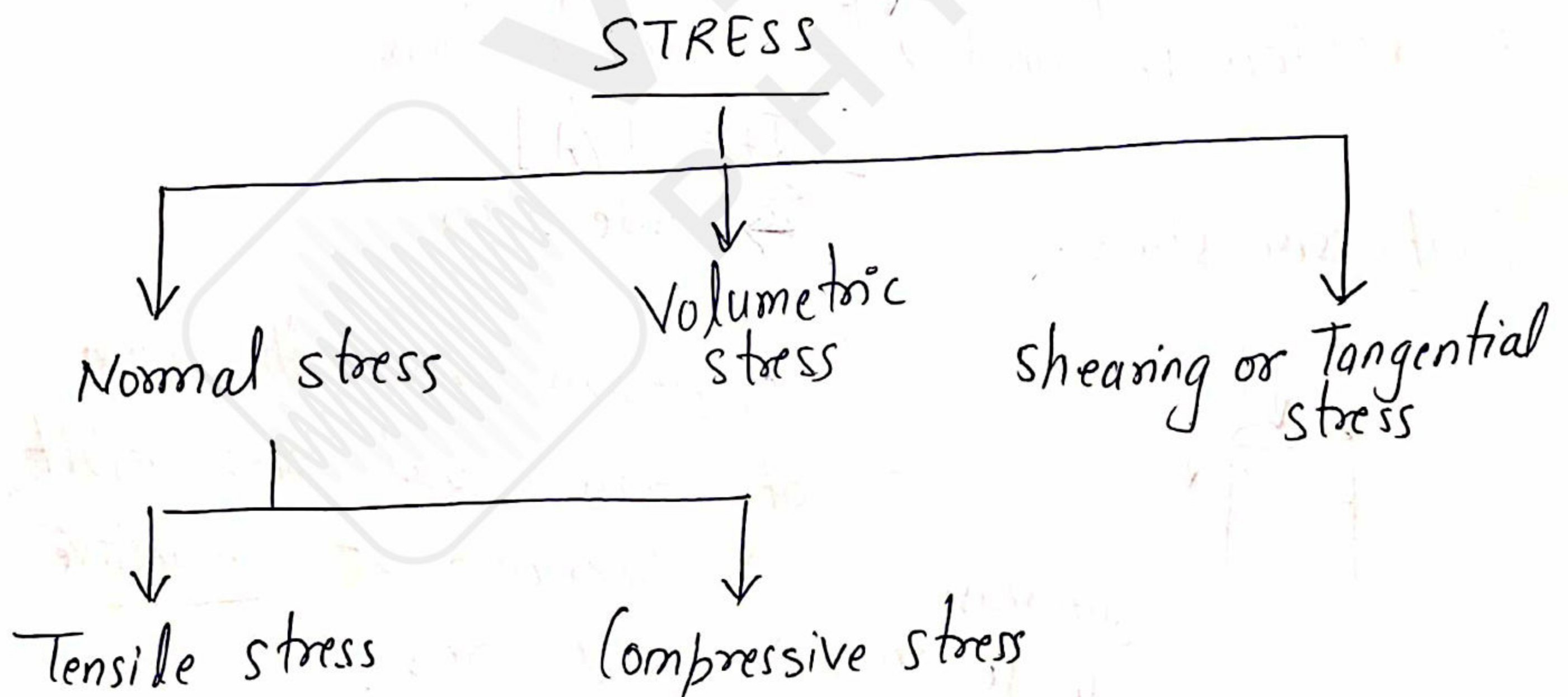
SI unit $\rightarrow$ Pa $\rightarrow$ N/m²

$\hookrightarrow$ pascal. [M L⁻¹ T⁻²]

Intensity of internal forces at a point
=
stress at point considered.

$$\Rightarrow \boxed{\text{Intensity} = \text{stress} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}}$$

* As internal force varies accordingly to maintain equilibrium:



Normal stress $\rightarrow$ Internal force perpendicular (Normal) to the section considered.

$$\sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_n}{\Delta A} \rightarrow \text{Normal force.}$$

(No change in volume)

Tensile stress:

$F \rightarrow$ Applied force



If increase in length in direction of applied force, (Tensile stress)

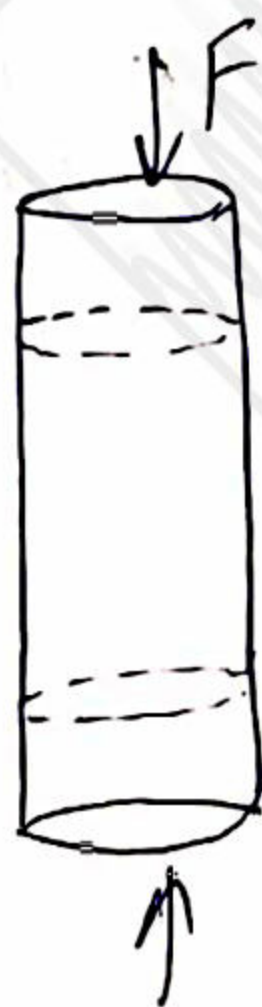
Here,

Restoring force per unit Area = Tensile stress

$$[\sigma_t = F/A]$$

$\rightarrow$ Tensile stress

Compressive stress:



decrease in length

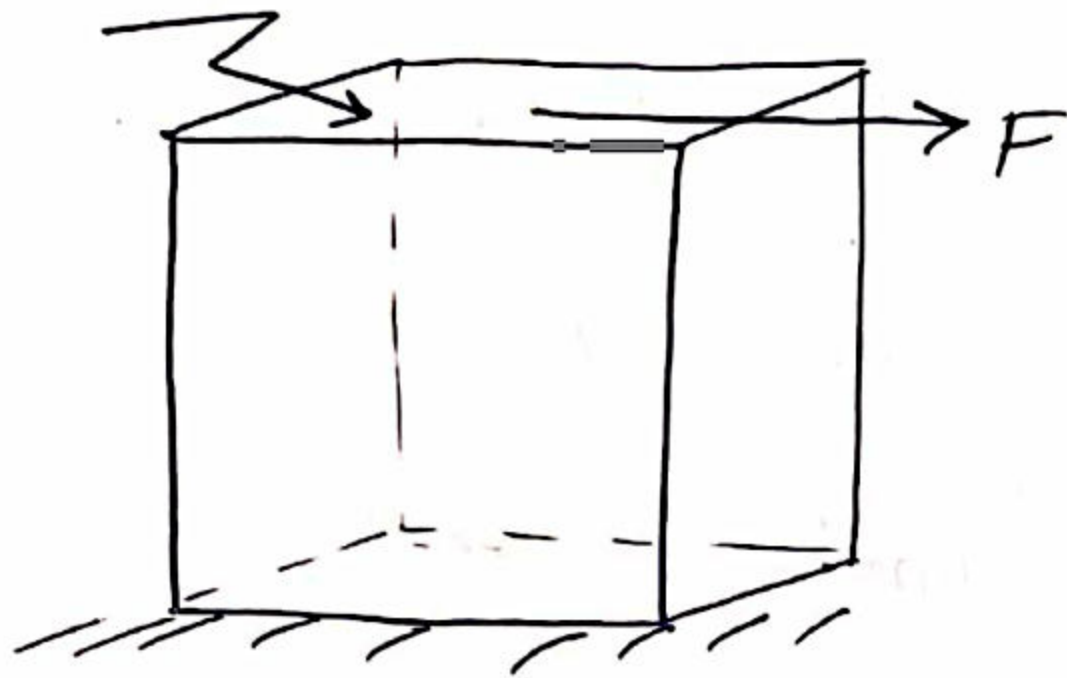
Compression under the action of two equal forces, result in generation of compressive internal stress.

$$[\sigma_c = F/A]$$

Compressive stress

Shearing stress:

