VISUAL PHYSICS

SHORT NOTES

CHAPTER

Circular Motion

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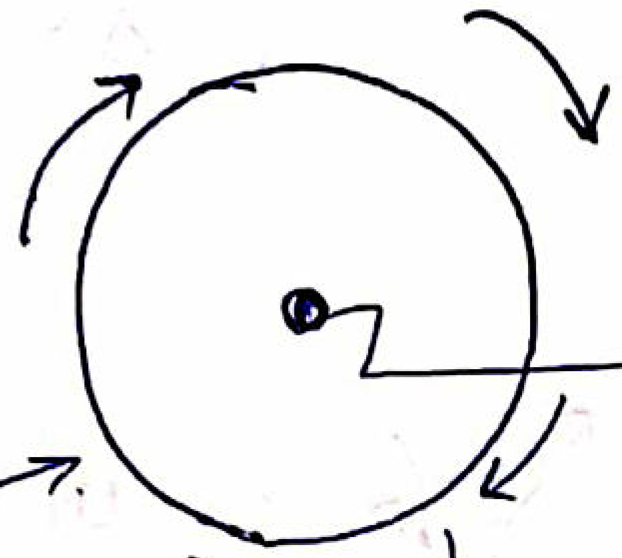
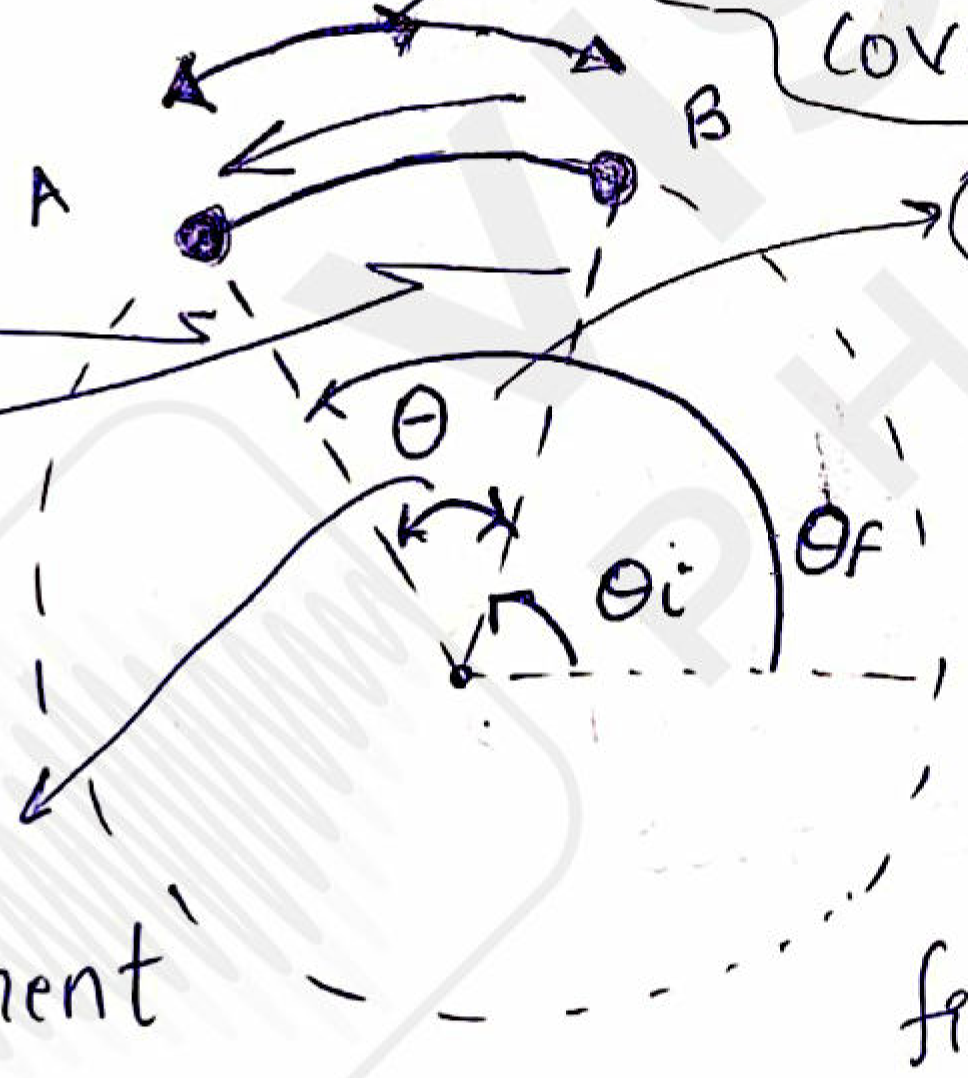
CIRCULAR MOTION



→ Motion, with no translational motion → Circular motion



$\theta_f - \theta_i = \theta$

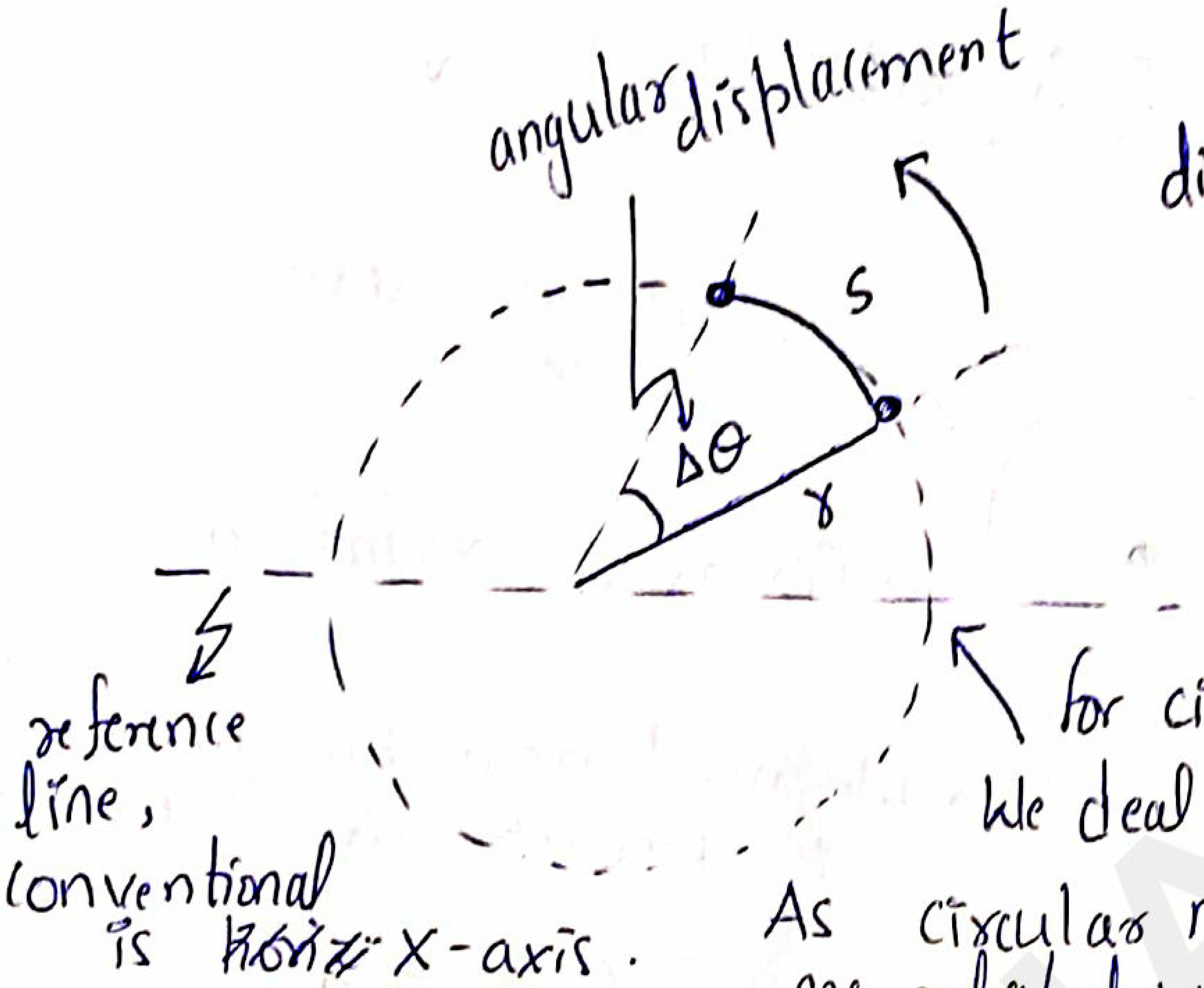
θ_f → final angular position

θ_i → initial angular position

$\boxed{\text{SI unit} \rightarrow \text{radian}}$

$\boxed{180^\circ \rightarrow \pi \text{ radians}}$

Here we are considering circular motion of a particle only.



