



VISUAL
PHYSICS

SHORT NOTES

C H A P T E R

Capacitors & Dielectrics



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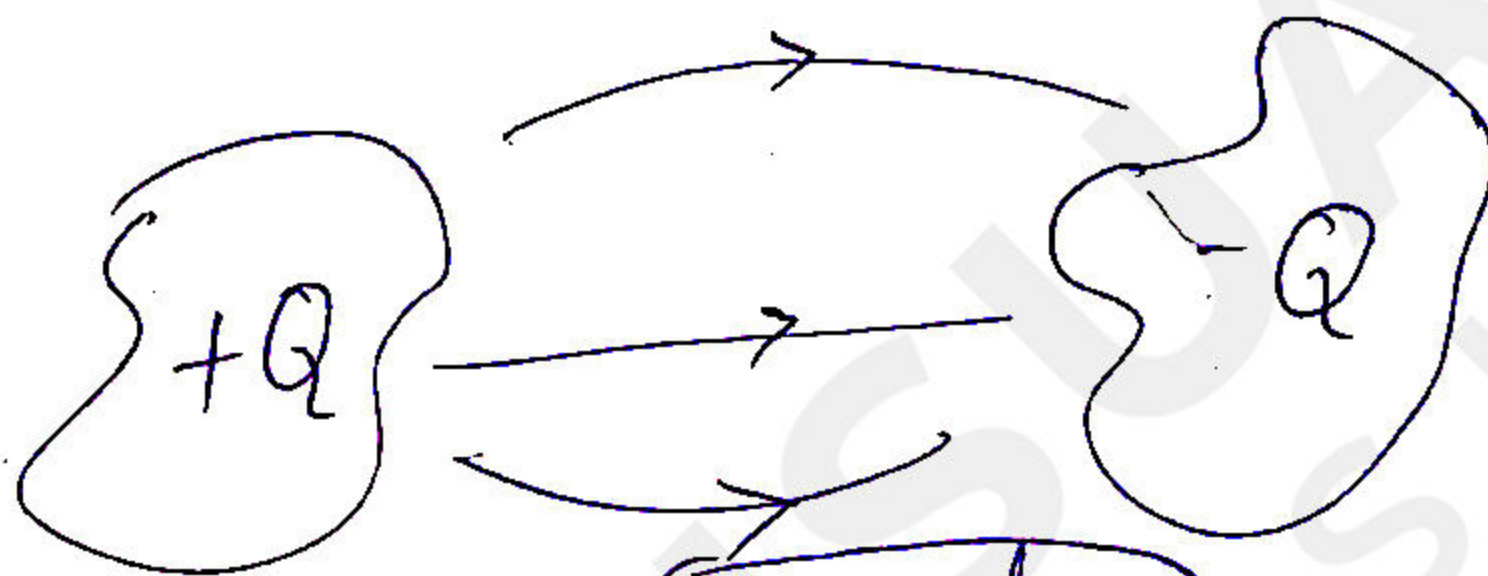


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CAPACITOR AND CAPACITANCE

Capacitor $\rightarrow$ Any two conductors separated by an insulator form a capacitor.

dielectric

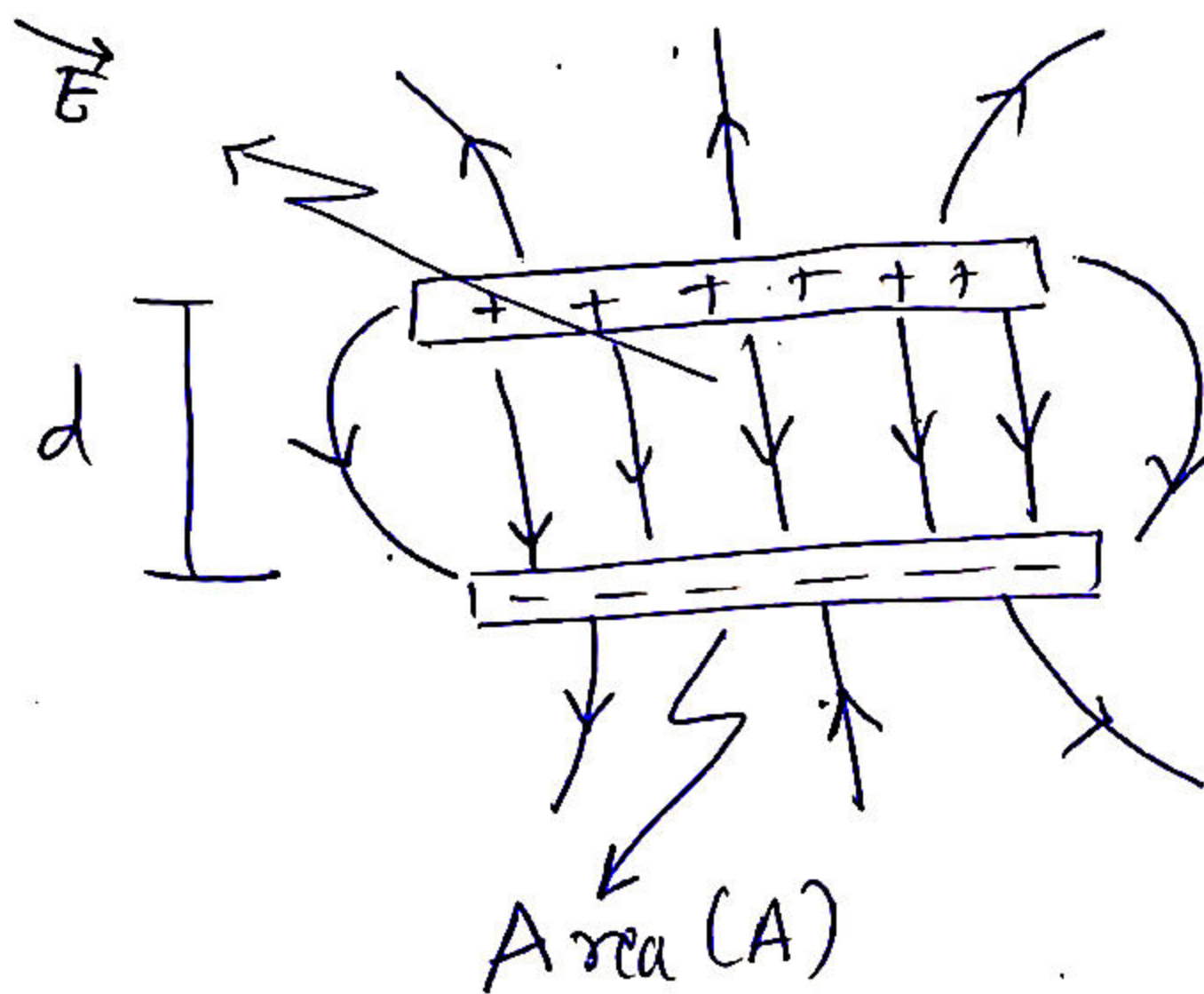


$Q = CV$ $\rightarrow$ Capacitance
potential difference between the conductors
 $\rightarrow$ charge stored in capacitor
depends on dimension of capacitor

Capacitance $\rightarrow$ means the ability of capacitor to store charge for a given potential difference.
Unit $\rightarrow$ Farad