Area
A



★ Force acting tangentially to the surface

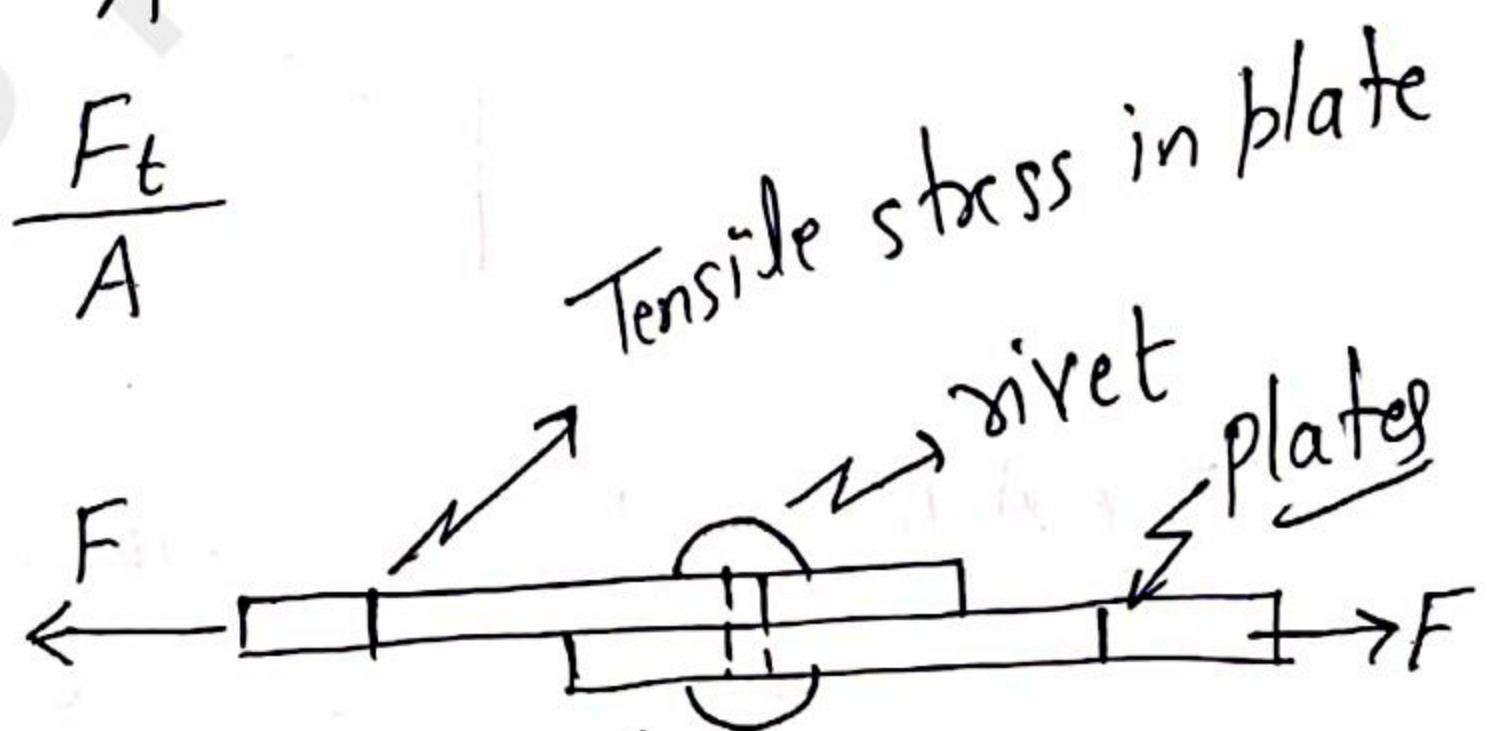
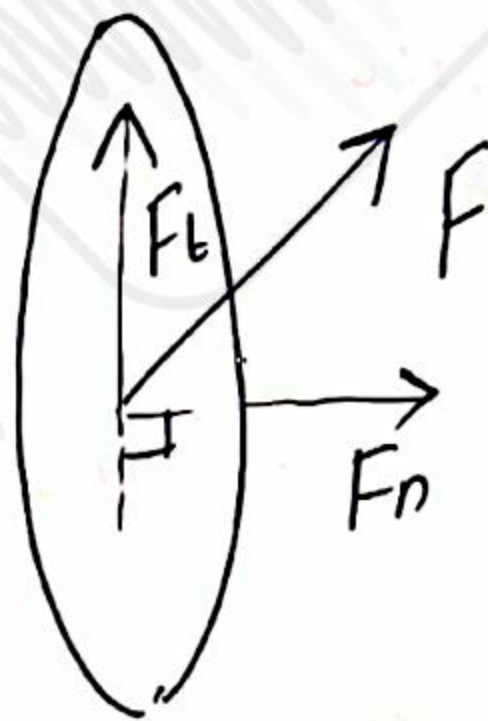
★ deformation along the force, no change in volume.

$$\Rightarrow \text{Tengential stress} = \tau = \Delta F / A$$

$$\text{Intensity, } \tau = \lim_{\Delta A \rightarrow 0} \Delta F / \Delta A$$

So, Normal stress = $\frac{F_n}{A}$

Shearing stress = $\frac{F_t}{A}$

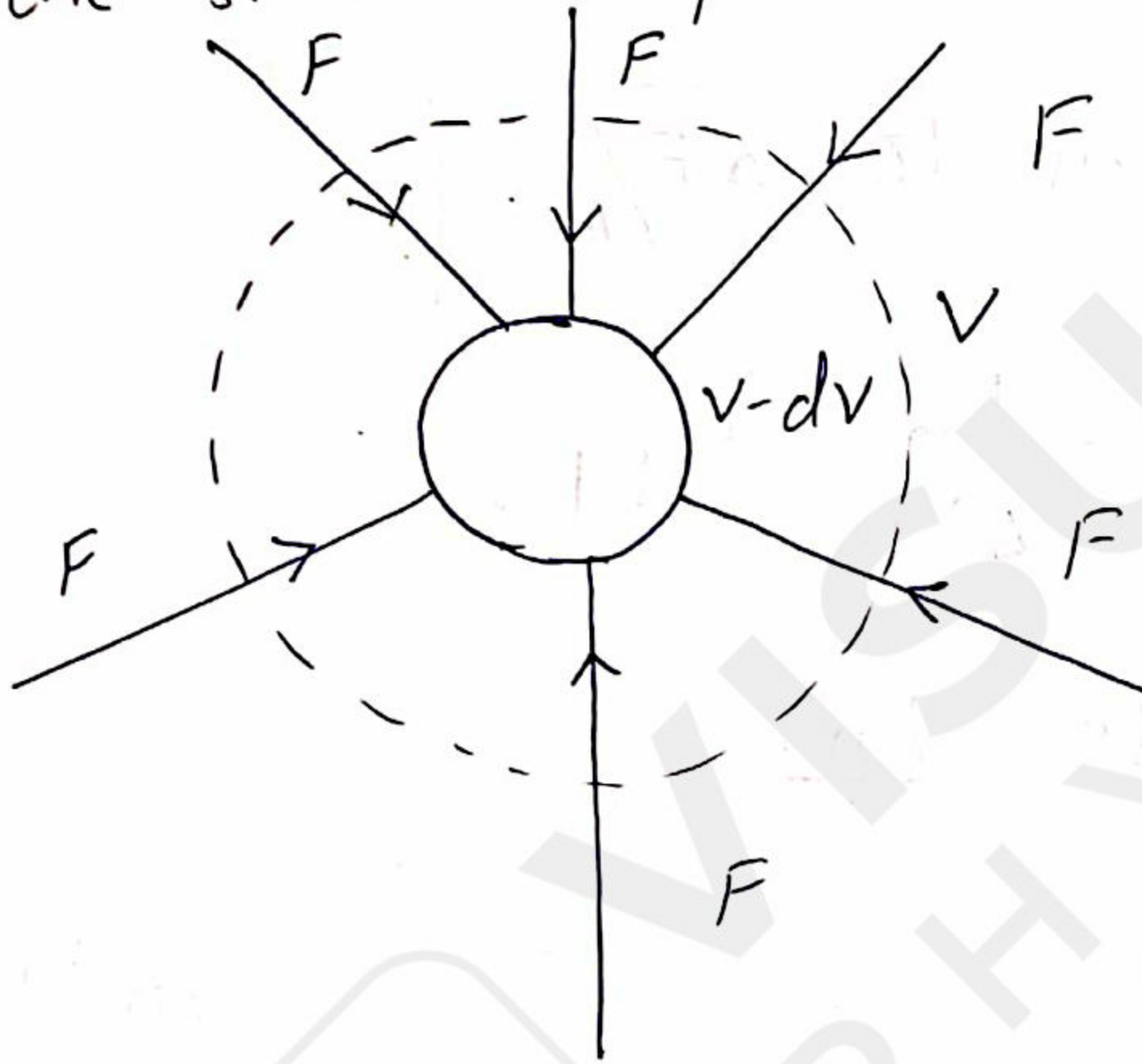


Hence bearing stress in rivet

Volumetric stress:

* When solid body undergoes a change in volume without any change in its geometrical shape

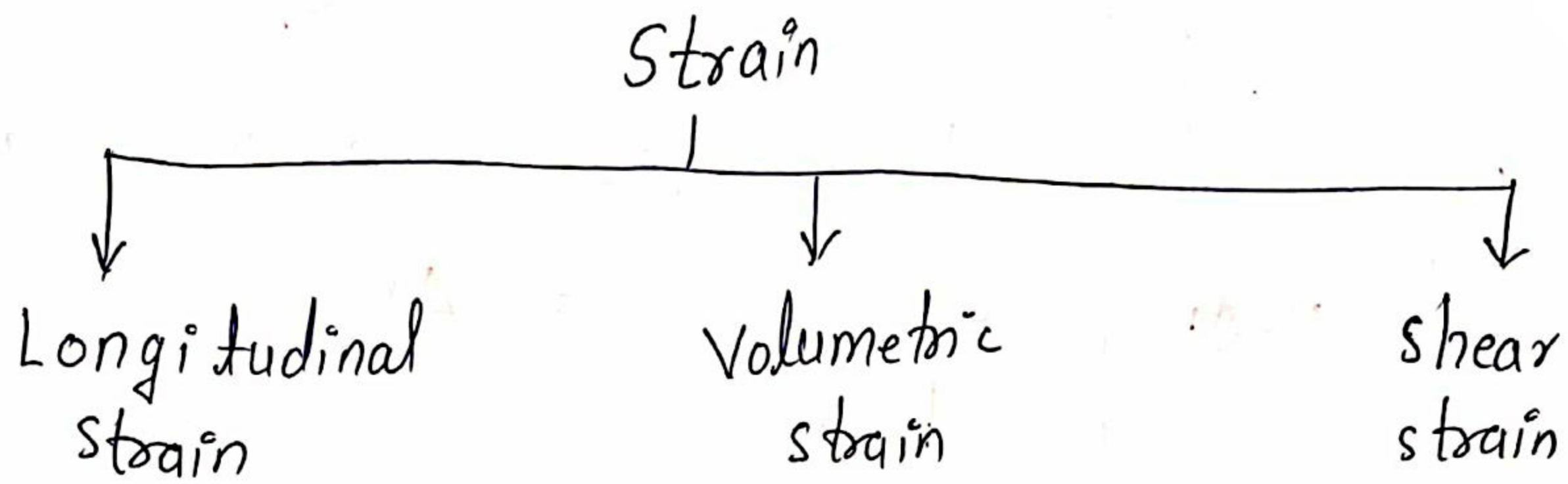
* If deforming force acts on body from all sides the stress develop = volumetric stress.



