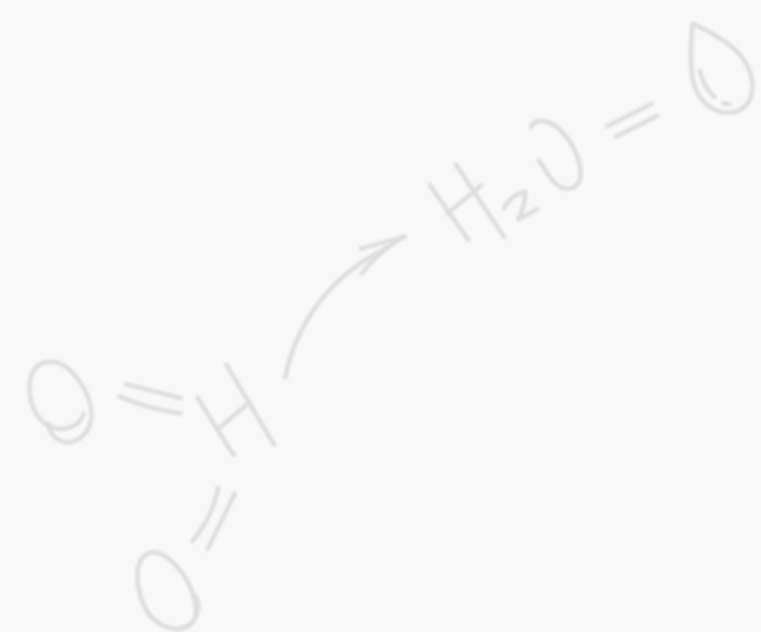




VISUAL
PHYSICS



V I S U A L P H Y S I C S

SHORT NOTES



C H A P T E R

CoM, Momentum & Collision



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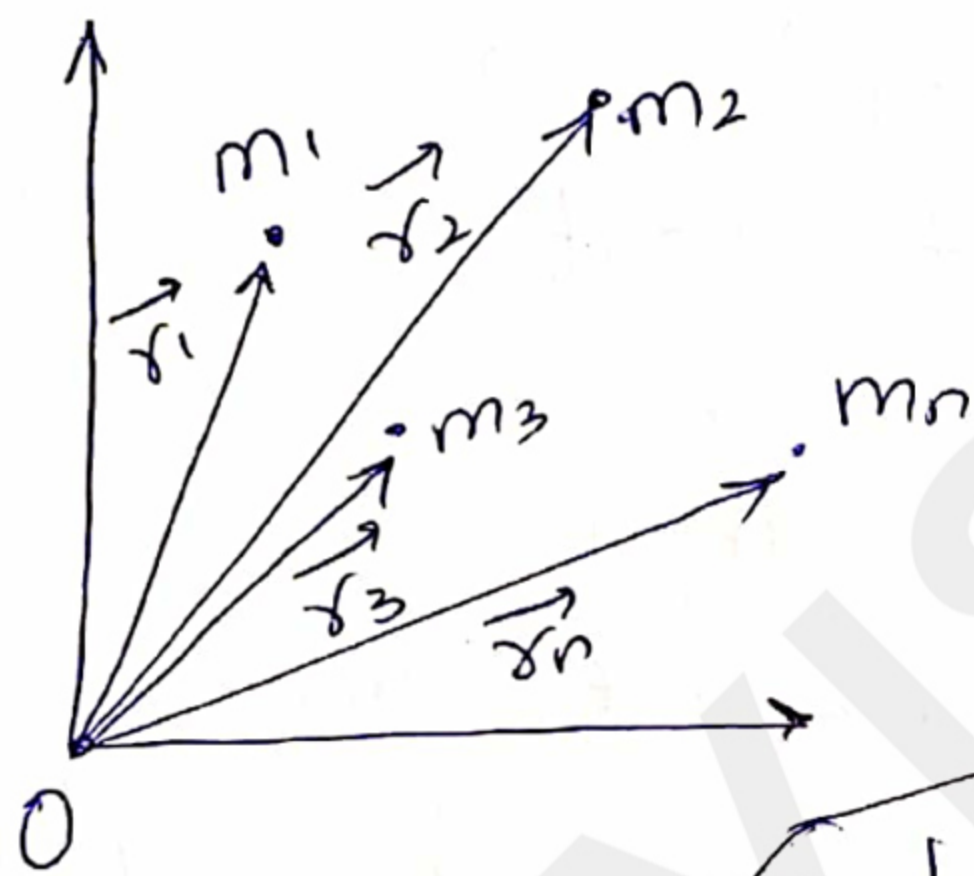
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Centre of Mass, Conservation of Linear Momentum & Collision.

Centre of mass:

↳ An imaginary point where we can consider the whole mass of the body at.



$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{(m_1 + m_2 + \dots + m_n)}$$

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M}$$

centre of mass from given reference point

Net mass of body

In terms of components

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{M}, \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{M}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots + m_n z_n}{M}$$

$m_1, m_2, \dots, m_n \rightarrow$ discrete mass

Centre of Mass of Continuous Mass

$$x_{cm} = \frac{\int x dm}{\int dm}, \quad y_{cm} = \frac{\int y dm}{\int dm}, \quad z_{cm} = \frac{\int z dm}{\int dm}.$$

Centre of mass → (need not to be on object) Used to study the motion of body of given dimensions

As till now we used newton's law and all for point object but what about continuous object?

→ A point which behave as a single point mass
So that we can apply all the laws of motion on it. And COM satisfy the real situation behavior

$$\Rightarrow \boxed{F_{net} = M a_{cm}}$$

↓ Centre of mass acceleration

Centre of gravity → where we ^{can} assume all the gravitational force to act on.

COM $\Leftrightarrow$ Centre of Mass

COM of symmetric object lie at its centroid.

if some part is removed.

$$x_{cm} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}, \quad y_{cm} = \frac{m_1 y_1 - m_2 y_2}{m_1 - m_2}$$