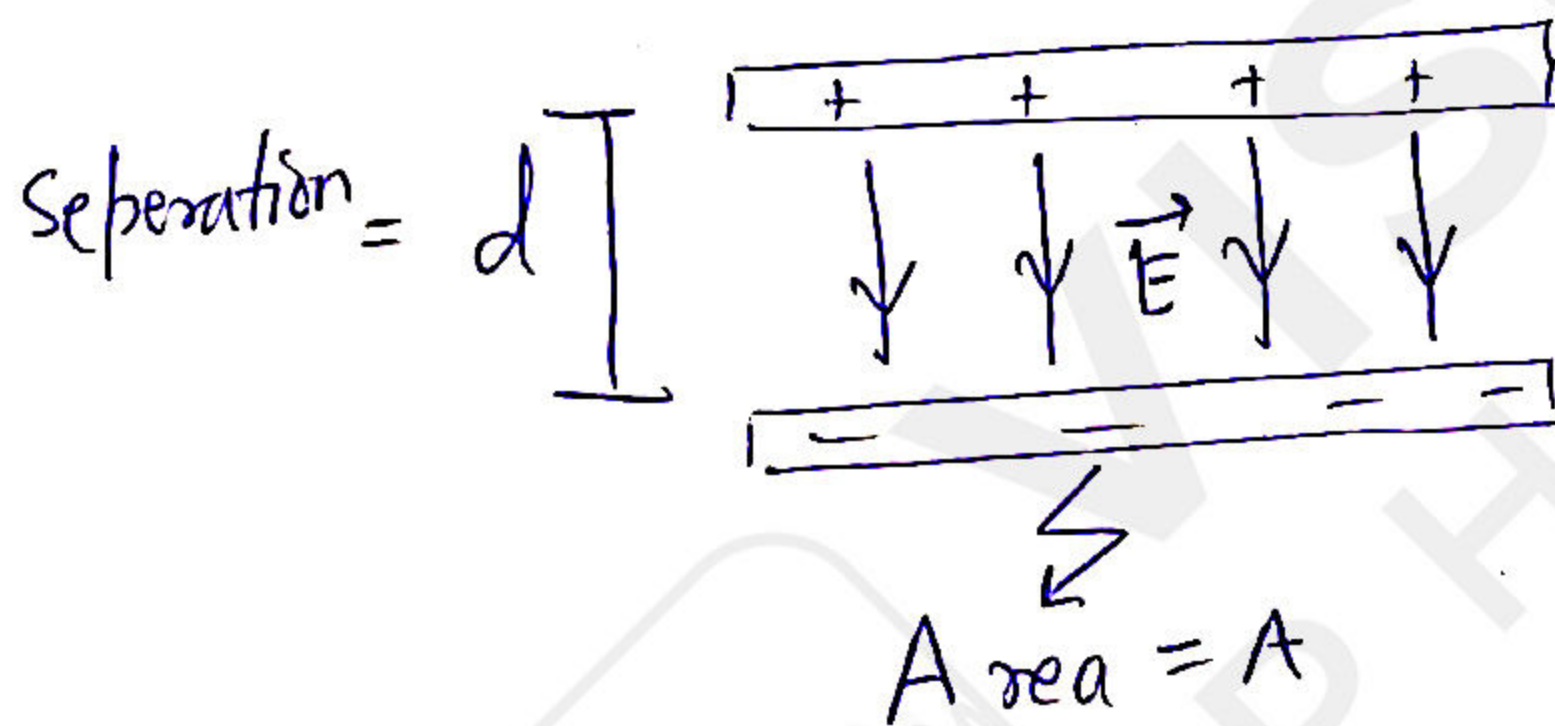
* A single conductor also acts as a capacitor by assuming that the other conductor is placed at infinity.

Parallel plate capacitor:



→ fringe effect for the case when d (separation) is less

When the separation is sufficient: (no fringe effect)



$$\vec{E} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$
$$\Rightarrow \boxed{\vec{E} = \frac{\sigma}{\epsilon_0}}$$