STRAIN: When deforming force is applied on a body, there is change in configuration.

Body is said to be strained or deformed.

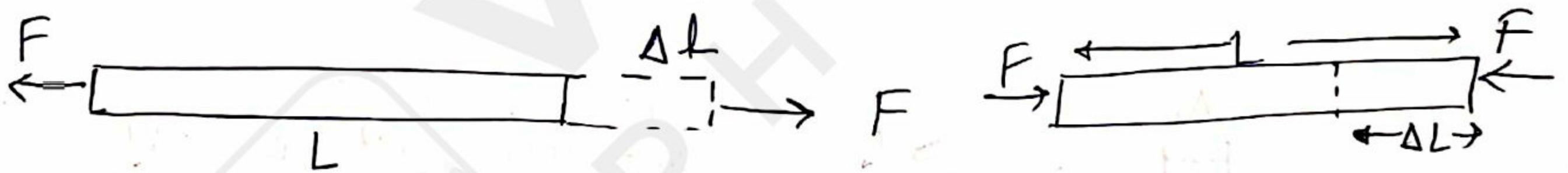
$$\text{Strain} = \frac{\text{change in configuration}}{\text{Original configuration}}$$



Longitudinal strain:

Deforming force causes a change in length of the body.

$$\epsilon_l = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

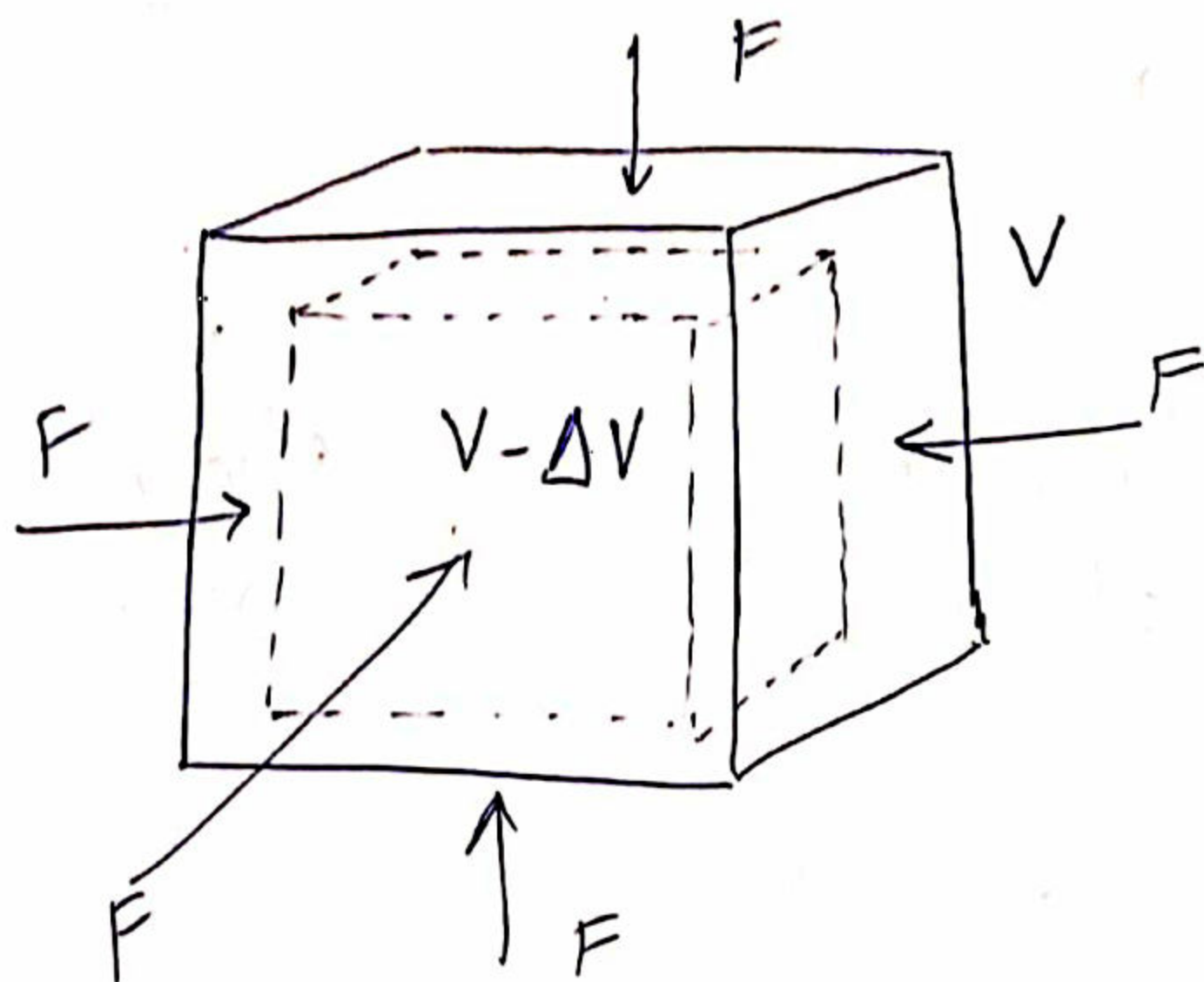


here, both compression & extension is considered

- The specimen must be of uniform cross-section
- material must be homogeneous
- The load must be axial.

Volumetric strain:

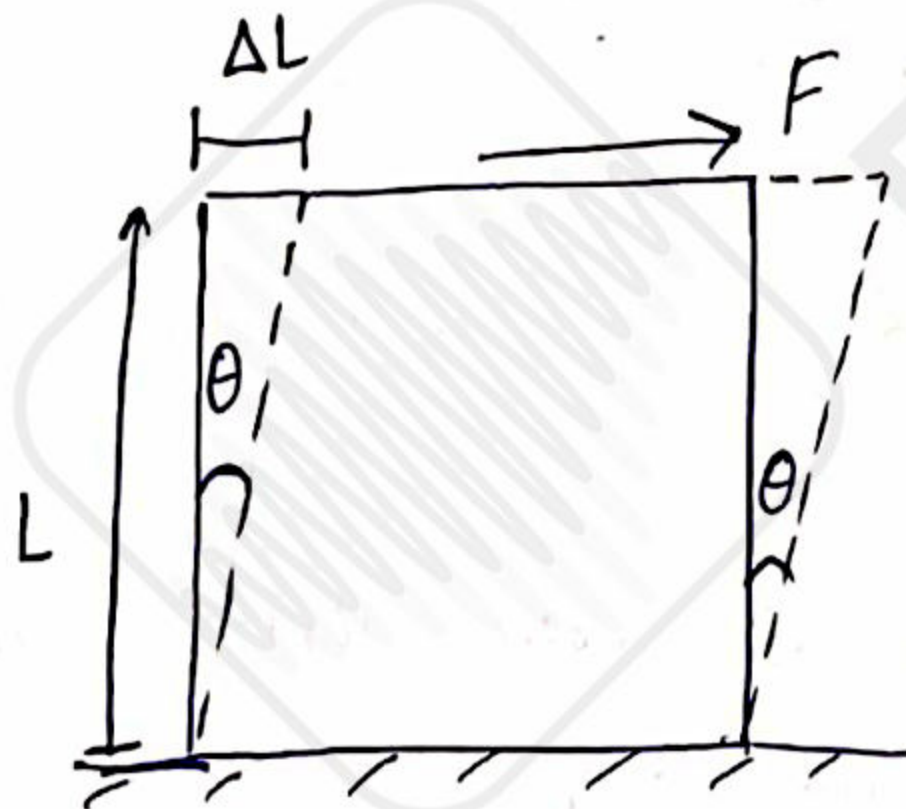
when deforming force produce change in volume of the body.



$$\begin{aligned}\epsilon_v &= \text{Volumetric strain} \\ &= \frac{\Delta V}{V}\end{aligned}$$

Shear strain:

Deforming force causes a change in shape of the body, without changing its volume.



$$\text{shearing strain} = \theta = \frac{\Delta L}{L}$$

- Angular change between two perpendicular faces is shearing strain.
- shearing strain never accompanied by volume change.

★ A change in shape/size i.e. dimensions need not necessarily imply strain.
e.g. When body is heated to expand, but strain is zero.

→ Unless & until internal elastic forces operate to bring the body to original state, no strain exists

ELASTIC LIMIT:

→ Upper limit of deforming force up to which, if deforming force is removed, the body regain its original form completely.