dimensionless

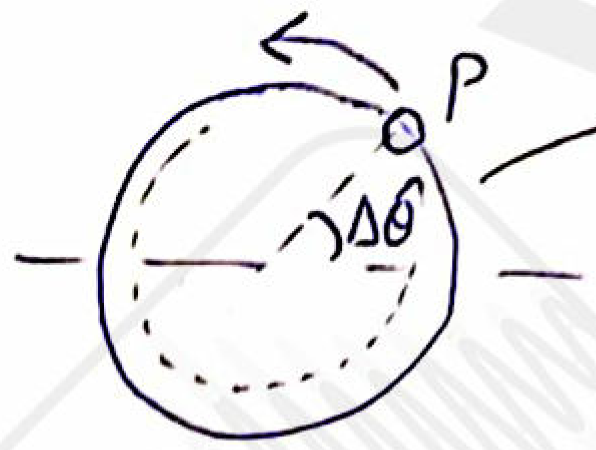
$$\Delta\theta = \frac{s}{r} \text{ radians}$$

for circular motion we deal ~~th~~ with '<u>radians</u>'

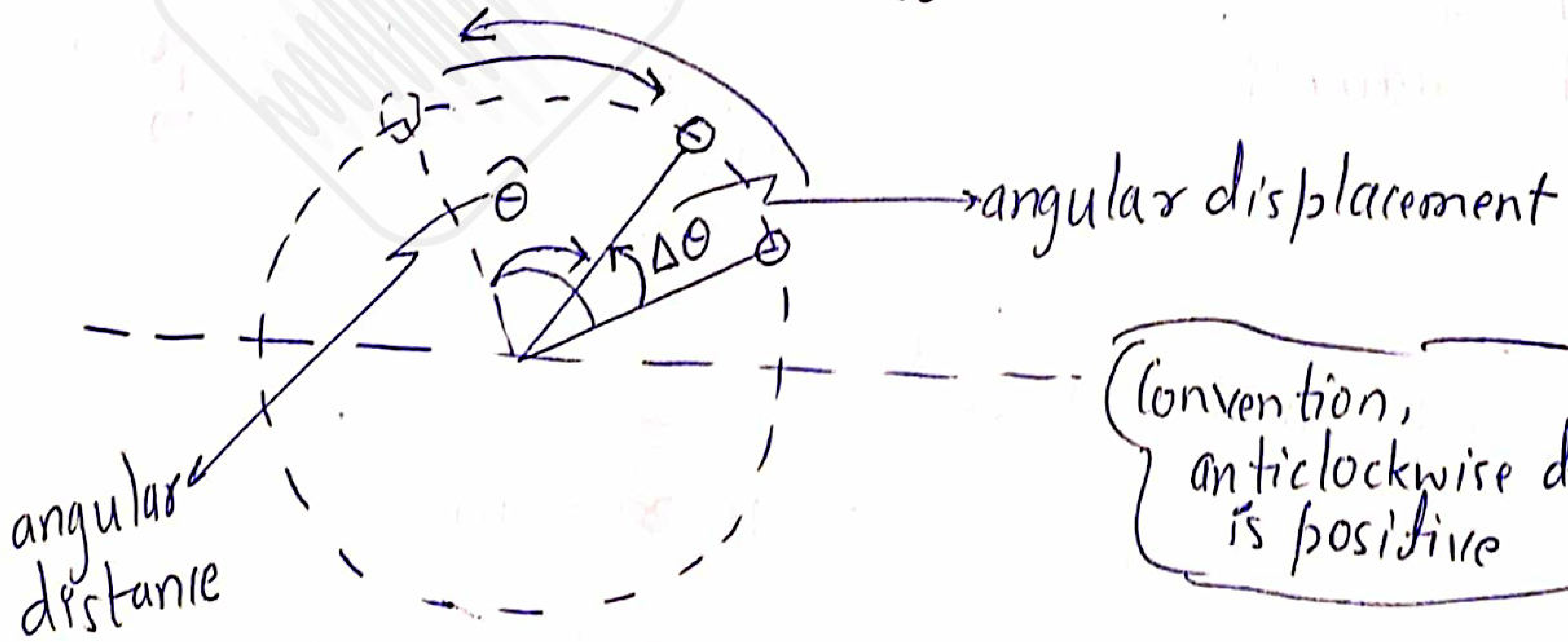
As circular motion parameters are related with particle translational sense.

$\Rightarrow$ for complete circle, angular displacement $\neq 0$

means:

☆ 

so net angular displacement when particle reach p again $= 2\pi + \Delta\theta$



Convention, anticlockwise direction is positive

For $\Delta\Theta \longrightarrow$ Vector?

$\swarrow$ direction $\quad \searrow$ magnitude

But

$\Delta\theta \longrightarrow$ for large value $\Rightarrow$ it is not Vector

$\Delta\theta \longrightarrow$ for small value $\Rightarrow$ it is vector

(As they obey cumulative law of vector addition

→ So In general angular displacement is not Vector.

So if '$\Delta\Theta$' in 'Δt' time.

dimensional formula

average angular velocity $= \dfrac{\Delta\theta}{\Delta t} = \Delta\omega_{avg}$. rad/s $[T^{-1}]$

average angular speed $= \dfrac{\widehat{\Theta}}{\Delta t} = \widehat{\omega}_{avg}$ rad/s $[T^{-1}]$

$\widehat{\omega}_{avg} \neq |\omega_{avg}|$ $\quad \Delta\Theta \longrightarrow$ magnitude, direction $\neq$ Vector for large $\Delta\Theta$

$\widehat{\theta} \neq |\Delta\theta|$

same $\Delta\omega_{avg} \neq$ Vector

$\Delta t \to 0$

$\Rightarrow \dfrac{d\theta}{dt} = \omega$ → instantaneous angular velocity

$\dfrac{d\hat{\theta}}{dt} = \omega_s$ → $\omega_s = |\omega|$

as $d\hat{\theta} = |d\theta|$

→ instantaneous angular speed

$d\theta \to 0$

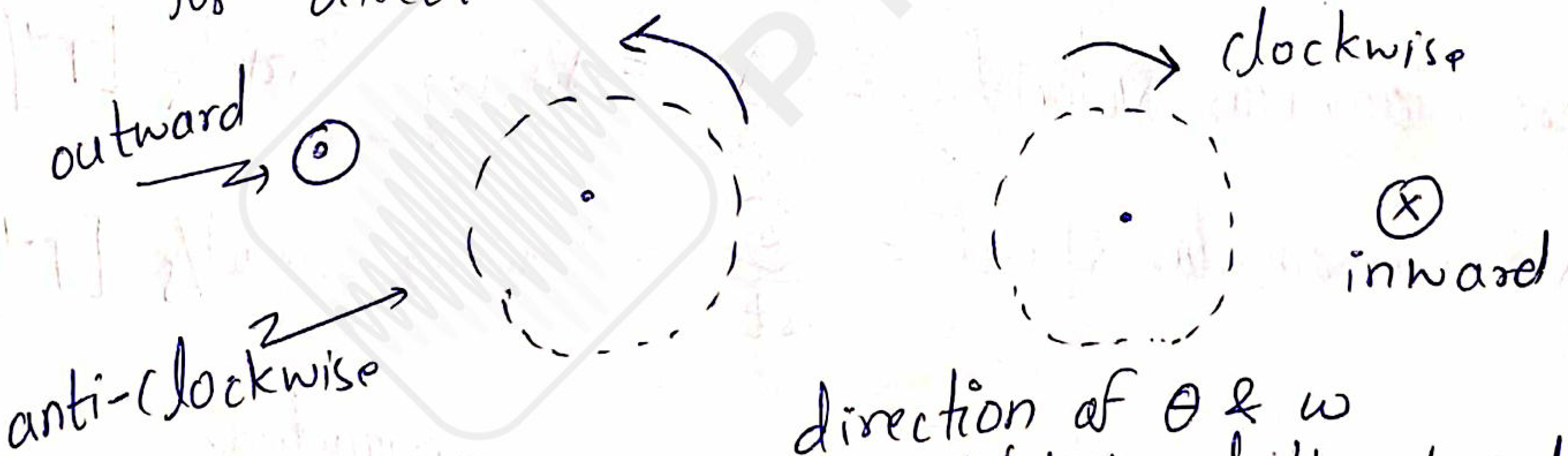
so, $d\theta \to$ vector

so $\omega \to$ vector

$T \to$ Time period of revolution → Time to cover 2π

rev/sec $\to (2\pi)$ rad/s $\Rightarrow \boxed{T = \dfrac{2\pi}{\omega}}$

for direction of $d\theta$ & ω



direction of θ & ω using right hand thumb rule.