$$V = \text{potential difference} = E d$$

$$V = \frac{\sigma}{\epsilon_0} d$$

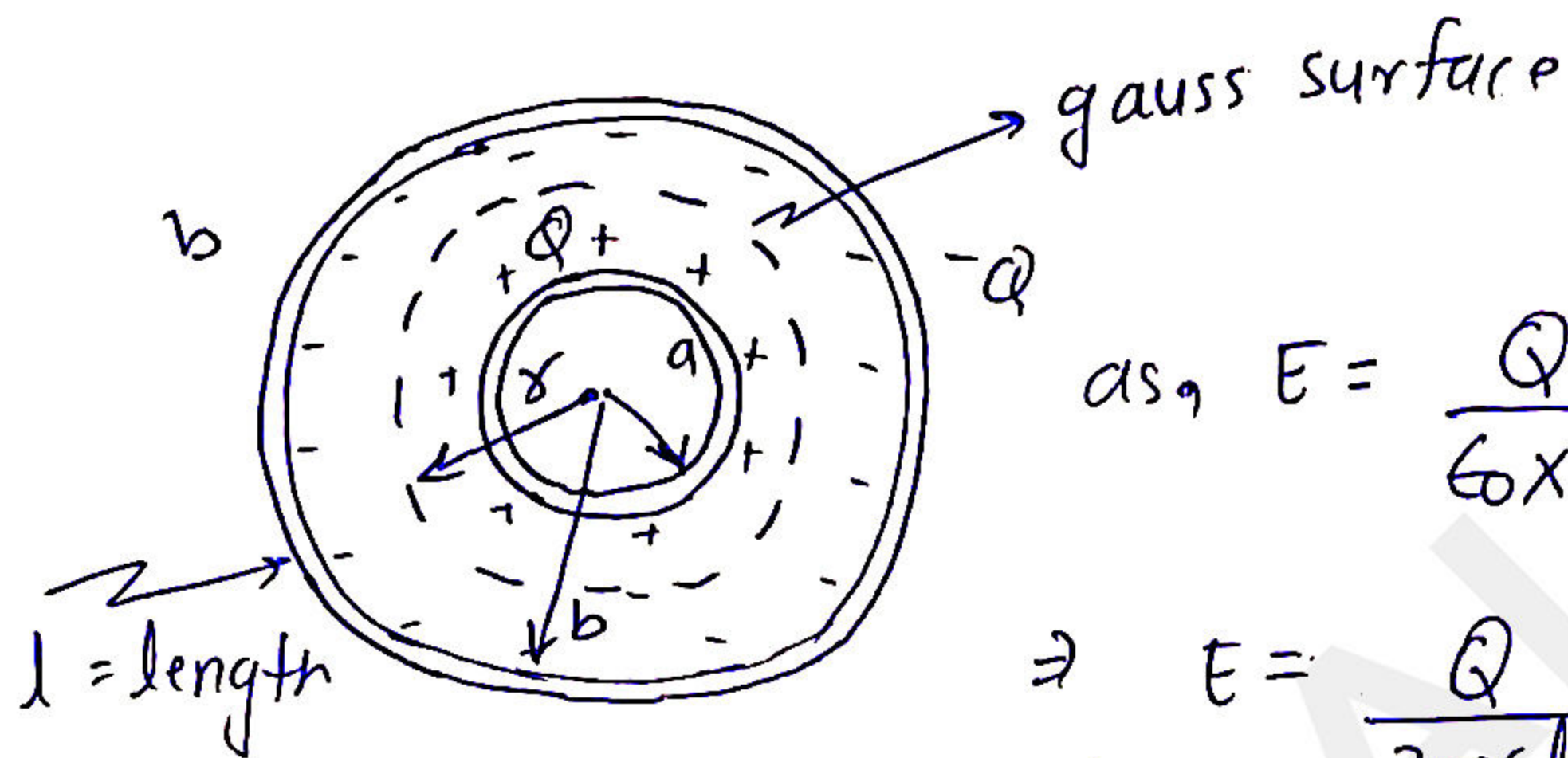
$$\left(\sigma = \frac{Q}{A} \right)$$

$$V = \frac{Q d}{A \epsilon_0}$$

$$\text{as } Q = C V$$

$$\Rightarrow \boxed{C = \frac{A \epsilon_0}{d}}$$

Cylindrical Capacitor



as, $E = \frac{Q}{\epsilon_0 \times A}$ from gauss law

$$\Rightarrow E = \frac{Q}{2\pi r l \epsilon_0}$$

So, $dV = - \int_a^b E dr$

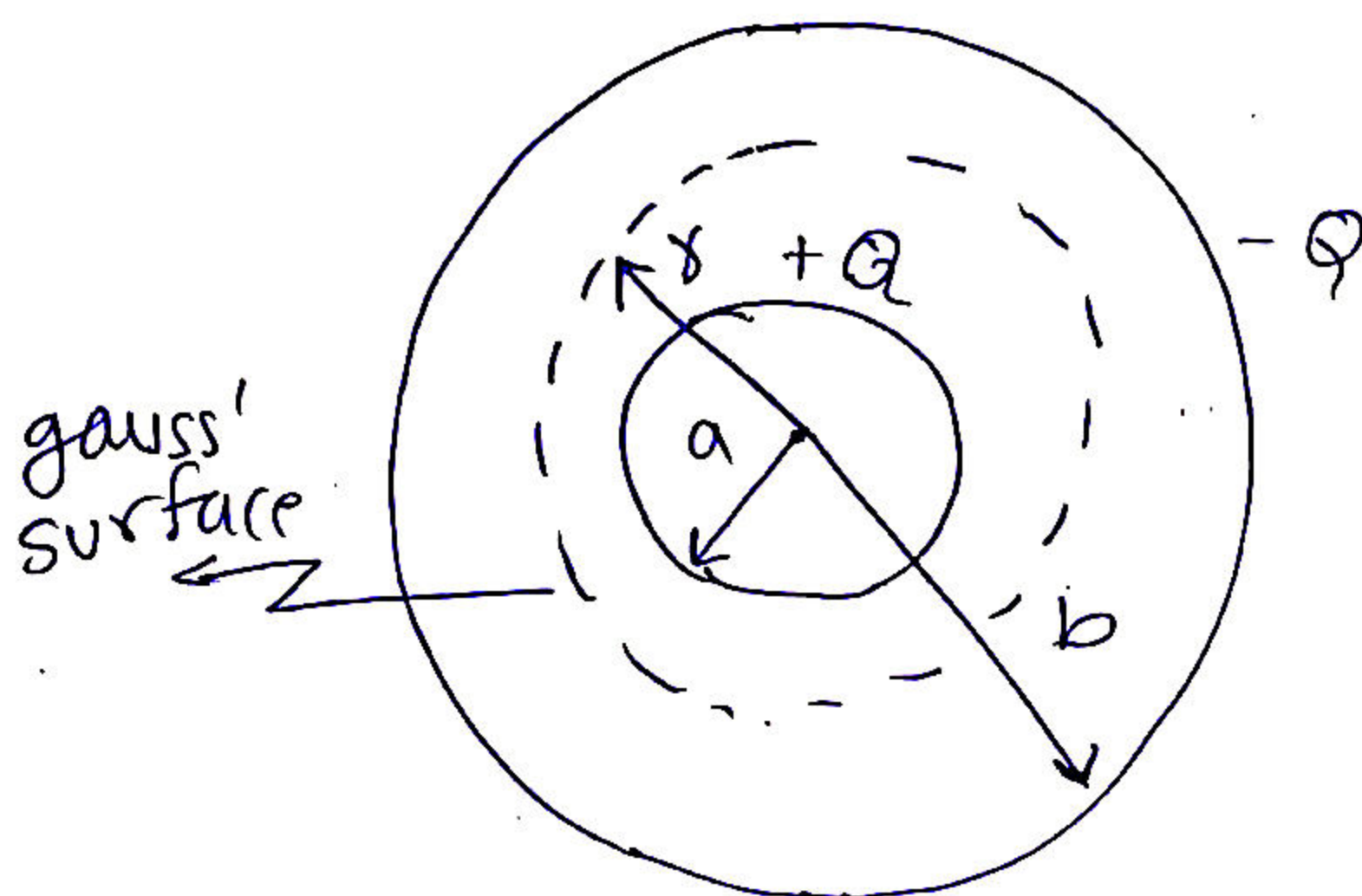
potential difference $\rightarrow \Delta V = V = - \frac{Q}{2\pi \epsilon_0 l} \int_a^b \frac{dr}{r}$

$$V = \Delta V = \frac{Q}{2\pi \epsilon_0 l} \ln\left(\frac{b}{a}\right)$$

as, $Q = C V$

$$\Rightarrow C = \frac{2\pi \epsilon_0 l}{\ln(b/a)}$$

Spherical Capacitor:



$$\text{flux} = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\int dV = - \int \vec{E} \cdot d\vec{r}$$

$$V = \Delta V = - \int_a^b \frac{Q}{4\pi\epsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_a^b$$

Potential difference

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{b} - \frac{1}{a} \right)$$

$$V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

SO, $C = \frac{Q}{V}$

where $b \rightarrow \infty$

$$\Rightarrow \boxed{C = \frac{4\pi\epsilon_0 (ab)}{(b-a)}}$$

$$C = \frac{4\pi\epsilon_0 a}{\left(1 - \frac{a}{b}\right)}$$

Capacitance of isolated sphere

$$\xrightarrow{b \rightarrow \infty} \boxed{C = 4\pi\epsilon_0 a}$$

Energy stored in a charged capacitor

As energy stored $\rightarrow$ work done in storing charge Q in capacitor of capacitance C .

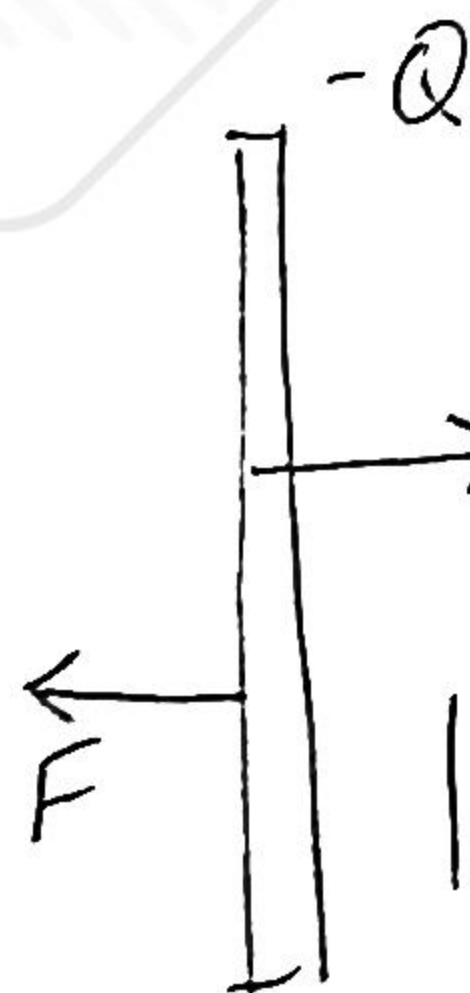
$$\text{as } W = \int_0^Q dw = \int_0^Q (dq) V$$