→ beyond this deforming force, body loses its property of elasticity & gets permanently deformed

Hooke's Law:

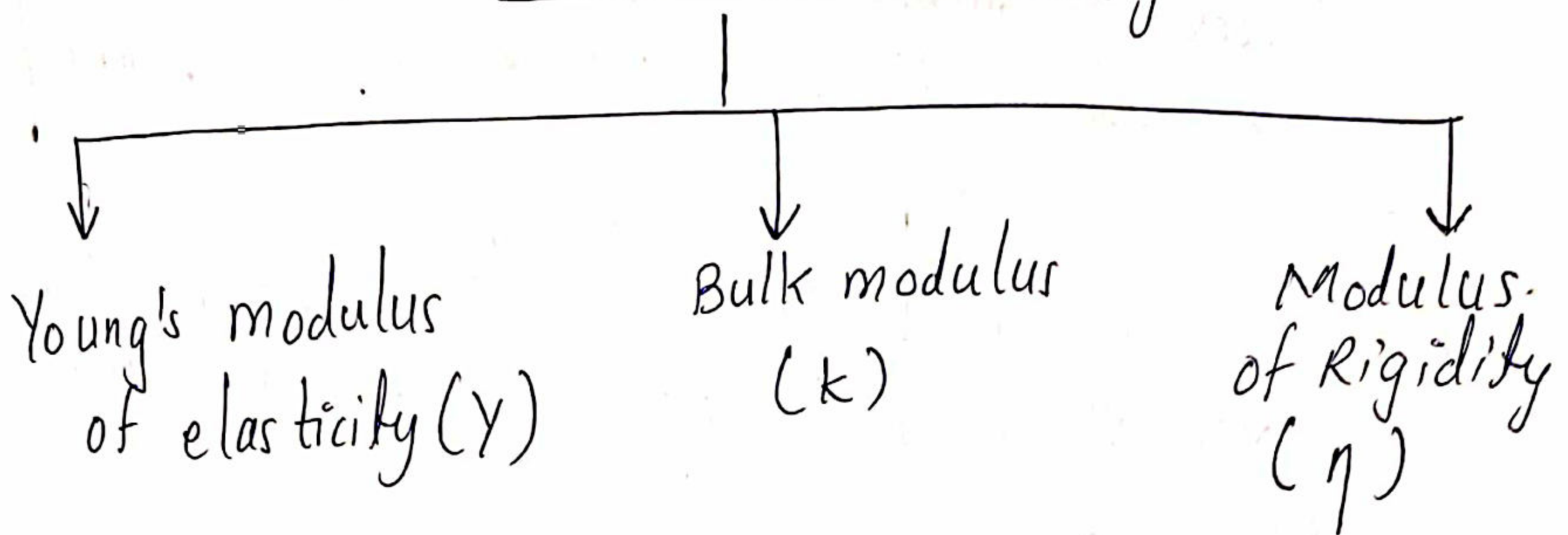
it is applicable up to ^{proportional} limit.

According to this:

stress produce $\propto$ strain

$$\text{Stress} = (\text{Modulus of Elasticity}) \times \text{strain}$$

Modulus of Elasticity



Young's Modulus: $\rightarrow$ Normal stress is acting

$$\text{So, } \boxed{Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}}$$

$\rightarrow$ greater the force $\Rightarrow$ larger the deformation

$$Y = \frac{F/A}{\Delta L/L}$$

$$\Rightarrow F = \left(\frac{Y A}{L} \right) \Delta L$$

$$\text{as } \Delta L \propto \frac{1}{A}, \Delta L \propto \frac{1}{Y}$$

$\rightarrow$ thicker rod produces less deformation
and a stiffer material produces less deformation.

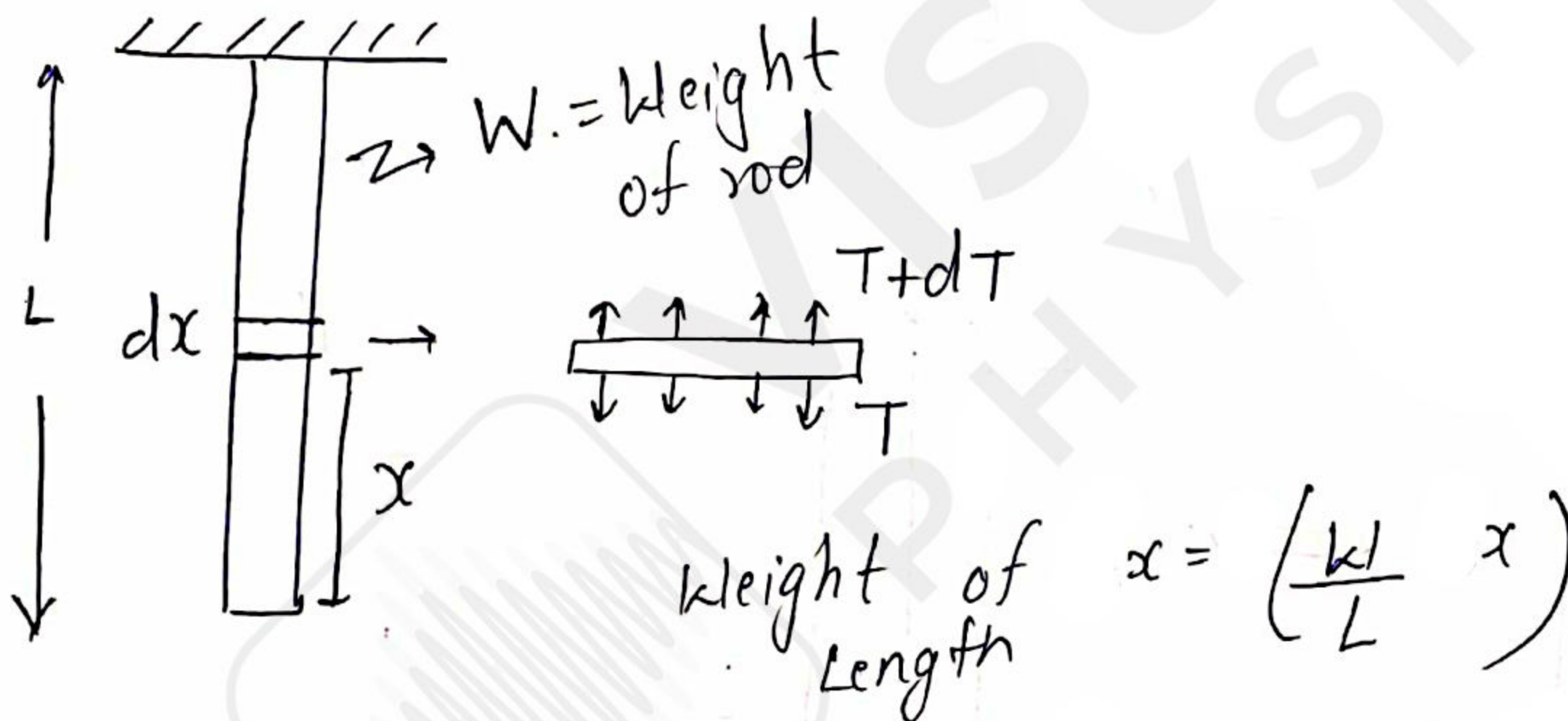
→ As $F \propto \Delta L$

so, $k = \frac{YA}{L}$, $F = K \Delta L$

↙ stiffness constant

Hence, uniform cross section may be considered as an elastic spring $k = YA/L$

→ Longer the rod, lesser the stiffness and thicker the rod.