$\Rightarrow$ moving right hand in direction of motion, thumb gives direction

$\Delta\omega$ → change in angular velocity, Δt → in Δt time

$$\boxed{\alpha_{avg} = \frac{\Delta\omega}{\Delta t}}$$ rad/s² $[T^{-2}]$ → dimension

α_{avg} → average angular acceleration

direction of α

if ω increasing → direction is same as of ω direction

if ω is decreasing → direction is opposite as of ω

now $\Delta t \to 0$, $\Delta\omega \to d\omega$

α_{avg} → instantaneous angular acceleration $= \alpha$

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

Angular equations:

As circular and translational motions are so much similar

like $\Delta x \leftrightarrow \Delta\theta$, $v \leftrightarrow \omega$, $a \leftrightarrow \alpha$

similar

$V = u + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = ut + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$V^2 = u^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$
$\Delta x = V_{avg}\, t$, $\Delta V = a_{avg}\, t$	$\Delta\theta = \omega_{avg}\, t$, $\Delta\omega = \alpha_{avg}\, \Delta t$

$\omega \rightarrow$ final angular velocity

$\omega_0 \rightarrow$ initial angular velocity

As $\theta = \frac{s}{r} \Rightarrow s = r\,\theta$ → angular displacement

now $v = \frac{ds}{dt} = \frac{d}{dt}(r\,\theta)$

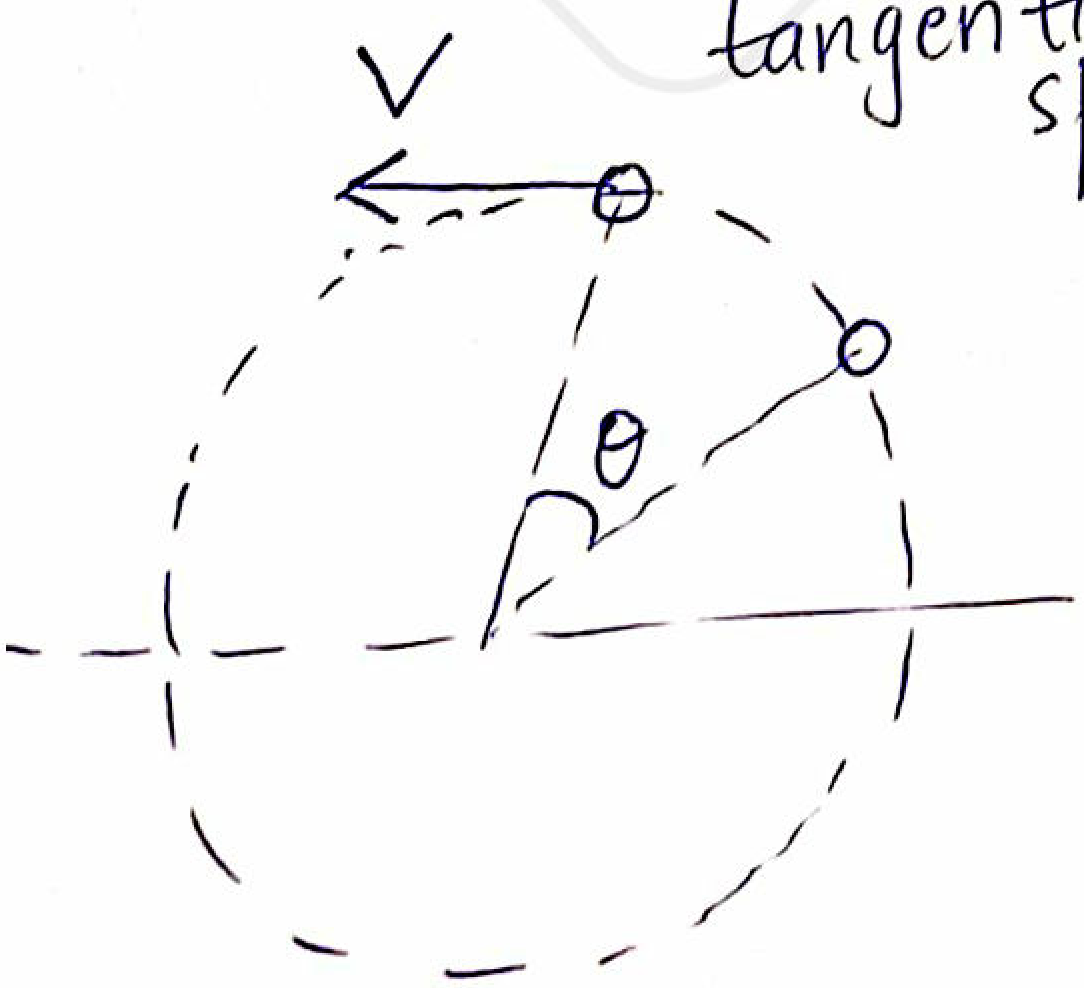
tangential speed