$$\text{as } q = CV \quad \Rightarrow \quad W = \frac{1}{C} \int_0^Q q dq$$

$$\text{so, } V = \frac{q}{C}$$

$$W = U = \frac{Q^2}{2C}$$

energy stored in capacitor



E_+ electric field due to plate $+Q$

$$|F| = QE_+ = \frac{Q\sigma}{2\epsilon_0} = \frac{Q^2}{2A\epsilon_0}$$

$$\text{As } \sigma = Q/A$$

Energy density

Energy/unit Volume
per

$$\frac{Q^2}{2C} \leftarrow \frac{U}{V} \rightarrow \text{volume} = Ad$$

energy density = $\frac{Q^2}{2CAd}$

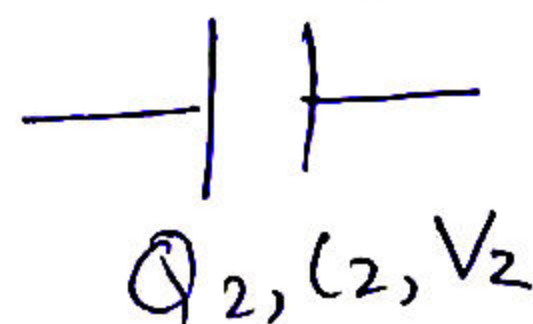
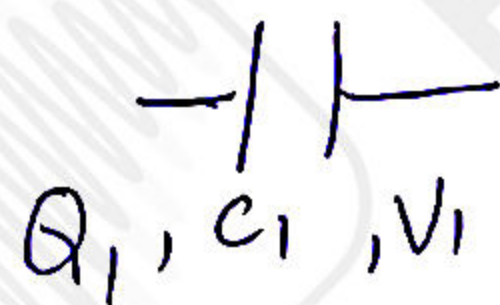
So, energy density

$$\frac{\epsilon_0 A}{d}$$

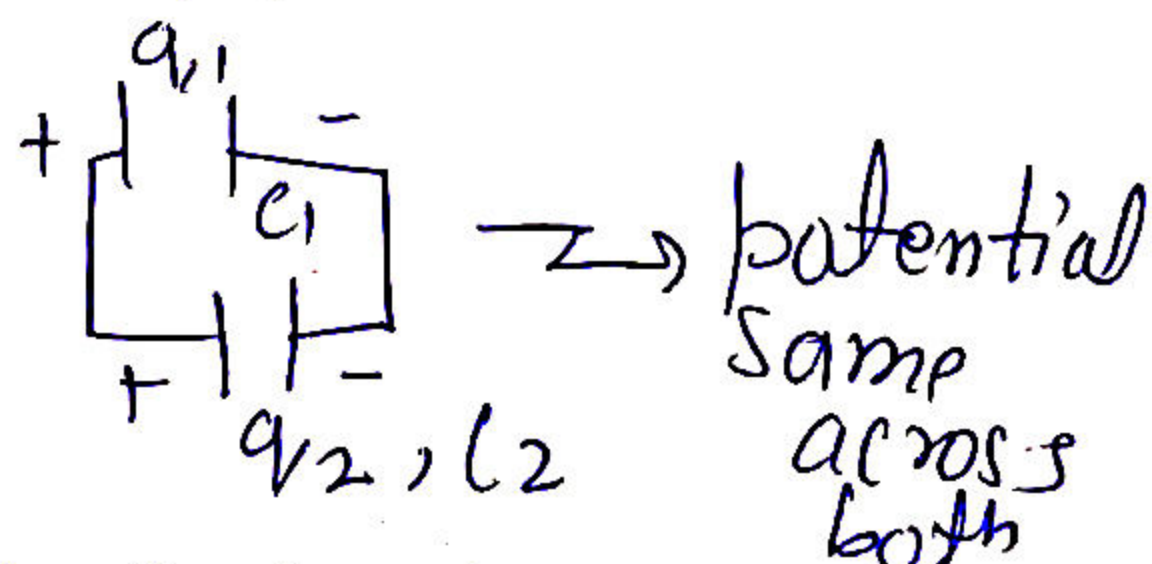
$$u = \frac{U}{V} = \frac{1}{2} \epsilon_0 E^2$$

will be same for all type of capacitors

Energy loss during Redistribution



$U_i = \text{Initial energy}$
 $= \frac{1}{2} \frac{Q_1^2}{C_1} + \frac{1}{2} \frac{Q_2^2}{C_2}$



$U_f = \text{final energy} = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} \frac{(Q_1 + Q_2)^2}{(C_1 + C_2)}$

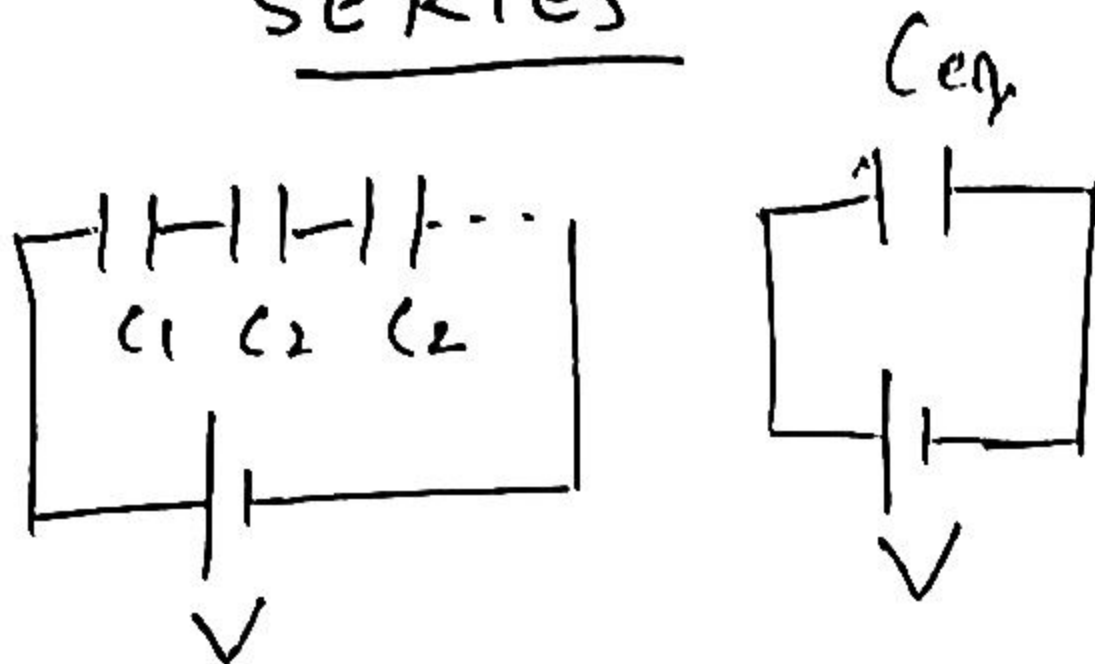
$\frac{Q}{(C_1 + C_2)} \rightarrow \text{net charge} = Q_1 + Q_2$

$$\Delta U = \text{Energy loss} = U_i - U_f$$

$$\Rightarrow \boxed{\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2}$$

COMBINATION OF CAPACITANCE:

SERIES



$$V = V_1 + V_2 + \dots$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$$

→ charge same on each capacitor

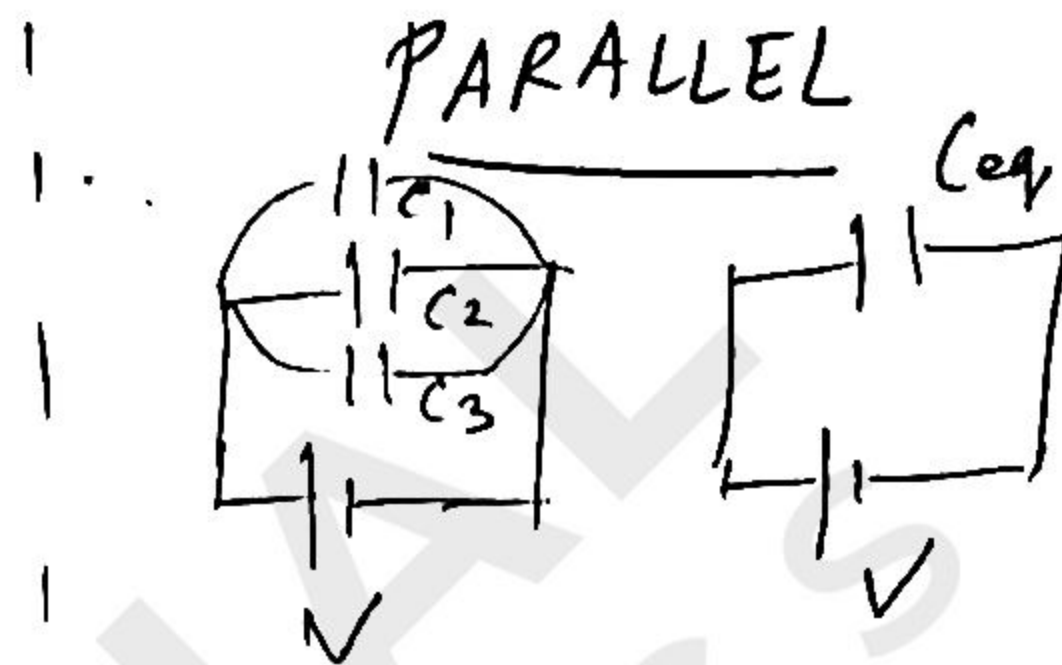
→ Energy

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \dots$$

$$U = \frac{Q^2}{2} \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots \right)$$

$$\boxed{U = \frac{Q^2}{2C_{eq}}}$$

PARALLEL



$$q = q_1 + q_2 + \dots$$

$$C_{eq} V = C_1 V + C_2 V + \dots$$

$$\boxed{C_{eq} = C_1 + C_2 + \dots}$$

→ potential across each capacitor same

→ Energy:

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \dots$$

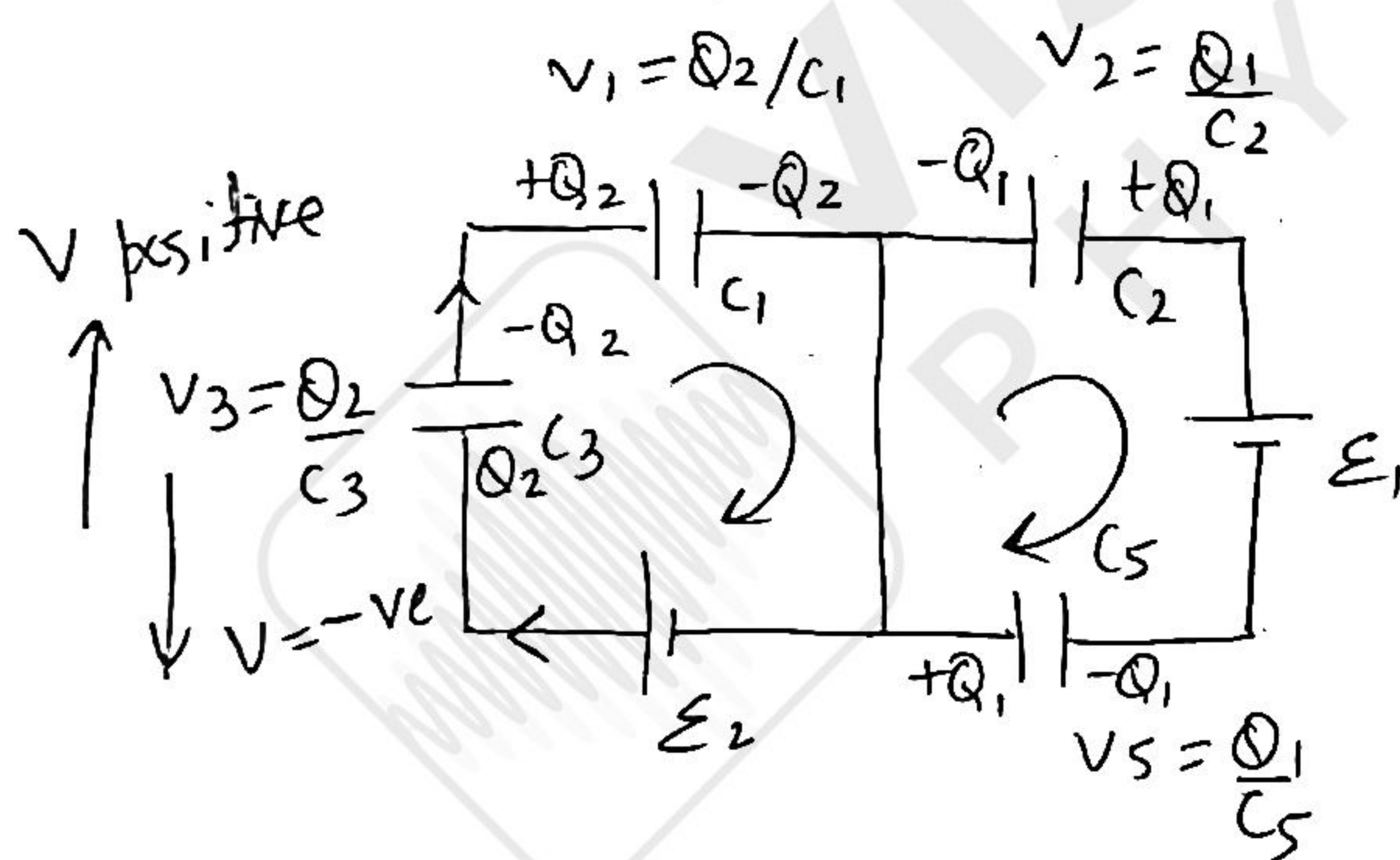
$$U = \frac{1}{2} (C_1 + C_2 + \dots) V^2$$

$$\boxed{U = \frac{1}{2} C_{eq} V^2}$$

$$V_1 = \frac{C_2 V}{C_1 + C_2}, \quad V_2 = \frac{C_1 V}{C_1 + C_2} \quad \Bigg| \quad Q_1 = \frac{C_1 Q}{C_1 + C_2}, \quad Q_2 = \frac{C_2 Q}{C_1 + C_2}$$

Kirchhoff's rule:

- Electric charge will remain conserved in every isolated region
- The algebraic sum of potential changes in each closed loop must be zero.



going from (+)vely charged plate → negative plate
 potential difference = -ve (vice versa)

DIELECTRIC!