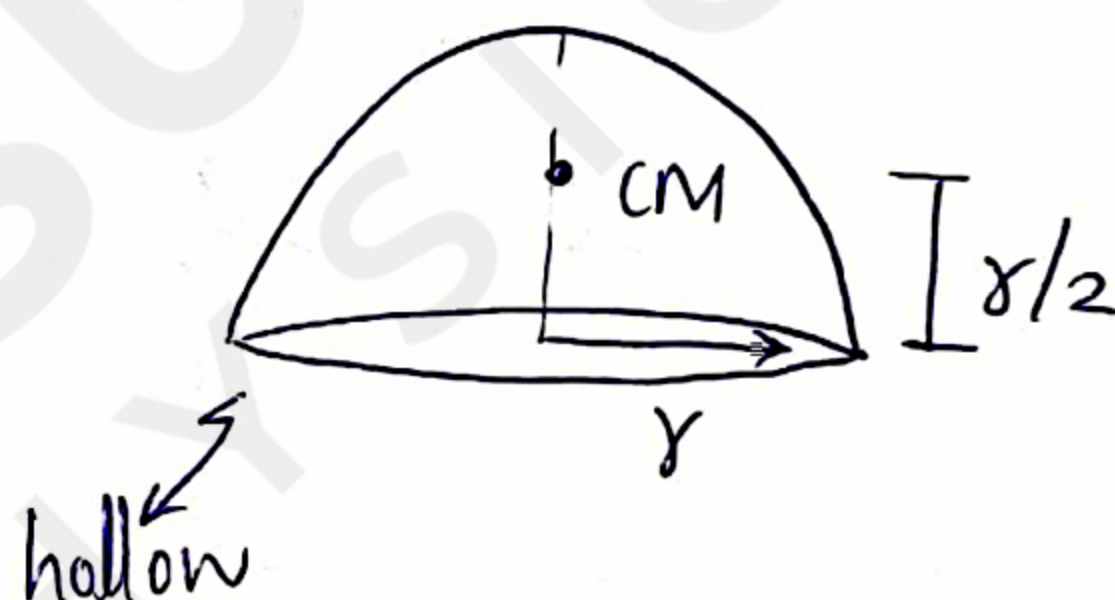
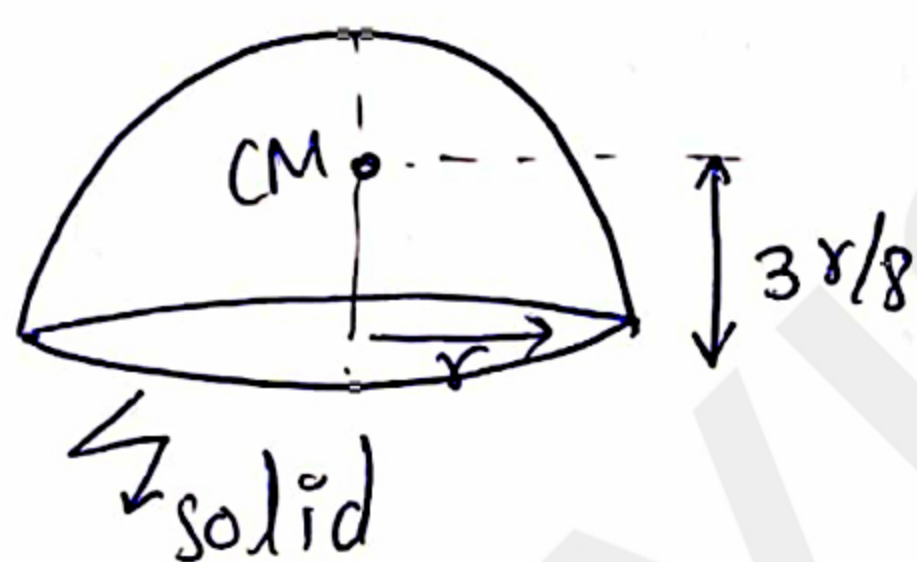
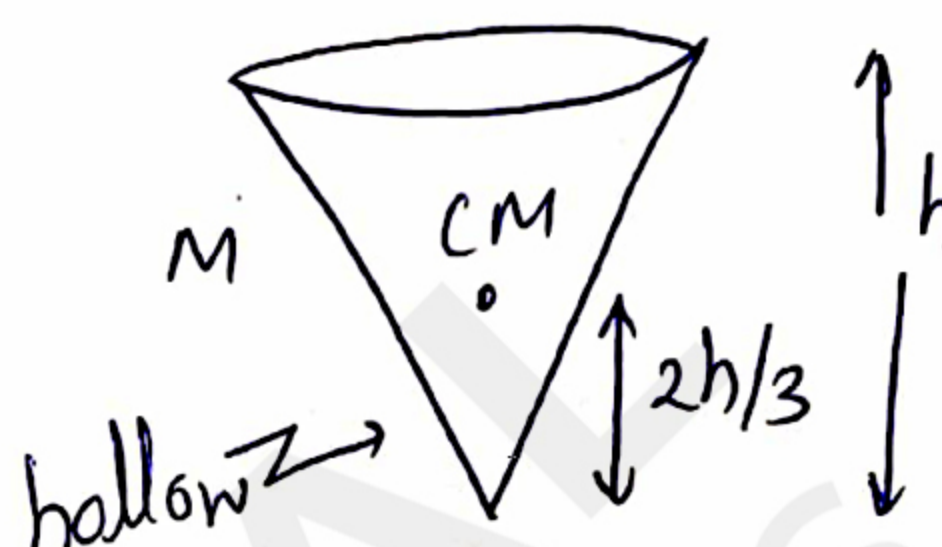
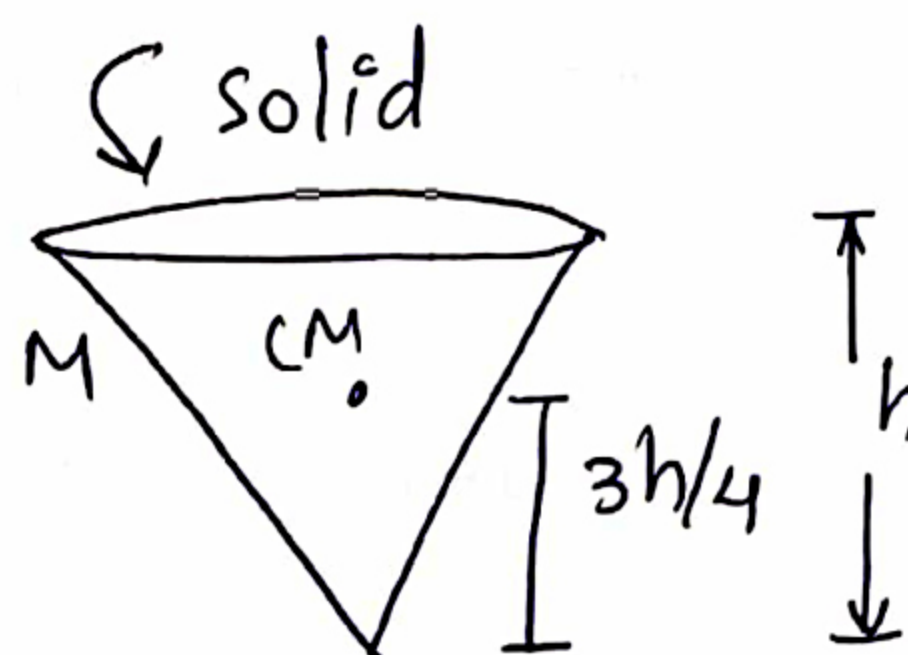
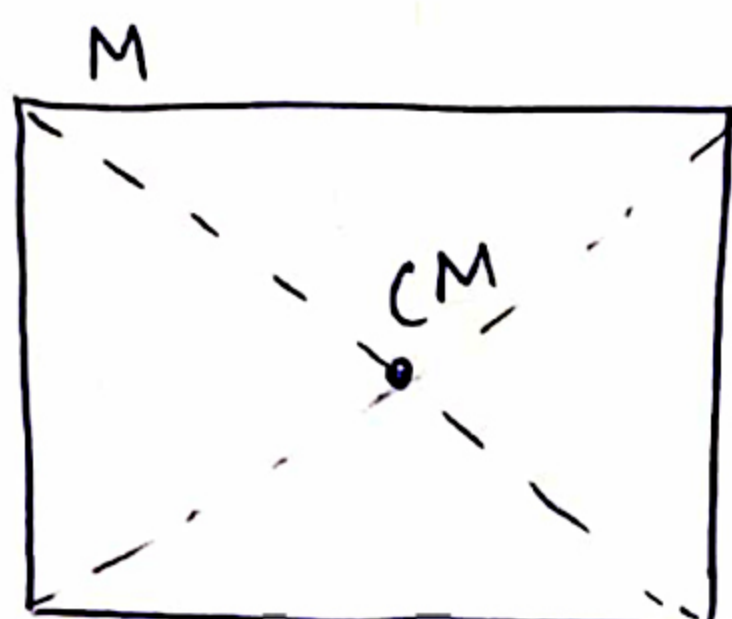
$$z_{cm} = \frac{m_1 z_1 - m_2 z_2}{m_1 - m_2}$$