$\Rightarrow$ if $r \rightarrow$ constant

$v = r\frac{d(\theta)}{dt}$

$\boxed{V = r\omega}$

$\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$ ↳ for direction



$\omega \rightarrow$ angular velocity

Constant

uniform circular motion

$\Rightarrow \omega =$ Constant

$\Rightarrow |\vec{v}| =$ speed $=$ Constant

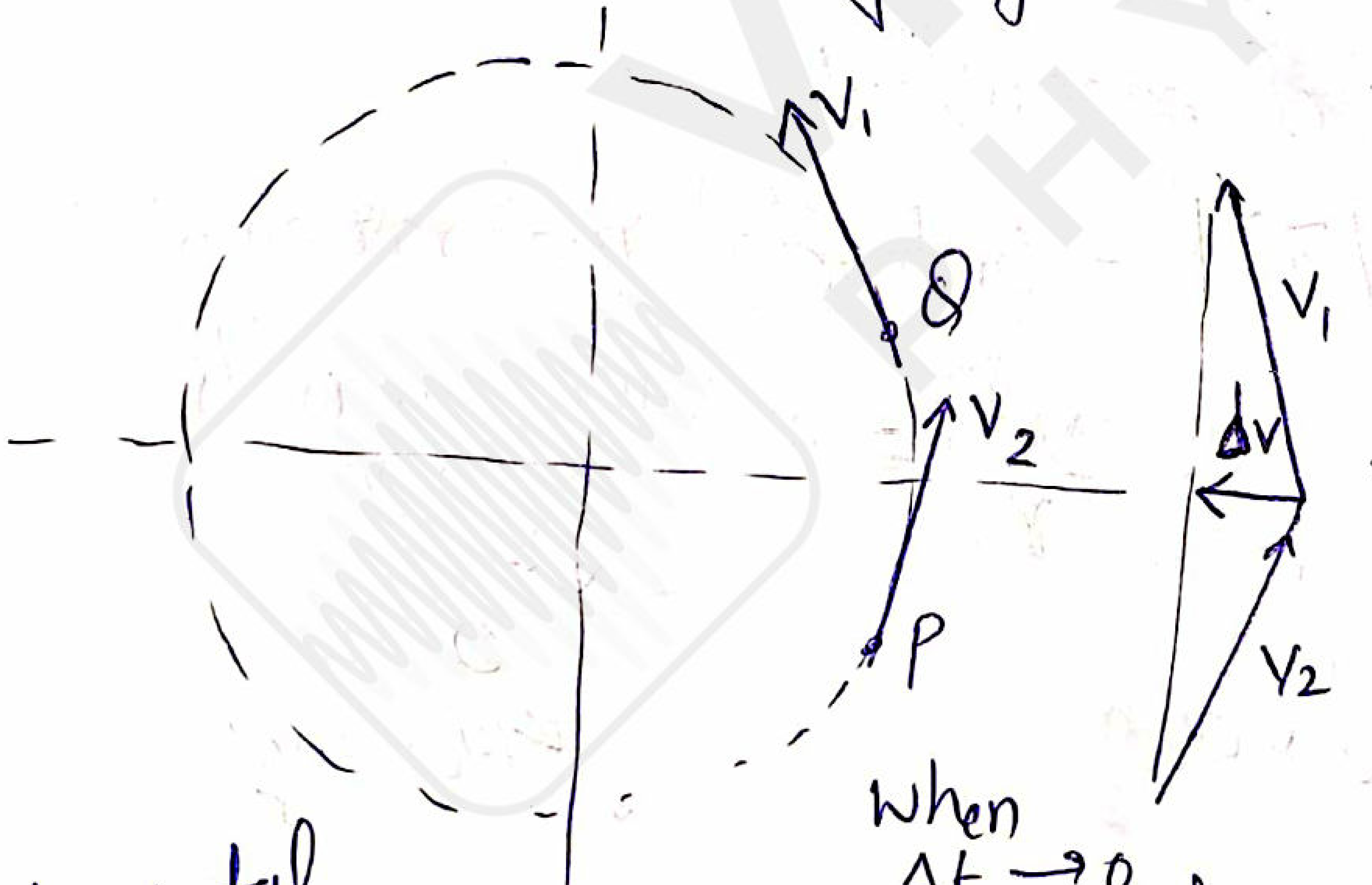
tangential

not Constant

non-uniform circular motion

$\omega \neq$ Constant

as direction is changing $\Rightarrow \vec{v} \neq$ constant



$\Rightarrow dV \rightarrow$ change in velocity direction is towards centre

when $\Delta t \rightarrow 0$

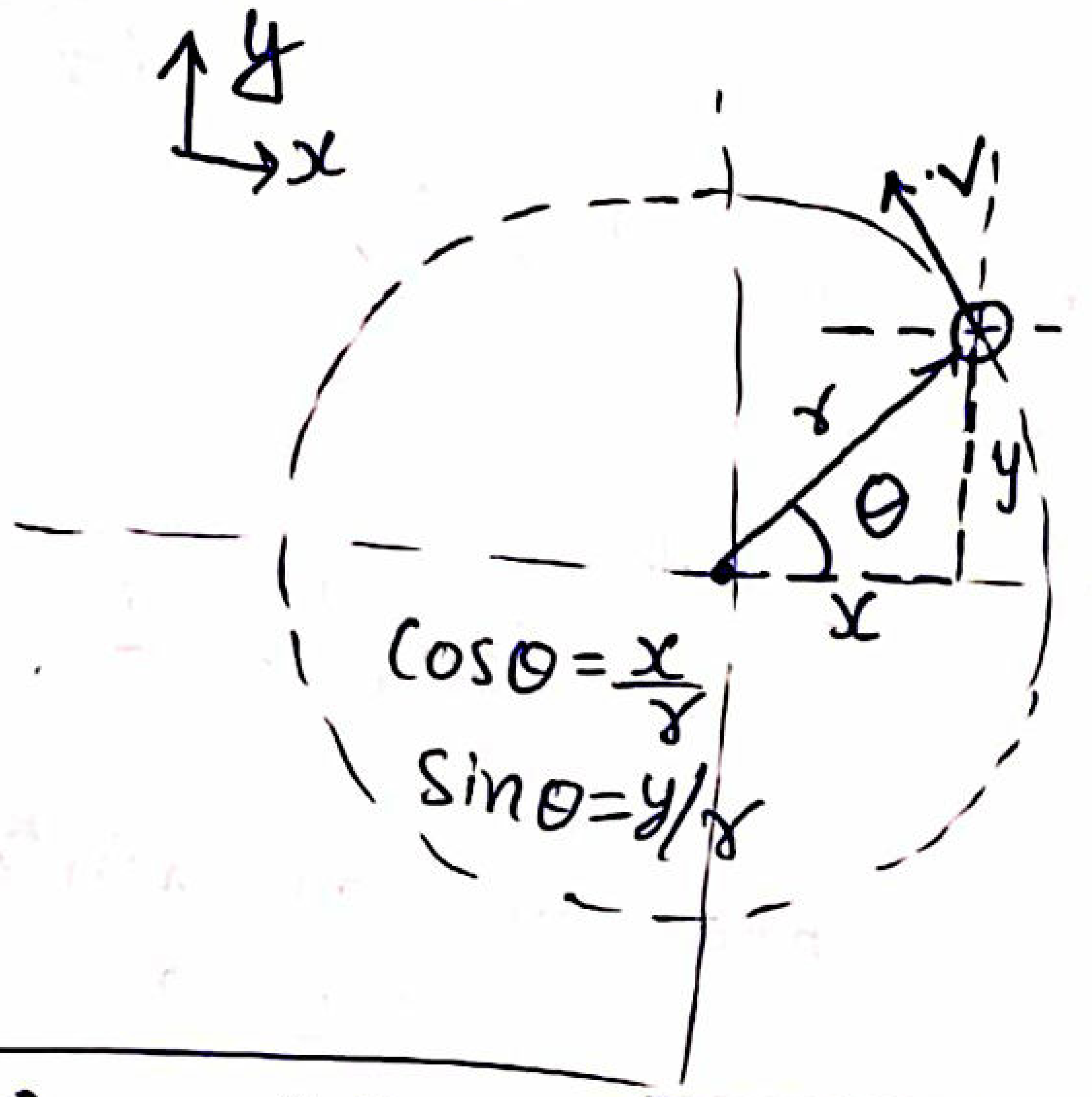
$\Delta V \rightarrow dV$

centri-petal

towards centre

means centripetal acceleration

As acceleration is directed towards centre.



$$\Rightarrow \vec{v} = -v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}$$

in component form.

$$\vec{v} = -v\left(\frac{y}{r}\right)\hat{i} + v\left(\frac{x}{r}\right)\hat{j}$$

$$\vec{v} = \frac{v}{r}\left(-y\hat{i} + x\hat{j}\right)$$

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{v}{r}\left(-y\hat{i} + x\hat{j}\right)\right]$$

$$\vec{a_c} = \frac{v}{r}\left[-\underbrace{\frac{dy}{dt}}_{v\cos\theta}\hat{i} + \underbrace{\frac{dx}{dt}}_{-v\sin\theta}\hat{j}\right]$$

$$\boxed{\vec{a_c} = \frac{v^2}{r}\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]}$$