$$\text{Stress} = \frac{w}{A} = \frac{wx}{AL}$$

$$\frac{dL}{L} = \frac{\text{stress}}{Y}$$

so, for 'dx' part, change in length be $d\delta$

$$\frac{d\delta}{dx} = \frac{wx}{ALY}$$

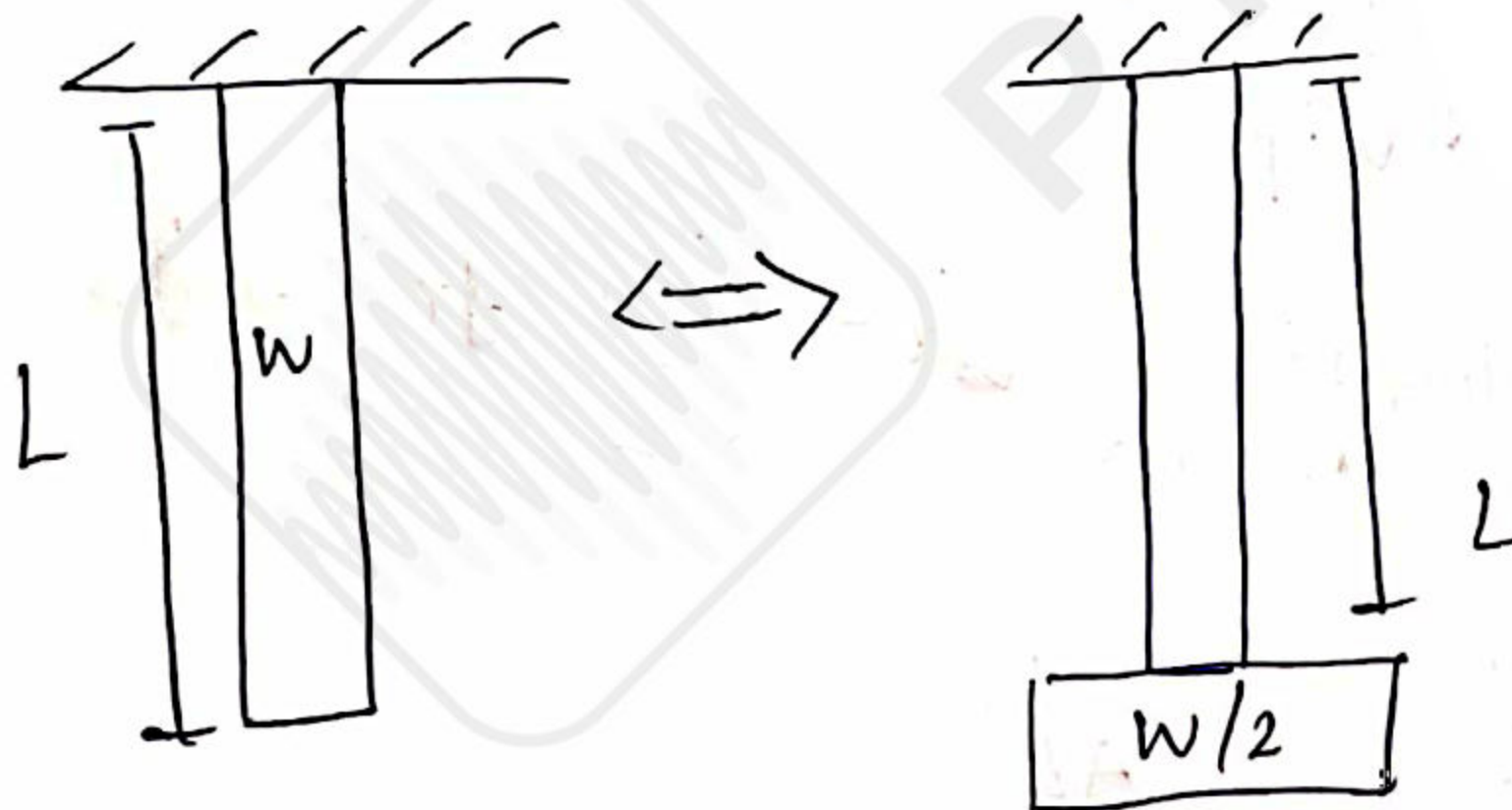
$$\int d\delta = \int \frac{w}{YAL} x dx$$

$$\Rightarrow \delta = \frac{w}{YAL} \left[\frac{x^2}{2} \right]_0^L$$

$$\boxed{\delta = \frac{wL}{2AY}}$$

OR if we consider rod massless, but having $(w/2)$ weight at end, so extension produced would be same

$$\boxed{\delta = \frac{(w/2)L}{AY}}$$



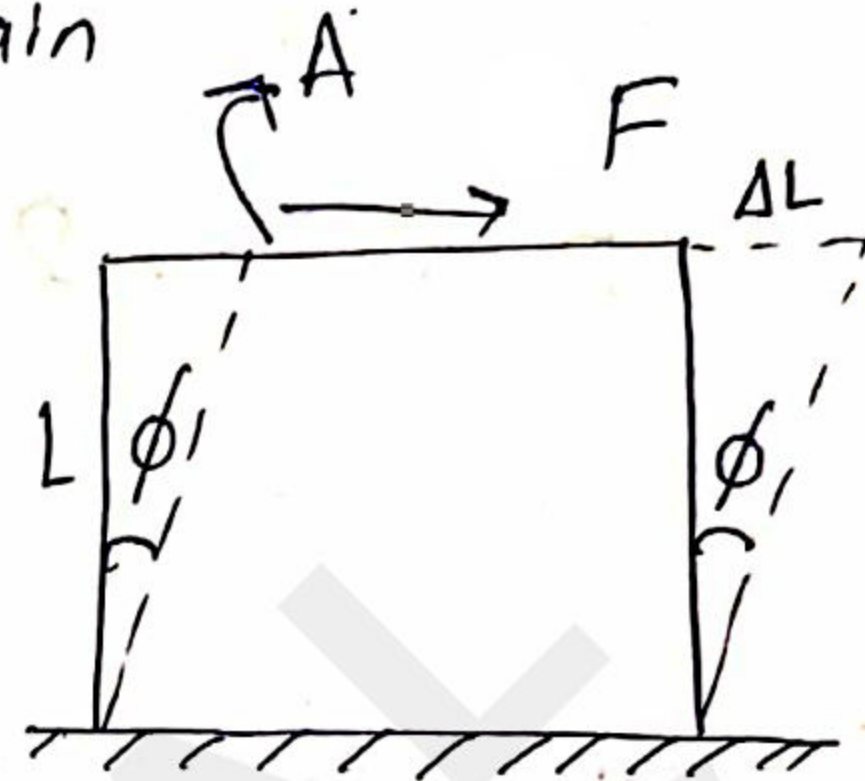
Modulus of Rigidity:

$$\text{Tangential stress} = F/A$$

And shearing strain = $\phi = \frac{\Delta L}{L}$

$$\eta = \frac{\text{Tangential stress}}{\text{shearing strain}}$$

$$\eta = \frac{F}{A \phi}$$



BULK MODULUS:

$p \rightarrow$ Uniform compression stress or expansion stress

$\Delta V \rightarrow$ change in volume

$V \rightarrow$ Initial volume.

$$P \propto \frac{\Delta V}{V}, \quad P = B \left(\frac{\Delta V}{V} \right)$$

$$B = \frac{P V}{\Delta V}$$

for compression

$$B = \frac{-P V}{\Delta V}$$

* Negative sign shows increase in pressure (P) cause decrease in volume (ΔV)

Compressibility:

↳ Reciprocal of bulk modulus of elasticity is called Compressibility.

$$B_{\text{solids}} > B_{\text{liquid}} > B_{\text{gases}}.$$

$$k = 1/B$$

↳ Compressibility

↳ Ability to get compressed.