if considered uniform lamina

$$x_{cm} = \frac{x_1 A_1 - x_2 A_2}{m_1 - m_2}, \quad y_{cm} = \frac{y_1 A_1 - y_2 A_2}{m_1 - m_2}$$

$$z_{cm} = \frac{z_1 A_1 - z_2 A_2}{m_1 - m_2}$$

★ m_1, A_1, x_1, y_1 & z_1 are the values for the whole mass while m_2, A_2, x_2, y_2 and z_2 are the values for the mass which has been removed.



Motion of COM:

$$\vec{V}_{CM} = \frac{m_1 \frac{d\vec{x}_1}{dt} + \dots + m_n \frac{d\vec{x}_n}{dt}}{m_1 + m_2 + \dots + m_n}$$

$$\vec{V}_x = \frac{m_1 \vec{V}_{x1} + m_2 \vec{V}_{x2} + \dots}{m_1 + m_2 + \dots + m_n}$$

$$\vec{V}_y = \frac{m_1 \vec{V}_{y1} + \dots + m_n \vec{V}_{yn}}{m_1 + \dots + m_n}$$

$$\vec{V}_z = \frac{m_1 \vec{V}_{z1} + \dots + m_n \vec{V}_{zn}}{m_1 + \dots + m_n}$$

$$\boxed{\vec{P}_{\text{total}} = [M_{\text{total}}] \vec{V}_{\text{cm}}}$$

Acceleration of Centre of Mass:

$$\vec{a}_{\text{cm}} = \frac{d}{dt} \vec{V}_{\text{cm}} = \frac{m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + \dots + m_n \frac{d\vec{v}_n}{dt}}{M}$$

$$\vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{ext}}}{M} = \frac{\text{Net external force}}{M}$$

So if there is no net external force, $\vec{a}_{\text{cm}} = 0$

* if internal forces are there the discrete particle will move under its influence, but

$$\vec{a}_{\text{cm}} = 0$$

Now, the COM will act as individual particle at which we can apply all the formulas of Newton's law.

Momentum:

for a given particle $\xrightarrow{\quad}$ Velocity $\vec{v}$
 $\xrightarrow{\quad}$ Mass $\rightarrow M$

momentum, $\boxed{\vec{p} = M\vec{v}}$

$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \rightarrow \text{change of momentum of system}$

external force

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

$\rightarrow$ Net external force = 0

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$\frac{d}{dt} [m_1 \vec{v}_1 + m_2 \vec{v}_2] = 0$$