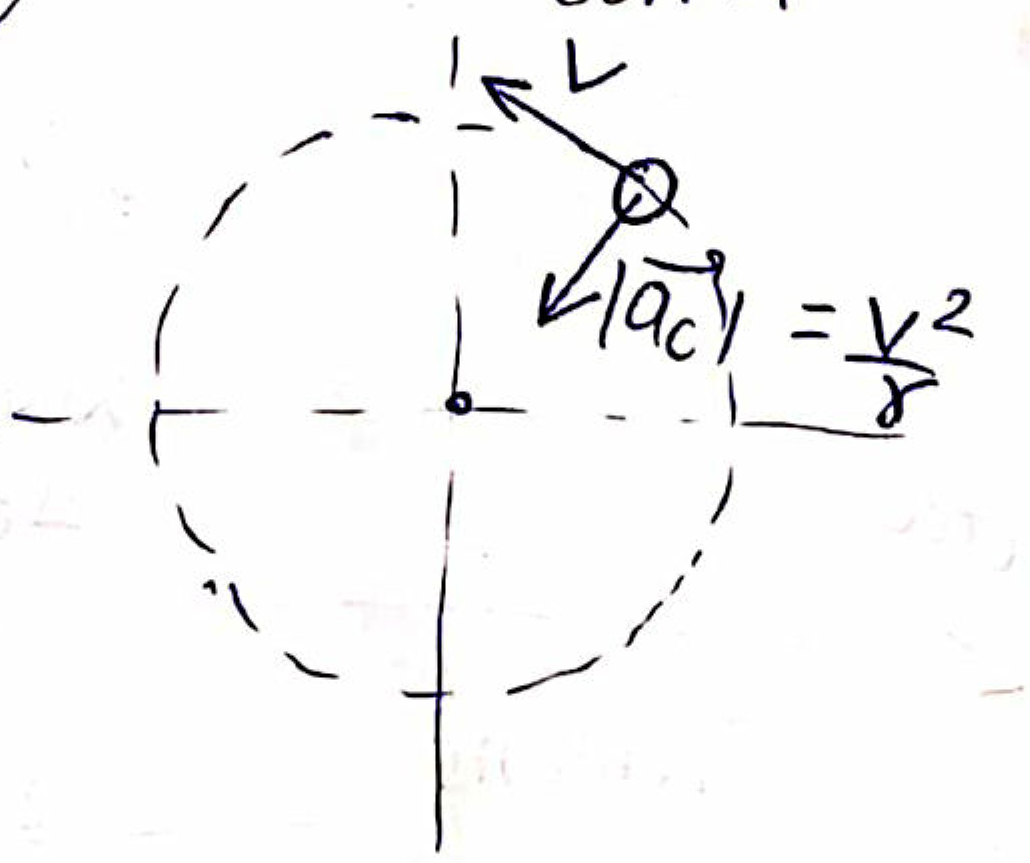
$$\Rightarrow |\vec{a_c}| = a = \frac{v^2}{r}$$

as $|-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}| = 1$

so in terms of magnitude

$|\vec{a_c}|$ = centripetal acceleration = $\frac{v^2}{r}$ = always towards centre

always towards radial direction



for non-uniform circular motion

$v = \omega r$ → angular velocity

tangential velocity $\Rightarrow \frac{dv}{dt} = |\vec{a_t}| = \frac{d}{dt}(\omega r)$

$\Rightarrow |\vec{a_t}| = r\alpha$

tangential acceleration

angular acceleratio

so a_{net} → two direction

centripetal acceleration

tangential acceleratio

responsible to change direction of velocity

$\frac{v^2}{r}$

responsible for change in tangential velocity magnitu

v ← change

$$|\vec{a}_{net}| = \sqrt{|a_t|^2 + |a_c|^2}$$

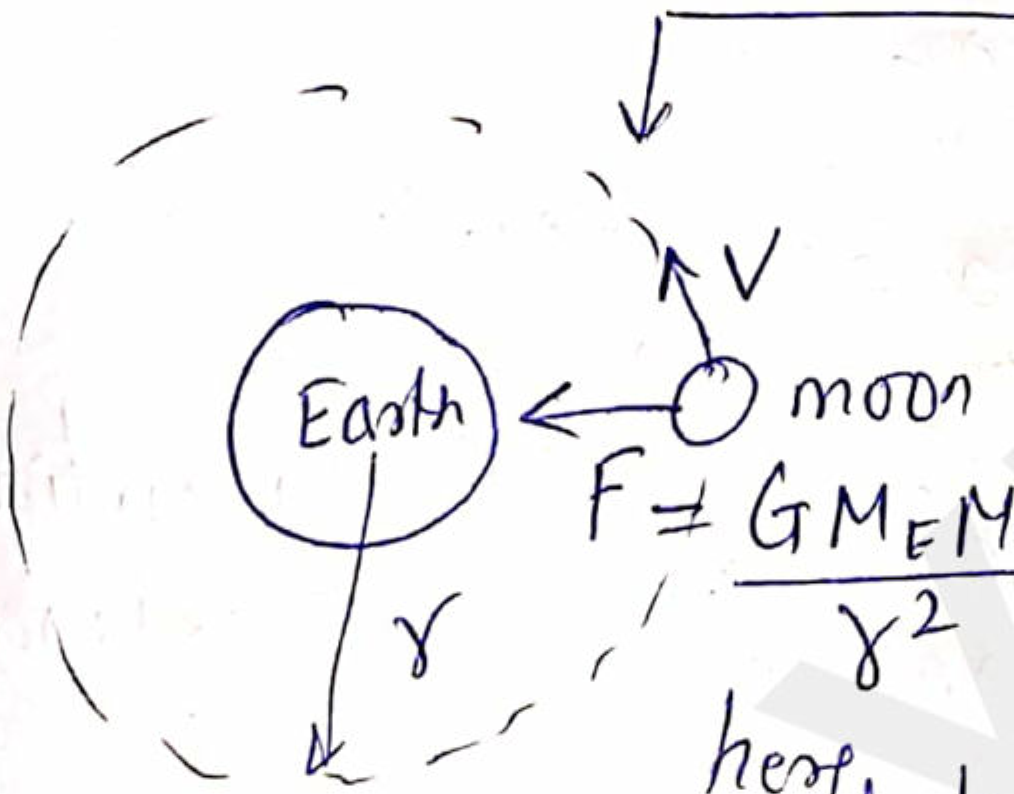
$$= \sqrt{(\alpha r)^2 + \left(\frac{v^2}{r}\right)^2}$$

from $\boxed{\vec{F} = m\vec{a}}$

acceleration is there because of Some external force

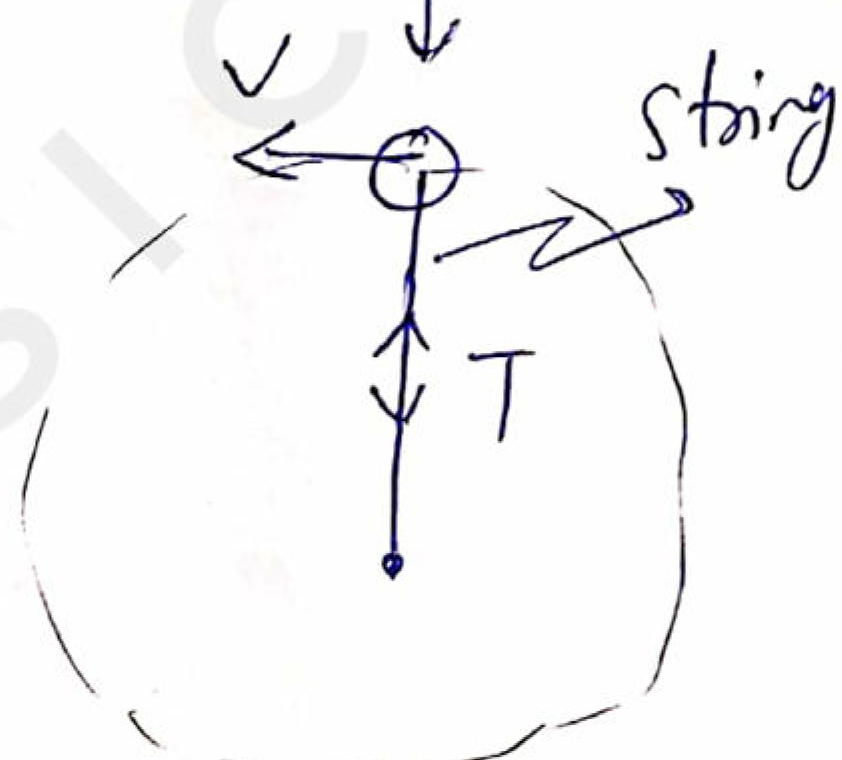
so if there is centripetal acceleration

there should be centripetal force → that provides acceleration



$$F = \frac{G M_E M_m}{\gamma^2} = m\vec{a_c}$$

here gravitational force ↳ provide Centripetal force



Here tension in string provide centripetal force

⇒ if force acts perpendicular to motion → It tends to change direction of motion.

Centrifugal force

away from centre

force radialy outwards

It is psuedo force

as actualy the force is not there.

It is there in the non-inertial frame of reference.