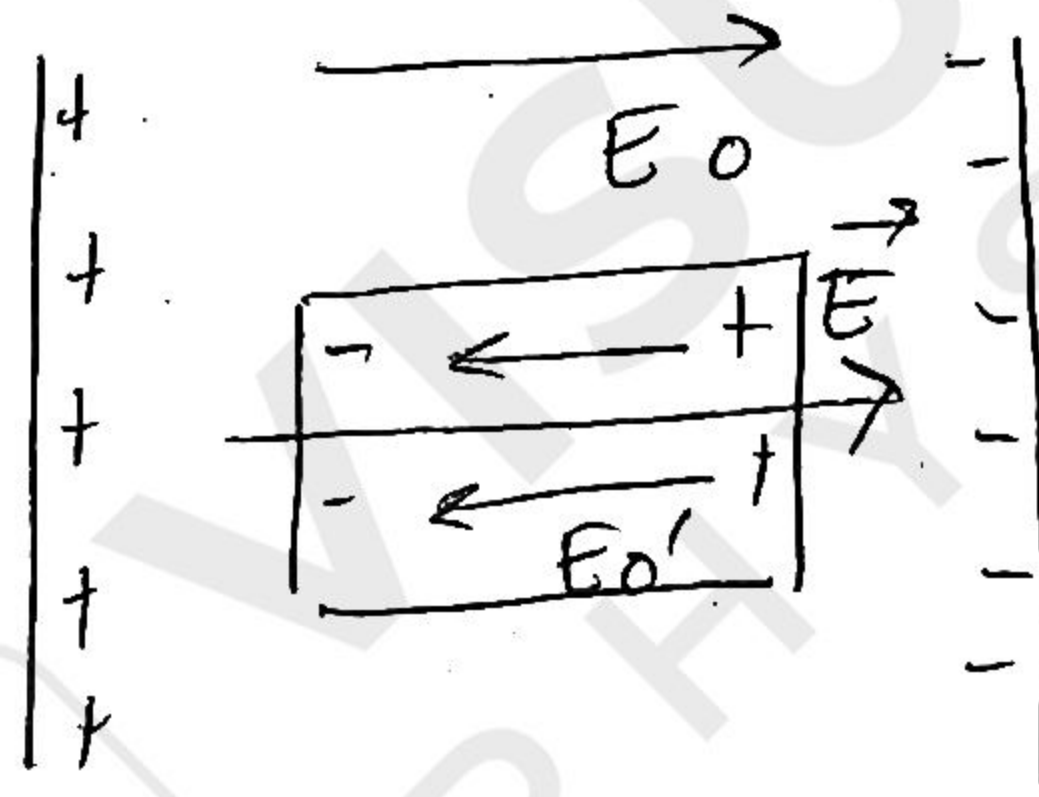
polar molecules

$$k = \frac{C}{C_0}$$

non-conductor up to certain value of field.
→ if field exceeds limiting value, called dielectric strength, dielectric loses insulating property and start conducting.

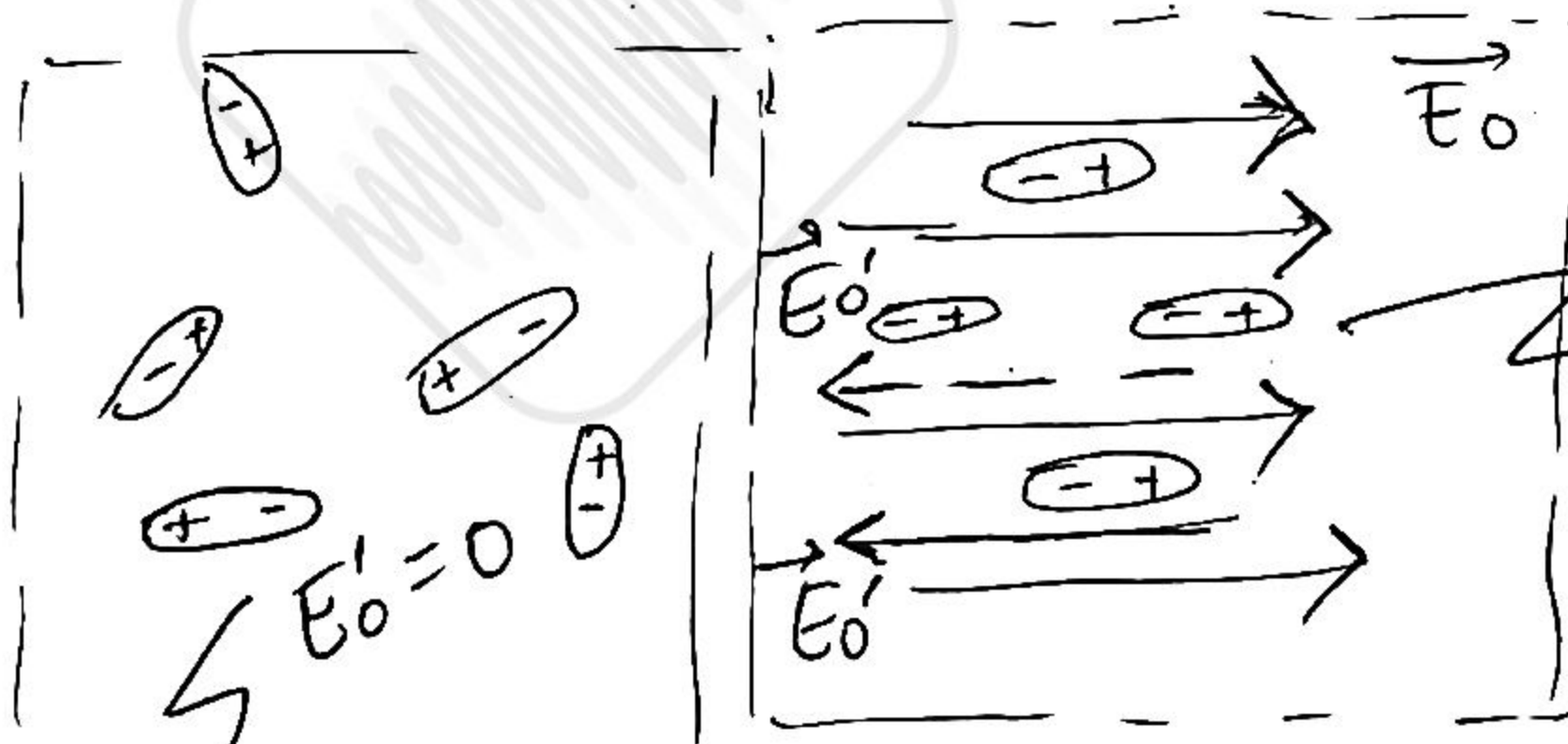
Capacitance when air or vacuum is present in plates between

Capacitance with dielectric in between conductors



$$\text{so, } \vec{E} = \frac{\vec{E}_0}{k}$$

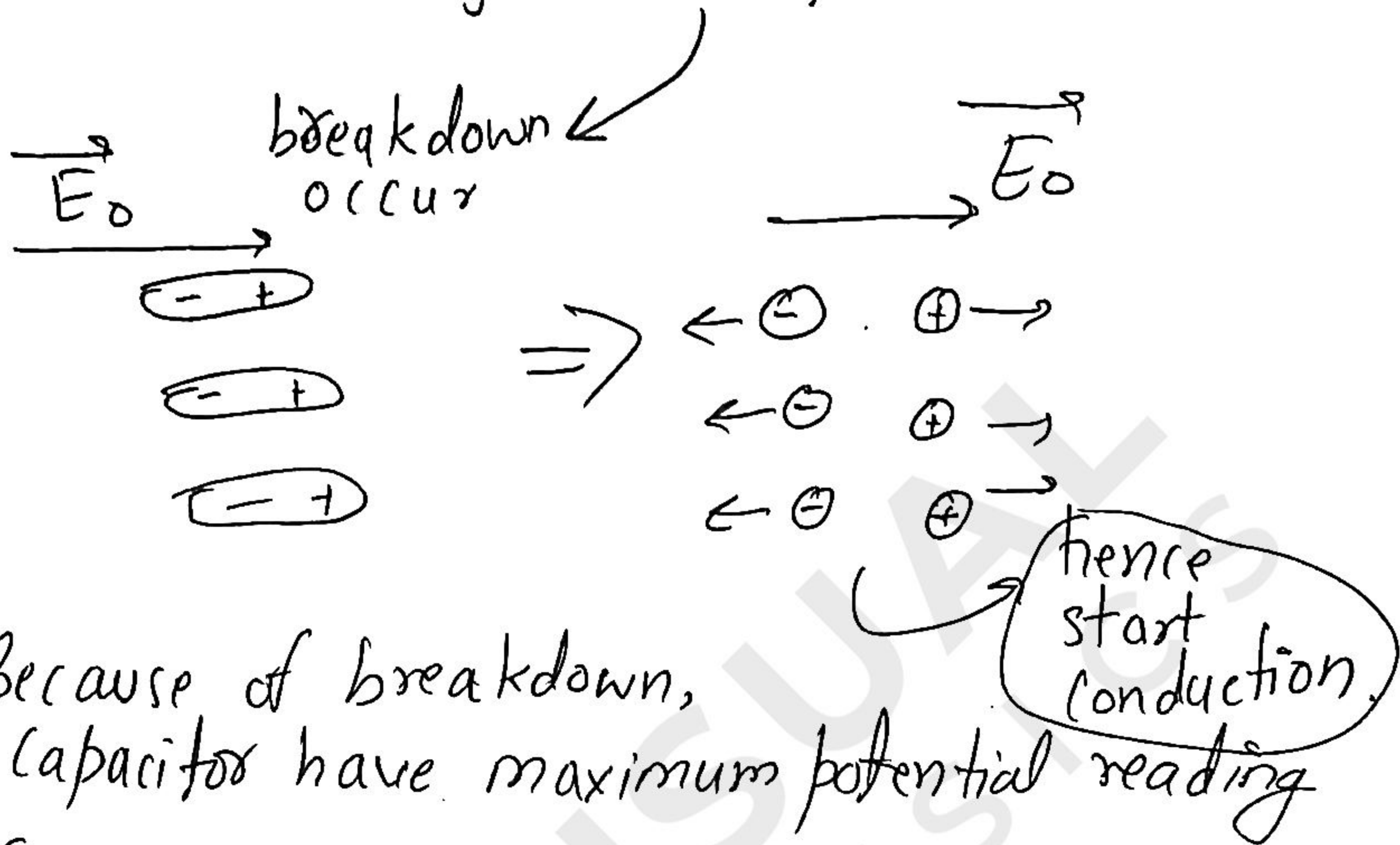
electric field reduce by 'k' factor.