$$\frac{d}{dt} [\vec{p}_1 + \vec{p}_2] = 0$$

$\xrightarrow{\quad}$ $\vec{p}_{\text{total}} \rightarrow$ total momentum

$$\Rightarrow \vec{P}_{\text{tot}} = \text{Constant}$$

$$\boxed{\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}}$$

$$\boxed{\Delta \vec{p} = \vec{p}_f - \vec{p}_i}$$

$$p = mv$$

as kinetic energy = $\frac{1}{2} mv^2$

$$K.E = \frac{1}{2} m \left[\frac{p}{m} \right]^2$$

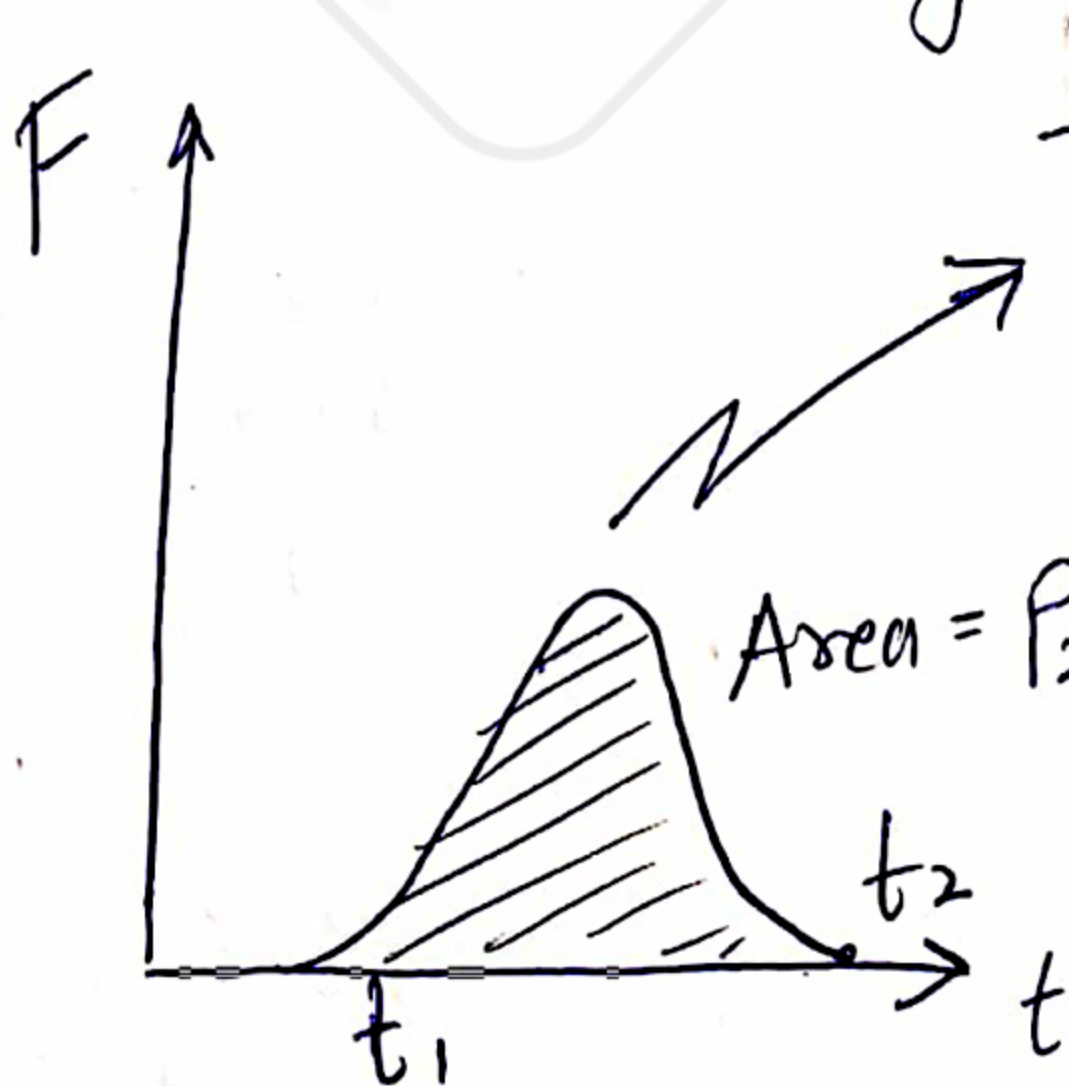
$$\boxed{K.E = \frac{1}{2} \frac{p^2}{m}}$$

Impulse:

Impulse of force F acting on body for time $(t_2 - t_1)$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt = \int_{t_1}^{t_2} m \frac{d\vec{v}}{dt} dt = \int_{v_1}^{v_2} m d\vec{v}$$

$$\vec{J} = \text{change of momentum} = \Delta \vec{p}$$



$$\text{Area} = p_2 - p_1$$

$$\vec{J} = \int \vec{F} dt$$

Area Under $F-t$ graph gives Impulse
= change in Momentum

Impulsive force → A force of relatively higher magnitude and acting for relatively shorter time.

$F_{ext} = 0$

COLLISION:

Momentum is always conserved in any type

Collision is an isolated event in which a strong force acts between two or more bodies for a short time.

- In collision particle may or may not come in physical contact
- time of collision Δt is negligible.

line of impact: the line passing through the common normal to the surfaces in contact during the impact is called line of impact.