As it is psuedo force → No reaction pair

So in the rotation frame of reference

we add an radial outward force

$\frac{V^2}{r}$ acceleration

Same as centripetal magnitude

centrifugal acceleration

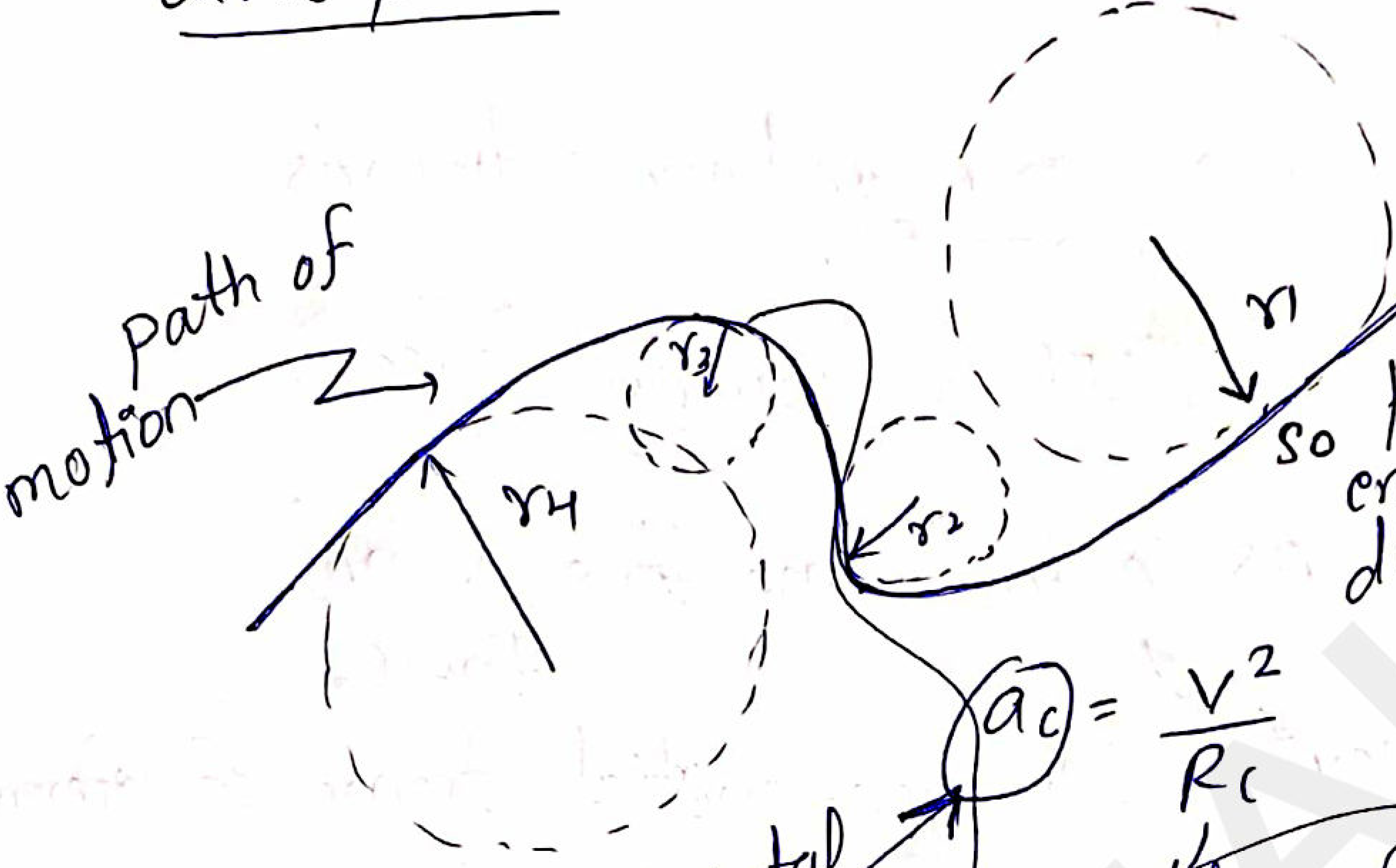
when we work in non-inertial frame.

Never exists in inertial frame

Centrifugal force and centripetal force do not form action-reaction pair.

it is there from different frame of reference.

Curve path



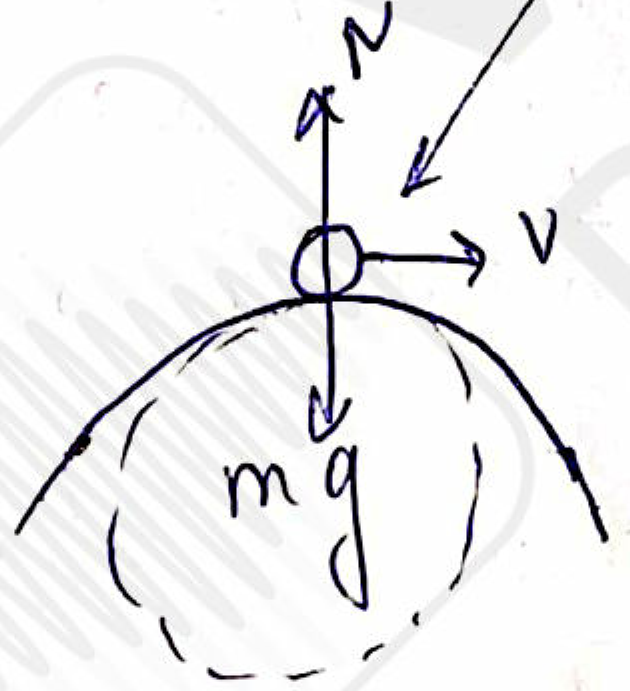
So particle encounter differen R_c at different point