Analogy of rod as spring

As for spring, $F = kx$

$x \rightarrow$ Extension

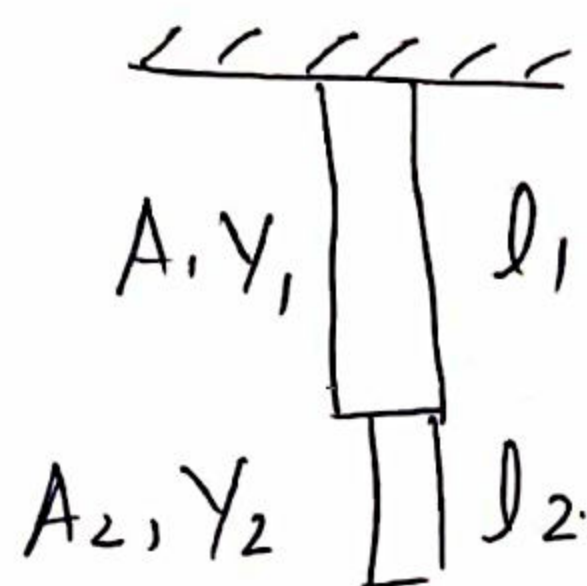
Similarly rod,

$$F = \left(\frac{AY}{L} \right) x$$

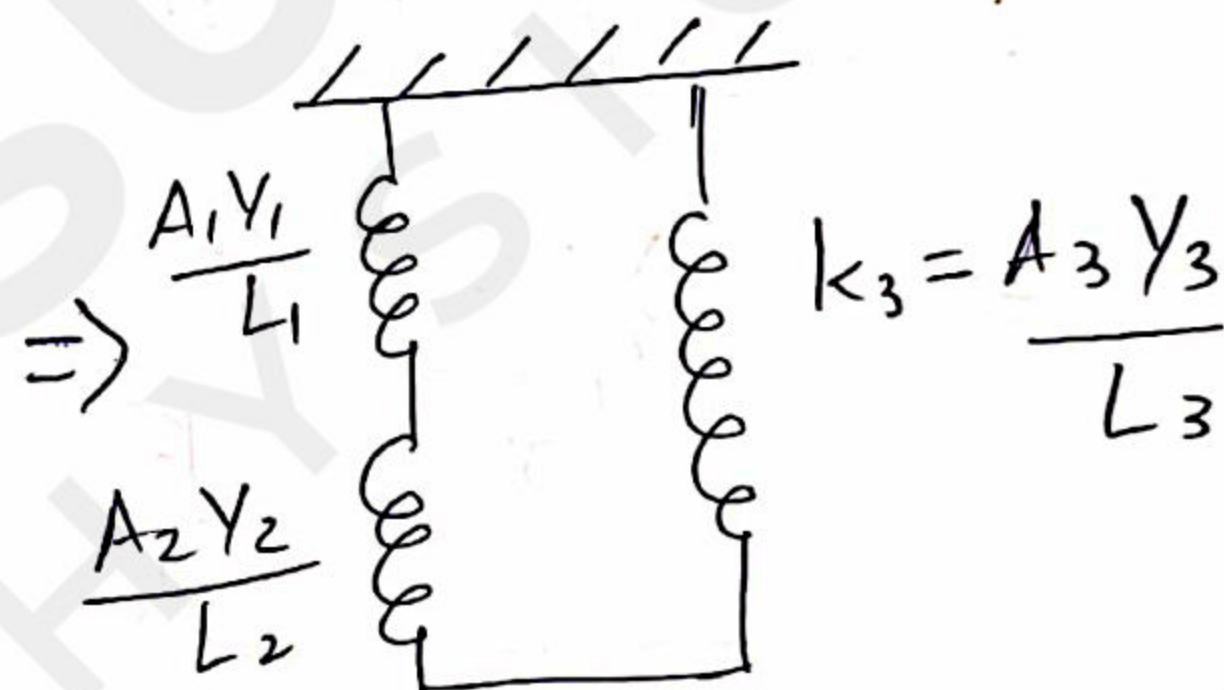
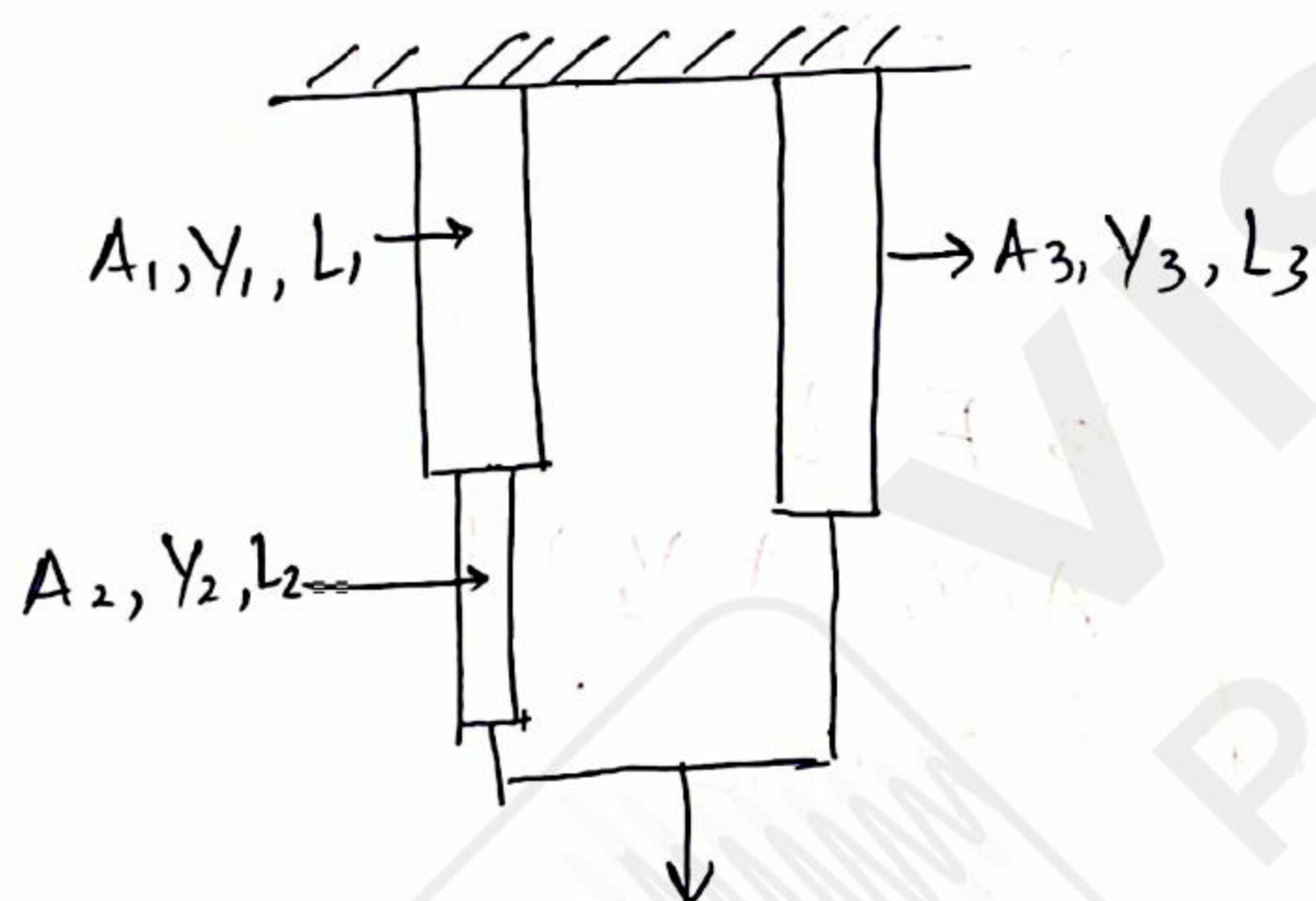
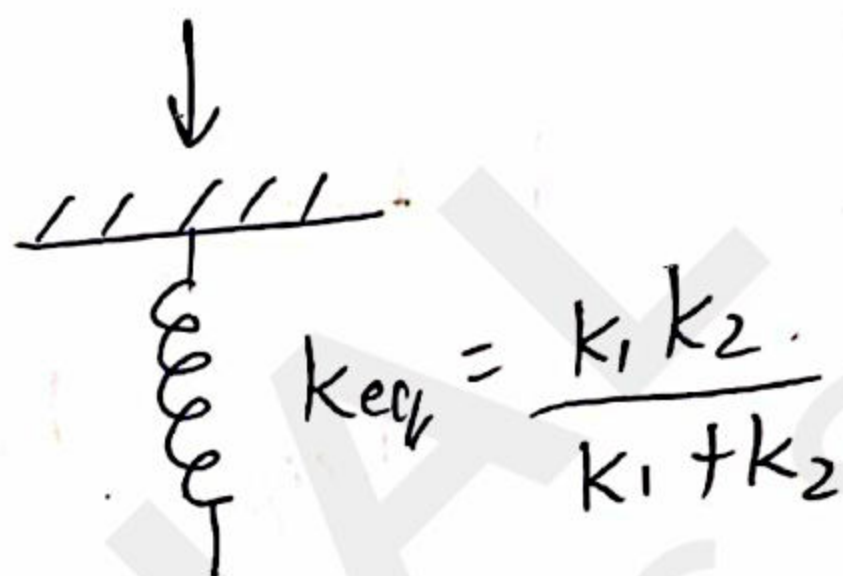
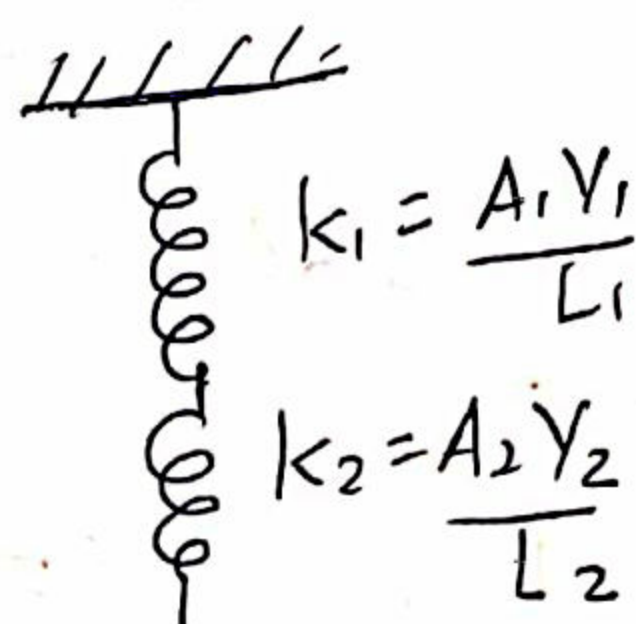
↳ change in length

so, $k = \frac{AY}{L}$

$$F = kx$$



$\Rightarrow$



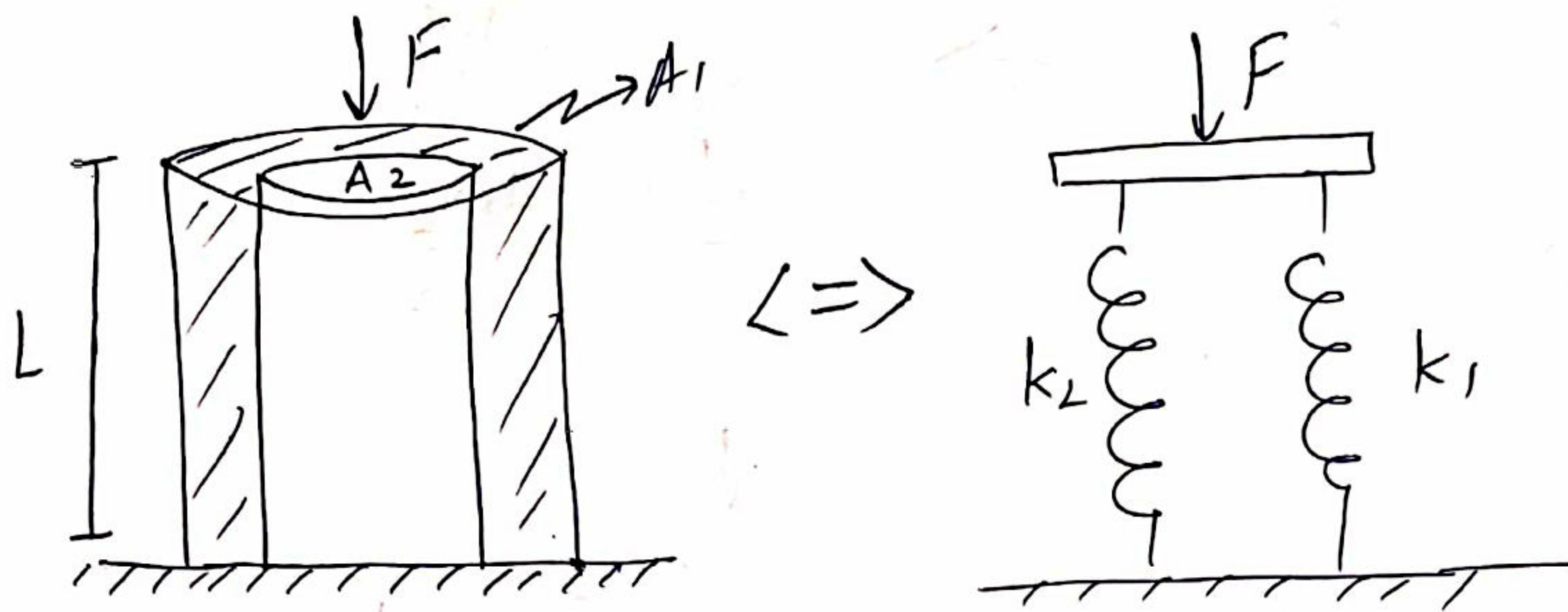
$A_s, \quad k = \frac{YA}{L}$

$\Rightarrow \boxed{kL = YA = \text{Constant for constant 'A'}}$

Hence, if we cut the rod,

$\boxed{k_1 L_1 = k_2 L_2}$

BARS OF COMPOSITE SECTION:



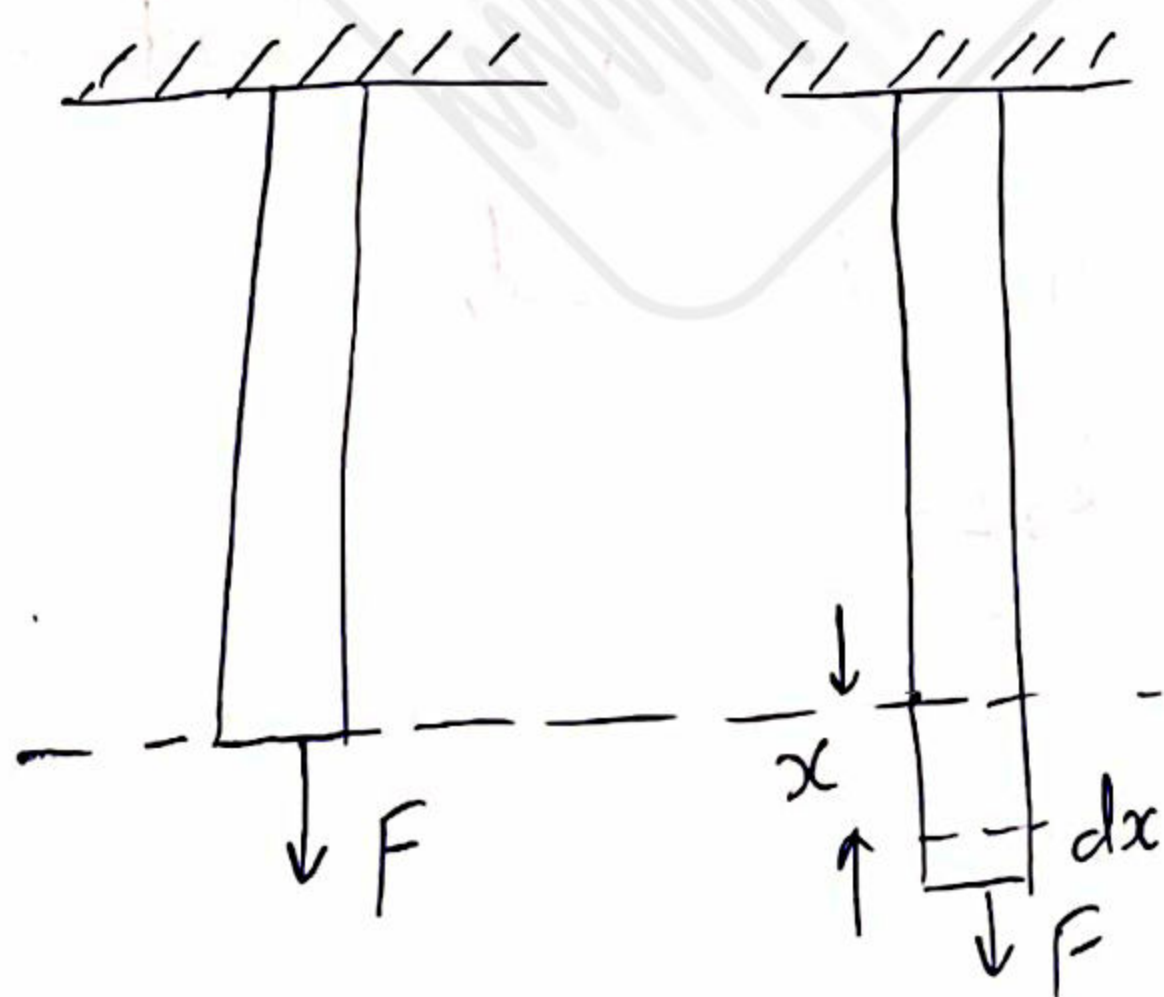
$$k_{eq} = k_1 + k_2 = \frac{A_1 Y_1}{L} + \frac{A_2 Y_2}{L}$$

change in length is same $= x$

$\Rightarrow$

$$x = \frac{F}{k} = \frac{F L}{(A_1 Y_1 + A_2 Y_2)}$$

Energy stored in deformed body:



$$F = \left(\frac{YA}{L} \right) x$$

$$\int dw = \int F dx$$

$$w = \frac{YA}{L} \int_0^{\delta} x dx$$

$$w = \frac{1}{2} \frac{YA}{L} \delta^2$$

$\delta \rightarrow$ Net extension/compression

$$\text{Energy stored} = W = U = \frac{1}{2} \frac{YA}{L} \delta^2$$

Similar result if considered spring analogy

$$U = \frac{1}{2} k_{eq} \delta^2$$

$$k = \frac{YA}{L}$$

$$\Rightarrow \boxed{U = \frac{1}{2} \left(\frac{YA}{L} \right) \delta^2}$$

$$\text{Now, } U = \frac{1}{2} \frac{YA}{L} \delta^2$$

$$\delta = \frac{\text{stress}}{Y} = \frac{FL}{AY}$$

$$\Rightarrow U = \frac{1}{2} \frac{YA}{L} \left(\frac{FL}{AY} \right)^2 = \frac{1}{2} \frac{F^2}{AY} L = \frac{1}{2} F \delta$$

$$U = \frac{1}{2} \frac{F}{A} \times \frac{F}{AY} AL$$

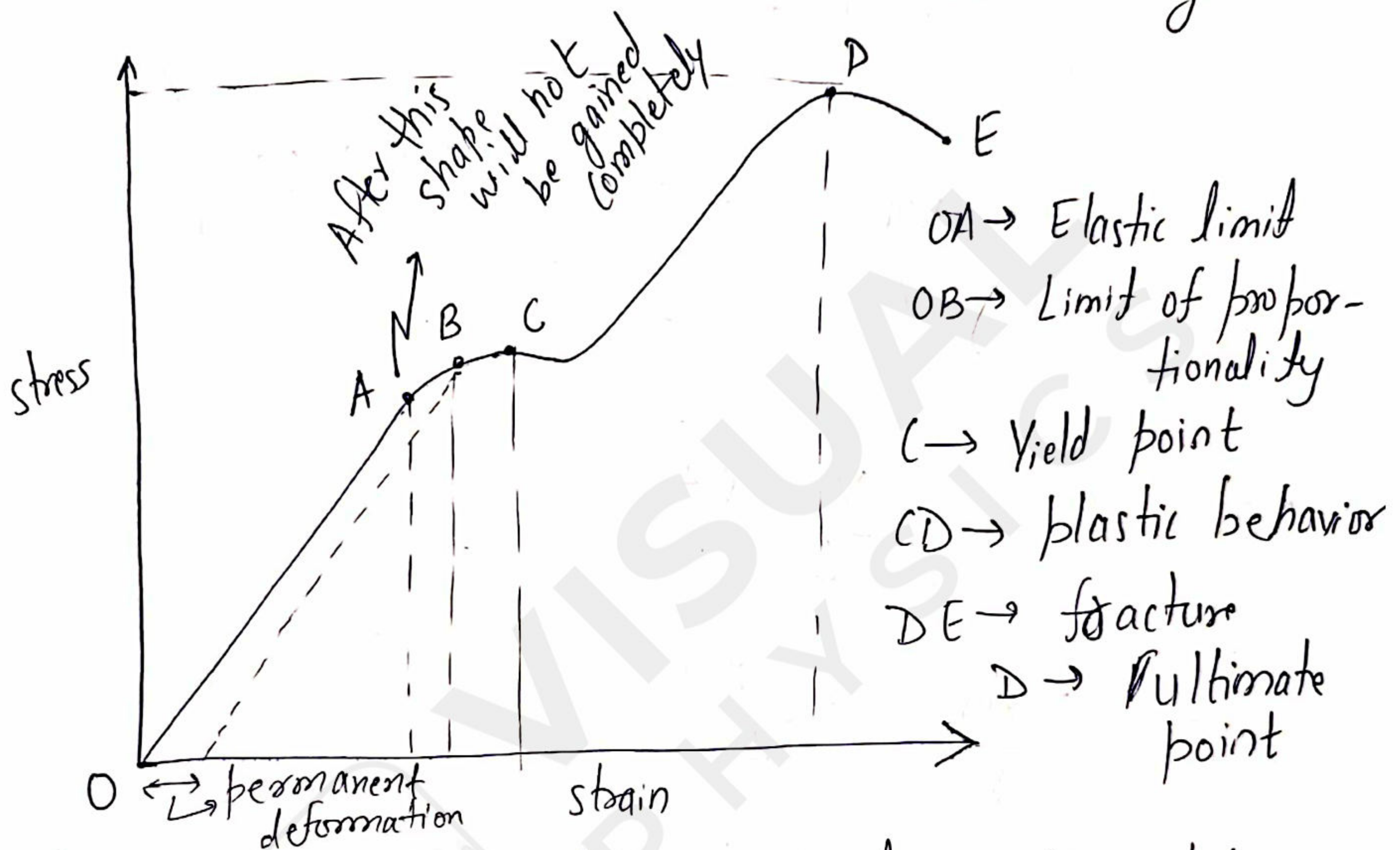
$$\Rightarrow U = \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{Volume}$$