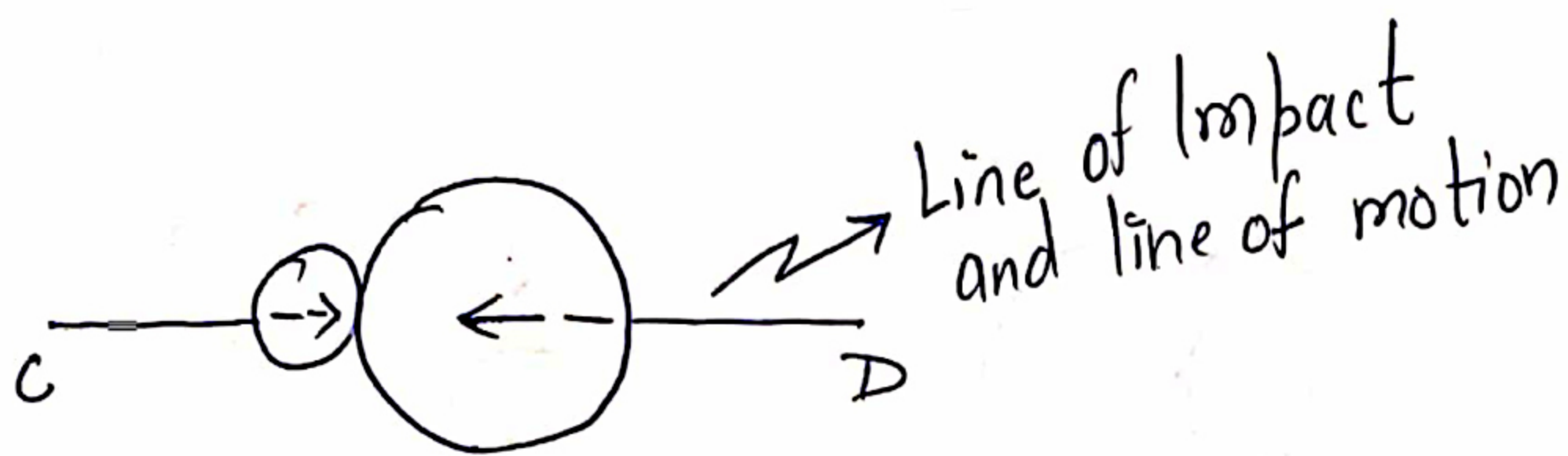
Classification of collisions

on basis of line of impact

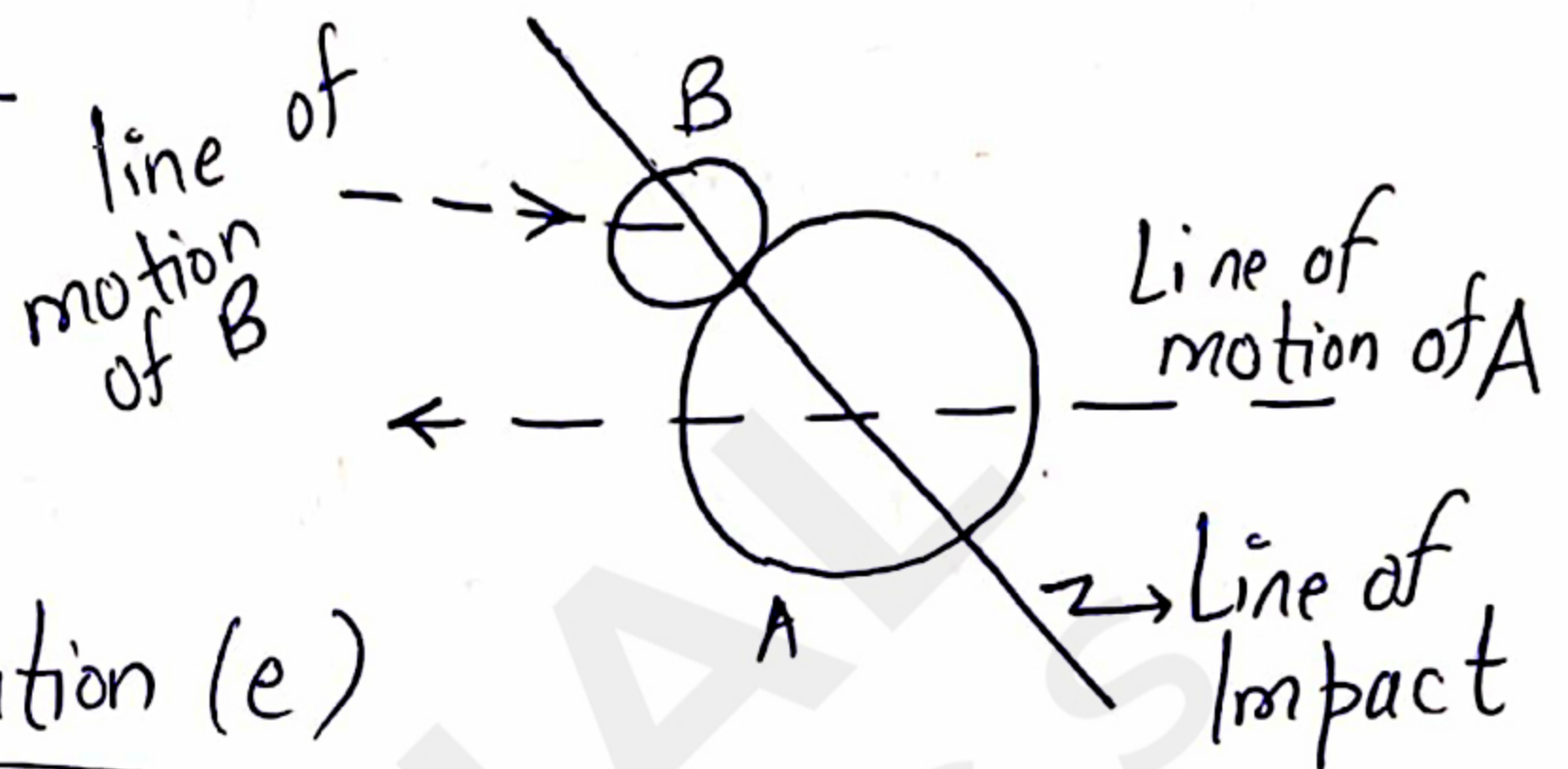
- Head-on collision
- Oblique collision

on the basis of energy

- elastic
(no mechanical energy lost)
- inelastic
(energy lost as heat)



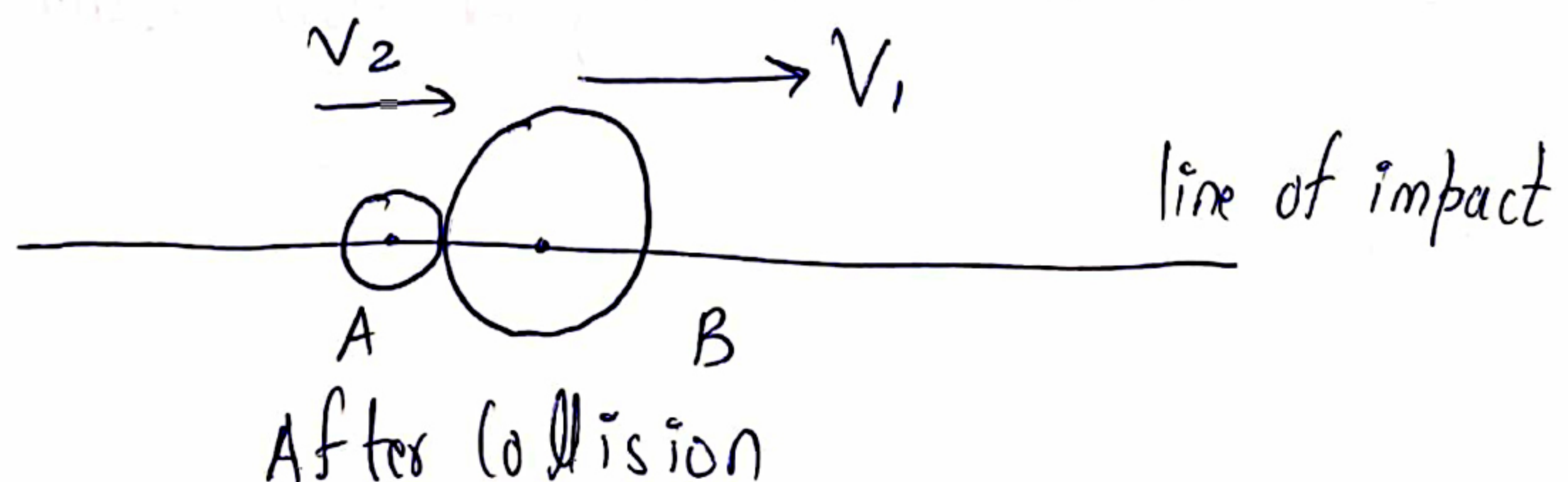
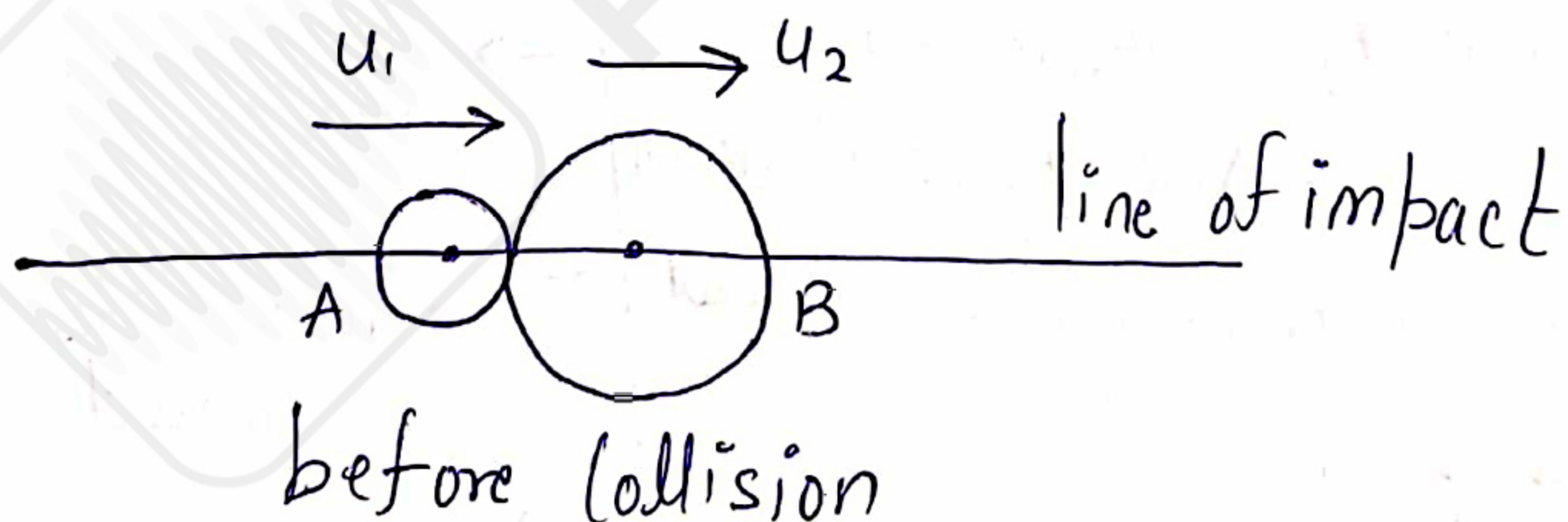
Head-on Collision