dielectric medium

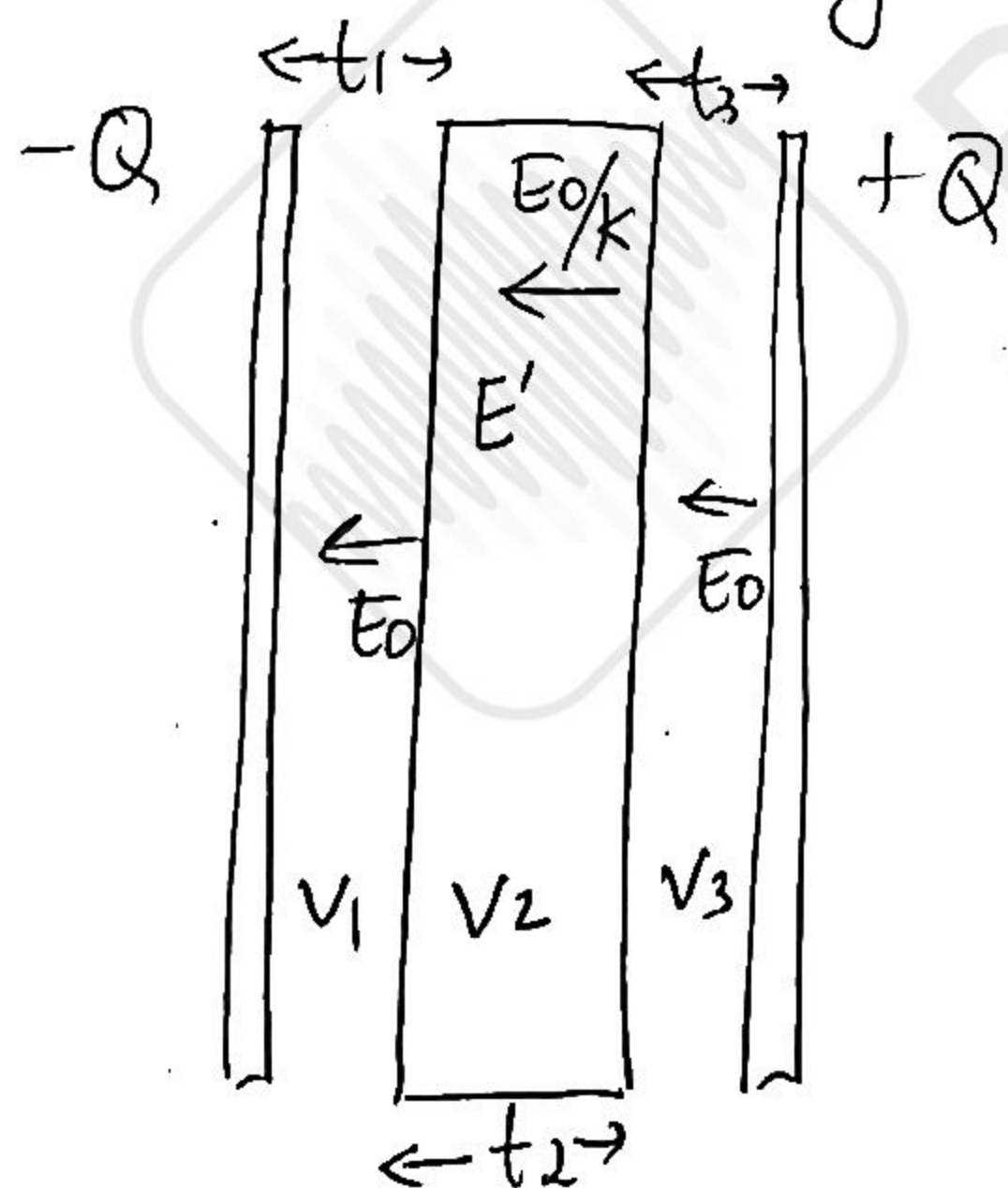
polar molecules alignment.

So, if dielectric material is subjected to a sufficient strong electric field



Because of breakdown, capacitor have maximum potential reading

If conduction occurs by breakdown, capacitor can't store charge.



$$E_0 = \frac{V}{\epsilon_0} = \frac{Q}{A \epsilon_0}$$

$$E' = \frac{E_0}{k} = \frac{Q}{k \epsilon_0 A}$$

$$V = V_1 + V_2 + V_3$$

$$= \frac{Q}{\epsilon_0 A} t_1 + \frac{Q}{k \epsilon_0 A} t_2 + \frac{Q}{\epsilon_0 A} t_3$$

$$V = \frac{Q}{\epsilon_0 A} (t_1 + t_2) + \frac{Q}{k \epsilon_0 A} t_3$$

$$\text{if } t_1 + t_2 + t_3 = d$$

$$t_3 = t$$

$$\Rightarrow \boxed{t_1 + t_2 = (d - t)}$$

So,

$$V = \frac{Q}{\epsilon_0 A} (d - t) + \frac{Q}{k \epsilon_0 A} t$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 A}{(d - t) + \frac{t}{k}}}$$