$$a_c = \frac{v^2}{R_c}$$

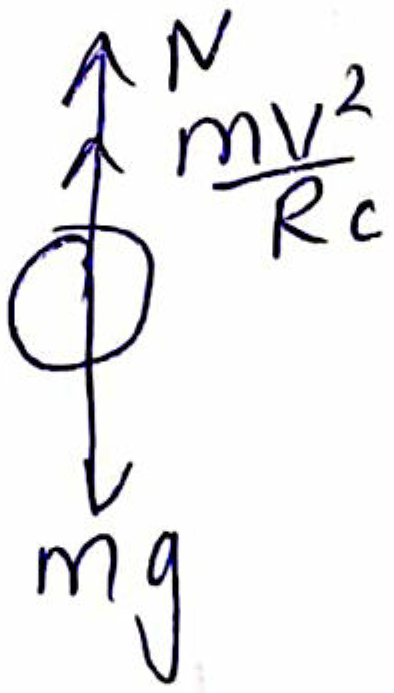
radius of curvatur

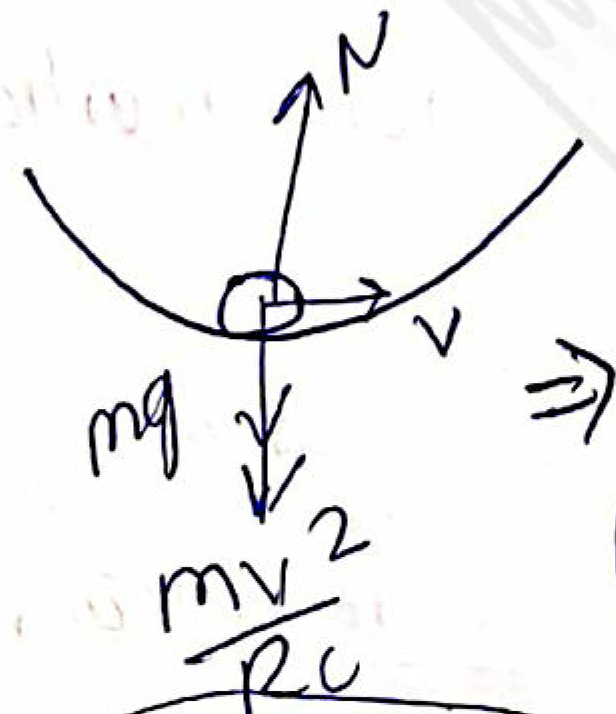
different at different point of path

So centripetal acceleration is different at different point



from non-inertial frame





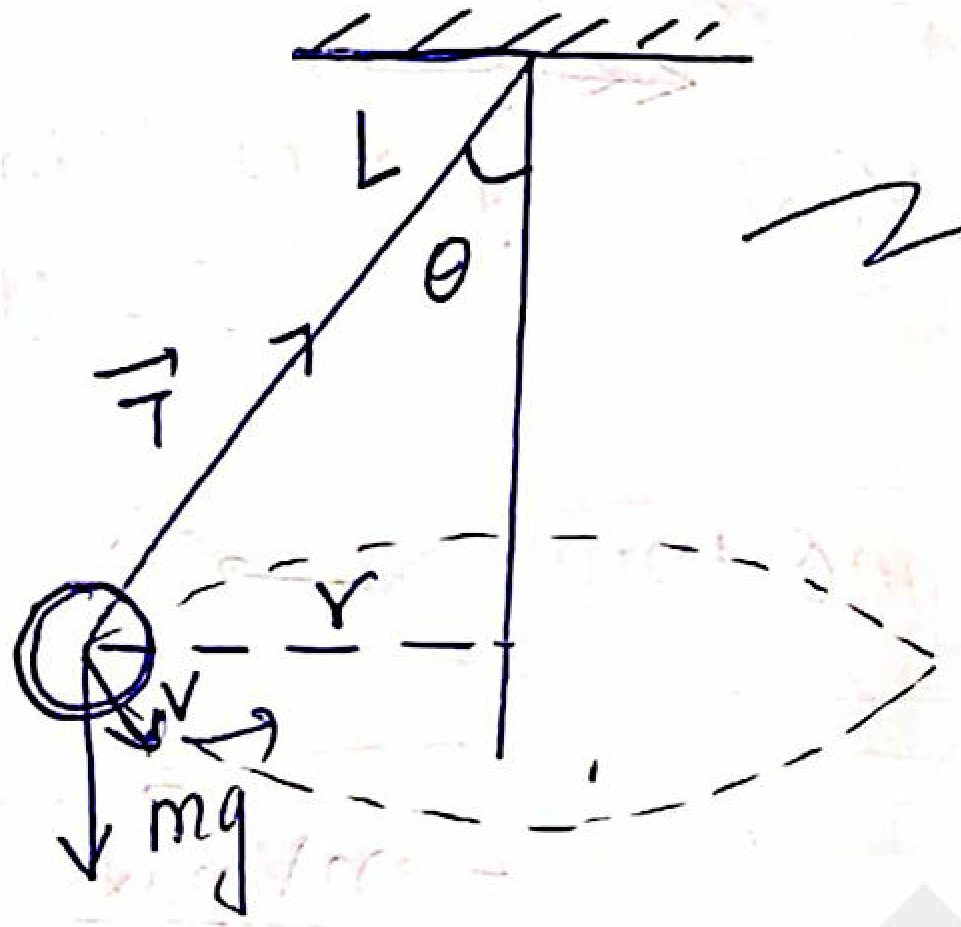
$$\Rightarrow N = mg - \frac{mv^2}{R_c}$$