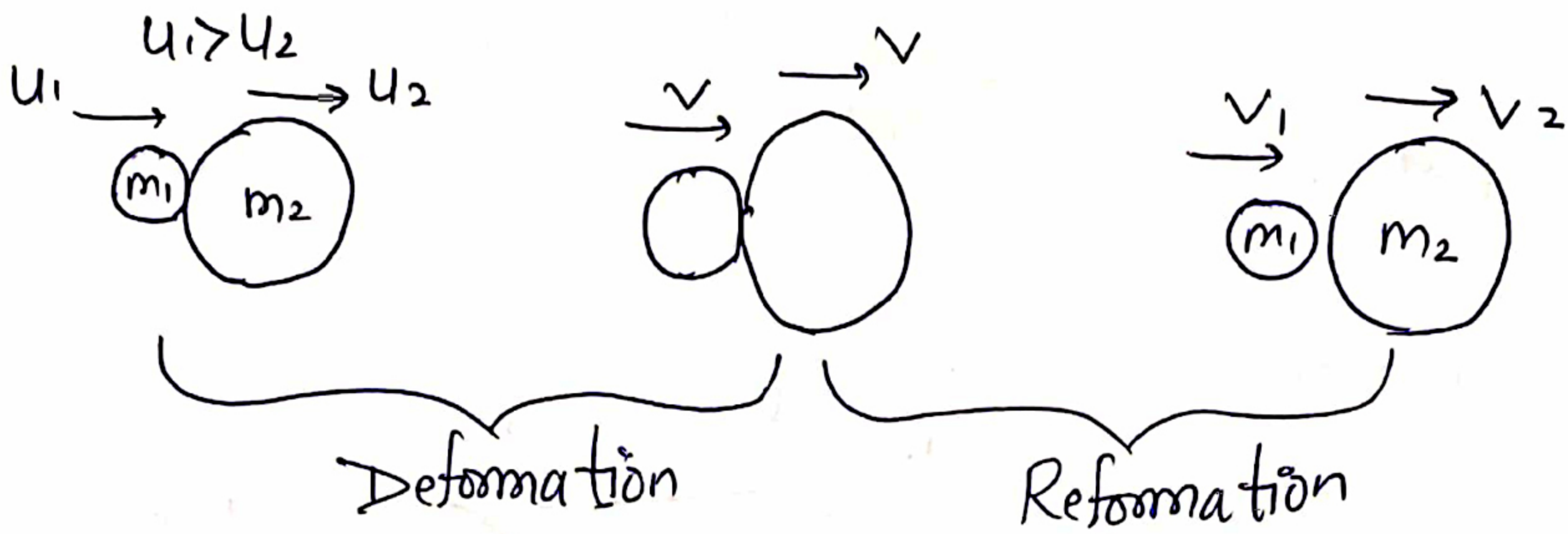


Oblique collision

Coefficient of Restitution (e)

$$e = \frac{\text{Velocity of Separation along line of Impact}}{\text{Velocity of approach along line of Impact}}$$



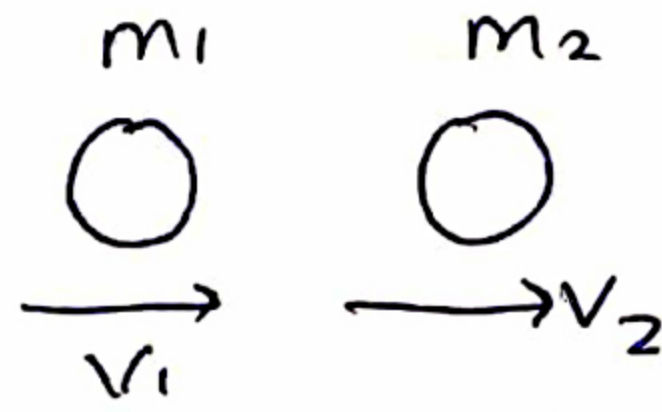
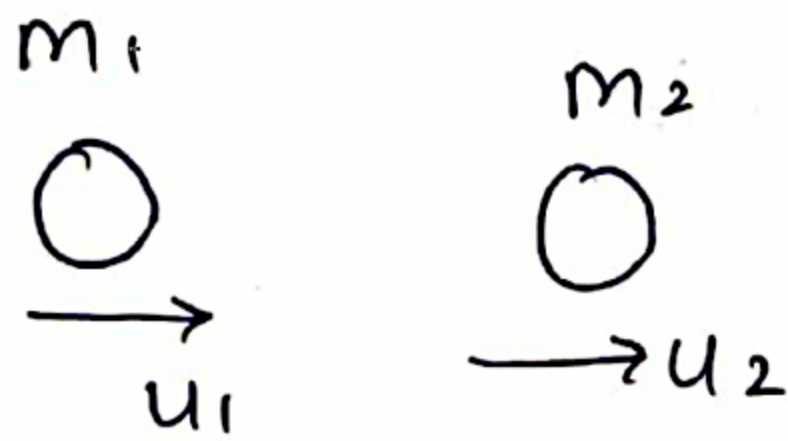


$\therefore F_{\text{ext}} = 0$, Hence net momentum is conserved

if $\underline{e = 1} \rightarrow$ perfect elastic collision

$\underline{e = 0} \rightarrow$ perfect Inelastic collision

<u>perfect elastic collision</u>	<u>perfect Inelastic collision</u>
$\rightarrow$ Velocity of separation = Velocity of approach	$\rightarrow$ Velocity of separation = 0
$\rightarrow$ Kinetic energy is conserved	$\rightarrow$ Kinetic energy is not conserved
$\rightarrow e = 1$	$\rightarrow e = 0$
$\rightarrow$ Momentum conserved	$\rightarrow$ momentum conserved