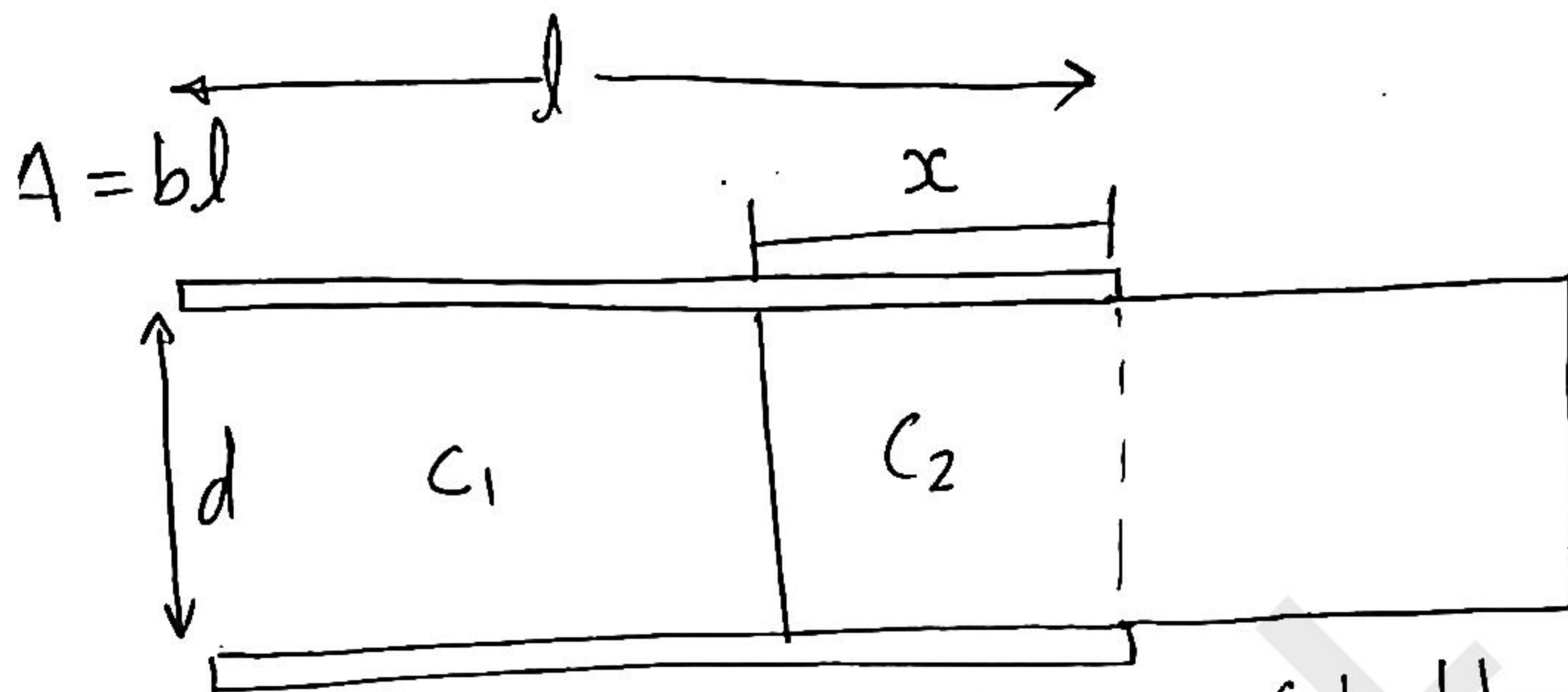
→ $C \rightarrow$ independent of position of slab

$$\rightarrow \text{if } k \rightarrow \infty, C' = \frac{\epsilon_0 A}{(d - t)}$$

$$\text{if } t \rightarrow d$$

$$C' = \frac{k \epsilon_0 A}{d} \quad (t = d)$$

Force on dielectric:



$$C = \frac{\epsilon_0 A_1}{d} + k \frac{\epsilon_0 A_2}{d}$$

(battery not connected)

$$C = C_1 + C_2$$

$$A_1 = b(l-x), \quad A_2 = bx$$

$$\Rightarrow C = \frac{\epsilon_0 b}{d} [l + (k-1)x]$$

$$U(x) = \frac{1}{2} \frac{Q^2}{C}$$

$$= \frac{Q^2}{2 \frac{\epsilon_0 b}{d} [l + (k-1)x]}$$

$$= \frac{Q^2 d}{2 \epsilon_0 b [l + (k-1)x]}$$

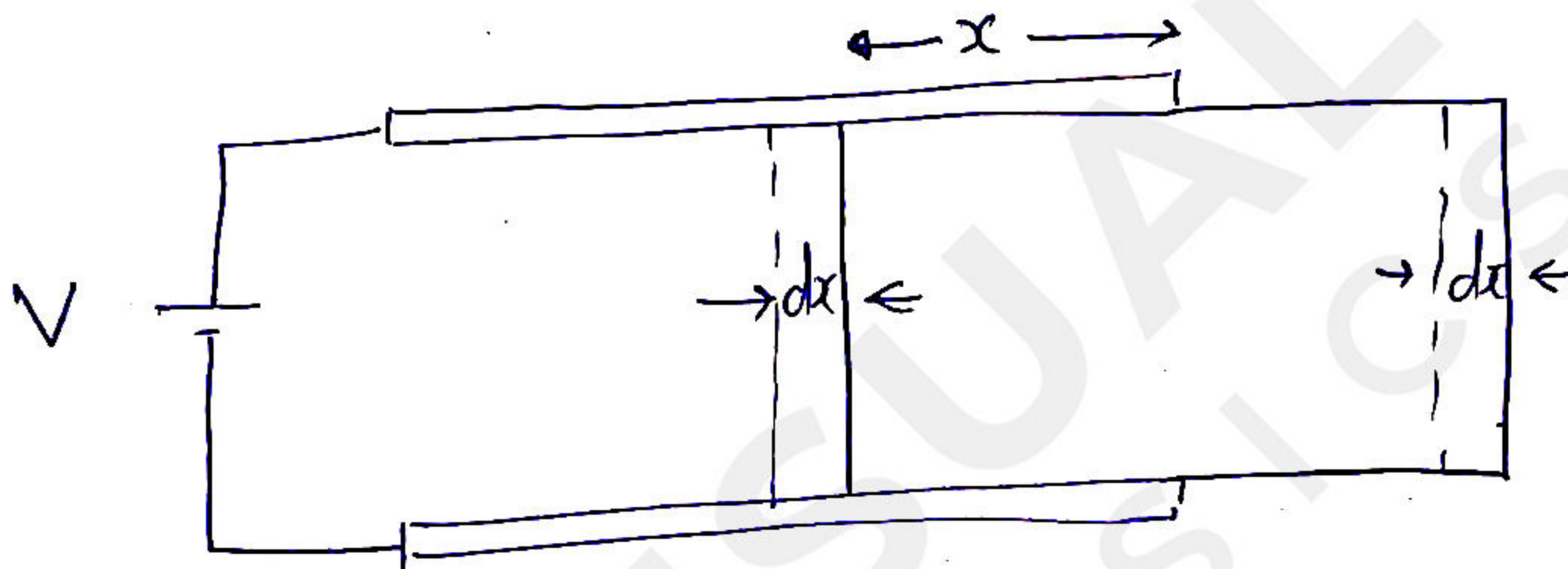
$$F(x) = - \frac{d}{dx} U(x)$$

$$F = \frac{Q^2 d}{2 \epsilon_0 b} \frac{(k-1)}{[l + x(k-1)]^2}$$

When $x = l$

$$F(l) = \frac{Q^2 d}{2 \epsilon_0 d} \frac{(k-1)}{[l + l(k-1)]^2}$$

$$F(l) = \frac{Q^2 d}{2 \epsilon_0 d} \frac{(k-1)}{l k}$$



$$C = \frac{\epsilon_0 b}{d} (l + (k-1)x)$$

$$U = \frac{1}{2} C V^2 \quad dU = \frac{1}{2} (dC) V^2$$

$$dC = \frac{\epsilon_0 b}{d} (k-1) dx$$

extra
charge given
when $C \rightarrow C + dC$
now

$$dQ = dC V = \left[\frac{\epsilon_0 b}{d} (k-1) dx \right] V$$

$$dU_{\text{bat}} = -dQ V = - \left[\frac{\epsilon_0 b}{d} (k-1) dx \right] V^2$$

energy decreased
by battery.

$$dU = \frac{1}{2} \left[\frac{60b}{d} (k-1) dx \right] v^2$$

$$dU_{\text{sys}} = dU + dU_{\text{bat}}$$

$$dU_{\text{sys}} = -\frac{1}{2} \left[\frac{60b}{d} (k-1) dx \right] v^2$$

$$\text{as } F(x) = -\frac{d}{dx} U(x)$$

$$\Rightarrow \boxed{F(x) = \frac{1}{2} \left[\frac{60b}{d} (k-1) \right] v^2}$$