more R_c → more normal reaction

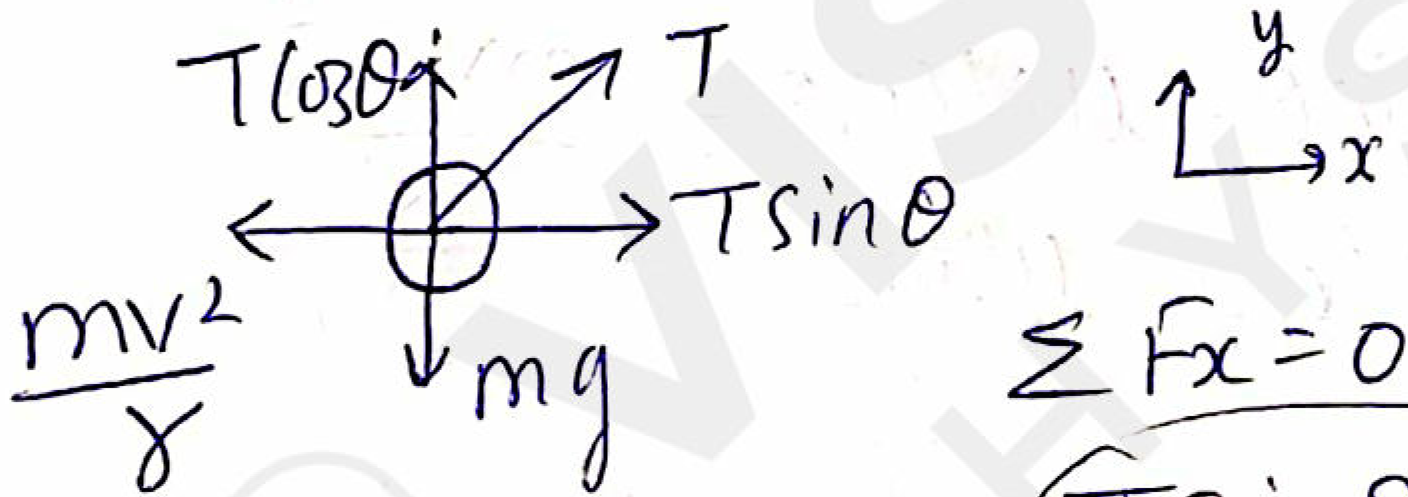
$$N = mg + \frac{mv^2}{R_c}$$

more R less Normal reaction

⟶ Conical pendulum



from → non inertial frame



$\Sigma F_x = 0$

$$T\sin\theta = \frac{mv^2}{r}$$

& $\Sigma F_y = 0$

$$\Rightarrow mg = T\cos\theta$$

$$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$$

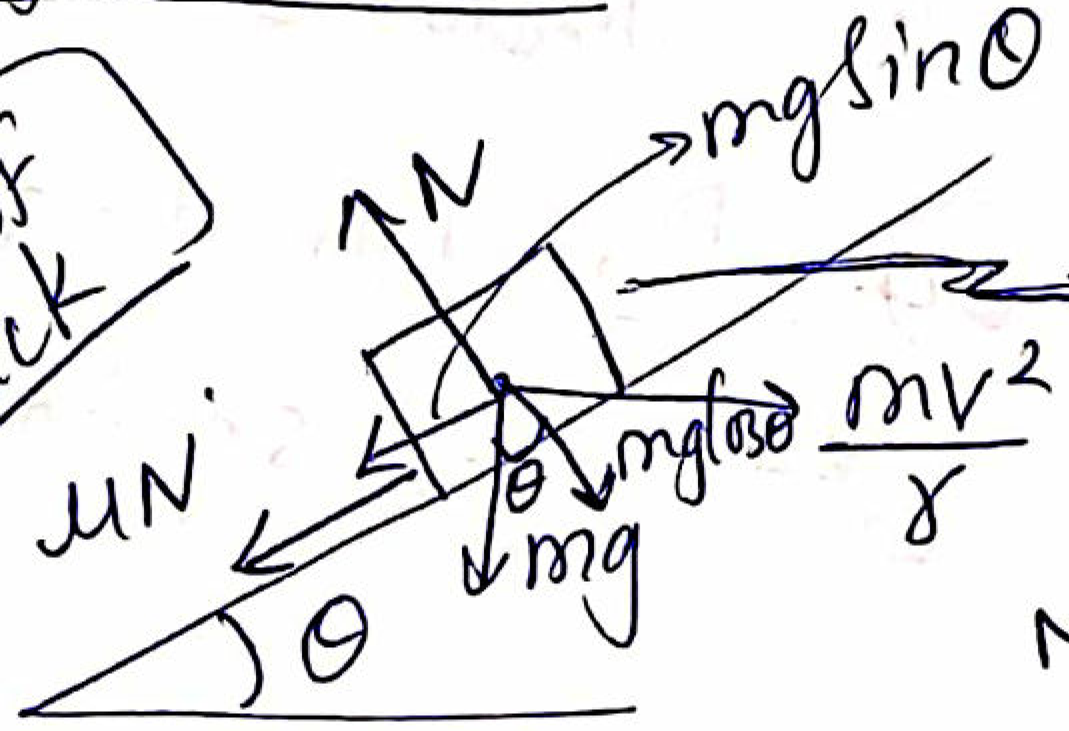
$$\Rightarrow v = \sqrt{rg\tan\theta}$$

independent on mass of Pendulum bob

Turning of vehicle

designed speed of track

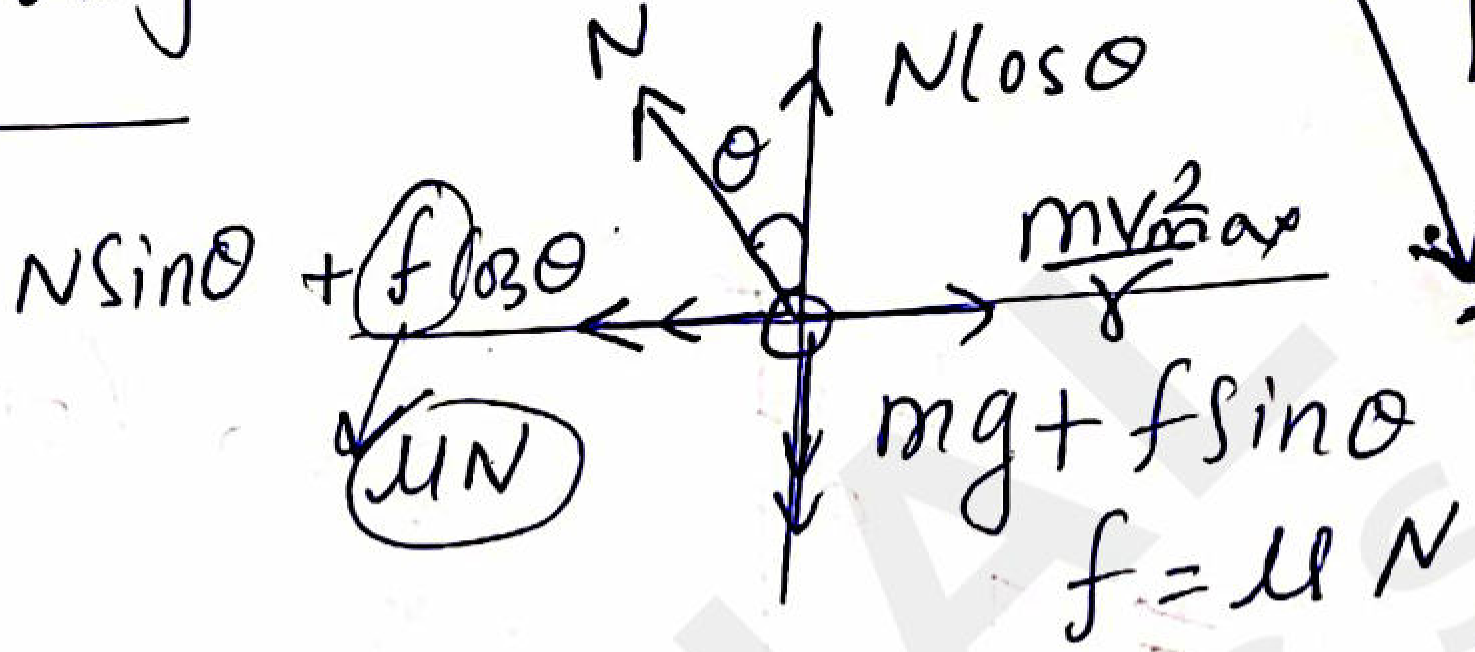
V_{max} → max speed



from non-inertial frame

Vehicle tends to move out

friction act down along the track



As

$\Sigma F_y = 0$

$\Rightarrow N\cos\theta = \mu N\sin\theta + mg \longrightarrow (i)$

an $\Sigma F_x = 0$

$\Rightarrow N\sin\theta + \mu N\cos\theta = \dfrac{mV_{max}^2}{R} \longrightarrow (ii)$

(ii) ÷ (i)

$$\frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu\sin\theta} = \frac{\frac{mV_{max}^2}{R}}{mg}$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{V_{max}^2}{Rg}$$

$$\Rightarrow \boxed{\frac{V_{max}^2}{Rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}}$$

When $\theta = 0$, road horizontal

$$\boxed{\frac{V_{max}^2}{Rg} = \mu}$$

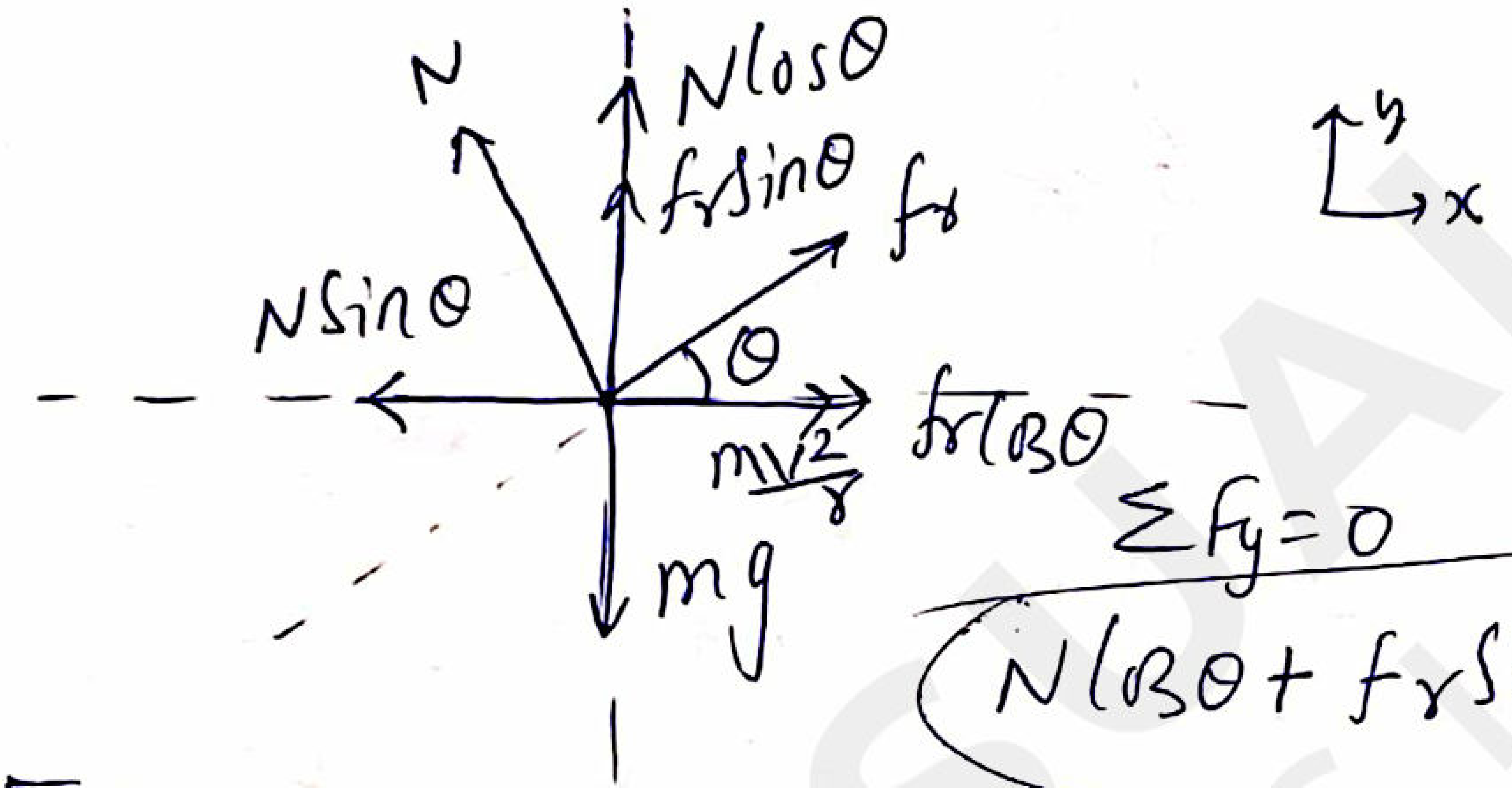
independent of mass

Speed less than designed speed.

in this case car tends to move down

↓

friction -upwards along the track



$\Sigma f_y = 0$

$$N\cos\theta + fr\sin\theta = mg \quad \text{(i)}$$

$\Sigma f_x = 0$

$$N\sin\theta = \frac{mv^2}{r} + fr\cos\theta \quad \text{(ii)}$$

on solving (i) & (ii)

$$\boxed{\frac{v^2}{rg} = \frac{\tan\theta - \mu}{1 + \mu\tan\theta}}$$