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2} \right) u_1 + \frac{(1+e)m_2}{(m_1 + m_2)} u_2$$

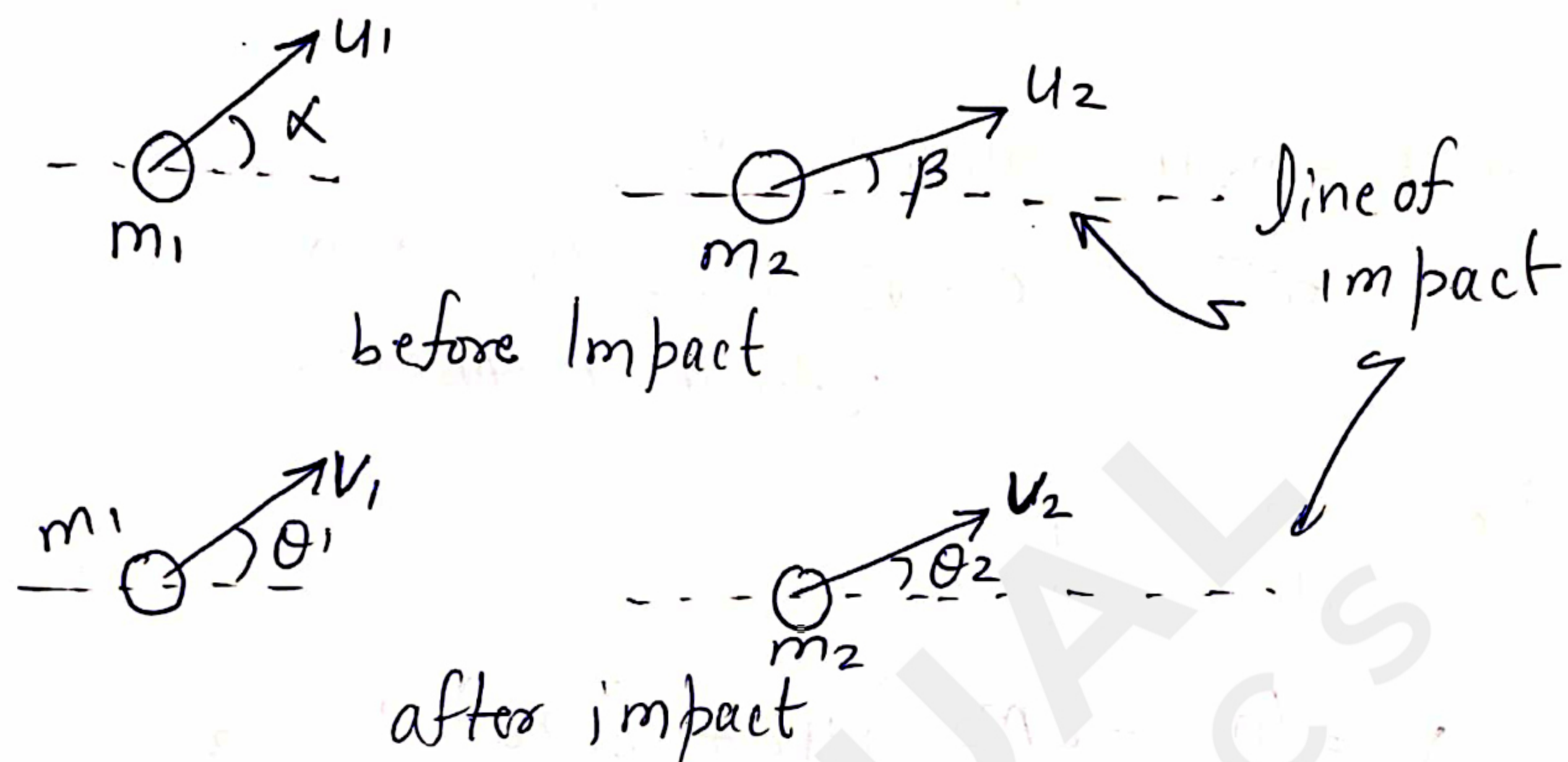
$$v_2 = \left(\frac{(1+e)m_1}{(m_1 + m_2)} \right) u_1 + \frac{(m_2 - em_1)u_2}{(m_2 + m_1)}$$

for elastic collision, $e = 1$

$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2 u_2}{(m_1 + m_2)}$$

$$v_2 = \frac{2m_1 u_1}{(m_1 + m_2)} + \frac{(m_2 - m_1)u_2}{(m_1 + m_2)}$$

Collision in Two-Dimensions (oblique)



Momentum is conserved:

so in x-direction:

$$m_1 u_1 \cos \alpha + m_2 u_2 \cos \beta = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$$

In y-direction:

$$m_1 u_1 \sin \alpha + m_2 u_2 \sin \beta = m_1 v_1 \sin \theta_1 + m_2 v_2 \sin \theta_2$$

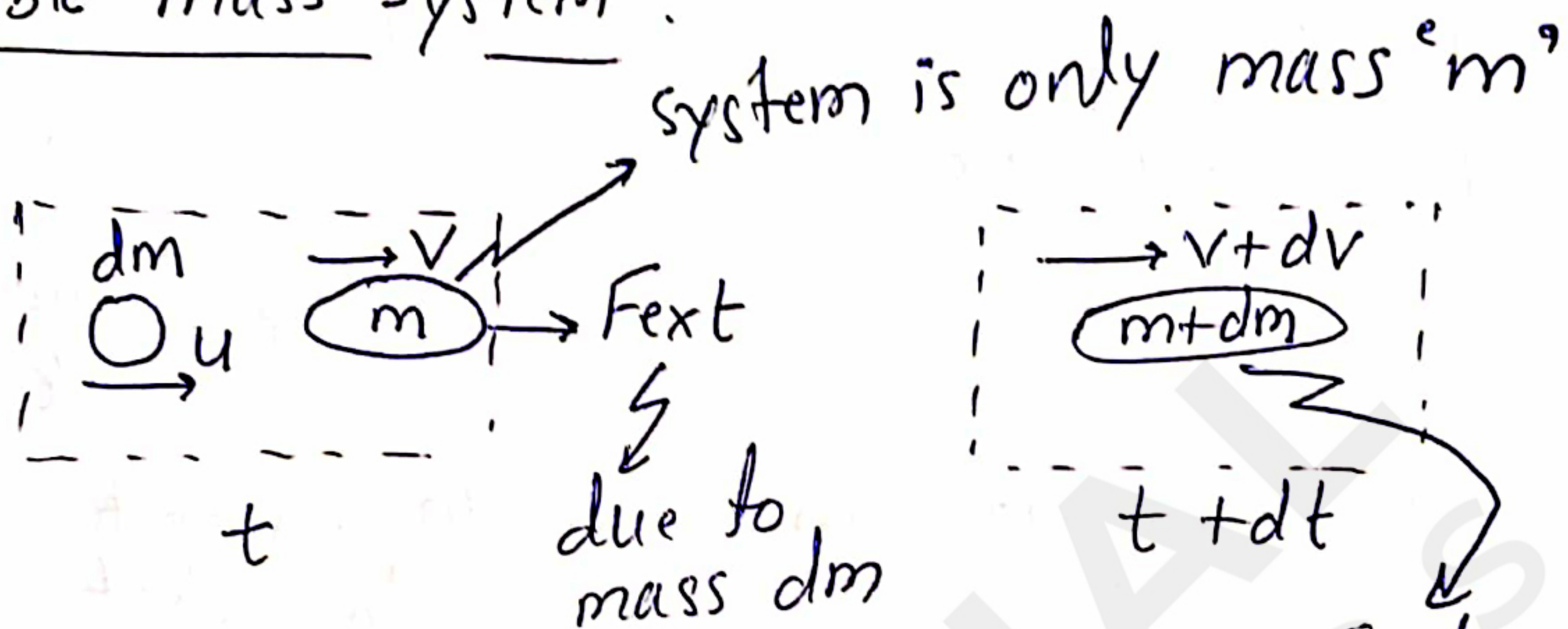
$$e = \frac{v_2 \cos \theta_2 - v_1 \cos \theta_1}{u_1 \cos \alpha - u_2 \cos \beta}$$