$$\text{or } U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{Volume}$$

$$\boxed{\text{Strain Energy density} = \frac{U}{\text{Volume}} = u = \frac{1}{2} \times Y \times (\text{Strain})^2}$$

$$U = \frac{1}{2} \times \text{stress} \times \text{strain}$$

→ Area Under stress strain Curve
give Energy density



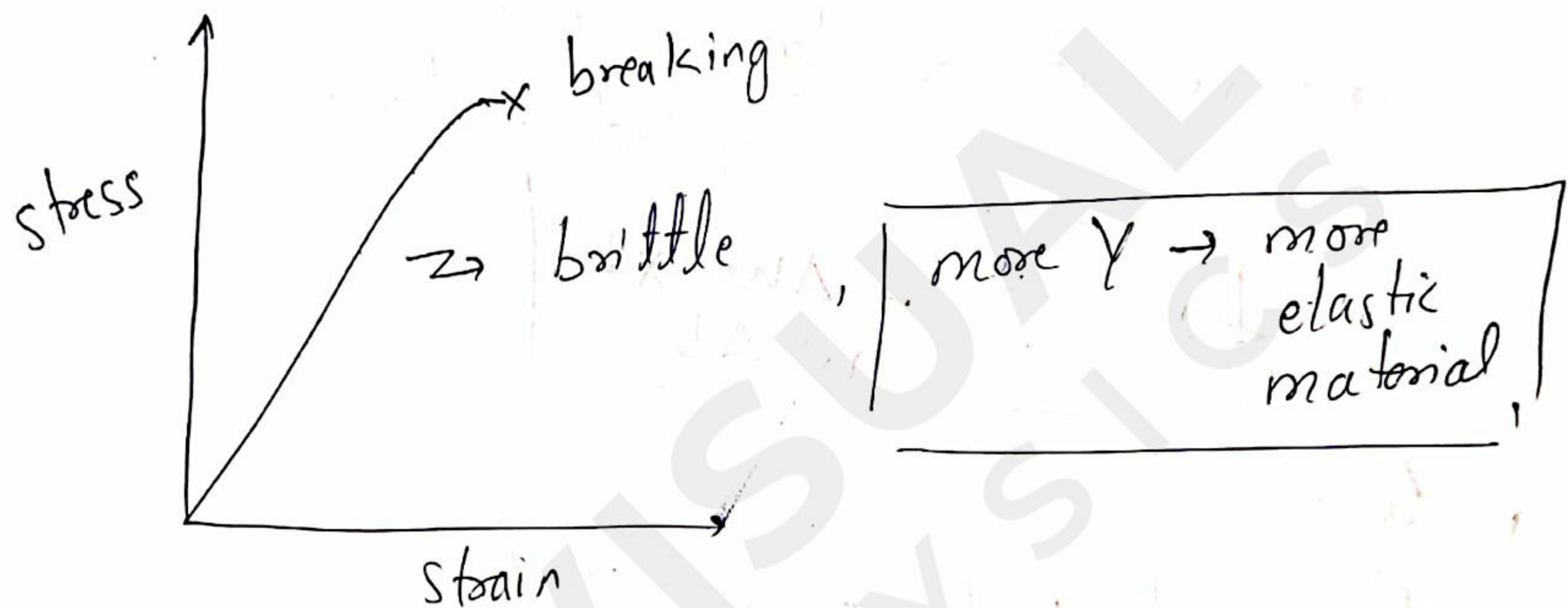
→ In Yield region, strain 15-20 times those take place up to proportional limit.

→ Yield point → close to elastic limit for most purposes the two may be taken as one

Working stress is lower than breaking stress

$$\text{factor of safety} = \frac{\text{breaking stress}}{\text{working stress}}$$

- Steeper curve indicates a stiffer material.
- long graph parallel to the strain axis, indicates ductile material.
- Absence of yield point or plastic zone refer to brittle material.



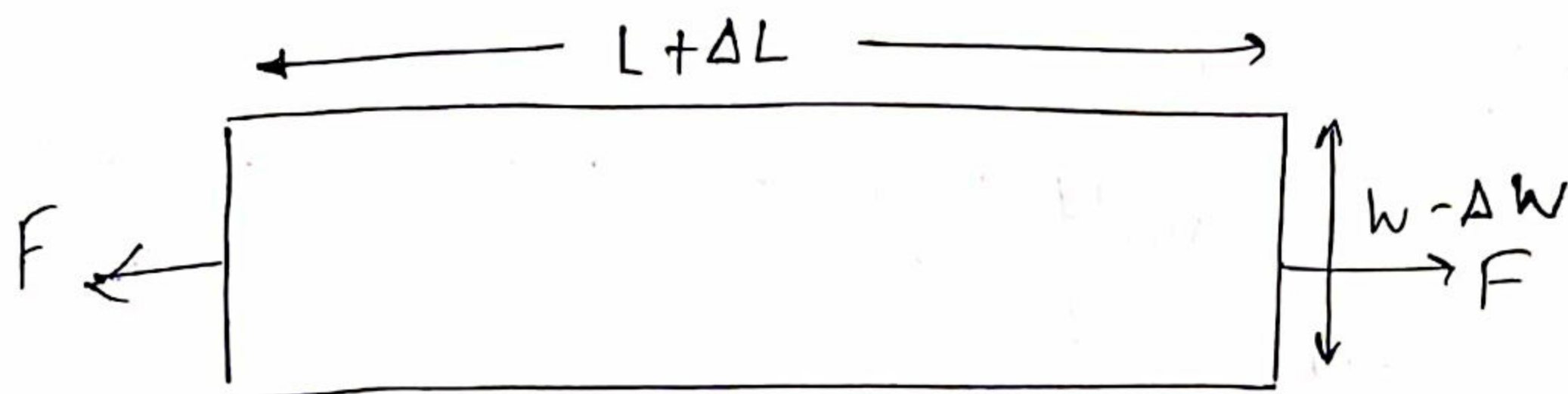
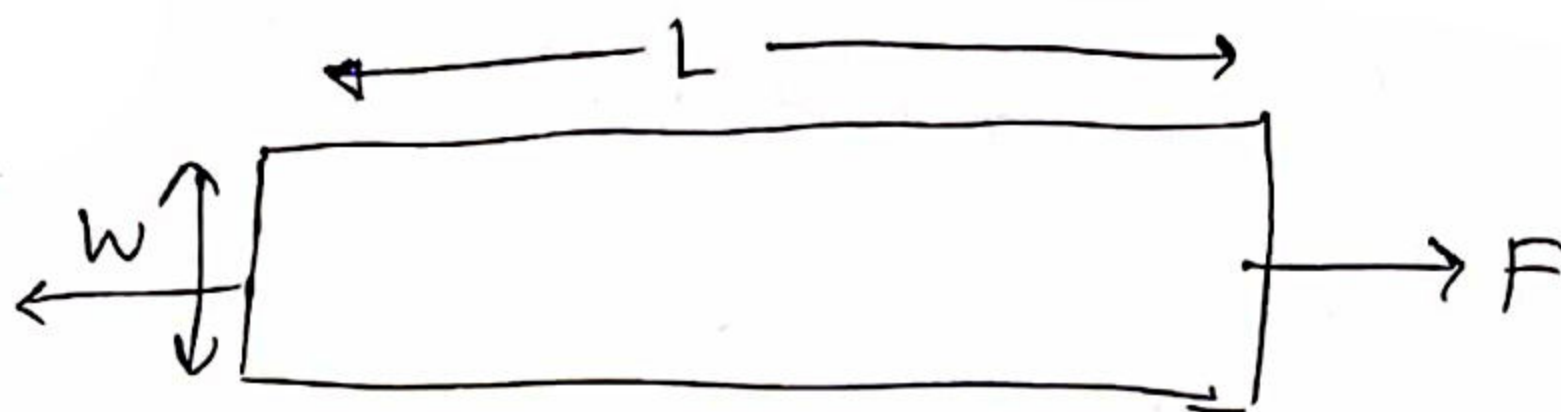
- More the area under stress vs strain curve implies more tougher material. Means more energy per unit volume is required to break the material.

Poisson's Ratio :

As during stretching & compression.

As volume remains constant.

- ⇒ change in longitudinal causes change in lateral dimension.



$$\text{poission's Ratio} = \frac{\text{lateral strain}}{\text{longitudnal strain}}$$

$$\sigma = - \frac{\Delta w}{w} \times \frac{L}{\Delta L}$$

poission's Ratio

→ property of material.

→ does not depend on size & shape of material.

Usually ν is positive.

→ means if $L \uparrow$, $w \downarrow$

But for some material

$\nu \rightarrow$ negative

→ means if $L \uparrow$, $w \uparrow$

e.g. polymers foams