if $m_1 = m_2$

$$\boxed{\theta_1 + \theta_2 = 90^\circ}$$

→ Angle between the objects after collision

Variable mass system :



$$(F_{ext}) dt = (m + dm)(v + dv) - mv - dm u$$

Impulse on system

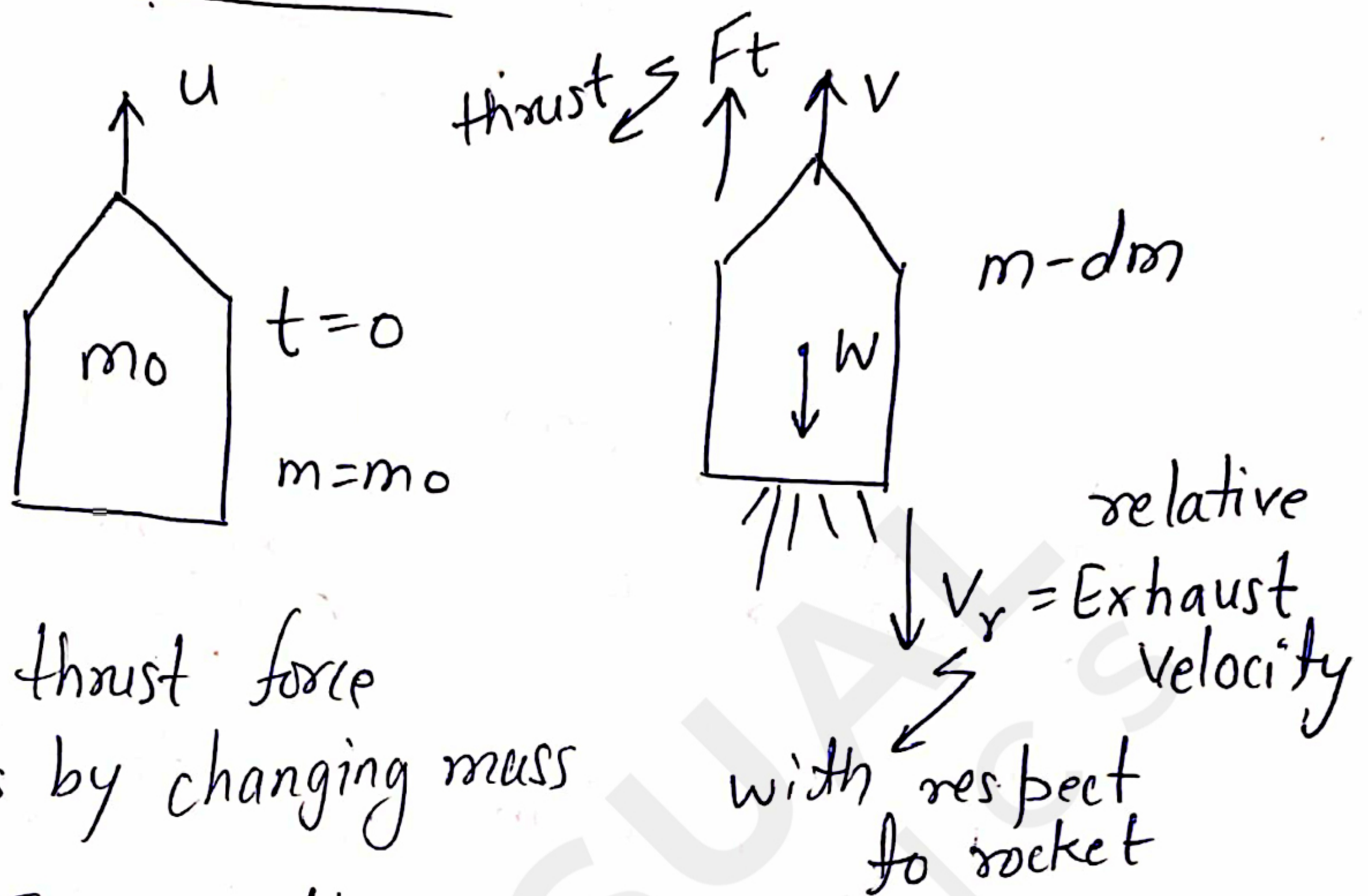
$$\Rightarrow (F_{ext}) dt = m dv - (u - v) dm$$

relative velocity $v_{rel} = u - v$

thrust [force due to change of mass]

$$F_{ext} + \left(v_{rel} \frac{dm}{dt} \right) = m \frac{dv}{dt}$$

Rocket propulsion



now thrust force is by changing mass

$$F_t = v_r \left(\frac{dm}{dt} \right)$$

now net force $F_{\text{net}} = F_t - W$

$$\Rightarrow F_{\text{net}} = v_r \left(\frac{dm}{dt} \right) - mg$$

$$a = \frac{F_{\text{net}}}{m} = \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{v_r}{m} \left(\frac{dm}{dt} \right) - g$$

$$\int_u^v dv = v_r \int_{m_0}^m \frac{dm}{m} - g \int dt$$

$$v - u = -v_r \ln\left(\frac{m_0}{m}\right) - g t$$

$$\Rightarrow \boxed{v = u - g t - v_r \ln\left(\frac{m_0}{m}\right)}$$